

A VISIBILITY BIAS MODEL FOR AERIAL SURVEYS OF
MOOSE ON THE AOSERP STUDY AREA

by

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ALBERTA OIL SANDS
ENVIRONMENTAL RESEARCH PROGRAM

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Minister of the Environment
222 Legislative Building
Edmonton, Alberta

and

The Hon. L. Marchand
Minister of State for the Environment
Fisheries and Environment Canada
Ottawa, Ontario

Sirs:

Enclosed herein is the final report on "A Visibility Bias Model for Aerial Survey of Moose on the AOSERP Study Area".

The report was prepared as a result of the derivation of a visibility bias model to be used in conjunction with aerial surveys, and was initiated by the Terrestrial Fauna Technical Research Committee of the Alberta Oil Sands Environmental Research Program, under the Canada-Alberta Agreement of February 1975 (amended September 1977).

Respectfully,



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A VISIBILITY BIAS MODEL FOR AERIAL SURVEYS
OF MOOSE ON THE AOSERP STUDY AREA

DESCRIPTIVE SUMMARY

BACKGROUND AND PERSPECTIVE:

This project was a continuation of research begun in 1976. Earlier studies had been designed to test the moose census procedures employed, analyze census results, and determine statistically reliable moose population estimates. The project TF 1.1.1 was designed to develop an analysis model which would produce statistically reliable estimates of visibility bias and total population.

The design was applied to an aerial census of a white-tailed deer study area in west-central Alberta and the data from this census was utilized to illustrate the application of the visibility bias model.

ASSESSMENT

The report entitled "A Visibility Bias Model for Aerial Surveys of Moose on the AOSERP Study Area" which was prepared by R.D. Cook (Department of Applied Statistics, University of Minnesota) and J.O. Jacobson (Interdisciplinary Systems Ltd.) has been reviewed by the Alberta Oil Sands Environmental Research Program, and by external reviewers. In view of the value of the document, the Alberta Oil Sands Environmental Research Program recommends that the report be published and made available.

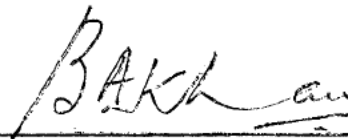
The model outlined in this report provides distinct advantages over standard estimating procedures commonly used in aerial surveys by: (1) providing statistically reliable estimates of visibility bias and absolute density; and (2) providing realistic estimates of the variance of the estimation. The report is comprehensive and well written, and includes the derivation of the model,

an example of model application to a set of data, and conclusions associated with the application and interpretation of the model.

The content of this report does not necessarily reflect the views of Alberta Environment, Environment Canada, or the Alberta Oil Sands Environmental Research Program. The mention of trade names for commercial products does not constitute an endorsement or recommendation for use.



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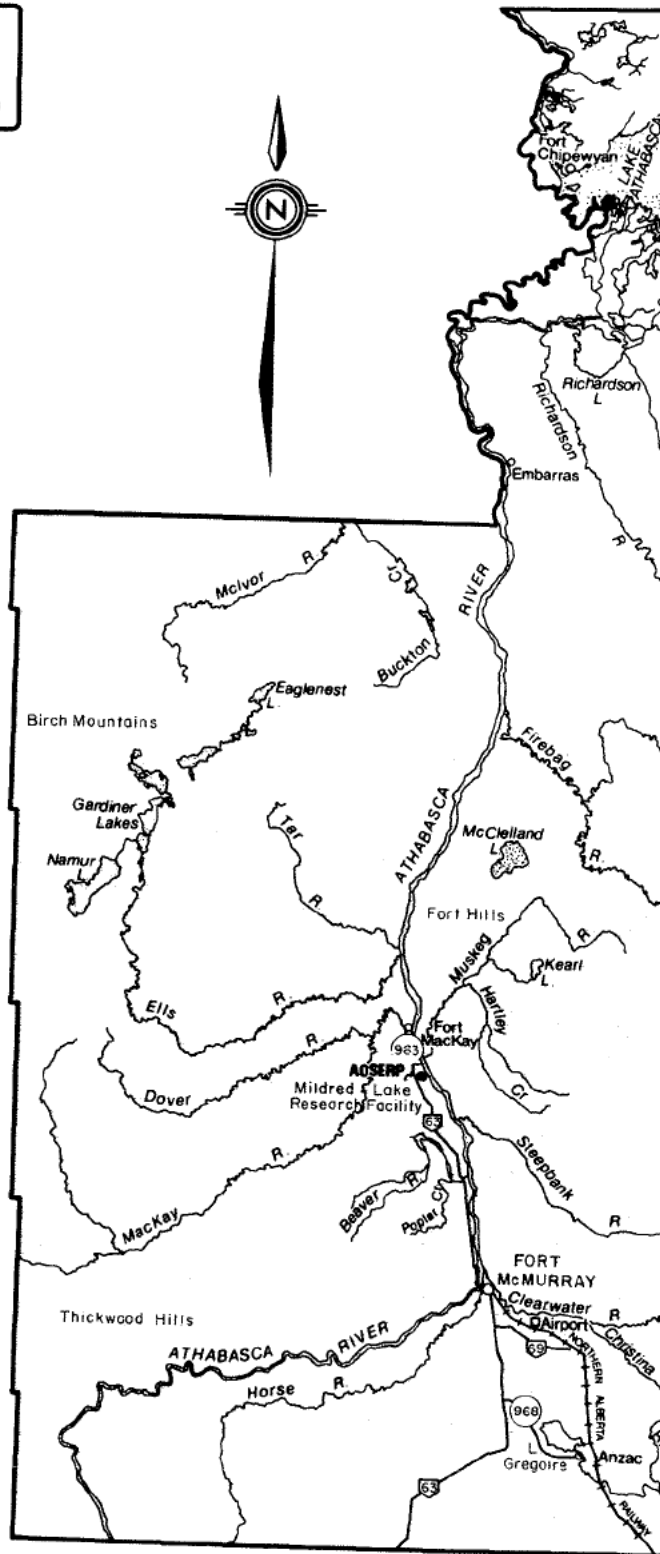
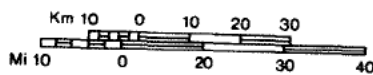
ABSTRACT

Commonly employed aerial survey techniques are known to contain substantial bias due to a failure to observe all animals. This paper details the derivation of a visibility bias model, appropriate to quadrat or transect surveys, which requires a simple set of assumptions and procedures that are reasonable and easily met under most survey conditions. Chi-square tests are outlined to validate model assumptions and goodness of fit. The model provides statistically reliable estimates of visibility bias and absolute density, and realistic estimates of population variance, none of which are generally available from standard estimating procedures. The importance of a statistically sound sampling design and a consistent and well-defined census technique, in conjunction with the visibility bias model, is emphasized.

ACKNOWLEDGEMENTS

This research project TF 1.1.1 was funded by the Alberta Oil Sands Environmental Research Program, a joint Alberta-Canada research program established to fund, direct, and co-ordinate environmental research in the Athabasca Oil Sands area of north-eastern Alberta. (Figure 1).

Data for the example used to illustrate application of the model were provided by Alberta Recreation, Parks and Wildlife, Fish and Wildlife Division. The comments made in Dr. G.E.J. Smith's review were appreciated.



1. Alberta Oil Sands Environmental Research Program Study Area.

1. INTRODUCTION

Aerial surveys have become a common technique to obtain population estimates of a variety of wildlife species. These surveys have usually been conducted using linear transects or quadrats; quadrats are the preferred technique for most big game surveys (Evans et al. 1966; Laws et al 1975). Occasionally, the total study area may be small enough to permit a complete search. Usually, however, quadrat surveys involve randomly choosing a number of small units (quadrats) within the study area and systematically searching each unit. Stratification is often used in combination with quadrat sampling to increase survey precision.

Even though aerial surveys for big game have been in use since 1942 (Sangstad 1942) and many technical aspects are well-developed, it is generally recognized that the potential for substantial bias exists because of the failure to observe all animals. This source of error, referred to as "visibility bias", is generally held to be the main cause of aerial survey inaccuracies.

The magnitude of visibility bias depends on a large number of variables, including animal behaviour and dispersion, observers, weather, habitat type, equipment, and methodology. Many of these variables (e.g., snow cover and observer abilities) affect only the general level of visibility bias for the survey as a whole, while others, primarily the propensity for animals to occur in groups, cause the magnitude of visibility bias to vary within the survey. LeResche and Rausch (1974) conducted a controlled study to determine the effect of observer experience and snow conditions on visibility bias of moose surveys in Alaska. They found experienced observers detected 70, 61 and 40 percent of total moose under excellent, good and poor snow conditions, respectively. The effects of visibility bias can, therefore, be substantial even under excellent survey conditions.

Cook and Martin (1974) proposed a refinement of the quadrat sampling method and developed a model for estimating the magnitude of visibility bias (i.e., the probability of not observing an animal). Their model contains four main assumptions:

1. Animals occur within quadrats in groups of varying size;
2. Each animal has a probability p_i of being observed, and the p 's are independent and identically distributed random variables with mean \bar{p} ;
3. Conditional on observing at least one member of a group, the entire group is observed with certainty; and
4. To fail to observe a group, the observer must miss each animal individually and independently.

In addition, their model requires the specification of:

1. The distribution of the number of groups per quadrat; and
2. The distribution of group sizes.

Under these assumptions and specifications, estimates of \bar{p} , the average group size and the average number of groups were developed. While this technique has been used effectively for aerial survey of moose in Minnesota, Assumption 4 and Specification 2 are often difficult to satisfy.

In this paper, we have extended the original concept of visibility bias models to show that when the aircraft can accommodate two observers situated on the same side, it is possible through careful design to estimate the magnitude of visibility bias without the need for Assumptions 1 and 4, and Specification 2. Assumption 3 and Specification 1 are required throughout the procedure. A discussion of the basic model and its assumptions and application can be found in the original paper by Cook and Martin (1974). Additional discussion may be found in Patil and Rao (1978).

2. SURVEY DESIGN

The basic design requires that the two observers assume different roles during the conduct of the survey. Initially, let Observers 1 and 2 be designated as primary and secondary observers, respectively. In the detection of groups, the primary observer behaves as if he were the only observer present. The secondary observer confirms all sightings by the primary observer and records only those groups he detected that were missed by the primary observer. Once a group has been sighted, both observers may assist in the enumeration of the animals in order to meet Assumption 3. The secondary observer, however, must not aid the primary observer in the detection of groups. Essentially, the secondary observer's record is conditional on the record of the primary observer. This procedure is followed until approximately half the survey has been completed, at which time the two observers switch roles (i.e., Observer 2 becomes the primary observer). We refer to the part of the survey in which Observer i is the primary observer as the i th half, $i = 1, 2$.

Experience has shown that with some training the above procedure is reliable and easily followed. The use of headsets is desirable since they reduce aircraft noise and greatly facilitate communication between observers.

For obvious reasons the observers must be situated on the same side of the aircraft. This tends to make the design more costly than the more typical one in which the observers are situated on opposite sides since more time will be needed to search each quadrat. Of course, estimation of visibility bias requires additional assumptions when the observers are on opposite sides.

If the survey area contains multiple habitats it is desirable to stratify. There are two reasons stratification is necessary. First, species of big-game generally exhibit seasonal habitat preferences. Stratification may therefore be expected to increase precision relative to simple random sampling. Second,

the chance of detecting a group of a specified size can be expected to exhibit variability between broad habitat types while it should remain relatively constant within types. The effect of visibility bias can be expected to be most severe in dense habitat. If stratification is used the observer's protocol described above should be used within each stratum.

Finally, the design requires that the quadrats be selected at random within a stratum. This is consistent with established sampling theory and, as will be seen later, is necessary for estimation of the total population size.

3. BASIC MODEL AND ESTIMATION

The basic model requires no assumptions about the sampling scheme beyond those detailed above. Also, the model considers each possible group size and each stratum separately. For notational convenience, we shall initially suppress any reference to group size. No confusion will result if it is remembered that, unless indicated otherwise, the following discussion is for a particular group size and stratum.

3.1 BASIC MODEL

For groups of a specified size, let

- α_i = probability that a group is recorded by Observer i , $i = 1, 2$;
- N_i = total number of groups in the i th half of the sampled area;
- N = total number of groups in the complete survey area;
- X_{ij} = total number of groups recorded by Observer i in the j th half of the sampled area, $i = 1, 2$, $j = 1, 2$.

We first derive the basic model from the survey design conditional on N_1 and N_2 . Assuming that each of the N_1 groups in the first half of the sampled area is independently observed by the primary observer with probability α_1 , X_{11} may be regarded as a binomial random variable with parameters N_1 and α_1 , $B(N_1, \alpha_1)$.

The secondary observer records only those groups that the primary observer failed to detect. In effect, this means that, given $X_{11} = x_{11}$, there are $N_1 - x_{11}$ remaining groups available for possible detection by the secondary observer. Thus, proceeding as before, the conditional distribution of $X_{21} | X_{11} = x_{11}$ is $B(N_1 - x_{11}, \alpha_2)$. The joint distribution of

This is the basic model on which we base estimation of N , α_1 , α_2 and p . It is applicable to a complete or partial survey of the study area. Note that if it is assumed that the distribution of groups over quadrats is multinomial with equal cell probabilities then $\lambda_i = f_i$, $i = 1, 2$, where f_i is the known sampling fraction for the i th half of the sampled area.

Of course, the likelihood associated with (2) does not allow for estimation of all parameters unless $\lambda_i = f_i$. However, α_1 , α_2 , p and $\beta_i = \lambda_i / (\lambda_1 + \lambda_2)$ may be estimated from the conditional distribution of \underline{X} given $X_{..} = x_{..}$. Then, given these estimates, N may be estimated from the marginal distribution of $X_{..}$. This is the approach we adopt.

3.2 ESTIMATION OF VISIBILITY BIAS PARAMETERS

From (2), the conditional distribution of \underline{X} given $X_{..} = x_{..}$ is

$$M_4 \left[x_{..}; \beta_1 \alpha_1 / p, \beta_1 \alpha_2 (1 - \alpha_1) / p, \beta_2 \alpha_2 / p \right] \quad (3)$$

From this a little algebra will establish that the conditional maximum likelihood estimates, $\hat{\alpha}_1$ and $\hat{\alpha}_2$, of α_1 and α_2 are

$$\hat{\alpha}_1 = \frac{x_{11} x_{22} - x_{12} x_{21}}{x_{11} x_{22} + x_{22} x_{21}} \quad (4)$$

and

$$\hat{\alpha}_2 = \frac{x_{11} x_{22} - x_{12} x_{21}}{x_{11} x_{22} + x_{11} x_{12}} \quad (5)$$

provided that the denominators are non-zero. Note that the numerators of (4) and (5) are the same. If the common numerator is zero while $x_{ij} > 0$, $i = 1, 2$, $j = 1, 2$, then α_1 and α_2 are not identifiable and separate estimates cannot be obtained. Note also that $\hat{\alpha}_1$ and $\hat{\alpha}_2$ in combination with (3) and the assumption $\lambda_i = f_i$ may be used to construct a standard chi-square goodness-of-fit test for the model.

The maximum likelihood estimate, $\hat{\beta}_1$, of β_1 is simply $\hat{\beta}_1 = x_{.1}/x_{..}$. The large sample covariance matrix of $(\hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}_1)$ deriving from (3) is given by the inverse of the Fisher information matrix, $\underline{U} = (u_{ij})$:

$$\begin{aligned}
 u_{11} &= \left[\frac{1 - \beta_2 \alpha_2}{\alpha_1 p} + \frac{\beta_1 \alpha_2}{(1 - \alpha_1) p} - \frac{(1 - \alpha_2)^2}{p^2} \right] x_{..} \\
 u_{12} &= -x_{..} / p^2 \\
 u_{22} &= \left[\frac{1 - \beta_1 \alpha_1}{\alpha_2 p} + \frac{\beta_2 \alpha_1}{(1 - \alpha_2) p} - \frac{(1 - \alpha_1)^2}{p^2} \right] x_{..} \\
 u_{33} &= x_{..} / \beta_1 \beta_2 \\
 u_{13} &= u_{23} = 0
 \end{aligned} \tag{6}$$

Note that $\hat{\beta}_1$ and $(\hat{\alpha}_1, \hat{\alpha}_2)$ are asymptotically independent and, thus, the variance of $\hat{\beta}_1$ is the usual variance associated with the parameter estimate from a binomial distribution, $\text{Var}(\hat{\beta}_1) = \beta_1 \beta_2 / x_{..}$.

This estimation procedure is conditional on $X_{..} = x_{..}$. However, from (3) it is apparent that the distribution of the conditional total $X_{.1} | X_{..} = x_{..}$ is $B(x_{..}, \beta_1)$ and, thus, conveys no information about α_1 or α_2 . For this reason it may be more

reasonable to base estimation of α_1 and α_2 on the product binomial model obtained by conditioning on $X_{.1}$ and $X_{.2}$. The estimates of α_1 and α_2 derived from the product binomial model are the same as those given in (4) and (5). The large sample covariance matrix for $(\hat{\alpha}_1, \hat{\alpha}_2)$ from the product binomial model is obtained using u_{11} , u_{12} and u_{22} of (6) with β_i replaced by $\hat{\beta}_i$. Thus, when variance estimates are obtained by substituting parameter estimates, the two models lead to identical results.

It should be clear from the previous discussion that it is desirable to have $\lambda_i = f_i$. Since the distribution of $X_{.1}|X_{..} = x_{..}$ is $B(x_{..}, \beta_1)$, a test of the hypothesis $H: \beta_1 = f_1/(f_1 + f_2)$ may furnish some insight about the reasonableness of assuming $\lambda_i = f_i$.

The maximum likelihood estimate, \hat{p} , of $p = 1 - (1-\alpha_1)(1-\alpha_2)$ is

$$\hat{p} = 1 - x_{12}x_{21}/x_{22}x_{11} \quad (7)$$

Note that this is a simple function of the cross-product ratio for 2×2 contingency tables. The asymptotic variance of \hat{p} for the multinomial model can be quickly determined from the asymptotic variance of the cross-product ratio (Bishop et al 1975),

$$\text{Var}(\hat{p}|x_{..}) = \frac{(1-p)^2 p}{x_{..}} \left[\frac{1}{\alpha_1 \beta_1} + \frac{1}{\alpha_2 \beta_2} + \frac{1}{\alpha_2 (1-\alpha_1) \beta_1} + \frac{1}{\alpha_1 (1-\alpha_2) \beta_2} \right] \quad (8)$$

3.3 ESTIMATION OF N

Conditional on $\lambda = \lambda_1 + \lambda_2$ (i.e., given the sampled quadrats), the distribution of $X_{..}$ is $B(N, \lambda p)$. The distribution of λ is the sampling distribution associated with the sum of the cell probabilities, q_i , for the randomly selected quadrats. If it can be assumed that the individual cell probabilities for all

quadrats are equal, ($\lambda_i = f_i$, $i = 1, 2$) then the unconditional distribution of $X_{..}$ is simply $B(N, fp)$ where $f = f_1 + f_2$. In this case a straightforward estimate, \hat{N} , of N is

$$\hat{N} = x_{..} / \hat{fp} \quad (9)$$

where \hat{p} is obtained from Equation (7).

A first approximation to the variance of this estimate can be found by writing $\text{Var}(\hat{N}) = E \text{Var}(\hat{N} | x_{..}) + \text{Var} E(\hat{N} | x_{..})$. Approximating $E(\hat{N} | x_{..})$ by $x_{..} / fp$ we obtain

$$\text{Var} E(\hat{N} | x_{..}) \doteq N(1-fp)/fp.$$

Next, using (8)

$$\begin{aligned} E \text{Var} (\hat{N} | x_{..}) &\doteq E x_{..}^2 \text{Var}(\hat{p} | x_{..}) / f^2 p^4 \\ &\doteq N x_{..} \text{Var}(\hat{p} | x_{..}) / fp^3. \end{aligned}$$

Thus,

$$\text{Var}(\hat{N}) \doteq N \left[\frac{x_{..} \text{Var}(\hat{p} | x_{..})}{fp^3} + \frac{1-fp}{fp} \right] \quad (10)$$

When the assumption $\lambda_i = f_i$ is not warranted, the use of (9) as an estimate of N requires some additional justification.

The unconditional expectation of $X_{..}$ is

$$EX_{..} = E_{\lambda} E(X_{..} | \lambda) = E_{\lambda} Np\lambda = Npf.$$

Thus, when $\lambda_i \neq f_i$ the method of moments leads to the same estimate of N .

A first approximation to the variance of N when $\lambda_i \neq f_i$ may be obtained by using the same argument that led to Equation (10):

$$\text{Var}(\hat{N}) = E_{\lambda} \text{Var}(\hat{N} | \lambda) + \text{Var}_{\lambda} E(\hat{N} | \lambda)$$

The first term on the right-hand side of this expression may be approximated by the right-hand side of (10). For the second term we obtain

$$\text{Var}_{\lambda} E(\hat{N} | \lambda) = \text{Var}_{\lambda} (N\lambda/f) = \frac{N^2}{f^2} \text{Var}(\lambda)$$

where $\text{Var}(\lambda)$ is from the finite sampling distribution of the sum of the individual quadrat probabilities. Thus, when $\lambda_i \neq f_i$

$$\text{Var}(\hat{N}) \doteq \frac{N^2}{f^2} \text{Var}(\lambda) + N \left[\frac{x_{..} \text{Var}(\hat{p} | x_{..})}{f_p^3} + \frac{1-f_p}{f_p} \right] \quad (11)$$

Aside from $\text{Var}(\lambda)$, this variance may be estimated by substituting estimates for unknown parameters. To obtain an

estimate of $\text{Var}(\lambda)$ assume, temporarily, that for each of the n sampled quadrats it is possible to observe $r_i = q_i / \sum_j q_j$, where q_j is the true probability associated with the j th sampled quadrat. Let $s_r^2 = \sum_i (r_i - 1/n)^2 / (n-1)$.

Then

$$\begin{aligned} (n-1) \text{Es}_r^2 &= E \sum_j (q_j - \bar{q})^2 / (\sum_j q_j)^2 \\ &= (n-1) E \left[s_q^2 / (\sum_j q_j)^2 \right] \\ &\doteq (n-1) \text{Es}_q^2 / f^2 \end{aligned}$$

Thus, $\text{Es}_q^2 \doteq f^2 \text{Es}_r^2$. From finite sampling theory it is known that $\text{Var}(\lambda) = n(1-f) \text{Es}_q^2$. Combining this with the previous result we obtain

$$\text{Var}(\lambda) \doteq nf^2 (1-f) \text{Es}_r^2 \quad (12)$$

Of course, the r_i cannot be observed and Es_r^2 cannot be estimated directly. However, r_i can be estimated by the observed proportion, 0_i , of groups of all sizes in the i th quadrat. (We assume that the r_i and hence the q_i are independent of group size.) Let $s_0^2 = \sum_i (0_i - 1/n)^2 / (n-1)$. Then it is easily verified that conditional on the sampled quadrats,

$$(n-1) E s_0^2 = (n-1) s_r^2 + \sum_i \frac{r_i(1-r_i)}{t}$$

where t is the total number of groups of all sizes observed.

Thus, s_r^2 may be estimated by

$$\hat{s}_r^2 = s_0^2 - \sum_i \frac{0_i (1-0_i)}{t(n-1)} \quad (13)$$

Combining this with (12) we obtain

$$\hat{\text{Var}}(\lambda) = nf^2 (1-f) \hat{s}_r^2 \quad (14)$$

as an estimate of $\text{Var}(\lambda)$.

4. MODEL APPLICATION

The design was applied to an aerial census of a 5,130 km² white-tailed deer study area in west-central Alberta from 14 December 1977 to 23 January 1978 (Jacobson and Cook 1978). The study area is agricultural land mixed with aspen parkland, contained an abundance of roads and section lines and was well-suited to a quadrat census. It was divided into four strata on the basis of wooded cover, and square-mile sample quadrats were randomly selected within each strata.

Sample quadrats were systematically searched by two experienced observers seated on the right side of a Cessna 182 aircraft flying at an average ground speed of 125 km/h and an altitude of 70-100 m AGL. Snow cover ranged from 20-30 cm and all censuses were flown between 1000 and 1600 hours on days with clear or lightly overcast skies and winds less than 15 km/h. Continuous air time never exceeded 2.5 hours.

A total of 250 white-tailed deer were observed in 94 groups on the high density strata; these data will suffice to illustrate the application of the visibility bias model.

The high stratum contained the highest deer densities, and with at least 50 percent wooded cover was also considered to have the highest visibility bias. Sampling consisted of 57 of the 149 total quadrats; 23 were designated as first half and 34 as second half. Thus, $f = 57/149 = 0.383$, $f_1 = 0.154$ and $f_2 = 0.228$.

The data, summarized by observer and group size, are presented in Table 1. Estimation of α_1 , α_2 and p for the first three group sizes was carried out as previously indicated. Variance estimates were obtained by substituting the parameter estimates in equations (6) and (8). These estimates are presented in the first three rows of Table 2. Notice from Table 2 that $\hat{\alpha}_1$, $\hat{\alpha}_2$ and \hat{p} increase with group size. This was expected since larger groups should have a higher probability of being observed than smaller groups.

Table 1. White-tailed deer aerial census data, Red Deer, Alberta, 1977-78.

Group Size	First Half		Second Half		Total
	Primary Observer (x_{11})	Secondary Observer (x_{21})	Primary Observer (x_{22})	Secondary Observer (x_{12})	
1	8	4	5	3	20
2	8	4	15	3	32
3	5	2	10	2	19
4	6	0	6	0	12
5	3	1	2	1	7
6	1	0	2	0	3
7	none observed		none observed		0
8	1	0	none observed		1

Table 2. Estimate of parameters and variances for the 1977-78 Alberta deer survey.

Group Size	$\hat{\alpha}_1$	$\hat{\text{Var}}(\hat{\alpha}_1)$	$\hat{\alpha}_2$	$\hat{\text{Var}}(\hat{\alpha}_2)$	\hat{p}	$\hat{\text{Var}}(\hat{p})$	\hat{N}	$\hat{\text{Var}}(\hat{N})$	Estimated Increase ^a
1	0.47	0.069	0.44	0.079	0.70	0.082	74.7	1134.5	25.5
2	0.56	0.036	0.62	0.027	0.83	0.018	100.4	472.9	46.1
3	0.66	0.044	0.77	0.026	0.92	0.008	54.0	128.0	13.3
4	0.89	0.012	0.88	0.014	0.98	0.0004	31.8	53.0	4.6
5	0.89	0.012	0.88	0.014	0.98	0.0004	18.6	32.2	1.6
6	1	0	1	0	1	0	7.8	12.6	0.3
8	1	0	1	0	1	0	2.6	4.3	0.03

^a Estimated increase in $\text{Var}(\hat{N})$ when the assumption $f_i = \lambda_i$ is relaxed, $\hat{N}^2 \text{Var}(\lambda)/f^2$.

The data for group sizes 4 and 5 required special treatment. Notice from Table 1 that the secondary observers did not observe any groups of Size 4 while they did observe one group of Size 5 in each half of the survey. Using Equation (7), we find $\hat{p} = 1.0$ for groups of Size 4 while $\hat{p} = 0.83$ for groups of Size 5. This is at odds with the accepted fact that larger groups have a higher probability of being observed than smaller groups. Generally, some smoothing will be required for larger group sizes. This could be accomplished by using a functional relationship which connects the visibility bias parameters with group size. Attempts at establishing such a relationship using available data have been unsuccessful. Alternatively, data for larger group sizes could be pooled in an *ad hoc* manner to yield combined estimates of the visibility bias parameters. The fourth and fifth rows of Table 2 present the estimates of the visibility bias parameters obtained by pooling the data for groups of Size 4 and 5.

For group sizes greater than 5, α_1 , α_2 and p were taken to be 1 because: (1) there is no evidence in available data that groups of Size 6 or more may be missed, and (2) the true probabilities of observing groups of Size 6 or more should be greater than about $p = 0.98$, the estimated probability for groups of Sizes 4 and 5.

Before \hat{N} and $\text{Var}(\hat{N})$ can be obtained, a judgement about the distribution of groups over quadrats needs to be made: Is it reasonable to assume that the cell probabilities associated with the individual quadrats are equal? The validity of this assumption may be checked by testing the hypothesis $H: \beta_1 = f_1 / (f_1 + f_2) = 0.4035$ where the sum of $X_{.1}$ over all group sizes is $B(94, \beta_1)$. (It is assumed that the distribution of groups over quadrats is the same for all group sizes.) Over all group sizes, $\hat{\beta}_1 = 0.4574$ and the Pearson chi-square statistic has a value of 1.14 with one degree of freedom. Thus, the assumption of equal quadrat probabilities ($\lambda_1 = f_1$) appears reasonable for these data.

Under the assumption that $\lambda_i = f_i$, the Pearson chi-square statistics, each on one degree of freedom, for the model given in (3) have the values 3.21, 0.11, 0.10 and 1.19 for Group Sizes 1, 2, 3, and 4 and 5 combined, respectively. Thus, the model seems to fit these data reasonably well.

Estimates of the total number of groups of each size from Equation (9) are given in the 8th column of Table 2. The 9th column in Table 2 gives the estimated variances obtained by substituting the parameter estimates in (10). For comparative purposes we have given in the 10th column of Table 2 the estimated increases, $\hat{N}^2 \hat{\text{Var}}(\lambda)/f^2$ (see Equation (11)) in these variances when the assumption $\lambda_i = f_i$ is relaxed. The frequency distribution of the number of groups per quadrat needed to calculate $\hat{\text{Var}}(\lambda)$ is given in Table 3. As anticipated, the estimated increases are relatively small.

With the information in the last three columns it is straightforward to construct an estimate \hat{T} of the total number of animals, T :

$$\hat{T} = \sum_s \hat{S} \hat{N}_s = 726$$

where the summation is over group sizes and \hat{N}_s denotes the estimated number of groups of size s . An estimate of the variance of \hat{T} is:

$$\begin{aligned} \hat{\text{Var}}(\hat{T}) &= \sum_s s^2 \hat{\text{Var}}(\hat{N}_s) \\ &= 6,561 \text{ when } \lambda_i = f_i \\ &= 6,998 \text{ when } \lambda_i \neq f_i \end{aligned}$$

Table 3. Distribution of the number of groups per quadrat, Red Deer, Alberta deer survey.^a

Number of Groups	Frequency	Estimated Quadrat Proportion (0_i)
0	20	0.00
1	12	0.011
2	7	0.021
3	8	0.032
4	7	0.043
5	2	0.053
6	1	0.064

^aParameters: $n = 57$; $t = 94$; $f = 0.383$.

For comparative purposes, the standard finite sampling estimate of T is 654 with estimated variance 6,053, indicating that the survey missed approximately 10 percent of the deer population existing on the high stratum of the study area.

5. CONCLUSIONS

Successful aerial census programs for ungulates are dependent on three main factors: a statistically sound sampling design, a consistent and well-defined census technique, and careful application and interpretation of visibility bias.

The tools are available in the literature (primarily from Cochran 1963, Seber 1973, and Jolly 1969) to develop statistically reliable sampling designs for aerial censuses. Application of these designs has been adequately discussed in prior AOSERP publications (Jacobson 1978, Cook and Jacobson in prep.) and in a companion study conducted for Alberta Fish and Wildlife (Jacobson and Cook 1978). The proper utilization of these designs and techniques must be a pre-condition of any future aerial census program on the AOSERP study area.

The model outlined in this paper provides distinct advantages over standard estimating procedures commonly used in aerial surveys. First it provides statistically reliable estimates of visibility bias and absolute density and secondly, it provides realistic estimates of the variance of the estimator. The major points associated with the application and interpretation of this model can be summarized as follows:

1. The model provides realistic estimates of the effects of visibility bias when the sampling protocol outlined here is followed. It is noteworthy that overall visibility bias was intuitively expected to be greater than the 10 percent calculated for this example. This is due to the normal tendency of researchers to associate visibility bias with the probability of missing individual animals rather than missing groups of animals.
2. The sampling procedures, data collection and data analysis are relatively easy to implement. Random sampling and stratification are commonly used in

existing aerial surveys. Experience indicates that with some training the protocol of two observers on the same side of the aircraft, regularly switching positions, is a reliable procedure that is easily followed. The design is appropriate to either quadrat or transect sampling. If stratification is required, sampling intensity in each strata should be adequate to provide data for both halves of the study through group Size 5. The analytical procedures have been computerized, in conjunction with other Alberta and Manitoba studies, and a user's manual is being finalized (Jacobson et al. in prep.)

3. The simplified set of assumptions required by this model is reasonable and easily met under most survey conditions. The two chi-square tests outlined in the procedure allow a check on whether cell probabilities are distributed uniformly within strata and, if so, whether the model fits the group size data generated by the survey.
4. The importance of effective stratification is built into the model. If the survey includes a diversity of habitats and stratification is not done, or is done poorly, the assumption of equal quadrat probabilities ($\lambda_1 = f_1$) will not be valid and the variance estimates will be increased.
5. The variance of the population estimates may, in some cases, be larger than those provided by the standard existing procedures. It must be remembered, however, that standard estimating procedures are usually not accurate or precise; they provide no information on what proportion of the population is observed, and they provide no information on the additional variance associated with this visibility bias.

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