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A COMPUTER SIMULATION OF A CHILD EXPERIENCING DIFFICULTIES WITH PROPORTION PROBLEMS: AN APPLICATION IN TEACHER EDUCATION

BY

Peter Balding

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF EDUCATION

DEPARTMENT OF SECONDARY EDUCATION

EDMONTON, ALBERTA

SPRING, 1987

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled A COMPUTER SIMULATION OF A CHILD EXPERIENCING DIFFICULTIES WITH PROPORTION PROBLEMS: AN APPLICATION IN TEACHER EDUCATION submitted by Peter Balding in partial fulfilment of the requirements for the degree of Master of Education.

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Date: 27 May, 1987

ABSTRACT

The purpose of this study was to design a computer based model of a child experiencing difficulty with proportion problems and investigate its effect on prospective mathematics teachers. Instruction must deal specifically with incorrect responses of individual students. Reinforcement of this need to deal with wrong answers by virtue of exposure to a large number of student errors, in a short period of time, was seen as significant. It was felt that such exposure would not only contribute to the improvement of the analysis skills of the participants but would also increase their awareness of the importance of such skills.

A LOGO program called RATIOERROR was developed that generated errors consistently for a defined type of possible student error to proportion questions. Participants were given both pre and posttests on their ability to analyze such errors along with questionnaires concerning their thoughts on the usefulness of incorrect answers to the teaching process. Prospective teachers used the program for approximately two hours, practising teachers for one hour.

The computer model was tested with twenty-two prospective teachers and ten practising teachers. In

all cases there was a marked increase in the ability to discriminate various actual student errors.

Most participants indicated a greater specific awareness to the usefulness of an incorrect answer in the teaching of proportion problem solutions. They came to view both student and teacher as integral parts of the development of mathematical procedures. There was an increased focus amongst the participants on the concrete actions possible when confronted with student errors. A corresponding shift away from abstract judgements about student ability was also noted. RATIOERROR was felt by most to help sharpen their diagnostic skills while increasing their awareness of the number of errors possible in proportion questions.

This research indicates that the ability of the computer program to present virtually all classifiable error patterns makes it a practical aid in the education of mathematics teachers.

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TABLE OF CONTENTS

CHAPTER PAGE

CHAPTER PAGE

CHAPTER PAGE

LIST OF FIGURES

FIGURE		PAGE

CHAPTER I

THE PROBLEM

INTRODUCTION

Computers have come to play an important role in education and the field of educational research. Software is often used to collect and/or help analyze data. As well, the computer is being increasingly utilized as a means of delivering instruction, as a tool to assist the student, and as a simulator of a situation to be examined. All such uses should be subject to study.

In the past instructional programs were usually designed with the intent of individualizing learning by means of drill-and-practice sessions, or through actual lesson presentations. This kind of computer-assisted instruction was generally in the form of a frame by frame presentation of information, where particular pages, or screens if you will, were shown. These screens were sometimes delivered according to the programmer's preconceived notions of what the student should be doing, and sometimes according to the students' responses to various questions. In each case however, the program was not interacting with the student. Rather, it was choosing various paths within itself, based on a minimal knowledge of that student.

Programs that can determine errors a student has made and analyze the route taken to arrive at those errors, rather than blindly follow a particular path, can have a profound effect on how both the teacher and the student visualize the student as a learner. In order to accomplish such an analysis, the program must contain or be able to develop a model of the student's mathematical actions and react accordingly in combination with the student's responses.

While we may present a variety of problems to students that deal with a specific concept in varied ways, variety, in and of itself, is not always enough to expedite learning. When it is not, we must consider the students' errors.

If we grant that children develop their own algorithms, we can better understand the trouble they have adapting to the procedures given by their teacher. In fact, the new and old procedures may co-exist. Keeping this in mind, we have to adjust ourselves to children's seemingly irrational answers generated from those theorems-in-action (Vergnaud, 1982b), often invoked despite their inappropriateness to the situation. Teachers must grasp what the students' primitive conceptions look like and the errors and misunderstandings which may result.

Given a framework within which to place a conception, we are not restricted to judging an answer as only right or wrong. If the answer is incorrect we should be able to determine how the student developed it. Aside from simple computational mistakes there are no incorrect answers, only the following of incorrect paths will usually lead to a wrong answer. This set of steps can be judged as incorrect only when inappropriate to the problem at hand.

The field of error analysis is in its infancy as far as the regular classroom teacher is concerned, with much of the work left to be accomplished by educators. The problems to be solved are not those of developing techniques for writing computer programs that may perform such analysis, but are of understanding students' misconceptions of particular knowledge domains and determining the various styles of learning within those domains. Any instruction must deal specifically with incorrect answers of the individual student. Is it possible to reinforce the understanding of this need in those who will be teaching children? Can a computer program that simulates a student experiencing difficulty with proportion problems help in this regard? It is the intention of this study to find the answer to these questions.

STATEMENT OF THE PROBLEM

The following questions exemplify the areas of concern of this study:

Is the representation of an error model an adequate generalization of students experiencing problems with proportion questions?

Will teacher candidates' perceptions of the role of incorrect procedures in the teaching of mathematics change if exposed to computer based simulation of such errors?

Will teacher candidates exhibit an increased ability to identify and analyze the algorithms used by children?

What will teacher candidates indicate as their perceived usefulness of the idea of children correctly following incorrect procedures?

Will experienced teachers and teacher candidates demonstrate similarities after exposure to instruction concerning:

1. the ability to identify and analyze children's algorithms

2. speculation as to the cause of student difficulties concerning proportion problems

3. the usefulness of an incorrect answer to the teacher

4. whether it is better to focus on the correct or incorrect procedure when helping a student experiencing difficulties with proportion problems

5. whether mathematical rules should be developed by the child or given out by the teacher?

PURPOSE OF THE STUDY

The purpose of this study is to develop and investigate the effect of a computer-based model of a child who is experiencing difficulties with proportion problems on the perceptions of the causes and usefulness of such knowledge of errors on prospective mathematics teachers. An attempt will be made to compare the results of the inexperienced teachers with a small group of practising teachers by analyzing their respective responses to an analysis of proportion errors, as well as categorizing their answers to various questions concerning the usefulness of knowledge of such errors.

A basic premise of this study is that not only can a representation of an error-prone student be accomplished, but also that interaction with such a program will focus the user on the need to analyze student errors and work with those errors, rather than to simply consider a wrong answer as a cue to review a pre-given set of rules.

DEFINITION OF TERMS

Algorithm refers to a list of processes specifying a sequence of operations which lead to the answer to a problem.

Bug refers to a student's misconception as to how a problem is solved.

CAI (Computer Assisted Instruction) refers to a method of using a computer to present instructional materials.

Computer-Based Model or Simulation refers to a model of an individual or situation or part thereof embedded within a computer program.

Error Model refers to a reproduction of what is believed to be the student's way of arriving at an incorrect solution to a problem.

Expert System refers to problem-solving programs that require an ability to model an expert's decision making process.

Functional refers to two magnitudes of different types related to each other so that the values of one correspond to the values of the other in a single valued manner.

ICAI (Intelligent Computer Assisted Instruction) refers to using a computer to present instructional materials that in some way develop a model of the user based on interaction.

Knowledge Base refers to a collection of facts, inferences and procedures that people know.

<u>Measure Space</u> refers to a relationship between quantiites.

<u>Perceptions of the Role</u> refers to the ideas about a topic held as inherent to that topic.

<u>Procedure</u> refers to a series of steps followed in a particular order.

<u>Procedural Network</u> refers to a number of connected computer procedures representing a model of a particular activity.

<u>Proportion</u> refers to an expression relating the equality of two ratios.

<u>Ratio</u> refers to the comparison of one number with another.

<u>Scalar</u> refers to a single value which relates two magnitudes of a single type through an arithmetic operation.

<u>Student Model</u> refers to a computer model of a particular aspect of student performance.

<u>Theorem</u> refers to a mathematics statement that has been proved or is conjectured.

DELIMITATIONS

In the interpretation of this study the following delimitations will have to be considered:

1. The study is confined to education students at the University of Alberta.
2. Experienced teachers were drawn from a group interested in the LOGO computer language.
3. There will be no followup concerning actual teaching methods of the subjects involved in the study.

LIMITATIONS

In the interpretation of the data, the following limitations will have to be considered:

1. The students were volunteers chosen on the basis of interest.

2. The teachers were participants in a LOGO workshop.

3. The results of the study may be restricted to proportion questions.

4. Exposure to the computer program is of a short duration (approximately two hours for the prospective teachers and one hour for the practising teachers). Such a short exposure may not demonstrate enough of a change to be significant.

IMPORTANCE OF THE STUDY

For computer programs to have an appreciable positive impact on learning they will have to fulfill a need that can not be easily met by other media. This study will attempt to evaluate, using criteria outlined in Chapter III, a microcomputer implementation of a program to encourage the view of the child as a developer of procedures used in some ways appropriately and in some ways inappropriately when encountering difficult material. If successful the results should serve as a beginning for further research into the use of the computer in the remediation of problems students encounter in the solution of proportion questions.

OUTLINE OF THE THESIS

The present chapter has introduced the background and significance of the study. Chapter II will organize previous work done on children's solving of proportion problems. It will also outline the development of various Intelligent Computer Assisted Instruction programs and their characteristics. The purpose of this review was to arrive at a suitable conjunction of the two areas in terms of a coherent and managable computer program that simulates a student experiencing difficulties with proportion type problems. Chapter III will describe the sample, the project goals, the project design, the computer program and how the data was collected and analyized. Chapter IV will portray the results of the use of the computer program by the groups tested. Chapter V will contain a summary of the study, some conclusions reached and suggestions for further research.

CHAPTER II

REVIEW OF THE LITERATURE

INTRODUCTION

The purpose of this review is to explore two different fields that hold significant implications for the study at hand - proportions, in conjunction with their development, and Intelligent Computer Assisted Instruction. Because of their diversity, each area will be treated separately, in the hope that the possibilities of their convergence will be realized. An understanding of the problems inherent in their mutual dependence related to the present research is necessary to consider the program developed. Special attention will be paid to the work of Gerard Vergnaud in the area of error analysis of proportion problems. The emphasis on Intelligent Computer Assisted Instruction will be on those programs, in educational settings, whose basis is a model of a person with little expertise in the area of interest.

PROPORTIONS

A proportion is a statement representing the equality of two ratios. Problems involving proportions prove difficult for many students and much research has recently been undertaken to find a solution to these difficulties (Kieren and Southwell, Noelting, Karplus, Stage and Pulos).

RESEARCH ON PROPORTIONS

Inhelder and Piaget (1958) developed concrete exercises involving a fulcrum and the projection of shadows to determine the child's conception of ratio. Piaget believed that children will fall back to additive strategies when confronted with difficult problems. According to Piaget, proportional reasoning develops from the additive to an ungeneralized but organized strategy to a formal law.

Kieren (Kieren and Nelson, 1978) studied natural numbers in ratio form using the concept of a machine which packs objects into boxes. Using this concept, it was hypothesized that three levels of operator development exist. First, there is is the 1:2 level where the child's concept is dominated by 1/2, second there is a level at which competence in the handling of unit operators (1/n, n/1) is displayed and finally there exists a level of ability to handle all forms of operators. Kieren's pattern problem (Kieren and Southwell, 1979) was designed with only pairs of numbers, one being the result of a transformation of the other. After several examples of such pairs, the subjects were to complete other pairs. As a result of this study, which compared pattern problems to the problem packing machine mentioned above, it was suggested that a partitioning strategy led to correct but often clumsy results, while seeing the pattern represents more sophisticated thought and is thus more subject to error.

Noelting (1980) has studied proportional reasoning where hypothetical orange juice flavor, which was determined by the concentration of orange juice in water, was examined. Two interrelated types of development were posited: a qualitive change corresponding to the onset of new strategies and a quantive change within a stage leading to an increasing consolidation of that strategy. Development is gradual. He also found that problems with integral ratios were easier than problems with only non-integral ratios.

Hart (1981) determined that students develop strategies establishing a relationship within a ratio and extend it to the second ratio by addition. While these solutions do lead to successful outcomes in simple problems, they become cumbersome when the problems contain non-integer ratios. Hart has also shown that students rely on their own informal procedures even when they have learned the formal mathematical algorithms necessary for the solution of a problem.

Quintern and Schwartz (1982) found that mixture problems are more difficult than other proportion problems possibly because of the understanding of a physical context inherent in such problems.

Karplus, Pulos, and Stage (1983a) duplicated Noelting's basic research but added detailed information about the subjects' use of integral ratios. The problems presented compare the sweetness of two recipes for lemonade. The problems require recognizing equality or determining the amount necessary to achieve equality. The main difference between pupils' approaches was between proportional and additive methods. The researchers believe that the student first tests the within, and then, the between ratio to see if they are integral and then compares them with the other ratios. This analysis seems to suggest that early attempts at proportional reasoning consists of the child reducing to unit ratios. Karplus, Pulos and Stage (1983b) have found that the relative frequency with which the types of comparison and various strategies are used is affected greatly by the context and numerical content of the problem, and even of the immediately preceeding problem. The choice of a scalar or a functional strategy was influenced by the context of the problem.

Streefland (1984) proposed that the logical status of ratio is more complex than that of such elementary ideas as length, mass, area, volume, number, adding, subtracting, multiplying, or dividing. The crucial question is how to steer the pupil to the correct solutions of ratios. He contends that a ratio table, a means of organizing the information presented in a problem by means of a table headings, can be used as a means to organize the student's spontaneous solution strategies.

Gerard Verngaud (1983) has examined those problems whose solution involves multiplication or division. He maintains that by providing a system of classification, it will be easier to interpret the procedures that students have used to arrive at their solutions to multiplicative problems. Such a framework also provides for the designing of research problems based on related mathematical problems.

Vergnaud has developed the idea which he has termed isomorphism of measures - all instances where two measure-spaces are directly proportional to one another. Examples would include rate and partition problems. Vergnaud's research group has found that most of the procedures that students used, even when they were incorrect, had a physical interpretation. His conclusions (1982b) include:

1. concept development is very slow.
2. complex concepts will not develop unless complex situations are met.
3. children develop their own 'theorems-in-action.' By this Vergnaud means the handling of higher level relationships (which we call theorems) during real problem-solving situation.

Vergnaud (1983) has also found that scalar procedures are more frequently used than are function procedures in the correct solution of proportion problems.

CORRECT PROCEDURES

Proportional reasoning seems to become more readily implemented with the developing relationship of maturity and experience with proportion. As the student grows older and has more experiences, the strategies that are used to solve problems involving ratios becomes more nearly correct and sophisticated. As a result of this interaction, increasingly difficult proportional problems fall within the realm of student solution.

Tourniaire and Pulos (1985) point out that solving proportional problems can be seen as the ability to overcome a series of successive hurdles. As was pointed out by Kieren in 1981, first the child must be able to progress form those problems involving 1:2 ratios to those consisting of 1:n ratios. Next is the solution of non-unit problems. Finally as suggested by Karplus, Pulos et al, the leap must be made between integer and non-integer proportional problems.

Building-up strategies consist of determining a relationship with a ratio and extending it to the second ratio through addition (Hart, 1981). While these strategies are initially successful, with non-integer ratios they are seldom effective except for a few students.

Multiplicative strategies involve a multiplicative relation being established between the terms within a ratio and that relation being extended to the second ratio. Such strategies imply a choice between scalar and functional solutions to proportional problems.

Karplus, Pulos and Stage (1983) have suggested that this choice is the result of the context presented in the problem. They also believe that progress in the solution of proportional problems comes with the reduction of rates to unit variables.

Vergnaud (1983) has outlined five correct ways of solving proportion problems. By using the idea of a measure space he is able to present these as well as incorrect ways in a succinct and easily analyizable form. For example, given the problem:

> If a car travels 'a' km. in 'b' hours, how long will it take to travel 'c' km.?

M1	M2
a	b
c	x

These correct procedures are as follows:

1. Scalar: The student calculates (y = c/a)

or (y = a/c) and then calculates (x = y*b) or (x = b*y) or (x = b/y).

2. Function: The student calculates (b/a = y) or (a/b = y) and then (x = y*c) or (x = c/y).

3. Unit Value: The student preforms the same calculations as in the function relation but explains that (b/a) is the unit value, making the procedure scalar in character.

4. Rule of Three: The student calculates (b*c)/a or (c*b)/a.

5. Scalar Decomposition: The student tries to decompose magnitude c as a linear combination of multiples or fractions of a.

INCORRECT PROCEDURES

Hart (1981) and Karplus (1983) have found that one frequent error strategy is to ignore part of the data in the problem. Additive strategies are often used in an attempt to deal with non-integer ratios. Typically according to Karplus (1983a), more elementary strategies are used as a fall-back method on the more difficult problems, whether that difficulty is due to the structure of the problem or to its content.

Vergnaud (1983) has developed seven error categories based on his idea of measure space. These are as follows:

1. Erroneous scalar: The student uses a scalar ratio or difference and gives it as the answer or applies it to b.

2. Erroneous function: The student uses a function ratio or difference and either gives it as the answer or applies it to c.

3. Erroneous scalar and function: The student makes a calculation b*c, forgetting or canceling division by a, or makes a combination of erroneous scalar and function operations.

4. Inverse: The student uses the inverse of the correct ratio.

5. Erroneous product: The student multiplies c and a, or b and a.

6. Erroneous quotient: The student divides c by b, or divides b by c.

7. Others: Procedures that do not fit into the above classifications.

The problems that students encounter in the mathematics curriculum involve complex structures and concepts. Vergnaud (1983) suggests that these concepts can only develop through problem solving experience and that such development is slow. If we can base the work of our students within a conceptual framework we can analyze likely sources of errors in order to assist them in their necessarily slow development of understanding.

Children can be seen as developers of mathematical theorems. This informal mathematics is, as Vergnaud (1983) points out, very powerful. That which the child is able to accomplish intuitivaly far surpasses their knowledge of formal mathematics. Learning is the result of a need to generalize, to remove the details and find common patterns (Davis, 1984). Encouraging this activity by error analysis, by helping the child find those patterns which are successful in the solution of problems, is one approach to the teaching of mathematics.

INTELLIGENT COMPUTER ASSISTED INSTRUCTION

Intelligent computer-assisted instruction (Barr and Feigenbaum, 1982) involves computer programs that attempt to model the user, either directly through the incorporation of mistakes that are made, or indirectly, through the comparison of responses to those of an expert.

The main components of such systems are (Barr and Feigenbaum, 1982):

1. Problem-solving expertise - the knowldege that the program tries to convey to the user.

2. The student model - an indication of what the student does and does not know.

3. Tutoring strategies - how the material should be presented.

As Barr and Feigenbaum (Barr and Feigenbaum, 1982, page 229) further point out:

> "Not all of these components are fully developed in every system. Because of the size and complexity of ICAI programs, most researchers tend to concentrate their efforts on the development of a single part of what would constitute a fully usable system."

Some of the issues that the developer of ICAI must concern themselves with include (Pliske and Pstoka, 1986):

1. The representation of knowledge in the program.

2. Whether a system can consistently provide adequate solutions to the problem at hand.

3. The theoretical model of the expert and/or student.

4. The level of detail needed in response to the user's error or request for help.

5. The problem of usually only one conceptual organization of knowledge.

6. The user interface often makes little use of simulations of mental models or pictorial representations.

Harris and Owens (1986) further point out that:

1. Knowledge representation is often atheoritical, influenced more by the coding system favored by its originator than research.

2. Performance criteria are vague.

3. Building in the appropriate amount of uncertainty into models not inherently deterministic is difficult.

4. Workload assessment techniques are needed that can quantify cognitive performance.

5. Stress in some tasks (especially military) will affect human-machine dynamics.

John Self (1979) discusses the following difficulties with reference to the student model of an ICAI system:

1. Representation - multiple representations are to be preferred since different representations such as frames, procedural networks and graphs, have advantages in different situations.

2. Content - given a user's answers to questions, the program must hypothesise an explanation on what is usually incomplete or ambiguous evidence.

3. Creation - are the student characteristics to be pre-stored or created by the teaching program?

4. Change - designing programs that learn from their interaction with the user.

5. Growth - determining what changes to the student model should be made.

6. Execution - the possibility of run-time errors.

7. Comparison - devising an appropriate teaching strategy by comparing the student model to an expert model.

8. Use for planning - develop a global rather than step-by-step teaching strategy.

9. Use for monitoring - as more effective learning is likely with student participation how can the student model be combined with the user responses to predict appropriate outcomes?

10. Efficiency - the cost of implementing a program and its response time to queries.

The preceeding concerns highlight the difficulty involved in developing useful ICAI programs. By examining some of the more significant work of the past, with relation to those attributes of ICAI described above, we should be able to come to a better understanding of the research currently being undertaken and the place of the present study within its scope.

HISTORY

Early research on ICAI systems was concerned primarily with the representation of the knowledge domain of an expert in that domain. This expertise enabled the programs to respond to a large number of questions.

SCHOLAR (Wyer, 1983) was created by Jaime Carbonell and Allan Collins. Its purpose was to tutor students about some simple geographical facts concerning South America. It allows both program and student initiated conversations through the posing of questions. The program itself can be used to teach material other than geography as the knowledge is not embedded within the tutor. SCHOLAR also "knows" about the extent of its own knowledge.

SOPHIE (Brown, J.S. and Burton, R., 1982) undertakes to teach problem-solving skills related to an electronics labratory where there is malfunctioning electronic equipment that needs to be repaired. The students take measurements of various devices and develop a theory as to what is wrong. SOPHIE analyzes the students' conclusions and where there are logical errors can provide other examples and suggestions. SOPHIE is provided with a natural-language interface so that the student may input commands in ordinary English forms.

From what was seen as a large database that contained all the facts needed to be taught, a shift was made to the replication of human reasoning. Problem solving methods were modelled on human reasoning patterns to make their line of reasoning more understandable to those using them.

WEST (Burton, R. and Brown, J.S., 1982) is a program based on a children's mathematical game called "How The West Was Won." It is conceived of as a computer coach, which while allowing the student to play the game, will interrupt with advice or suggestions only when it determines that the student could use some help to optimize his performance. The program has been used at an elementary school level where the coached group was shown to use a greater variety of mathematical expressions then the uncoached group.

GUIDON (Clancey, 1983) is a system that uses a set of diagnostic rules developed in the MYCIN program as the knowledge base with which to guide students' learning of the diagnosis of a patient who is suspected of having an infection. It makes use of lengthy discussions which do not rely on the student's last input.

BUGGY (Brown and Burton, 1978) attempts to explain arithmetic errors by analyzing the mistakes students make. This is based on Brown's assumption that students are very good procedure followers, but that they sometimes follow the wrong procedures. BUGGY presents examples of incorrect behaviours and asks the teacher to diagnose the error. BUGGY has been the basis for much of the educational research presently taking place, including this study. As teachers tend to concentrate on whether the problems a student solves are right or wrong, difficulties encountered may appear to be greater than they actually are. Because of the procedural interrelationship of arithmetic operations, a student can get many problems wrong by having one basic misconception in an important algorithm that is being used to solve different questions.

The BUGGY system is strictly diagnostic, with no attempt being made to tutor the participants. The program was used with both student teachers and school age pupils in order to increase their ability in diagnosing subtraction errors. Also, arithmetic errors of a large population of school children were diagnosed and classified by the researchers.

A diagnostic model representing a student's procedural knowledge must be built. The technique used to represent diagnostic models is a procedural network which is a collection of procedures where the control structure between procedures is made explicit. BUGGY was an attempt to incorporate such a model in a computer program.

There are, as is noted by researchers, several complications with this simple paradigm for diagnosing a student. One complication is that the student who has developed a novel bug (as opposed to one that arose out of a combination of 'primitive' bugs) will not be diagnosed. Another is that students do make 'random' mistakes (presumably as many while following an incorrect procedure as a correct one) that could erroneously lead to the exclusion of his bug or the inclusion of another bug that happened to coincide with a student's 'randomness.' Finally, blindly considering all possible combinations of bugs can lead to an explosion of possibilities.

CURRENT RESEARCH

Current research in ICAI, at the elementary and secondary school levels, is generally based on suggestions imp t in the original BUGGY program. BUGGY has spawned many interesting research designs but such designs have tended towards mathematically simple operations devoid of a meaningful problem context.

The most immediately obvious example of a BUGGY related research program is DEBUGGY (VanLehn, 1982), an extension of the BUGGY system. DEBUGGY has been used to analyze students' subtraction errors in an attempt to understand the procedures they are applying. Many more "bugs" than were originally posited have been discovered by the program. At the moment, there are hundreds of documented "bugs", a most unfortunate situation as far as being of use to the average teacher. While the program has been used in the classroom to analyze students' errors, remediation has been limited to informing teachers that individual students do indeed have a problem.

VanLehn has used DEBUGGY (Brown and VanLehn, 1980) to arrive at what he calls Repair Theory. This theory tries to explain errors (other then arithmetic slips), by arguing that when a student has unsuccessfully applied a procedure, he will attempt a repair. This attempt will often lead to an impasse that the student will try to surmount by further patching of the algorithm.

Tatsuoka at the University of Illinois (Tatsuoka and Eddins, 1985) has also used the idea of "bugs" to analyze students learning of the concept of signed number operations and how erroneous rules are formed.

At a tutorial level, Attisha and Yazdani at the University of Exeter, have developed two microcomputer-based systems for diagnosing childern's subtraction (Attisha and Yazdani, 1983) and multiplication (Attisha and Yazdani, 1984) errors respectively. The systems are designed to allow the student to interact with the computer to improve their performance. Unfortunately this interaction seems to consist solely of a statement as to what the student has done to arrive at his answer. In other words, a description of the incorrect procedure or procedures the child has chosen is written to the screen.

Another, less direct, off-shoot of Brown and Burton's original work has been the work of Sleeman (Sleeman, 1982) at the University of Leeds. At this point though, he has come to a different conception of the learning model students use (Sleeman, 1984) than has VanLehn. Sleeman's Leeds Modelling System (LMS) is an ICAI system that analyzes what he t[illegible] mal-rules ("bugs") in the solving of algebraic expressions. He has used the program with 14 and 15 year-olds. Sleeman argues that Vanlehn's Repair Theory proposes a specific mechanism common to all students, whereas he believes that the problem is one of misgeneralization; that is, students infer several rules consistent with an introductory, simple example, not just the teacher-given algorithm. This initial knowledge, according to Sleeman is extremely influential, and given the notion of learners as active theory builders, who form theories and then test them out, an algorithm that generally works will usually be applied to a similar situation. Once a mal-rule is created it may be very difficult to eliminate. Sleeman further suggests that while Repair Theory may be appropriate for the explanation of the application stage of mathematics, misgeneralization is more plausible in explaining rule acquisition.

A self-improving quadratic tutor was developed by Tim O'Shea (O'Shea, 1982) at the University of Leeds. A set of production rules was established to envelop the teaching strategy. The tutor learns about the utility of a teaching strategy by looking at the student's responses. Changes were made to these rules after selecting a particular educational objective. After evaluating the program's resulting performance, updating of the production rules was undertaken. Students were successful in discovering the rules of quadratic equation solving through the representation of a variety of computer generated examples.

An ICAI system for teaching equation solving (Lantz, Bregar and Farley, 1983) uses production rules to build the expert module, while a tutorial interface controls the rule-based problem solver so that it can emulate a student's steps in the solution of a problem. The system can check ahead to see if the student is on the right track, or back up if the student has gone astray. In a sense, as the authors note, their system acts like a coach. While the program is sensitive to errors, especially those concerning operator understanding and selection, it is not based on "bugs." The system, while flexible, is limited to linear equations of no more than two variables. This expertise limitation is indicative of one of the major stumbling blocks of ICAI - the restriction of the knowledge domain. This constraint occurs in order to account for virtually every conceivable student misunderstanding within a domain.

From the above summary, it is evident that much of what can be deemed as specifically educational research in the ICAI field is concerned with the way children learn. Actual remediation or tutoring has generally not been stressed either at the student or teacher level. It is interesting to note that most of the more ambitious examples of tutoring are taking place in the context of learning programming languages in computer science departments. Systems such as PROUST (Johnson and Soloway, 1985), which tutors beginning PASCAL programmers, and GREATERP (Anderson and Reisner ,1985), which does the same for LISP, have been developed. This evolution is occurring not only because authors have access to large amounts of computing power and expertise, but because they believe that such programs make much more efficent use of development time than do standard CAI programs.

CHAPTER III

DESIGN FOR THE STUDY

INTRODUCTION

This section will focus on the description of the sample, deliniation of the project goals, description of the project design, ethical considerations, the computer program to be used, monitoring instruments and finally the analysis of data.

THE SAMPLE

The prospective teacher sample for the project consisted of volunteer students enrolled in the Faculty of Education at the University of Alberta. It is expected that because of the volunteer nature of the project some confounding influences may have been introduced.

Of the total of twenty-two participants in this phase of the experiment, seven were mathematics majors and fifteen were mathematics minors with a variety of majors, such as business education, physical education and science education.

The ten experienced teachers who participated in the study were taking part in a LOGO workshop concerning list processing. They were all mathematics teachers of varying degrees of experience, training and grade level placement.

PROJECT GOALS

The project goals which follow are considered pertinent to the development of an attitude amongst educators, of children as theorem developers, who bring certain non-formal mathematical understandings to the formal mathematical problems they encounter in school. The means to this goal is assumed to involve the development of an understanding that the child's errors are not senseless but can offer us a way to recognize and treat difficulties encountered in certain mathematical situations.

The goals of the project can be outlined as:

1. Participant recognition that solutions to proportion problems are based on analyzable procedures.

2. Participant realization that by understanding how a child arrives at an incorrect solution the teacher has a formidable tool in assisting that student.

3. Development of a useful computer model of a student experiencing difficulties with proportion problems.

PROJECT DESIGN

The original project plan was not viewed as a rigid and fixed schedule of future events but as a tentative outline which could be adapted as the needs of the study become more clear through its implementation.

For each prospective teacher group participating in the study the following sessions were undertaken:

SESSION 1 - An introduction to the project.
- Attempts to analyze student errors on proportion problems.
- A questionnaire concerning concept development.

SESSIONS 2-4 - Small groups using the program.

SESSION 5 - Further attempts to analyze errors.
- A follow-up questionnaire and

debriefing of participants.

The experienced teachers, because they were participating in a day long workshop, while following the same type of schedule as the student teachers, were only able to spend one hour using the program. All other activities were of a length equal to that of the other group.

THE COMPUTER PROGRAM

RATIOERROR is a LOGO program which develops a student's wrong answer to a ratio question by following a path through a procedural network and asks the user to determine the route taken to arrive at that incorrect response. The program, as it is presently written, is meant to encourage the teacher to analyze a student's incorrect procedure.

The user is initially presented with a proportion question and a student's incorrect solution to it. There is an opportunity to create similar questions with the computer supplying the appropriate incorrect answer that the student would generate. When the user feels they have sufficient knowledge to state the error path they are tested by attempting to supply the student's answer to questions posed.

The various errors analyzed are based on the work of Vergnaud. He uses the concept of a measure space to describe a ratio question as follows:

M1	M2
A	C
B	?

An example would be: In A hours heating consumption is C litres of oil. What is the consumption in B hours?

Following are descriptions of the errors with tree diagrams of their representation.

Erroneous Scalar: use of a scalar ratio or difference to which B is applied.

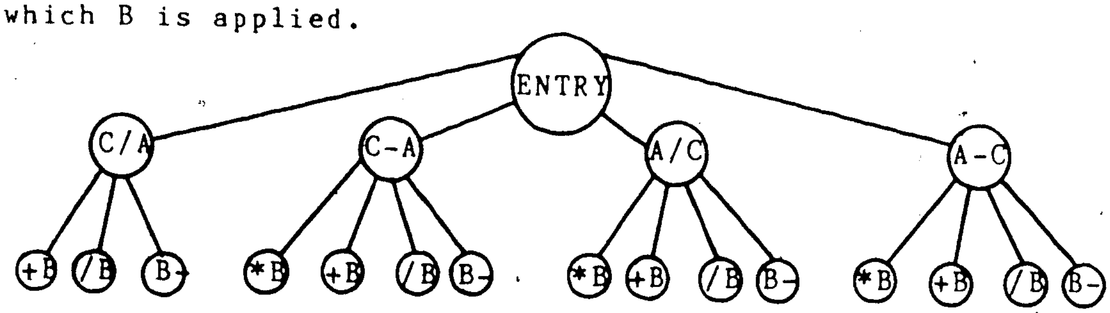

FIGURE 1a

Erroneous Function: use of function ratio or difference to which C is applied.

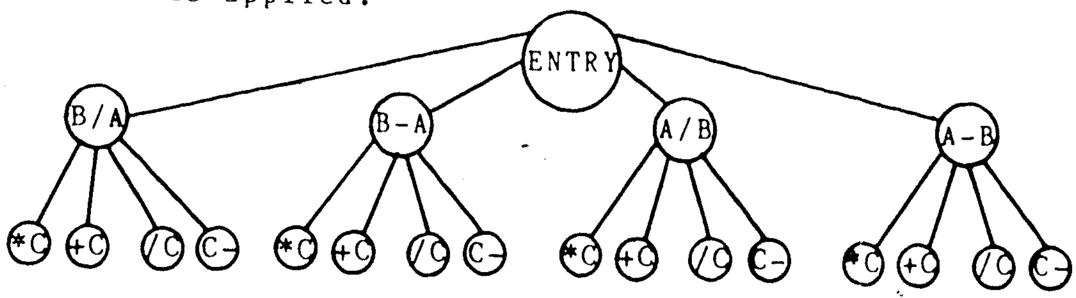

FIGURE 1b

Erroneous Product or Quotient: multiplies or divides which has no physical meaning.

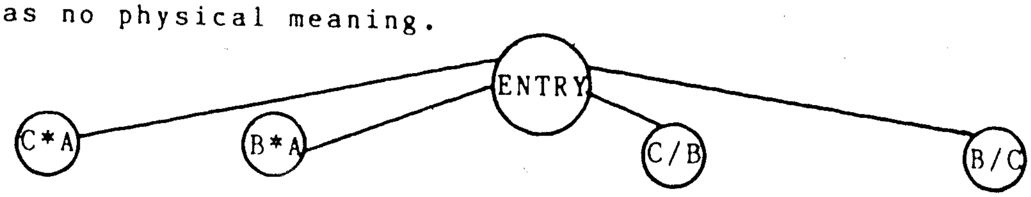

FIGURE 1c

Erroneous Scalar and Function: combination resulting in B * C.

<u>Computer System</u>

The program was written in IBM LOGO requiring 40K of memory and ran on IBM XT's operating under the JANET2 network. Each machine had 640 kilobytes of random access memory. The program was resident in memory and thus there was no use of the computer network except in downloading the program to each machine.

Program Description

The algorithm for the actual program presentation is described below.

1. Present a ratio question.

2. Choose an incorrect answer by randomly following an error path as outlined in figures 1a, 1b and 1c.

3. Present an opportunity to test solutions by allowing the user to assign values to variables in the original question.

4. Supply student's incorrect answer to the supplied variables.

    a. If requested, access help page.

    b. If requested, repeat [3] and [4].

5. Ask for a description of the error path hypothesized.

6. Present a new question.

7. Ask for the appropriate incorrect student response.

   a. If requested, access help page.

   b. If requested and second time through, provide error path.

   c. If answer incorrect, repeat [3] through [7].

   d. If answer correct and first time through, repeat [6] and [7].

8. Describe error path.

9. If not end of questions, goto [1].

LOGO Representation

The error paths, accessed randomly by the program, were represented by means of data structures called trees. Each tree is a list consisting of a number of nodes, all but the terminal nodes having four children. The first node of each tree is empty, that is, it does nothing. All other nodes specify a mathematical operation to be performed.

The method of representing trees in LOGO is simple and consistent. To build the trees used the following procedures adapted from Weston (1984) were employed.

```
TO TREE.CONSTRUCT :SUBJECT
   PRINT [WHAT IS THE ROOT NODE?]
   MAKE :SUBJECT TREE.BUILD READLIST
END

TO TREE.BUILD :NODE
   IF EMPTYP :NODE [OUTPUT []]
   OUTPUT (LIST :NODE
                TREE.BUILD GET.FIRST :NODE
                TREE.BUILD GET.SECOND :NODE
                TREE.BUILD GET.THIRD :NODE
                TREE.BUILD GET.FOURTH :NODE)
END

TO GET.FIRST :NODE
   PRINT :NODE
   PRINT [FIRST ...]
   OUTPUT READLIST
END

TO GET.SECOND :NODE
   PRINT :NODE
   PRINT [SECOND ...]
   OUTPUT READLIST
END

TO GET.THIRD :NODE
   PRINT :NODE
   PRINT [THIRD ...]
   OUTPUT READLIST
END
```

```
TO GET.FOURTH :NODE
   PRINT :NODE
   PRINT [FOURTH ...]
   OUTPUT READLIST
END
```

To traverse the trees once they were established, the following procedures were used.

```
TO EXPLORE :TREE :ANSWER
   IF EMPTYP :TREE [STOP]
   RUN FIRST :TREE
   IF :SOFAR = :ANSWER [GO "DONE]
   EXPLORE LEFT2.BRANCH :TREE :ANSWER
   EXPLORE LEFT1.BRANCH :TREE :ANSWER
   EXPLORE RIGHT1.BRANCH :TREE :ANSWER
   EXPLORE RIGHT2.BRANCH :TREE :ANSWER
   LABEL "DONE
END

TO LEFT2.BRANCH :TREE
   OUTPUT FIRST BUTFIRST :TREE
END

TO LEFT1.BRANCH :TREE
   OUTPUT FIRST BUTFIRST BUTFIRST :TREE
END

TO RIGHT1.BRANCH :TREE
   OUTPUT LAST BUTLAST :TREE
END

TO RIGHT2.BRANCH :TREE
   OUTPUT LAST :TREE
END
```

Each node was represented by a procedure that performed a mathematical operation and kept track of the path taken. An example of one such node is given below.

```
TO X.TIMES.Y :X :Y :X1 :Y1
   MAKE "SOFAR :X * :Y
   MAKE "PATH (LIST WORD ": :X1 "* WORD ":
:Y1)
END
```

MONITORING INSTRUMENTATION

Pretest

All participants attempted to analyze student errors to ten proportion problems and try to explain how a student could have arrived at such an error. These mistakes were taken from junior high students' errors to the problems given. It was developed with the assistance of a mathematics teacher and validated by two professors of mathematics education.

The test was designed to lead into and focus the questions to be answered on the questionnaire described below. As the pretest was basically a written interpretation of those problems which will follow when using the computer program, it was hoped that the carry-over from the pretest to the posttest would be small.

Pre-Project Questionnaire

An open-ended questionnaire concerned with the attitudes of those participating to the usefulness of analyzing student errors was given. This was supervised by two professors of mathematics education.

Video Tape Recording

A video recorder with a microphone was interfaced directly to the computer screen and a microphone put in operation to document group comments as they worked through the program. The tapes were analyzed in an attempt to clarify the perceptions of the users regarding error analysis.

Posttest

Similar to the pretest in content though with different questions.

Post-Project Questionnaire

Similar to the pre-project questionnaire in content though with different questions.

ANALYSIS OF DATA

The data acquired from the previously described instruments was used on four levels:

First, the information obtained from the video-tapes described the process of interaction with the computer program by the participants. Specifically an endeavor was made to classify comments and reactions to the errors. This classification was presented in terms of the significance of such errors to the teaching process in terms of their usefulness in remediation as well as to the view of the child as a theorem developer.

Second, an attempt was made to establish the effect of using the computer program on the stated attitudes of the subjects towards children's mathematical errors. Questionnaire responses were analyzed and broadly categorized so that a pattern might emerge regarding change in attitudes towards proportion errors in particular, and mathematical errors in general. These patterns were established by means of matrix representations of the responses.

Third, an effort was made to determine any change in the ability to discover causes of errors on proportion problems. Performance scores for each participant were analyzed in terms of both the pretest/posttest as a whole, and specific categories of errors. These specific scores were of use in evaluating relative difficulties encountered in analyzing certain types of errors. Due to the small size of the groups involved the data were not statistically analyzed.

Fourth, the computer program itself was evaluated in terms of some of the important considerations presented in Chapter II (Pliske and Pstoka, 1986, Harris and Owens, 1986 and Self, 1979) including:

1. Problem specification - the problem must be specified in terms of a comparison to the expert's natural process based on theoretical models.

2. Problem representation - must provide an effective representation of the chosen model.

3. Program content - given the user's answers to problems must be able to hypothesize an explanation consistent with the model in question.

4. Program interface - should be interesting with pictorial representations and

simulations of mental models of the user.

5. Learning outcomes - should satisfy learning objectives in a testable fashion.

6. Efficency - is not cost prohibitive and is search proficient.

CHAPTER IV

RESULTS OF THE STUDY

INTRODUCTION

Chapter IV will deal with the implementation of the research design, paying particular attention to the groups involved and the computer program used. Actual scores on the tests of error analysis of student mistakes will be examined, some of the comments made by those prospective teachers while dealing with the student simulation will be brought forth, and an analysis of the reponses to the questionnaires concerning views of proportion errors and their usefulness will be given. Following this, an examination of the computer program will be undertaken.

Nature of the groups

Two groups were tested using the program:

1. Prospective teachers whose major or minor is mathematics
2. Practising teachers, teaching at least some mathematics.

Procedure

Each group was given a pretest involving the analysis of student errors in proportion problems (Appendix 1) and a questionnaire concerning their attitudes towards such errors (Appendix 3). As a result of absences on the day that this phase was undertaken with the prospective teachers, 5 students did not write the pretest or fill in the initial questionnaire.

Following this initial testing, each group spent time using a computer program which simulates such student errors as was described in Chapter II. They then were given a posttest (Appendix 2), similar in content to that previously taken, concerning analysis of proportion errors, as well as a further questionnaire (Appendix 4) regarding attitudes towards student errors on proportion problems.

The prospective teachers, both mathematics majors and minors, spent two hours using the program. The practicing teachers, because participation was part of a LOGO workshop, were only able to spend approximately one hour using the program.

Each group was given 15 minutes to answer the error analysis questions on both the pretest and posttest. Few completed the pretest in the alloted time, while the majority of the prospective teachers were able to finish the posttest. They had as much time as they needed to fill out the open-ended questionnaires.

RESULTS OF THE STUDY

Error analysis

All groups showed improvement in their ability to analyze student errors of the types detailed by Vergnaud after spending time using the computer program.

The pre and post tests contained ten questions with five errors being of the scalar type and five of the function type. Question types were scattered throughout the test to minimize the effect of placement. Each group was given 15 minutes for each test.

As can be seen in Figure 2, experienced teachers received lower scores on both the pretest and posttest. This may have been due to their participation being a small part of a day long computer workshop whose primary purpose as far as they were concerned, was the discussion of list processing in LOGO. Their inital score of 38% overall improved to 62% in the posttest, after approximately one hour using the computer program.

Those prospective teachers who took the pretest, initially received a score of 50% solved. In the posttest, written several days after their use of the computer, they improved to an 83% solution rate. All participants in this group were volunteers, recieving no compensation in any form.

The five prospective teachers who did not write the pretest, but did use the computer program for two hours, scored 78% on the posttest.

The prospective teachers initially experienced more difficulties with errors of the functional type than those of the scalar type. After using the program, where they were exposed to numerous examples of ~~each, they became~~ equally adept at the solution of each.

Findings for the experienced teachers were dissimilar, with no inital differences between the diagnosis of scalar and function errors, nor were there any on the posttest.

SCORES ON ANALYSIS OF PROPORTION ERRORS

Prospective Teachers (Pretested)	Perspective Teachers (No pretest)	Experienced Teachers (Pretested)	Researcher's Comments
17 50 %	5	10 38 %	Interesti[illegible] note the lower levels o[illegible] icing teachers.
17 83 %	5 78 %	10 62 %	All groups improved significantly - those who did not t[illegible] the pretest did almo[illegible] well as those who did take it.

FIGURE 2

Participants' observations during program

All computer sessions involving the prospective teachers were video-taped by means of a direct connection of a video recorder to the computer monitor so that the computer screen was recorded at all times, in conjunction with the participants' reactions being recorded by a microphone as they worked through the program. Participants were generally in groups of two or three people.

There were many interesting reactions and changes amongst the participants as they actually worked through the program. There was an initial tendency to be disbelieving that any student could make some of the errors exhibited by the program. From this initial doubt there was a movement towards analysis - why was the student doing things this particular way. Finally the focus became remediation and teaching - how could the errors be overcome.

Most initial comments concerned the apparent strangeness of the errors, coupled with an inability to understand why a student would respond in such a manner.

> I like that - he subtracted hours from dollars. Its unbelievable.

There's no rationale for some of these errors because all we are doing is manipulating the number.

I don't know where these kids get these things. At least he's dividing using the same units. Some of them weren't.

From this initial sceptisim there was a progression towards trying to theorize about why a student may be making errors.

He probably tried it out once something like this and it worked and then ... hah!

See I don't believe a kid whose thinking as wrong as these kids are thinking are going to use the same error paths twice in a row.

Maybe the kid read the problem incorrectly.

So what the kid is forgetting is that it matters how many boys you had in the first place.

He's always going to put the difference down. He doesn't even have the concept of if you're given these two ratios - what you're given is going to determine ...
Yeah, but if you read the question you might ask where they get them from. I could never understand ratios I'd always have them backwards and screw them up.

Finally there was an attempt to understand some of the underlying difficulties with the ratio concept that could be the basis of such errors as were being exhibited.

Now you cannot divide seconds by a whole -

> that's the amazing thing.
> That's why I tell them to put their units in. But then my cooperating teacher said no they get all mixed up on them. But then you can cancel them out and its easy doing anything.
>
> I taught the ratio proportions and I thought it was the actual interpretation. Once they got the ratio down they knew what to do with it. It was trying to get the right numbers in the right places.
>
> What do you do with a kid who does this thing for one and another thing for one?

<u>Participant questionnaires</u>

Participants were given pre and post questionnaires, with a number of questions concerning the importance of errors in proportion problems. Answers to the questions were analyzed and separated into appropriate categories. To facilitate analysis of the responses, answers were placed in only one category with the most likely category chosen.

Prospective teachers answered the post questionnaire from 5 to 7 days after responding to the pre questionnaire. Experienced teachers answered the post questionnaire later in the same day as they answered the pre questionnaire due to the fact that this was part of a LOGO workshop that the participants were taking.

Students' Difficulties with Proportion Problems

Prospective teachers (Figure 3) initially felt that most difficulties inherent in proportion problems could be seen as a lack of understanding on the student's part as to what is involved with the notion of ratio. They theorized that the concept may have been introduced too early, that it was hard for a student to visualize the relationship between ratios, or that the student did not know what to find. A significant minority believed that problems stemmed from the actual setting up of the ratio problem, such as rewriting the problem in mathematical form.

After spending approximately two hours using the computer program which simulated a student experiencing difficulties with proportion problems, the participants exhibited a marked tendency to blame such difficulties on the setting up of the problem, of deciding how the components related to each other. This is not all that surprising as the program was written in such a way as to emphasize this setting up (by means of the construction of a small chart) to follow what often seems at first like impossible errors by means of visualizing relationships between components of that chart. They also seemed to

STUDENTS' DIFFICULTIES WITH PROPORTION PROBLEMS
ACCORDING TO PROSPECTIVE TEACHERS

Too Abstract	Understanding	Setting Up	Other	Researcher's Comments
4	7	6	1	
introduced too early hard to visualize	what to find concept not understood relationship between ratios	don't know how to set up placing values rewriting in math form	carelessness	A marked shift to the idea that setting up a ratio problem is the most probable difficulty.
0	3	16	3	
	concept not understood what to find cross-mult. without understanding	rewriting in math form organizing information mixing up terms	solving condition no special difficulties not sure	

FIGURE 3

feel that this could be an important tool in the organization of information and making questions more meaningful to the student experiencing difficulties.

The experienced teachers' responses (Figure 4) to this particular area of the questionnaires changed little after their short exposure to the program. There was a slight tendency to concentrate on comprehension and abstraction even when describing an error pattern, where the prospective teachers looked at such patterns (after completing the computer program), as being more mechanical in nature. Experienced teachers still seemed to be more concerned with finding underlying causes of error patterns, rather than those patterns by themselves.

STUDENTS' DIFFICULTIES WITH PROPORTION PROBLEMS
ACCORDING TO EXPERIENCED TEACHERS

Too Abstract	Understanding	Setting Up	Other	Researcher's Comments
2	5	3	0	There was no significant movement in this catagory amongst the experienced teachers.
cannot see data in a "wholistic" manner equations are so abstract	just follow algorithm don't know what is being compared	not enough care taken in setting up		
1	4	3	2	
confusion over number rep.	developing an error sequence making comparisons	organizing data transposing	no response	

FIGURE 4

Usefulness of Incorrect Answer

Prospective teachers, in the pre-questionnaire (Figure 5), concentrated on the importance of the incorrect answer in helping remediate student errors. They were concerned with the methods to use establishing an overall view of the situation which could in some way help to bring the student up to the level of work that would be expected. The shift towards teaching strategy, as witnessed in figure #6, was due entirely to the influence of those who did not fill out the pre-questionnaire, they chose the identification of an improved teaching strategy as the focus brought about by looking at an incorrect answer. There seemed to be almost no movement in relation to this question, with respondents believing that incorrect answers either told them something about the success of their initial teaching strategy, how to help an individual student or the ability of the student. It was noted however that those who concentrate on student ability, after using the simulation, were more inclined to speak of it in terms of things that could [illegible]latively easily remediated, as opposed to their [illegible]ial inclination to describe such things as stages of thought and difficulty with abstract concepts. The pre-questionnaire contained vague comments

USEFULNESS OF INCORRECT ANSWER
ACCORDING TO PROSPECTIVE TEACHERS

Student Ability	Remediation	Teaching Strategy	Other	Researcher's Comments
4	9	3	1	There was some shift towards incorrect answers leading to new teaching strategies.
stage of thought concrete/ abstract	method used view of situation	evaluation of teaching strategy to teach	carelessness	
8	8	9	0	
inco[illegible] lear[illegible] probl[illegible] areas	method used identify where goes wrong	focus instruction identify poor teaching		

FIGURE 5

on the usefulness of the incorrect answer such as:

> The child didn't understand.
>
> Incorrect answers show how the children think (in concrete or abstract terms).
>
> Student ability.

On the post-questionnaire the concerns expressed through the examination of the incorrect answer were much more specific, including:

> Whether or not the student is thinking or just juggling around the numbers.
>
> When the student is confused in the algorithm.
>
> Detect comm n errors.

While practicing teachers (Figure 6) were split evenly between student ability, remediation and teaching strategy (with one no response), there was a significant shift after using the simulation program towards the usefulness of incorrect answers in the remediation process. On the pre-questionnaire typical responses included:

> Not much - the teacher would have to also look at the work and discuss it with the student.
>
> How far off the student's thinking is - at what level the student is at.

USEFULNESS OF INCORRECT ANSWER
ACCORDING TO EXPERIENCED TEACHERS

Student Ability	Remediation	Teaching Strategy	Other	Researcher's Comments
3	3	3	1	
level student is at	put the student back on track	evaluation of teaching	no response	There was a marked shift towards the importance of individual remediation as opposed to student ability.
0	8	2	0	
	possible errors error pattern student logic	where teaching has gone wrong		There seemed to be an increased tendency to focus on what to do rather than on judgment.

FIGURE 6

On the post-questionnaire the responses changed to reflect an increased concern with using the incorrect answer:

> We can learn what misconceptions a student has and be better able to correct the error.
>
> How far off the student is - ie. is it setting up the condition or is it in the process of calculation?

There seemed to be a greater interest than was exhibited by the prospective teachers as to the need to work individually with their pupils, with much less emphasis on changing teaching strategies. This is not surprising as most of these teachers would be expected to be fairly confident in their own teaching styles and abilities whereas those who have not had that experience would tend to be more concerned with their possible inadequacies as teachers.

Focus When Helping Students

As to how the student who is experiencing difficulties with proportion problems should be helped, prospective teachers who completed both the pre and post questionnaires (Figure 7) showed little variation in their focus, though most chose either to dwell on the incorrect

FOCUS WHEN HELPING STUDENTS EXPERIENCING DIFFICULTIES ACCORDING TO PROSPECTIVE TEACHERS

Correct Procedure	Incorrect Procedure	Both	Other	Researcher's Comments
5	7	5	0	Most of the apparent movement was attributable to those who did not do the pre-questionnaire stating that the incorrect procedure is what should be empasized.
more reinforcing student will remember	show what is wrong	equally important		
3	14	5	0	
learn rules properly	how they arrived at the answer focus on the problem	first, the error, then the correct procedure		

FIGURE 7

procedure or on both incorrect and correct. On the pre-questionnaire typical responses were:

> Concentrate on the correct response but do not neglect the reason for the incorrect one.
>
> I feel it is most important for the student to analyze his techniques to see where the mistake was made and to remedy the misunderstanding.

Those prospective teachers who did not do the pre-questionnaire were unanimous in believing that the focus should be on the incorrect procedure. Viewing the computer simulation seemed to focus their perceptions on the incorrect by reviewing the numerous errors committed. Responses to the post-questionnaire included:

> Focus on how they arrived at their answer.
>
> It is better to focus on how they arrived at their answer because then you will be able to understand where they need help.

Experienced teachers (Figure 8) showed no significant change regarding focus of remediation. Most felt the concentration should be on the incorrect procedure, with some believing that both must be examined. Typical comments for both pre and post-questionnaire included:

> Focus on how they arrived at their answer.
>
> Focus first on how they arrived at their answer. You must understand how they are

FOCUS WHEN HELPING STUDENTS EXPERIENCING DIFFICULTIES
ACCORDING TO EXPERIENCED TEACHERS

Correct Procedure	Incorrect Procedure	Both	Other	Researcher's Comments
1	5	3	1	No significant change was noted between the pre and post questionnaire for the experienced teachers.
concentrate	misperception can then be solved	how student arrived at response and direct to correct path	no response	
0	7	3	0	
	you must understand how they arrived at their answer	how they arrived at answer and then focus on correct algorithm		

FIGURE 8

It would seem that those teachers with classroom experience, unlike their inexperienced colleagues, felt that they already knew where to concentrate when there were problems. The majority believed that it was necessary to focus on the student's incorrect procedure, though a few felt there should be an equal emphasis on the incorrect procedure and the correct one.

Developer of Mathematical Rules

As a developer of mathematical rules, the teacher was chosen initially by 8 of the 17 prospective teachers (Figure 9) who completed the pre questionnaire. Responses included:

> The teacher should provide the appropriate rules after some concrete examples have been presented.

> The students should try to create the rules if they can; however, should they (the students), be too far off base then the teacher should define the rules for clarity.

After using the computer program simulating a student with difficulties, only 3 maintained this stance, with the majority opinion being that both the student and the teacher must develop rules in a joint effort. Responses included:

> Students should be guided because it is essential that students find patterns in problem solving by themselves - this facilitates learning and not rote memorization.

> Both are important. If students develop their own rules, they will be at their level of understanding, yet some teacher rules are necessary.

Experienced teachers (Figure 10) changed from generally believing that the student should develop

DEVELOPER OF MATHEMATICAL RULES
ACCORDING TO PROSPECTIVE TEACHERS

Teacher	Student	Both	Other	Researcher's Comments
8	4	5	0	
correct foundation with appropriate examples	try to create discover	combination joint effort		From a concentration on the role of the teacher, the participants came to view development as the work of both the student and the teacher, quite a few still held the student should be the primary developer.
3	7	12	0	
prescribed method students obviously need help	derive own won't mis-apply rules	joint effort guided by teacher developed by students, clarified by teacher		

FIGURE 9

DEVELOPER OF MATHEMATICAL RULES
ACCORDING TO EXPERIENCED TEACHERS

Teacher	Student	Both	Other	Researcher's Comments
1	6	3	0	A large shift away from the student as the developer of rules to both student and teacher as being equally involved.
by teacher	if a student understands a concept they will develop own should not give rules	guided by the teacher		
2	1	7	0	
given by teacher	developed by students within the parameters of the teacher	both guided by teacher developed by students, assisted by teacher		

FIGURE 10

rules so that they can truly understand a concept, to a joint rule development, guided explicitly by the teacher. Typical responses to the pre-questionnaire were:

> I think that there is a time for both approaches, however if students truly understand a concept they will develop their own rules.
>
> When introducing a problem, the teacher should always (ideally) let the students generate their own rules.

On the post-questionnaire responses included:

> I think that both situations are appropriate and guided development is a good approach.
>
> Developed by the students with assistance from the teacher.

Learned From Using the Computer Program

Prospective teachers generally felt that they had come to learn more about why errors in proportion problems were made (Figure 11), as well as the unexpected range of errors possible. Responses included:

> There are many different ways of making errors - we must be aware of them.
>
> That there are many common error paths.

Experienced teachers (Figure 12), while often

LEARNED FROM USING COMPUTER PROGRAM
ACCORDING TO PROSPEGTIVE TEACHERS

Mistakes Possible	Analyzing Skills	Value of Incorrect	Other	Researcher's Comments
9	13	1	0	Most felt it sharpened their skills and increased their awareness of possible errors.
range of errors.	why errors are made	now see value in looking at incorrect answer		
mistakes students tend to make	common errors			
	no units			
	illogical			

FIGURE 11

LEARNED FROM USING COMPUTER PROGRAM
ACCORDING TO EXPERIENCED TEACHERS

Mistakes Possible	Analyzing Skills	Value of Incorrect	Other	Researcher's Comments
2	3	4	1	
errors in setting up	analyzing errors may lead to best teaching approach	frequently we think we know why and we're off track - by trying to help we just confuse	nothing - I do this type of analysis with most of my students' tests	wide range of responses - a number emphasized importance of the incorrect answer when trying to help.

FIGURE 12

mentioning analyzing skills, were generally more taken with the usefulness of incorrect answers. Responses included:

> Frequently we think we know what errors a student is making and we may be totally on the wrong track. Helping them with what I think the error is may totally confuse the student.

> The mistakes made by students are often very obvious and quite different from what I thought they might be.

Analysis of the Computer Program

The computer program RATIOERROR used in the study was analyzed in terms of the six ~~catagories~~ outlined in the third chapter.

Problem Specification

The problem to be dealt with by the program was simulating a student experiencing difficulties with proportion problems. A model of those errors classified by Vergnaud was implemented. While no attempt was made to revalidate the appropriateness of these error paths, work of ~~junior high~~ students on test questions used to determine the answers to analyze, were readily classifiable according to this scheme.

The program chose various error paths at random with no effort to distribute errors as to their likelihood of occurrence. It could be said then that RATIOERROR is a good model of a different student through each iteration, solving ratio problems in one specifically incorrect way one time. Of course, the program itself does not have a reason for choosing its various paths, it is in effect just manipulating numbers, as it would sometimes appear to the observer that a student is.

Problem Representation

The problem was represented by means of a tree structure chosen for what to the researcher was its obvious suitability. Each node of the three trees involved had four children. This particular representation easily captured those errors outlined by Vergnaud.

Program Content

RATIOERROR was consistent in its provision of appropriate solutions to the types of errors generated. However, because some common errors will always generate the same answer, the program and the user will often offer different yet equally correct explanations. This was a problem when some users felt the program was not choosing errors randomly, when in fact the problem was different interpretations of the same incorrect answer.

Program Interface

Perhaps the weakest area of the computer program was its interface with users. There was no attempt to do more then minimally analyze user responses and provide rudimentary help screens. Pictorial representation was limited to a depiction of information in table form to encourage the idea of setting up the proportion problem.

While some users were easily bored with the program, most became quite engrossed for the short period of time involved.

Learning Outcomes

As outlined in previously in this chapter, using RATIOERROR improved the analysis scores of the participants markedly. From their comments both during the use of the program and afterwards, RATIOERROR generated an increased interest in the usefulness of such tasks in the teaching process.

Efficiency

RATIOERROR, being written in LOGO, was for its size, fairly straightforward to implement. IBM LOGO is quite fast for an interpreted LOGO and computer response times were adequate for the use envisaged.

CHAPTER V

SUMMARY, CONCLUSIONS AND FURTHER RESEARCH

THE STUDY

The purpose o[illegible] study was to design a computer based model of a child experiencing difficulty with proportion problems and investigate its effect on prospective mathematics teachers. Instruction must deal specifically with incorrect responses of individual students. Reinforcement of this need to deal with wrong answers by virtue of exposure to a large number of student errors, in a short period of time, was seen as significant. It was felt that such exposure would not only contribute to the improvement of the analysis skills of the participants but would also increase their awareness of the importance of such skills.

A LOGO program called RATIOERROR was developed that generated errors consistently for a defined type of possible student error to proportion questions as outlined by Vergnaud (1983). Participants were given both pre and posttests on their ability to analyze such errors along with questionnaires concerning their thoughts on the usefulness of incorrect answers to the teaching process. Prospective teachers used the program for approximately two hours, practising teachers for one hour.

CONCLUSIONS

OVERVIEW OF THE RESULTS

The computer model was tested with twenty-two prospective teachers and ten practising teachers. In all cases there was a marked increase in the ability to discriminate various actual student errors.

Most participants indicated a greater specific awareness to the usefulness of an incorrect answer in the teaching of proportion problem solutions. They came to view both student and teacher as integral parts of the development of mathematical [illegible]. There [illegible] an increased focus amongst the participants on the concrete actions possible when confronted with student errors. A corresponding shift away from abstract judgements about student ability was also noted. RATIOERROR was felt by most to help sharpen their diagnostic skills while increasing their awareness of the number of errors possible in proportion questions.

The major weakness of the program was the user interface. There was no attempt to incorporate a variety of problem representations. Pictorial depictions of problems, a common and useful method of solving proportion questions, was not employed due to time limitations. As well, no endeavor was made to adapt instruction implicitly presented to a model of the user developed through the implementation of a coach or expert advisor. Such an effort would be essential for any program being used over an extended period of time and/or with a larger domain of knowledge.

RATIOERROR proved to be, despite a somewhat mediocre user interface, a useful presentation. Its ability to present virtually all classifiable error patterns as delineated by Vergnaud makes it a potentially practical aid in the education of mathematics teachers.

Tree Representation

It is apparent from both a programming and instructional basis that representation of information by means of tree structures is a very powerful device. By following a path down a tree, a limited number of choices are available at any given level. The tree may have many levels but the choices at each level are severly limited. Thus it is possible to present a very complex situation in an understandable fashion.

As a method of instruction, tree diagrams offer a ready representation of complex relationships that otherwise may be overpowering to the student. By understanding that such relationships amount to simple choices at each level of the tree, the student is in a position to take control of the learning situation.

Not only is a tree structure useful as a guide to instruction, it can help us analyze a problem or a particular solution to a problem. The tree manifests its power as it becomes a tool in our problem-solving arsenal.

Implications

RATIOERROR is a simple model of a student experiencing difficulties with proportion problems. Much like BUGGY (Brown and Burton, 1978), it simulates a student with a 'bug and asks the user to discover the problem.

The discovery that errors, which at first may appear random, are in actuality the result of systematic procedures is a revelation to many prospective teachers. Those who used RATIOERROR felt they had benefited from using the program. A teacher who is not able to diagnose a difficulty with a particular type of question can only reteach the whole procedure. The attempt to remediate falls back to reexamining the entire concept involved in the solution of a problem. This reteaching of the complete procedure may leave the student with no idea as to why previous answers were judged incorrect.

A computer program that can model virtually all errors made in a certain type of problem allows its user to experience the full spectrum of errors in a relatively short time. This exposure is further enhanced by the consistency of the error over a number of questions. This consistency is not always available when dealing with actual students.

Another positive aspect of using a computer simulation in the education of prospective teachers is that diagnosis may be practiced without interfering in a detrimental way with actual students. While much has been written about the potentially harmful effects of acquiring knowlege through the use of machines, (Weizenbaum, 1976 and Dreyfus, 1979) such training as that exhibited by RATIOERROR avoids possible negative effects associated with untrained teachers interacting with students. By experiencing, diagnosing and describing errors made by a computer program, prospective teachers are allowed to think about the underlying causes of such errors in a non-threatening environment.

SUGGESTIONS FOR FURTHER RESEARCH

The following suggestions for further studies arise from the present research.

Replications

The present study had many restrictions placed on it. It should therefore be replicated with special emphasis on an adequate experimental design, increased sample size and the employment of a control groups, one with no instruction and one with instruction based on Vergnaud's theories.

Further Research

The following are possible areas for further research.

1. Would using the present program directly with students experiencing difficulties with proportion problems aid them in analyzing their own errors?

2. Pictorial representation is a particularly good use of computer resources. Would an emphasis on pictorial representation as opposed to a word problem approach have a greater influence on potential users of the program?

3. No attempt was made to follow up on the actual teaching practices of those involved. Do relatively short exposures to computer models of students experiencing difficulty have long term effects on teaching styles?

4. RATIOERROR was a relatively straight forward representation of a limited knowledge domain. Is it possible to implement other

student errors for other types of problems using a similar LOGO program?

FINAL COMMENTS

Further developmental research and error analysis, such as that undertaken by Vergnaud, will be needed if computer-based simulations of students, whether as subjects of study, or as the basis of a user interface model, are to be established. Educators must be involved in the development of educational computer programs of all types. Such development is not primarily a matter of representing a student model, but is rather a matter of understanding students' misconceptions of particular knowledge domains and determining the various styles of learning within those domains.

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APPENDIX 1

PRE AND POSTTESTS OF ANALYSIS OF STUDENT ERRORS

PRETEST: STUDENT ERRORS SOLVING PROPORTION PROBLEMS

Please describe how you think the student arrived at the answer shown.

1) A sheep's heart beats 15 times every 10 seconds. How many seconds does it take for 36 beats?

STUDENT ANSWER = 54 seconds.

2) On a hike Nancy takes 15 steps for every 9 steps that Grant takes. How many steps does Nancy take when Grant takes 36 steps?

STUDENT ANSWER = 42 steps.

3) During a poll, 16 out of 96 drivers were not in favor of seat belt legislation. If another 54 drivers were polled, how many additional drivers would not be in favor of legislation.

STUDENT ANSWER = 25 drivers.

4) The heights of two poles are in the ratio 5:2. The shorter pole is 8 meters high. What is the height of the taller pole?

STUDENT ANSWER = 3.2 meters.

5) A dripping faucet wastes how many liters of water in 11 hours if 12 liters are wasted in 2 hours?

STUDENT ANSWER = 21 liters.

6) Jenny earns $128 for working 32 hours. How many hours would she work to earn $70?
STUDENT ANSWER = 66 hours.

7) Find the actual distance on a map which would represent 64 km. if the scale is 3 cm. to 8 km.?
STUDENT ANSWER = 88 km.

8) A cake recipe requires 84 ml. of flour to make 3 layers. How much flour is required to make 21 layers?
STUDENT ANSWER = 1.33 ml.

9) The ratio of the amss of uncooked hamburger to the mass of cooked hamburger is 24:21. How much cooked hamburger can be prepared from 6 kg. of hamburger?
STUDENT ANSWER = 3 kg.

10) A penquin weighing 24 kg. eats .8 kg. of fish a day. Suppose that a girl weighing 72 kg. who "eats like a bird" ate the same proportion of food. How much would she eat in one day?
STUDENT ANSWER = 2.4 kg.

POSTTEST: STUDENT ERRORS SOLVING PROPORTION PROBLEMS

Please describe how you think the student arrives at the answer shown.

1) How much would an ad running for 6 days cost if a similar ad costs $100 for 4 days?

STUDENT ANSWER = $.24

2) On a hike Nancy takes 15 steps for every 9 steps that Grant takes. How many steps does Nancy take when Grant takes 36 steps?

STUDENT ANSWER = 21.6 steps

3) During a poll 16 out of 96 drivers were not in favor of sezt belt legislation. If another 54 drivers were polled, how many additional drivers would not be in favor of the legislation?

STUDENT ANSWER = 48 drivers

4) The heights of two poles are in the ratio 5:2. The shorter pole is 8 meters high. What is the height of the taller pole?

STUDENT ANSWER = 6 meters

5) A dripping faucet wastes how many liters of water in 11 hours if 12 liters are wasted in 2 hours?

STUDENT ANSWER = 132 liters

6) Jenny earns $128 for working 32 hours. How many hours would she work to earn $70?
STUDENT ANSWER = 90 hours

7) A sheep's heart beats 15 times every 10 seconds. How many seconds does it take for 36 beats?
STUDENT ANSWER = 32 seconds

8) A cake recipe requires 84 ml. of flour to make 3 layers. How much flour is required to make 21 layers?
STUDENT ANSWER = 12 ml.

9) The ratio of the mass of uncooked hamburger to the mass of cooked hamburger is 24:21. How much cooked hamburger can be prepared from 6 kg. of hamburger?
STUDENT ANSWER = 6.8 kg.

10) A pully device enables John to lift a load of 144 kg, with a pull of 24 kg. How much can be lifted if he and Harry can pull 42 kg.?
STUDENT ANSWER = 408 kg.

APPENDIX 2

PRE AND POST-QUESTIONNAIRES

PRE-QUESTIONNAIRE

1) What do you think is the most common difficulty in the solving of mathematical word problems?

2) What should a teacher do with a student who is having difficulty with proportion problems?

3) When introducing a concept should the teacher provide the appropriate rules or should the student develop their own rules?

4) Is it best to concentrate on the correct procedure when a student is having difficulty or on the student's incorrect response?

5) Describe how using a computer could help to overcome a student's difficulties with proportion problems.

6) In general why do students do so poorly on ratio questions?

7) What can a teacher determine from an incorrect answer?

8) Outline how you would attempt to help a student who is not able to solve proportion problems.

POST-QUESTIONNAIRE

1) What would you think is the most frequent type of error on mathematical word problems?

2) Are there any difficulties that students face that are peculiar to proportion problems?

3) Should mathematical rules be given by the teacher or developed by the student?

4) When would a student not use a correct rule?

5) Is it usually better to focus on the correct algorithm when helping a student or to focus on how they arrived at their answer?

6) Outline the type of computer programs that could help students who are experiencing difficulty with mathematical concepts.

7) How would you attempt to help a student who is not able to solve proportion problems?

8) What can we learn from an incorrect answer?

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