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AUTHOR – AUTEUR

Full Name of Author – Nom complet de l'auteur

Vamosi, Alexander Richard

Date of Birth – Date de naissance

April 29/1960

Canadian Citizen – Citoyen canadien

[X] Yes Oui [ ] No Non

Country of Birth – Lieu de naissance

Canada

Permanent Address – Résidence fixe

14762-47 Ave
Edmonton, AB.
Canada
T6H-5L4

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Name of Supervisor – Nom du directeur de thèse

Professor T. Tsushima

AUTHORIZATION – AUTORISATION

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LINKAGES BETWEEN THE HIGH-EMPLOYMENT DEFICIT AND INFLATION

by

(C) ALEXANDER R. VAMOSI

A THESIS

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(SIGNED) ...Alex Vamosi...........

PERMANENT ADDRESS:

14762 - 47 Avenue

EDMONTON, Alberta

Canada

T6H 5L4

DATED ...May 31..... 1985

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled LINKAGES BETWEEN THE HIGH-EMPLOYMENT DEFICIT AND INFLATION submitted by ALEXANDER R. VAMOSI in partial fulfillment of the requirements for the degree of MASTER OF ARTS.

T. Tomshima ....
Supervisor

William ? Phillips ...

..........A. Buse..........

DATE May 31/1985 ......

ABSTRACT

The purpose of this thesis is to investigate the relationship between the high-employment deficit and inflation. Although this study is mainly theoretical, an empirical chapter which attempts to quantify the impact of the high-employment deficit/ money stock ratio on the rate of inflation is included.

Tobin and Buiter's 1976 paper is extended by modelling inflationary expectations as a function of the high-employment deficit/ money stock ratio. Expectations formed in this way explicitly recognize the government budget constraint. A formulation such as this was first adopted by Jerome Stein; however, Stein's model excluded any consideration of government bonds.

When government expenditures are defined to include interest payments, a money-financed expenditure policy is found to be stable while a bond-financed expenditure policy is unstable. Also, there appears to be little qualitative difference in the short-run adjustment process between models which form inflationary expectations in terms of the high-employment deficit/ money stock ratio and those assuming static expectations. When government expenditures are defined separately from interest payments the issue of stability becomes an empirical question for both money-financed and bond-financed expenditure policies.

A simple simultaneous equation model is developed in order to quantify the impact of the high-employment deficit/money stock ratio on the rate of inflation. We find that in the United States, the impact is small and possibly negligible for the time period 1964-81.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF DIAGRAMS

ABBREVIATIONS and SYMBOLS

D = d/dt, time difference operator

M = stock of money defined as M1

B = outstanding stock of government bonds

G = real government expenditures, excluding interest payments

G* = real government expenditures, including interest payments

R = nominal rate of interest on government bonds

P = price level

H = high-powered money stock

W = real wealth, K+(M+B)/P

K = stock of capital

Y = real GNP (output)

DP/P = rate of inflation, continous time definition

DM/M = rate of growth in the nominal money supply, continuous time definition

DK/K = rate of growth in the stock of capital

CP = rate of inflation, discrete time version

CM, υ = rate of growth in the nominal money supply, discrete time definition

X = expected rate of inflation

Me = expected growth rate of nominal money supply

V = ((M+B)/P) - (M/P)

$f(K), YF$ = capacity output

$f'(K)$ = marginal product of capital. Rate of return on new capital stock

u = W/Y, wealth-income ratio

$uf(K)$ = desired long-run level of wealth

n_1 = speed of adjustment between the actual and expected rate of inflation

n_2 = speed of adjustment between the growth rate of the nominal money supply and expected inflation

z = degree of price flexibility in the economy

s = speed of adjustment between actual and desired levels of wealth

β = proportion of the high-employment deficit/ money stock ratio individuals expect to be monetized

λ = $(1-n_1-n_2)$

θ = proportional and marginal tax rate

Ω = real rate of return differential between existing capital stock and bonds

Γ = proportion of the high-employment deficit financed by money

α = propotion of the cyclical component of the deficit financed by money

κ = proportion of the actual deficit financed by bonds

τ = the money multiplier

δ = represents the relationship between the money multiplier and the business cycle

RN = net real rate of return

HEDEF = high-employment deficit

ADEF = actual deficit

HEDMS = high-employment deficit, money-stock ratio

ADEFMS = actual-deficit, money-stock ratio

CCDEF = cyclical component of the deficit

MFEXP = money-financed expenditure

BFEXP = bond-financed expenditure

GBC = government budget constraint

2SLS = two-stage least squares

IV = instrumental variables

MA1 = first-order moving average process

GAP = output-gap, potential minus actual

GNPD = gross national product price deflator

OLS = ordinary least squares estimation procedure

AUTO = estimation procedure correcting for first-order autocorrelation

LM = Lagrange multiplier test statistic

D.W. = Durbin-Watson test statistic

D.H. = Durbin's H test statistic

DV = dummy variable

I. LITERATURE REVIEW

INTRODUCTION

The purpose of this thesis is to study the relationship between government deficits and the inflation rate, both in a theoretical and empirical framework. In particular, the high-employment deficit rather than the actual deficit will be the focus of attention. Recently, the deficit and inflation rate relationship has had important implications for the continued success of the United States in controlling inflation. Exponentially growing deficits in the U.S. could lead to another round of inflationary expansion. By itself this may not be such a major concern; however, in the last twenty to twenty-five years each swing in the cycle has led to a new higher combined inflation and unemployment rate. That is, at the peak of the cycle, both inflation and unemployment reached higher levels than in previous cycles. Such a continued process may prove destabilizing in the long-run.

This chapter will attempt to lay the foundation on which successive chapters can be built. We first review the theoretical arguments for a deficit and inflation rate link. The problem of measuring the deficit is then addressed, with a special focus on the high-employment budget deficit (HED). In order to familiarize ourselves with the mechanics of macro-dynamic models which incorporate deficits, the government budget constraint literature is examined. We conclude with a summary of the major empirical investigations in this area.

1. LINKS BETWEEN INFLATION AND DEFICITS

It is widely believed that at least in the long-run, inflation is purely a "monetary phenomena". If we can control the growth rate of the money supply, in the long-run we will also control the inflation rate. If this is the case, why do we worry about deficits? In part, the answer lies in the government budget constraint (GBC). Eq. (1-1) is a typical representation of the GBC (7,12,13).

(1-1) $G-\theta Y+(1-\theta)B/P=(DH/P)+(DB/RP)$

where G= real government expenditures,

Y= real output,

P= price level,

B= number of outstanding government bonds, paying out a \$1 coupon rate per year, forever,

θ= marginal tax rate,

R= nominal rate of interest,

H= high-powered money stock,

D= d/dt, time difference operator.

This equation shows (for a given tax rate) that any discrepancy between expenditures and revenues leads to a creation or retirement of either government bonds or high-powered money. If a deficit exists some form of financing is needed. The important point to note is that if deficits are financed by high-powered money, there will be an increase in the money supply via the money multiplier. The above analysis implies that the link between deficits and inflation is an indirect link through the money supply.

It is important to observe that not all of the deficit needs to be financed by high-powered money creation. In fact, theoretically, none of the deficit has to be monetized. This is due to the presumably independent agencies such as the Central Bank of Canada or the Federal Reserve in the U.S. which set monetary policy. These agencies have the ability to control the degree of deficit monetization.

A commonly held view asserts that the monetary authority will react to stabilize interest rates, especially if they are rising. Deficits, by placing increased demand pressures on the financial markets, tend to raise the rate of interest. If the monetary authority reacts to this rising interest rate by increasing the rate of growth in the money supply, the rate of inflation will increase. In essence, this hypothesis suggests that the deficit is monetized to the extent that deficits raise the interest rate (20,35,50). Do deficits raise the interest rate and if so, can the monetary authority still conduct an independent policy?

The relationship between deficits and interest rates can be analysed in a familiar IS-LM setting (7). Initially, assume that the economy is in a position of equilibrium and that the government budget is in balance.

A bond-financed increase in government expenditures will shift the IS curve to the right, increasing output and the interest rate. The interest rate will rise because the shifting IS curve creates a disequilibrium in the financial markets. In order to meet a higher income level, the transactions demand for money increases. This implies by Walras' law the existence of an excess supply of bonds. To re-establish equilibrium, the interest rate will rise. The magnitude of the increase in the interest rate depends on the interest sensitivity of the demand for money. The more interest sensitive is the demand for money, the smaller will be the increase in the interest rate. By increasing borrowing costs, higher rates of interest crowd out private investment and expenditure demand. This is the first level of crowding-out discussed by Blinder and Solow in their seminal article, "Does Fiscal Policy Matter?" (7).

A second level of crowding out exists due to wealth effects. The need to finance the deficit in each time period requires further bond-financing. By increasing wealth, consumption demand is positively stimulated; the IS curve continues its rightward movement. However, increased wealth also stimulates an increase in the demand for money and bonds, thereby shifting the LM curve towards the left. These two shifts combine to raise the interest rate to an even higher level. The output effect is ambiguous as it depends on the relative magnitude of the opposing IS-LM curve shifts.

As Robert Barro has pointed out, the above wealth effects will not occur if the public does not perceive government bonds to be net wealth. In other words, if the public believes that the issuance of government bonds leads to an increase in their future tax liability, there will be no change in the interest rate; neither the IS nor LM curve is affected. The validity of this "Ricardian equivalence" hypothesis is an empirical question. No definite conclusions have yet been reached (9,16,24,31,50). An excellent counter argument to this hypothesis is given by Tobin (47).

Assuming that bond-financing will lead to somewhat higher interest rates, will the monetary authority react by increasing the money supply? Franco Modigliani does not believe that it will (33).

"There is no mechanical connection between running a deficit and creating money supply...Just think, for example, of what happened in the United States in 1982. The deficit was enormous; it was well over $100 billion. Yet the central bank expanded its monetary base by only about $10 billion. Furthermore, this increase was less than it had been in years when the government deficit was smaller."

Sargent and Wallace might agree with the Modigliani analysis as a short-run occurrence but possibly not as a long-run phenomena (39). In their analysis, the government is constrained by the public demand for bonds in two ways: by setting a limit on the desired real stock of government bonds relative to the size of the economy and by influencing the interest rate which must be paid on the debt. Two points should be observed. First, an implicit stock adjustment is taking place between desired and actual levels of bonds. Second, the interest rate is influenced by the need to entice individuals into holding newly issued government bonds. A rejection of the "Ricardian equivalence" hypothesis is therefore implicit in the Sargent-Wallace model.

A crucial assumption is that the above constraints, initially imposed on the fiscal authority, can be redirected onto the monetary authority. In the words of Sargent and Wallace, fiscal policy dominates monetary policy by setting its level of spending independently of the imposed public constraints. They thereby shift the constraints onto the monetary authority by placing a limit on their ability to control the money supply.

The authors show that if the growth rate of the interest payments on bonds exceeds that of output, the monetary authority cannot permanently control the rate of growth of the monetary base. A faster interest payment growth implies that the real stock of bonds grows faster than output. The public's demand for bonds, however, places an upper limit on the stock of outstanding bonds they are willing to hold relative to the size of the economy. This in turn implies a need to monetize the deficit. As well, if inflationary expectations are modelled in terms of anticipated future growth rates in the nominal money supply, even current inflation cannot be controlled.

To summarize, deficits can cause inflation indirectly if they raise interest rates which the monetary authority attempts to stabilize by increasing the money supply. Although this type of monetization can be resisted by the monetary authority in the short-run, under certain conditions it cannot be resisted in the long-run.

2. MEASURING THE DEFICIT

Some people might suggest that even if a relationship between the deficit and inflation can be found, causality runs from inflation to the deficit, not vice versa. These people are correct. The actual nominal deficit (ADEF) is an endogenous variable which is influenced by changes in the inflation rate and by deviations in output from its potential level.

Inflation affects the ADEF in two ways. First, certain expenditure programs and revenue sources are automatically influenced by inflation. Second, a higher rate of inflation implies higher nominal interest rates, thereby, increasing interest payments which must be paid on the debt each year.

The deficit also automatically responds to cyclical variations in output. The classic example is unemployment compensation which increases during downturns. It is argued that to determine the effect of discretionary fiscal policy, these automatic responses must be removed. When this adjustment is made, we arrive at a measure of the high-employment budget deficit (HEDEF).

The HEDEF is a measure of the budget deficit which would have occurred, had the economy been fully-employed. In the United States, in terms of the unemployment rate, full-employment figures have been periodically revised upwards from 4% in 1955 to 5.1% in 1981. In Canada, the cyclically adjusted unemployment rate is defined to be a level at which a high rate of capacity utilization is sustainable in the medium and long-run. At the present time, the cyclically adjusted unemployment rate is believed to be in the range of 6.5 to 7.5% (26). Many papers have dealt with the measurement problem. See, for example, (27,28,29,36, and 38) in the United States and (8,15, 26 and 37) in Canada.

Michael Parkin (37) suggests that both the cyclical and inflation components of the deficit should be removed before we can study the effects of the deficit on inflation. However, it is not readily apparent why the cyclical component needs to be removed. Jerome Stein (45) has developed a simple theory to justify the cyclical adjustment.

Stein defines the following:

ADEF= actual deficit,

HEDEF= high-employment deficit,

CCDEF= cyclical component of the deficit, which equals ADEF-HEDEF,

τ_t = money multiplier in time period t,

Γ_t = proportion of the high-employment deficit financed by high powered money in period t,

α_t = proportion of the cyclical component of the deficit financed by high-powered money in period t,

H_t = high-powered money,

M_t = stock of money defined as M1.

Stein hypothesizes that a fraction Γ of the HEDEF and a fraction α of the CCDEF is financed by high-powered money as written in Eq.(1-2) below.

(1-2) $H_{t+1}-H_t = \Gamma_t HEDEF_t + \alpha_t CCDEF_t$

By recognizing that the money multiplier is the ratio of the money supply to high-powered money, an approximate relationship of the percentage change in the money supply is derived in terms of high-powered money and the money multiplier.

(1-3) $(M_{t+1}-M_t)/M_t = (H_{t+1}-H_t)/H_t + (\tau_{t+1}-\tau_t)/\tau_t$

Substitution of Eq.(1-2) into Eq.(1-3) and the use of $H_{t+1}=M_{t+1}/\tau_{t+1}$ leads to Eq.(1-4).

(1-4) $\upsilon_{t+1}=\tau_t(\Gamma_t(HEDEF/M)_t+\alpha_t(CCDEF/M)_t)+(\tau_{t+1}-\tau_t)/\tau_t$

where υ is the percentage change in the nominal money supply.

Stein then argues that a negative relationship exists between the money multiplier and the CCDEF. This arises from the observation that during contractions the money multiplier decreases while the CCDEF increases. This inverse relationship, written as Eq.(1-5), is assumed to be linear. The coefficient is a positive number.

(1-5) $(\tau_{t+1}-\tau_t)/\tau_t = -\delta_t(CCDEF/M)_t$

By substituting into Eq.(1-4) we obtain Eq.(1-6).

(1-6) $\upsilon_{t+1}=\tau_t\Gamma_t(HEDEF/M)_t+(\alpha_t\tau_t-\delta_t)(CCDEF/M)_t$

If the term $(\tau_t\alpha_t-\delta_t)$ is close to zero, the cyclical component becomes unimportant. Stein tests this proposition by regressing the lagged actual deficit/ money stock (ADEFMS) ratio and the lagged high-employment deficit/ money stock (HEDMS) ratio on the actual growth rate of the nominal money supply. If the coefficient on the ADEFMS ratio is not significantly different from zero the cyclical component is also insignificant. Stein's finding confirms that the hypothesis of a significant cyclical component can be rejected. We shall now review the GBC literature in so far as it is related to this thesis.

3. THE GOVERNMENT BUDGET CONSTRAINT (GBC)

A review of the GBC literature is required because of the theoretical model developed in Chapter II. Understanding how the GBC affects models which incorporate capital growth and the inflation rate is of crucial importance in the arguments presented in that chapter.

As Blinder and Solow point out, when we explicitly recognize the GBC, the effectiveness of fiscal policy hinges on crowding-out effects caused mainly by increases in wealth. As crowding-out has been previously discussed, no further elaboration is made. Suffice it to say, in the GBC literature little if any attention is paid to the Ricardian equivalence hypothesis.

Recall Eq.(1-1) introduced in Section 1 of this chapter, which represents one form of the GBC. In this particular form, government bonds are assumed to be perpetuities paying interest of one dollar a year, forever.

Briefly, the GBC says that in each time period all government expenditures must be financed by its various available sources: tax revenues and new issuance of either money or government bonds. Three points can be developed from this simple constraint.

(1) Long-run stationary equilibrium requires a balanced budget.

(2) Models incorporating the GBC are stable if and only if the budget returns to a position of long-run equilibrium.

(3) The GBC implies that one degree of freedom is lost in terms of available policy variables. Therefore, if n policy variables exist, only (n-1) of these variables can be exogenously set.

In the GBC literature, long-run equilibrium is reached when both stock and flow equilibria prevail. Point number one follows directly from this definition. In stock equilibrium neither real money balances nor real outstanding bonds are growing. Consider now, as a great deal of the literature does, the two polar extremes of pure bond financing and pure money financing policies. When we ignore a population growth rate, this means that either DH or DB is initially set equal to zero. Given the initial policy, stock equilibrium requires that both DH and DB equal zero, thereby, implying a balanced budget.

If H and B are both allowed to grow, a moving as opposed to static equilibrium occurs in the long-run. Here, the nominal growth rate of each asset must be the same; i.e., (DH/H)=(DB/B). The key point to remember is that real asset holdings cannot be changing if stock equilibrium is a requirement. This condition must hold regardless of whether or not a stock equilibrium in which the budget is balanced or a moving equilibrium in which the deficit or surplus grows at a constant rate is studied.

The second point is a logical extension of the first. Whether a static or moving equilibrium is analyzed, stability implies that stock/flow equilibrium must be attained in the long-run.

The third point is a direct consequence of the GBC. The need to finance a deficit or destroy assets when a surplus exists implies that one policy variable must be endogenously changing. In Eq.(1-1) there are four policy variables: G, θ , DH and DB. Any three of these variables can be exogenously set; however, the fourth becomes endogenous in order to satisfy the constraint. Usually, G and θ are assumed exogenous, allowing one of either DH or DB to be endogenously determined.

3.1 FIXED PRICE MODELS

We next analyze the implications of the GBC in the cases of money-financed expenditures, bond-financed expenditures and open market operations. Our intention is to familiarize the reader with the mechanics of the GBC as well as to highlight the stability issues arising in dynamic models. The analysis, which follows closely that of Blinder and Solow (7), assumes fixed prices.

3.1.1 A MONEY-FINANCED EXPENDITURE POLICY (MFEXP)

Assume an initial equilibrium position in which the budget is in balance. A MFEXP policy initially creates a budget deficit which, after first round effects, decreases. The stimulus of increasing G will raise real output and thereby increase the amount of tax revenue received by the government. It is unlikely that output will rise by an amount sufficient to balance the budget after this first round effect. As such, the money supply continues to expand. In terms of an IS-LM diagram, the LM curve continues its rightward movement. As output continues to rise, the deficit becomes smaller and smaller. Eventually, the budget returns to balance at a new, higher equilibrium level of output. The system is stable because budget balance is regained and real balances stop growing. This is a typical fixed-price, closed economy conclusion in which a MFEXP policy is unambiguously stable.

It should be noted, however, that with the introduction of inflation and capital growth, the stability of a MFEXP policy is not as clear cut as this simple model suggests.

3.1.2 A BOND-FINANCED EXPENDITURE POLICY (BFEXP)

If government expenditures are bond-financed and we set DH=0, Eq.(1-1) can be rewritten in the form:

(1-7) $DB=R\{G-\theta Y+(1-\theta)B\}$ where the price level is normalized to equal to one.

Government bonds affect the constraint in two ways; first, through its positive effect on Y which raises tax revenues and second, by increasing after-tax interest payments made to the public. These two effects work in opposite directions. Stability depends crucially on which effect dominates. Equilibrium can only be attained if the increase in tax revenues exceeds the increase in after-tax real interest payments. Algebraically, $\theta Y_B>(1-\theta)$, where Y_B is the partial derivative of Y with respect to B; i.e, the change in Y due to a change in B, ceteris paribus. This condition will hold only if Y_B is positive. In other words, total crowding-out must not occur and output must rise by a degree sufficient to cover both the increase in government expenditures and the higher interest payments. This simple example illustrates the potential instability of pure bond-financing.

3.1.3 OPEN MARKET OPERATIONS

Starting from a position of equilibrium, if open market operations are pursued, $dM=-dB/R$. Initially, the stock of wealth remains unchanged but due to the expansion in money, the LM curve shifts to the right. This raises real output and creates a budget surplus. Also, real after-tax interest payments decrease which further increases the surplus. To re-establish equilibrium, budget balance must be restored. The stock of money will be decreasing if the money supply is endogenous. By lowering output and tax revenues the surplus decreases until equilibrium is re-established. No instability arises in this scenario.

If the stock of outstanding bonds changes endogenouly there will be a decrease in after-tax interest payments which puts further upwards pressure on the surplus. The system is unstable if the after-tax interest payment decrease outweighs the decrease in tax revenues caused by the falling output. Wealth effects are the source of the falling output; consumption demand decreases and the demand for money decreases. In the IS-LM paradigm, the IS curve shifts to the left while the LM curve shifts to the right.

3.2 OTHER FORMS OF THE GBC

Christ (13) attempts to reconcile the instability problem of bond-financed expenditures by respecifying the GBC. In this formulation, government expenditures are redefined to include purchases and interest payments gross of taxes; $G'=G+(B/P)$. The equilibrium version of the GBC is written as Eq.(1-8).

$$(1\text{-}8)\quad G'-\theta(Y+(B/P))=0$$

If G' is exogenously determined, any change in interest payments will be fully offset by a change in purchases (G). Thus, total government expenditures remain constant. Under the new specification, ceteris paribus, bond-financing will increase tax revenues as the outstanding stock of bonds increase. Christ shows that if inflation expectations are formulated statically, bond-financing is potentially stable. This result is in contrast to Tobin and Buiter's finding that bond-financing is unstable when inflationary expectations are static. Tobin and Buiter, hereafter T-B, specify a GBC which defines government expenditures to include purchases and interest payments net of taxes.

Consider a model in which government expenditures are not exogenous but change endogenously depending on the business cycle. William Scarth (44) develops a model which resembles Blinder and Solow's except for the inclusion of one equation.

(1-9) $DG=a(YF-Y)$, $a>0$ and YF equals potential output.

Thus, government expenditures vary endogenously if any deviation between actual output and the target level of output (potential output) exists. Recall the Blinder and Solow stability condition, $\theta Y_B>(1-\theta)$ where, ceteris paribus, $Y_B=\partial Y/\partial B$. Under Scarth's specification the stability condition becomes $(Y_B/Y_G)>(1-\theta)$ where ceteris paribus, $Y_G=\partial Y/\partial G$. It turns out that this condition will be satisfied if the interest elasticity of investment spending is low. Empirical estimates of this value suggest that instability is a legitimate concern.

3.3 OPEN ECONOMY MODELS

Although an open economy model will not be developed in this study, given the important rôle it plays in Canada, a few comments are in order.

The general concept of the GBC does not change in an open economy model: expenditures must still be financed by total revenues from all sources. On the expenditure side, when the exchange rate is pegged, purchases or sales of foreign exchange are made to maintain the pegged level. On the revenue side, interest payments from holding foreign bonds are taxable. Notice that under a pegged exchange rate regime, an exogenous money supply policy is possible only if one resorts to sterilization. This implies that the stock of outstanding government bonds becomes endogenous.

The difficulty in reaching any firm conclusions in open models is effectively demonstrated by comparing two papers; one by Scarth (43) and the other by Allen (1).[1] Both authors find that a BFEXP policy is unstable when the exchange rate is fixed. Initially, such a policy increases the domestic rate of interest and output. The interest rate effect creates a balance of payments surplus which leads to a higher level of reserve holdings. At the same time, to maintain control of the money supply, open market sales of bonds are undertaken. The deficit becomes larger as interest payments continue to increase. No mechanism exists to reduce the deficit, thus the system is unstable.

The authors also find that under a flexible exchange rate regime a MFEXP policy leads to a higher level of output and is stable. An ambiguity in the findings arises in the money-financed, fixed exchange rate and bond-financed, flexible exchange rate cases. Scarth finds both to be stable; however, Allen's analysis shows both cases are unstable.

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1 The two models examined view interest payments differently. In Scarth's model, foreign residents purchase domestic bonds but domestic residents do not purchase foreign bonds. In Allens' model the opposite procedure is used.

Given the assumption of perfect capital mobility, output and interest rate changes will not occur if a BFEXP policy is pursued under flexible exchange rates. This is a consequence of the exchange rate increase which raises import demand and lowers export demand. In this case the deficit can only be reduced if after-tax interest payments decrease. This cannot occur, because bond supply is increased to meet the increased government expenditure. The results derived by Scarth show that a balance of payments equilibrium is attained; therefore, the deficit must also be balanced in the long-run. However, there does not seem to be any mechanism which reduces the inital deficit caused by the higher level of government spending; bond-financing again is unstable. The reason Scarth does not obtain this result is because a formal stability analysis was not attempted. Instead, Scarth determined the long-run multipliers from which he inferred stability. Unfortunately, these multipliers are valid only if stability is proven before hand.

As a final case, a MFEXP policy is likely stable. Scarth shows that in the long-run output increases. This will re-establish budget balance by increasing the tax revenue of the government. Allen does not reach the same conclusion; however, his argument is not well developed. In his opinion, a MFEXP policy leads to a position where the government deficit just equals the balance of payments deficit. Because this position does not coincide with his definition of equilibrium, Allen argues that the policy is unstable. The process by which the economy arrives at this position in the first place is not explained.

4. REVIEW OF STABILITY ISSUES WHEN PRICES ARE ALLOWED TO FLUCTUATE

How equilibrium is defined in models with inflation is very important because a number of authors have used a concept in which a long-run steady state rate of inflation exists (14,45). If stock equilibrium is to be achieved, real balances must be constant in the long-run. For this condition to hold in conjunction with a steady state inflation rate, the growth rate of nominal balances, money and bonds, must equal the rate of inflation: (DM/M)=(DB/B)=(DP/P). Clearly, the nominal stock of both bonds and money are growing.

There are two ways in which a BFEXP or a MFEXP policy can be viewed. What has thus far been termed a pure money or pure bond-financed policy is one way. Here the government decides which financial asset will be held constant in nominal terms. As previously seen, if policy is defined in this way an equilibrium rate of inflation is not possible if real stocks must be constant in the long-run. This is consistent with the notion of a static equilibrium.

The second way a BFEXP or a MFEXP policy can be viewed is one in which the government decides the equilibrium rate of growth of money and bonds. Given this viewpoint, a positive rate of inflation is compatible with the notion of stock equilibrium as long as real balances are constant. In the steady state the nominal growth rate of financial assets is the rate of inflation. This viewpoint is consistent with the notion of a dynamic or moving equilibrium. In Chapter II the policies examined follow the first viewpoint in which one of the nominal assets is held constant.

CARL CHRIST

In Carl Christ's flexible price model inflation is a function of inflationary expectations and the Okun-gap. The Okun-gap or output-gap is the difference between actual output and potential output.

(1-10) $DP/P=z\{(Y/f(K))-1\}+X$

In Eq.(1-10), Y refers to actual output, f(K) to potential output and z shows the speed of adjustment between actual and potential output. It is important to note that Christ does not explicitly accommodate inflationary expectations, X; however, upon re-deriving the stability conditions, it is clear that static expectations is assumed since X is set equal to zero.

According to Christ, money-financing is unambiguously stable. Initially, output increases, thereby causing a condition of excess demand (Y>f(K)). In response to this demand the inflation rate rises. This raises the price level, which combined with the higher level of output decreases the initial budget deficit. If the system is stable, actual output reverts back to its potential level leaving tax revenues unaltered. The only way in which the budget will be balanced is if the price level adjusts to a higher level. This decreases real interest payments and offsets the initial rise in G.

In the bond-financed case, stability depends on the sign of $\partial P/\partial H$. If it is positive as empirical evidence suggests, bond-financing is unstable. If the system is stable, the long-run comparative static results suggest that the price level must rise by a greater magnitude than in the money-financed case. This is due to the initial budget deficit which is larger when a BFEXP policy is pursued. An increase in the outstanding stock of bonds to help finance the deficit, raises real after tax interest payments which pushes the deficit to a higher level. In the money-financing case, B remains constant and this extra burden on the deficit does not materialize.

TOBIN AND BUITER

Three forms of inflation expectations are considered by Tobin and Buiter (48); static expectations, myopic perfect foresight and adaptive expectations. However, only bond-financed expenditures are analyzed under the price flexibility assumption. In this section the stability conditions determined by Tobin and Buiter are reviewed. As well, the stability conditions for money-financed expenditures, under the assumption of static expectations and myopic perfect foresight are introduced. Since these two forms of expectations are limiting cases of adaptive expectations, the stability conditions can be derived for all three forms without explicitly considering the adaptive form.

Static Expectations (X=0)

Tobin and Buiter solve the IS-LM system in terms of R and P where

$R = R(B,K,X;M,G',\theta)$

$P = P(B,K,X;M,G',\theta)$.

Necessary and sufficient conditions for stability are as follows:

1) $I'(f''-R_K) < 0$

2) $-I'R_B P\theta f' > 0$

In the above conditions, $I'>0$, where $DK=I\{f'(K)-R+X\}$; i.e., investment demand, DK, is an increasing function of the rate of return differential between existing capital and bonds. The rate of return on capital is the marginal product of capital, $f'(K)$, whereas the rate of return on bonds is the real interest rate; the nominal interest rate minus expected inflation, R-X.

A sufficient condition which satisfies 1), is that $R_K > 0$ or close to zero. This condition may or may not be satisfied. R_K is the effect of an increase in capital stock on the short-run IS-LM solution for R.

However, condition (2) will not hold, since R_B is unambiguously positive. R_B is the effect of an increase in bonds on the nominal rate of interest in the short-run.

The mathematics of the T-B model are such that we can quickly determine the stability conditions for a MFEXP policy. We need to replace all R_B

variables with the term R_M, the short-run interest rate effect of an increase in the stock of money. R_M can be shown to be unambiguously negative. This guarantees the satisfaction of condition (2). However, money-financing is still potentially unstable since condition (1) may or may not be satisfied.

Myopic Perfect Foresight (X=DP/P)

A necessary but not sufficient condition for stability in the bond-financed case is that $(I'-(M+B)/P) > 0$. This term describes the effect which an increase in expected inflation has on aggregate demand. As X increases, the real rate of return on bonds decreases, thereby increasing investment demand which has become more attractive than bond investments. That is, the rate of return on bonds (R-X) falls relative to the rate of return on capital (f'(K)). Because of the expected capital losses on real financial assets, (M+B)/P, caused by an increase in X, individuals save more and therefore consume less. The two effects which have been described oppose one another. For stability the investment effect must outweigh the consumption effect.

Tobin and Buiter specify a portfolio stock adjustment process in which individuals attain their long-run desired level of wealth. The authors show that the speed of adjustment between desired and actual levels of wealth, ($\bar{s}$), must attain certain strict upper and lower bounds before stability is possible. What these bounds actually are is difficult to conceptualize but they cannot be too small or too large; otherwise, the model is unstable.

In the money-financed expenditure model, s must still be bounded by certain limits; however, a necessary condition for stability is that $I'-(M+B)/P<0$. This is exactly opposite to the condition required for stability in the bond-financed model. We can, therefore, conclude that simultaneous stability under either policy cannot occur. One of the two policies is unambiguously unstable. Also, given the limits on the value of s, it is theoretically conceivable that both policies are unstable. Thus, Tobin and Buiter do not arrive at any clearly unambiguous results. Instability is possible even when government expenditures are financed by money. When expectations are adaptively formed, the same general conditions of the perfect foresight case applies.

JEROME STEIN

The next model to be discussed is developed by Jerome Stein (45). The unique aspect of the model is the way inflation expectations are formed. Stein assumes that the expected rate of inflation is equal to the expected growth rate of the nominal money supply. As explained in Section 2, money supply growth depends on the high-employment deficit-money stock ratio (HEDMS); therefore, individuals are hypothesized to form expectations in terms of this variable.

$X=\{G-\theta f(K)\}/(M/P)$, where $G-\theta f(K)$ is the high-employment deficit.

Two points need to be made. First, no government bonds exist in this model; the only financial asset is money. In equilibrium, real money balances must be constant. This requires an equal growth rate between the nominal money supply and inflation: $DM/M=DP/P$. Second, inflation expectations are fully realized only in the long-run when equilibrium is re-established.

The equation modelling inflation is almost identical to that of Christ's except that the output-gap is no longer in terms of percentages; i.e., $(Y/f(K))-1$ is replaced by $Y-f(K)$. Changes in real money balances are a function of the growth rate of nominal money and the rate of inflation, $D(M/P)=(DM/M-DP/P)M/P$. Because the inflation rate is a function of the output-gap, changes in real money balances are also a function of the output-gap. In fact, if we substitute the HEDMS ratio for X in Eq.(1-10) and the actual deficit-money stock ratio (ADEFMS) for (DM/M), the growth rate of real money balances can be written entirely as a function of the output-gap. Stein's derivation shows a negative relationship between the growth rate of real money balances and the output-gap.

Stein attempts to illustrate the dynamic process which is set in motion when, for whatever reason, actual output deviates from its potential level and government expenditures are increased in an attempt to quicken the return to equilibrium. Initially, the increase in G raises output and decreases the gap. Since the output-gap is inversely related to the growth in real money balances, the lower gap leads to a rise in real money balances.

However, in the long-run, real money balances must decline because an excess aggregate demand would exist at the initial level of capacity output. The decline in real balances raises the real rate of interest and decreases capital formation. Real government purchases crowd out real capital formation in the long-run. This is one form of the debt burden argument discussed by Barro (5).

Also, in equilibrium, the combined effect of a lower capital stock, which implies smaller tax revenues, and a lower real money stock, is a larger HEDMS ratio. In turn, this implies a higher equilibrium rate of growth in the money stock and therefore a higher equilibrium rate of inflation.

Stability requires that the effect of an increase in capital stock on aggregate demand to be sufficiently small. This partial derivative is composed of three parts: a negative investment effect arising from a lower marginal product of capital, a positive consumption effect due to the increase in wealth and an increased demand for money caused by greater wealth. The net effect on aggregate demand is ambiguous. The lower investment demand and higher demand for money shifts the IS and LM curves to the left, lowering aggregate demand. The increased consumption demand shifts the IS curve to the right, increasing aggregate demand.

Blinder and Solow in their fixed-price model allow for capital growth in the second part of the paper. Stability of bond-financed expenditures requires that the negative investment demand of increased capital outweighs the positive effect of the increased consumption demand. In the Stein model, if this condition holds, $\partial Y/\partial K$ is unambiguously negative and stability is attained.

RUDIGER DORNBUSCH

Although Dornbusch does not model the output-gap, his model is similar to Stein's in that the area of concern is the effect of certain policies on the equilibrium stock of capital and inflation rate (14). Unlike Stein, government bonds are included in this model. Also, any deficit is simultaneously financed by money and bonds.

Consider the effect of an increase in government expenditures. Starting from a position of equilibrium, an increase in G will create an excess demand and an increase in the deficit. To restore goods market equilibrium the stock of capital must fall and the inflation rate must rise. The level of wealth is lowered by both effects. Therefore, consumption demand decreases which tends to re-establish goods market equilibrium. The capital stock decrease also lowers real output, which ceteris paribus, creates an excess demand. For stability purposes it is assumed that consumption demand is influenced more than output for a given change in the capital stock.

Because the deficit needs to be financed, the stock of capital must rise in order to create a larger tax base or the inflation rate must rise so that the revenue generated from money and bond creation is enlarged. This is best seen by considering that the steady state equilibrium rate of growth of nominal bonds and money equals the rate of inflation. This is written as:

$(DP/P)(M/P)=(1-\kappa)DEF$ and $(DP/P)(B/P)=\kappa DEF$

where κ is the proportion of the deficit financed by bonds and DEF is equal to the real budget deficit.

From these two conditions, we can readily see that an increase in the deficit implies that either the equilibrium rate of inflation has to increase or the equilibrium stock of capital has to increase. When combining this result with the finding that the inflation rate must rise or the stock of capital must fall to re-establish commodity market equilibrium, it is apparent that the equilibrium rate of inflation increases but the change in the capital stock becomes ambiguous.

Dornbusch shows that if the interest sensitivity of real money balances is low, the probability of capital decreasing is enhanced. Because it is composed partly of real money balances, wealth is a function of the nominal rate of interest. A rise in the nominal rate of interest lowers the demand for money and wealth. If the demand for money is insensitive to interest rate movements then the demand for wealth is also insensitive to interest rate movements. Therefore, to achieve a certain level of wealth reduction, the stock of capital must fall by a greater magnitude than would be necessary if the interest sensitivity of money demand was higher. In other words, there is an increased likelihood of a lower stock of capital.

The analysis by Dornbusch is important because it shows that once bonds are included in the GBC, the conclusion reached by Stein is no longer straight-forward. An expansion in government expenditures raises the equilibrium rate of inflation in both models; however, the stock of capital does not necessarily fall as suggested by Stein. This concludes the review of the major papers which have dealt with the GBC. The next section will review the various empirical studies which have tried to determine the relationship between deficits, monetization of deficits and inflation.

5. ECONOMETRIC STUDIES

ROBERT BARRO

In recent years, a number of articles have been written on the subject of deficit monetization. In particular, a money supply growth equation, first developed by Barro (3,4), has been the focus of attention. Although Barro's initial purpose was not in determining the significance of deficits, he nevertheless modified his original equation to incorporate this possible link (5). The deficit variable which is used by Barro is the actual nominal government deficit divided by the product of the GNP deflator and a trend-value of real G.N.P. Although this variable seems to adjust the deficit for its cyclical component bias, it is an imperfect adjustment because it implicitly assumes that both revenues and expenditures are affected to the same degree when deviations from trend output arise.

Barro finds that a variable representing deviations of real government expenditures from their "normal" level is more significant in explaining increased money supply growth than the deficit variable for the periods 1941-76 and 1946-76.

HAMBURGER AND ZWICK

Hamburger and Zwick (20) undertake a similar analysis but substitute a trended government expenditure variable in place of the deviations from trend variable used by Barro. They also replace expenditure and deficit data calculated in terms of the year ending in the fourth quarter, with annual averages. This adjustment is consistent with the method used to determine the growth rate of the money supply. For the period 1954-1976, the result found by Barro is reconfirmed; however, when the period 1961-1974 is analysed, the deficit variable becomes significant whereas the trended expenditure variable becomes insignificant. Hamburger and Zwick place more weight on these latter results because of the belief that monetary policy underwent a substantial shift in the early 1960's. During these years, interest rate movements were stabilized to a much greater degree than in the 1940's and 1950's.

ALLEN AND SMITH

Allen and Smith (2) present the most thorough study of Barro's equation. They introduce three changes. First, the growth rate of the money supply is replaced by the growth rate of the monetary base which is believed to be under the control of monetary authorities to a larger extent than the money supply. Second, the deficit variable is replaced by a DEBT variable, measured as the change in net federal debt divided by nominal trend GNP. Third, quarterly data is used in place of annual data. The authors are not only concerned about the significance of the variables but also about structural shifts that have occurred in monetary policy. They find that the expenditure coefficient changes for each of the break points considered (1969:4, 1974:1 and 1976:4) whereas the coefficient on the DEBT variable is not significantly altered.

GERALD DWYER Jr

Gerald Dwyer Jr. (16) employs a vector autoregressive model which includes the inflation rate, growth rates of nominal income, money supply, government debt held by the public and debt held by the federal reserve. An interest rate variable is also included. Dwyer's results indicate that government debt is not perceived to be net wealth and that deficits have no effect on federal reserve debt purchases. In fact deficts do not affect any of the macro-variables. These conclusions are, however, subject to some debate. Dwyer found that he could reduce the lag structure of the model from 8 quarters to 4 quarters; however, inspection of the results would lead one to doubt whether or not the lag structure should have been reduced even further.

Also, Dwyer does not include the deficit per se as a relevant explanatory variable. The deficit, for any time period, is the change not only in the outstanding stock of debt held by the public but also of the change in Federal Reserves debt holdings. Because of this separation, Dwyer's statement that the deficit does not affect Federal Reserve debt holdings is not accurate. Interestingly enough, Dwyer finds evidence that the debt acquired by the Federal Reserve does feed back onto inflation. Unfortunately, little attention is paid to this finding. Finally, the conclusion that the deficit does not affect inflation is incorrect. The outstanding stock of bonds held by the public may not affect inflation but the deficit cannot be defined in this way.

RICHARD FROYEN

Froyen (18) does not use any exotic statistical techniques; however, he designs a fairly complex reaction function which assumes the existence of four stabilization targets and three financial objectives. The analysis, which uses quarterly money-base data as the dependent variable, is again broken into different time periods. A full-employment deficit variable is significant in only one period, 1961:2-1969:1. Interestingly, this is the only time period in which the target inflation rate variable is also significant. The balance of payments and interest rate variables are found to be insignificant in explaining the money supply reaction function.

BARTH, SICKLES AND WIEST

Spline functions are used to test a money supply reaction function in which the response of the stock of money to inflation and unemployment is the major area of concern. Monthly data is incorporated for the period, Jan.1953 through Feb.1978. They found that the Federal Reserve responds consistently to unemployment but systematically to the inflation rate only in periods when it is high relative to the target rate. The authors find that changes in the interest rate and the balance of payments surplus are rather insignificant. The monetary authority responds significantly to total business sales and also to the full-employment deficit variable.

MICKEY LEVY

A major deficiency of most of the studies thus far considered is the lack of any formal theoretical model development. Levy (30) introduces a full IS-LM model which incorporates the GBC, inflation expectations and a money supply reaction fuction. For estimation puposes, Levy concentrates on the reaction function in which the stock of money, defined as the change in base money, is the dependent variable. The list of regressors includes the changes in inflationary expectations, capacity output, the interest rate, unemployment rate and outstanding publicly-held debt. The last three variables are regressed on the other variables in the system as the first stage of a two-stage procedure. Interestingly, the unemployment rate and the interest rate are not significant while inflation expectations and the outstanding debt are highly significant.

Levy has an interesting interpretation of why the interest rate variable is insignificant. He hypothesizes that the Federal Reserve expands the money base in response to nominal interest rate increases imbedded in inflationary expectations and to higher real interest rates caused by deficits. In other words, the interest rate is highly correlated to these variables. Thus the insignificance of the interest rate does not mean that it has no effect on the reaction function.

WILLIAM NISKANEN

William Niskanen (35) appears to be the only person to make an attempt to quantify the impact of deficits on the inflation rate. He specifies both a money supply reaction function as well as an equation explaining inflation. The growth rate of the nominal money supply is the dependent variable of the reaction function. In neither case are the results very encouraging. The R^2 value for the reaction function is much lower than found in previous studies. This indicates that the model may be poorly specified. Niskanen's findings suggest that at most 20% of the

U.S deficit has been monetized. However, the deficit variable included in the inflation rate equation is shown to be insignificant. The variable of interest is defined as (D/Y_{-1}) where D is the deficit in the current year and Y_{-1} is the nominal income in the previous year. No one else has used this specification. Theoretically speaking, it is odd that the ratio is comprised of two variables, one lagged and the other not. Also, Niskanen's model assumes that the deficit directly affects the rate of inflation. We found at the beginning of this chapter that if deficts do affect inflation, it does so indirectly via the money supply. This suggests a simultaneous equation model may be more appropriate.

Although the evidence is somewhat mixed, it would appear that the full-employment deficit is a significant variable and should be included in equations specifying money supply growth. Certain articles which do not find a link between deficits and inflation have not been as technically rigourous as one might expect. In addition, balance of payments and interest rate variables have been consistently rejected throughout the majority of studies reviewed.

II. THE TOBIN-BUITER MODEL REVISITED: AN EXTENSION

INTRODUCTION

In his book, "Keynesian, Monetarist and New Classical Economics", Jerome Stein has analyzed an IS-LM growth model. The innovative feature of his model centers around the formation of inflationary expectations. Stein assumes that individuals form their expectations of future inflation rates on the expected growth rate of the nominal money supply, which in turn is a function of the high-employment deficit/money stock ratio, (HEDMS). Algebraically, this is written as:

(2-1) $\lambda = M^e = [G - \theta\, f(K)]/(M/P)$,

where
- λ = expected rate of inflation,
- M^e = expected rate of nominal money growth,
- G = real government expenditures,
- θ = marginal tax rate,
- $f(K)$ = capacity output,
- (M/P) = real stock of money.

The purpose of this chapter is to integrate Stein's version of inflationary expectations with the model developed by Tobin and Buiter (hereafter T-B). There are three reasons for doing this:

(1) Stein did not include government bonds and consequently the interest payments made on these bonds in his analysis. This

omission neglects the fact that the issuance of bonds is an option often used by the governments, (both money and bond financing policies are examined in detail in this chapter).

(2) In the T-B model, there is a strong emphasis on portfolio adjustments between stocks of bonds, capital and money. The desired levels of stocks which individuals wish to attain in the long-run and the mechanism by which this is achieved is not modelled by Stein.

(3) By using the T-B framework, we are able to analyze any differences which arise from modelling expectations in terms of the HEDMS ratio as opposed to static, perfect foresight as well as adaptive expectations hypothesis.

This chapter is organized in the following way. The T-B model is introduced in Section 1. Each equation will be explained in detail. Section 2 examines the various modifications used in extending the T-B model. Section 3 presents the results of the extended model. Both long-run and short-run analyses are offered as well as a comparison of the stability conditions found in the extended model and the T-B model. In the major part of Section 3, constant full-employment is assumed; however, unemployment is considered briefly. Section 4 briefly summarizes the findings of this chapter.

1. THE TOBIN-BUITER MODEL

Tobin and Buiter base their analysis on the following equations:

(GT1) $G' - \theta f(K) = (DM+DB)/P$

(GT2) $G+R(1-\theta)B/P - \theta f(K) = (DM+DB)/P$

(LLMB) $M/P = L[R(1-\theta),\ 1/u]uf(K)$

(LLMM) $B/P = uf(K) - L[R(1-\theta),\ 1/u]uf(K) - K$

(2-2) $W = uf(K)$

(2-3) $K = g(R),\ dg/dR = 1/f''(K) < 0$

(2-4) $Y/N = f(K/N)$ where: $N=1,\ f'(K) > 0,\ f''(K) < 0$

(IS) $I[f'(K)-(R-X)]-s[uf(K)-W]-X(M+B)/P + (DM+DB)/P = 0$

(LM) $M/P=L[R(1-\theta),\ (Pf(K))/(M+B+PK)][K+(M+B)/P]$

where: $I'>0,\ L_1<0,\ L_2>0,\ uL>L_2$.

All variables are defined in the list of abbreviations which is found at the beginning of this thesis.

Following T-B, the government budget constraint (GBC) is modelled in one of two ways, with either a (GT1) or (GT2) specification being used. The only difference between (GT1) and (GT2) is the treatment of real, after-tax interest payments. (GT2) separates gross government expenditures into two parts, real purchases (G) and real after-tax interest payments $R(1-\theta)B/P$. Modelling the GBC in this way implies that the government can exogenously set the level of G. In the (GT1) version the two components are not separated. Gross government expenditures (G') are exogenously determined by the government, implying that any endogenous change in interest payments must be offset by changes in G. In other words, any change in B, P, or θ automatically sets in motion a response in G which keeps G' constant.

One further elaboration needs to be made. Tobin and Buiter treat government bonds as bills of short-term maturity; in their own words value of interest payments is equal to the outstanding stock of bonds (B) multiplied by the nominal rate of interest (R). This is different from the usual assumption that all government bonds are consols paying a coupon rate of $1.00, forever. If government bonds are consols, nominal interest payments are equal to the outstanding stock of bonds in a given year; i.e., B. Apparently, T-B's reason for choosing one method over the other is one of taste rather than any theoretical justification. Either method is appropriate; therefore, for consistency we follow T-B.

A good portion of the T-B analysis is presented graphically. In their long-run analysis, GT and LLM curves are developed. The LLM curve is a long-run counterpart to the short-run LM curve. We will discuss the derivation of the LLM curve in greater detail below. For now, the reader is alerted to the notational confusion which may exist with referece to the LLMB and LLMM equations. LLMB refers to the long-run LM curve when a BFEXP policy is followed. In this case, financial market equilibrium is written in terms of the real money stock. LLMM refers to the long-run LM curve when a MFEXP policy is pursued. Here, financial market equilibrium is written in terms of the real outstanding stock of bonds. The purpose of presenting the LLM curves in two different ways is for diagramatic simplification.

The GT curve represents the combinations of R and P which allows the government budget to be in balance. This is written as (DM+DB)/P=0. If the GBC is specified as (GT1), then P has no role in determining budget balance. Therefore, the slope of this curve is equal to zero, (See Diagram 1A on page 55). If the GBC is specified as (GT2), then an increase in P, given an initial balanced budget, decreases real interest payments and creates a surplus. To maintain a budget balance, R must rise so that real interest payments increase. This curve is upwards sloping (see Diagram 1B on page 55). The derived slopes are presented in Section 3.1.

Eq.(2-4) is considered to be a well-behaved, constant returns to scale neoclassical production function, in terms of capital (K) and labour (N). Because a growing population is disregarded in this analysis, T-B set N=1; thus Y=f(K). The slope of the production function, f'(K), which is also the marginal product of capital, is positive throughout. As capital stock increases the marginal product of capital decreases, f"(K) < 0. In other words, the law of diminishing marginal returns holds.

The term f(K) can be regarded as capacity output. In Section 3 we assume that actual output, Y, is equal to capacity at all times. That is, the economy always operates on a point along the f(K) curve. In models of unemployment, only in equilibrium, Y=f(K), can a point along the f(K) curve be attained.

The negative relationship which exists between the rate of return of bonds, R, and the capital stock, K, is illustrated by Eq.(2-3). This inverse relationship arises from a crucial assumption: capital assets and government bonds are perfect substitutes. An increase in the rate of return on bonds implies that government bonds become a more attractive investment in individuals' portfolios than capital assets. Therefore, more government bonds are held relative to capital assets.

Eq.(2-2) states that in long-run equilibrium, the stock of wealth, W, held by individuals is a constant proportion, u, of capacity output f(K). The term uf(K) is the desired level of wealth individuals wish to attain in the long-run. In the short-run, the actual level of wealth will not equal the desired level of wealth. Tobin and Buiter assume a stock-adjustment process where the actual level of wealth approaches the desired level of wealth at a speed of adjustment, s. This is illustrated in Eq.(2-5) below.

(2-5) $DW=s[uf(K) - W]$

The desired level of wealth increases (decreases) if the level of capital increases (decreases). This is due to the positive relationship between capacity output and the level of capital stock.

The LLMB and LLMM equations describe the proportion of wealth held by individuals in the long-run, in the form of real money and real government bonds, respectively. The two equations are the long-run LM curves used by T-B. Only one of the two equations is required in the analysis, depending on whether money or bond financed expenditure cases are being studied. For example, if an increase in government expenditures is financed by bonds, then LLMB is the appropriate equation since M is held constant. The GBC will determine the amount of new bond issues as well as the level of capacity output required to balance the budget. Once the new capacity level is known so is the equilibrium stock of capital. LLMB thus needs to be satisfied given the new equilibrium stocks of bonds and capital. In a similar fashion, LLMM is the appropriate portfolio equation when government expenditures are money-financed.

LLMB states that individuals hold a certain fraction of their desired wealth, uf(K), in terms of real money balances (M/P). In the long-run, this fraction is a function of the after-tax rate of return differential R(1-θ). The income-wealth ratio is written as 1/u, in the long-run. Therefore, since u is assumed constant throughout, the long-run proportion of wealth held in the form of real money balances is unaffected by the income-wealth ratio. However, in the short-run, the ratio is written as Y/W which is clearly not constant and therefore affects financial market equilibria, so we change the (LLM) specifications to (LM).

The after-tax real rate of return differential is determined by subtracting the expected real rate of return of money, -X, from the expected real after tax rate of return on bonds and capital, $[R(1-\theta)-X]$. Therefore, $[R(1-\theta)-X]-[-X]=R(1-\theta)$.

Finally, we can derive LLMM by recognizing that $B/P=W-K-M/P$. Substituting Eq.(2-2) for W and LLMB for M/P leads to LLMM.

The final two equations used by T-B describe the IS and LM curve relationships respectively. Consider first the IS equation which T-B derive by equating investment flows with savings flows, both private and public; the sum of private and public savings equals the rate of capital accumulation.

Capital accumulation is described by the term $I[f'(K)-(R-X)]$. It is a function of two rates of return. The rate of return obtainable by investing in new capital goods is given by the marginal product of capital $f'(K)$. Since we are assuming that capital and bonds are perfect substitutes, (R-X) is the real rate of return on existing capital or existing bonds. Investment depends on the difference between these two rates of return. If the differential increases, investment increases and vice versa; i.e., I'>0.

Private savings are composed of two parts. The term s[uf(K)-W] in the IS equation describes the stock adjustment of the actual level of wealth to the long-run desired level of wealth. This occurs at the speed s, which should not be confused with the marginal propensity to save.

The third term, X(M+B)/P, represents the expected capital losses in financial assets, (M+B)/P, due to inflation. To compensate for these losses and maintain their level of wealth, individuals must increase their savings by this amount.

The final term, (DM+DB)/P, is the rate of saving or dissaving which can be attributed to the government. A negative value implies a budget surplus, thus the government saves, while a positive value implies a deficit, the government dissaves.

The short-run LM curve is identical to the LLMB curve except that the income-wealth ratio is not constant. Therefore, the LM curve is a function of both the real after- tax rate of return differential and the income-wealth ratio. Because the income-wealth ratio does not remain constant in the short-run, a second equation parallelling the LMM curve is not needed. LM is a complete representation of short-run financial market equilibria under either money or bond-financed expenditure cases.

The demand for money is inversely related to the after-tax rate of return differential, $L_1<0$, and directly related to the income-wealth ratio, $L_2>0$. Following T-B, we assume $uL>L_2$. By recognizing that L_2 is the change in the demand for money with respect to the income/wealth ratio, and that u is the income wealth/ratio, the above expression can be written as an elasticity. In particular, $uL>L_2$ says that the elasticity of the demand for money with respect to the income-wealth ratio is less than unity. Without this assumption many of the LM curve movements become ambiguous. Before describing the additions to the T-B model, a brief discussion of the long-run equilibrium conditions follows.

As illustrated in Chapter I, stock equilibrium must be fulfilled in the long-run. In the T-B model stock equilibrium requires three conditions:

(1) The budget must be in balance so that the nominal growth of financial assets is zero.
(2) Individuals must attain their desired income-wealth ratio.
(3) The inflation rate must equal the expected inflation rate which is zero.

The first and third conditions imply that the growth rate of real financial assets is equal to zero. These two conditions are a direct result of the two policies analyzed, pure money-financed and pure bond-financed expenditures. For example, under pure money-financing DB=0. It is conceivable that real money-balances would stop growing if DM/M = DP/P; however, the real growth rate of bonds would never equal zero since a positive rate of inflation, given a constant stock of bonds, implies the following:

$$D(B/P) = -(B/P)DP/P < 0.$$

D(B/P) will equal zero if and only if the rate of inflation equals zero. This then implies that D(M/P) = 0 if and only if DM/M=0. To summarize, under either a pure money-financed expenditure (MFEXP) or a pure bond-financed expenditure (BFEXP) policy, the budget must be in balance and the rate of inflation must equal zero. If the expected rate of inflation does not equal zero, there will be continual nominal increases in financial assets and condition (2) cannot be realized. The second condition also implies that the rate of capital growth must equal zero. This can be seen by the (IS) equation. The last three terms all equal zero by conditions (1) through (3); therefore, the first term which is equivalent to DK must also equal zero. The long-run equilibrium conditions are written as DK=DW=DM=DB=DP=X=0.

2. EXTENSIONS OF THE TOBIN-BUITER MODEL

As stated at the beginning of this chapter, the T-B model is extended by changing the formation of inflationary expectation. In this extended framework, the HEDMS ratio, Eq.(2-1), becomes the key variable from which individuals form their expectations of inflation. Since two versions of the GBC are considered, it is necessary to include two equations of inflationary expectations which correspond to GT1 and GT2. Eq.(2-6) corresponds to a GT1 specification, while Eq.(2-7) corresponds to a GT2 specification.

(2-6) $X = \beta\,[G' - \theta f(K)]/(M/P)$; $0 < \beta < 1$

(2-7) $X = \beta\,[G + R(1-\theta)B/P - \theta f(K)]/(M/P)$; $0 \leq \beta < 1$

The coefficient "β" is the proportion of the HEDMS ratio individuals expect to be financed by money. Stein does not include this coefficient in his model (implicitly $\beta = 1$) because there are no government bonds; therefore, bond-financed expenditures cannot be analysed. If creating money is the only financing policy option available, given θ, it is reasonable to set β=1. However, if a bond-financing policy option is also available, individuals can reasonably expect a combination of the two options to be used at any point in time.

For "β" to be constant, an important assumption is required. The government and monetary authority must behave consistently over time.

It is possible to assume that "β" is endogenous, by modelling a reaction function. For example, the monetary authority may have a different policy of money supply control when inflation is high than when inflation is low. If the monetary authority tightens monetary control during periods of high inflation, then "β" is expected to decrease; i.e., bond-financing will be substituted for money financing. However, notice that even if "β" is specified to be endogenous, the reaction function must be stable over time. Random changes in policy would create a situation in which economic agents consistently choose the wrong β; i.e., they can never predict what portion of the HEDMS ratio will be monetized.

3. THE EXTENDED MODEL RESULTS

In this section, the results of the extended model for four cases are presented. Bond and money financed expenditure policies are examined under different GBC specifications. Constant full-employment is assumed throughout this section, with the exception of Section 3.1.3 which offers a brief description of the long-run comparative static results under the assumption of unemployment.

3.1 LONG-RUN ANALYSIS

To begin the analysis, long-run comparative static results are considered. A graphical depiction is offered with the aid of a GT curve and a LLM curve. The GT curve represents combinations of R and P which balance the government budget. The LLM curve depicts combinations of R and P which satisfy long-run portfolio equilibrium; i.e., individuals hold their desired levels of money, bonds and capital. Derivation of the slopes of these curves is given in Table 1 (pg 51). The long-run multipliers are summarized in Table 2 (pg 53). The double stars in the bracket refer to derivations identical to those found by Tobin and Buiter. This occurs because the long-run comparative static results are unaffected by expected inflation which is the only modification in the model.

From Table 1, we observe that the slope of the GT1 curve is horizontal, while the slope of the GT2 curve is positive. The horizontal GT1 can be explained very easily as P has no effect on budget balance under this specification. To maintain an exogenous G', any changes in R or P must be offset automatically by changes in G. On the other hand, GT2 has a positive slope because any increase in R creates a budget deficit which, ceteris paribus, can only be offset by increases in P. We also notice that the LLM curve slope is positive when a BFEXP policy is maintained and negative when a MFEXP policy is pursued. The signs of the slopes can be explained by considering explicitly the two policy options. Consider first a BFEXP policy in which case (LLMB) is the appropriate long-run LM curve. Any increase in the nominal rate of interest creates an excess supply of money since the transactions demand for money falls. The right-hand side of the (LLMB) expression is, therefore, lower. Portfolio equilibrium is maintained, given a constant nominal stock of money, if the price level rises and thereby reduces the excess supply of money. Thus, the slope of LLMB is positive. Notice that once the money market is in equilibrium, by Walras' law, both the bond and capital markets are in equilibrium. Bonds and capital can be viewed as one market by the assumption of perfect substitutability.

TABLE 1

SLOPES OF GT-LLM CURVES

GT1(**)

$(dR/dP)=0$

GT2

$(dR/dP)=(1-\epsilon)RBf''/\{(1-\theta)Rf''-P\theta f'\}P > 0$

LLMB(Bond-Financing) (**)

$(dR/dP)= -Mf''/P^2\{ufL_1(1-\theta)f''+uLf'\} > 0$

LLMM(Money-Financing)

$(dR/dP)= -Bf''/P^2\{(uf'-1)-Luf'-L_1(1-\theta)uff''\} < 0$

If a MFEXP policy is pursued, (LLMM) is the appropriate long-run LM curve. In this case we focus on equilibrium in the bond market. If the nominal rate of interest for whatever reason rises, the demand for bonds increases. The right-hand side expression of (LLMM) becomes larger. To maintain portfolio equilibrium, given a constant nominal stock of bonds, the price level falls, thereby increasing the real stock of outstanding bonds. Thus, the slope of LLMM is negative. Again, if the bond market is in equilibrium, by Walras' law, the money market is also in equilibrium.

Before considering the specific cases in more depth, notice that the long-run interest rate multipliers in Table 2 (pg 53), when GT1 is the assumed GBC, are identical under either a BFEXP policy or MFEXP policy. This implies an identical long-run increase in output regardless of which policy is pursued. Recall from Eq.(2-3), $dK = 1/f''dR$. Dividing by dG' yields the long-run capital stock multiplier of an increase in G'. Since dR/dG' is the same under either policy, $dR/dG'=f''/\theta f'$, the capital stock multiplier will also be the same. This implies equal increases in capacity output. In fact, substitution of dR/dG' yields the following capital stock and output multipliers:

$$dK/dG' = 1/\theta f' > 0 \text{ and}$$

$$dY/dG' = f'(dK/dG') = 1/\theta > 0$$

This output multiplier is identical to the one derived by Blinder and Solow and T-B in their fixed price models. Because they specified the GBC in the form of Eq.(1-1) rather than (GT1), Blinder and Solow found this result only in the case of a MFEXP policy.

TABLE 2

LONG-RUN MULTIPLIERS

BFEXP (GT1) (**)

$(dR/dG') = f''/ f'\theta < 0$

$(dP/dG') = -\{L_1 uff''(1-\theta)+uLf'\}P^2 / \theta f'M < 0$

BFEXP (GT2)

$(dR/dG) = M/\{P^2(DET1)\} \lessgtr 0$

$(dP/dG) = -\{f''L_1(1-\theta)uf+uLf'\} /f''(DET1) \lessgtr 0$

MFEXP (GT1)

$(dR/dG') = f''/\theta f' < 0$

$(dP/dG') = P^2\{uf'L+L_1(1-\theta)uff''-(uf'-1)\} / \theta f'B > 0$

MFEXP (GT2)

$(dR/dG) = -B/\{P^2(DET2)\} < 0$

$(dP/dG) = -\{uf'L+L_1(1-\theta)uff''-(uf'-1)f''\} / f''(DET2) > 0$

$DET1 = \{(\theta f'/f'')-(1-\theta)B/P\}(M/P^2)-\{(1-\theta)RB/P^2\}\{L_1(1-\theta)uf+(uLf'/f'')\} \lessgtr 0$

$DET2 = -\{(\theta f'/f'')-(1-\theta)B/P\}\{B/P^2\}$

$\qquad -\{(1-\theta)RB/P^2\}\{L_1(1-\theta)uf+(Luf'/f'')-(uf'-1)/f''\} > 0$

3.1.1 MONEY-FINANCED EXPENDITURE POLICIES (MFEXP)

From Table 2 we find that a MFEXP policy will decrease R and increase P in the long-run, regardless of how the GBC is modelled. The reasoning behind this finding is straight-forward. The initial rise in expenditures (G' or G) creates a budget deficit which must be eliminated before equilibrium can be obtained. Capacity output must rise in order to increase the revenue intake of the government; therefore, the stock of capital must increase in the long-run. The increase in capital stock is brought about by a decrease in the interest rate.

Individuals will invest in new capital stock only if the rate of return on new capital becomes higher relative to the rate of return which can be earned on bonds; i.e., $f'(K)>R$. Initially, the lower R stimulates investment demand. As the stock of capital increases, the marginal product of captital, $f'(K)$, falls until at equilibruim the two rates of return are equal; i.e., $f'(K)=R$.

On the other hand, the price level increases in order to restore portfolio equilibrium. Portfolio disequilibrium occurs because an excess supply of bonds is created by the decline in R. As R decreases the attractiveness of holding bonds decreases relative to money and capital. To decrease the excess supply of bonds, the price level rises and equilibrium is restored.

Notice that in the case where GT2 is assumed, since $(1-\theta)B$ is fixed, as R decreases and P increases, the expression $R(1-\theta)B/P$ is lower in the long-run. Evidently, this decrease in real after-tax interest payments is not enough to offset the initial increase in G; thus, a deficit still exists. Therefore, capacity output must increase if the budget is to be balanced. These results are conditional on the stability of the system. Diagrams 1A and 1B illustrate the movement of the GT curves needed to obtain the above results.

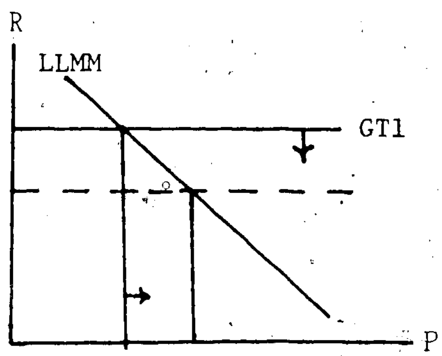

Diagram 1A.

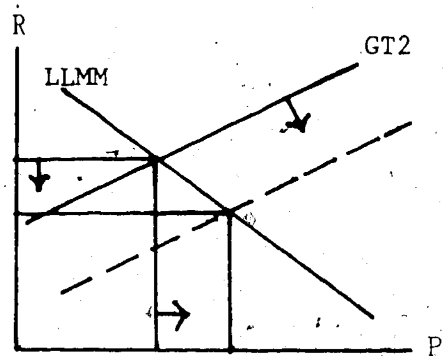

Diagram 1B.

3.1.2 BOND-FINANCED EXPENDITURE POLICIES (BFEXP)

The long-run comparative static results depend critically on how the GBC is specified. If GT1 represents the GBC, the conclusions are straight forward; both R and P decrease in the long-run. However, if we model the GBC as GT2, the long-run multipliers will be different depending on the relative slopes of GT2 and LLMB. A steeper GT2 slope leads to an increase in R and P while a flatter GT2 slope leads to a decrease in R and P. These three cases are illustrated in diagrams 2A, 2B, and 2C on page 56.

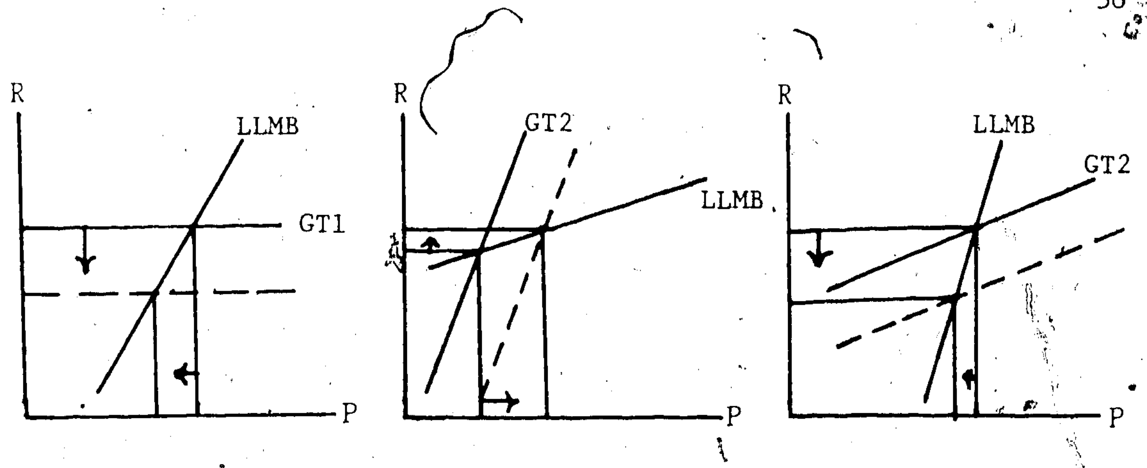

Diagram 2A. Diagram 2B. Diagram 2C.

Assume initially that the GBC is represented by GT1. The economic rational of a decrease in R and P is much the same as the two MFEXP cases. The decrease in R is required to stimulate capital stock investment. As the stock of capital increases, capacity output and total tax revenues increase, thereby eliminating the initial budget deficit.

The decrease in R causes a switch from bonds to money and capital. To restore portfolio equilibrium, given a constant money stock, the price level must start falling. Consequently, this increases the real money stock by enough to satisfy the increased money demand.

When the GBC is specified as GT2, the long-run results depend critically on the value of L_1. L_1 is the interest sensitivity of the demand for money. It can be shown that as L_1 approaches to zero, the

LLMB slope is steeper than the GT2 slope: R and P both decrease as previously found. The economic rational described above applies to this case as well.

However, as L_1 approaches infinity, the GT2 curve will be steeper than LLMB. Thus, P and R both increase in the long-run. The likelihood of this occuring appears rather small for the following reason. Larger equilibrium values of G and R, ceteris paribus, increase the deficit. Because of the negative relationship between R and $\dot{K}$, K decreases. This lowers capacity output and the tax revenue paid to government; again, the deficit increases even further. Presumably, the outstanding stock of bonds is also larger since this is the method chosen to finance the deficit. Each term which appears in the GT2 equation raises the deficit except the increase in P. Therefore, an increase of the price level must be of a magnitude sufficient enough to outweigh the combined effect these other variables have on the deficit.

At first glance, the above conclusion does not appear intuitively plausible. However, under the extreme assumption of a flat LM curve and LLM curve (L_1 equal to infinity), such an outcome may occur. If a flat LM curve is assumed, then the interest rate remains constant which in turn implies a constant capital stock. Two of the destabilizing elements, an increase in R and a decrease in K, are eliminated. Therefore, it becomes possible for large price increases to maintain a balanced budget. It should be noted that $L_1 = \infty$ is the "liquidity trap" case which is relatively uninteresting.

3.1.3 LONG-RUN RESULTS UNDER THE ASSUMPTION OF UNEMPLOYMENT

In this section, we utilize a heuristic argument as opposed to the formal, mathematically derived results of previous sections. Short-run unemployment is introduced by allowing deviations between actual and capacity output. In other words, an Okun gap is present. The actual rate of inflation becomes a function, as expressed in Eq.(2-8) of the Okun gap and expected inflation. In this framework, stock equilibrium prevails if real stocks cease to grow; DK=DP=DM=DB=X=0 and y=f(K).

(2-8) $DP/P = z(Y-f(K)) + X; \qquad z > 0$

The Okun gap (Y-f(K)), is defined as the difference between actual output and capacity output. The closer these two values are, the greater the demand pressure facing the economy and the higher is the rate of inflation. The coefficient z is a measure of the degree of price flexibility within the economy or more appropriately, the speed of adjustment between actual and capacity output.

Jerome Stein analyses an economy operating well below capacity. In his words "a severe depression" exists. The adjustment process returning the economy to full-employment may be of considerable length. To shorten this process, Stein assumes a MFEXP policy is initiated. His analysis confirms that the path to an equilibrium is indeed quicker but at the cost of a lower steady state stock of capital and a higher permanent rate of inflation.

However, the preceding conclusion cannot be reached in the extended T-B model. Consider an initial position in which the high-employment deficit (HED) is in balance; i.e., $G' = \theta f(K)$. Since actual capacity is below potential, the actual budget position of the government is one of deficit, $G' > \theta Y$. If G' is now exogenously increased to close the gap the high-employment budget also falls into a deficit position. In equilibrium $G' = \theta f(K)$; therefore, in this model, the stock of capital must increase to raise tax revenues sufficient to balance the HED.

Stein does not reach the extended model conclusion because a positive rate of inflation and therefore a non-zero deficit, is consistent with the notion of stock equilibrium in his model. There are no government bonds in Stein's model. Thus, stock equilibrium is reached when $DK = 0$ and $DP/P = DM/M$; i.e., the nominal rate of money stock growth equals the rate of inflation.

Stein also introduces a stability analysis of his model which may be misleading. A linearization procedure is generally followed during the process of deriving stability properties. The dynamic equations are linearized around their respective equilibrium points. Stablility properties derived in this way can only be legitimately used to analyze small deviations in the neighbourhood of equilibrium. A deep recession, Steins' starting point, does not fall into this category. Thus, the stability conditions which Stein derives, since they are local in contrast to global, may not be sufficient to return the economy to equilibrium.

In the Stein model, the most that can be said is that a MFEXP policy starting from a position of deep-recession will lead, in the steady state, to a lower capital stock, a higher inflation rate and a higher deficit if global stability conditions are satisfied. In the extended model, such a policy will lead to a higher capital stock with zero inflation and a balanced budget, again, only if global stability conditions are satisfied.

3.2 THE SHORT-RUN DYNAMIC ADJUSTMENT PROCESS AND THE LONG-RUN STABILITY CONDITIONS

To facilitate the short-run dynamic analysis, the IS-LM slopes as well as various short-run multipliers are derived, (See appendix on page 84). These are found in Table 3 (pg 62) and Table 4 (pg 63).

Inspection of Table 3 yields some interesting results. Given a GT1 specification of the GBC, the usual upwards sloping LM and downwards sloping IS curves are found. However, substitution of GT2 for GT1 allows for the possibility of an upwards sloping IS curve. The slope depends crucially on the coefficient β, the proportion of the deficit individuals expect to be monetized. For the IS curve to be negatively sloped, the following condition must hold:

$$\{M/B(1-\beta)\}\{(I'-B/P+\theta B/P) / \{I'-B/P- M/P\} >\beta$$

where: $0 < \beta < 1$

Clearly, without the term $M/B(1-\theta)$, the above expression is greater than one; however, $M/B(1-\theta)$ is a value lying between zero and one. It follows that the left-hand side term may be a positive fraction, in which case β cannot take on its full-range of values if a downward sloping curve is desired. An attempt to set an upper bound on β follows.

Each term on the left-hand side can be rewritten as a ratio of real GNP. United States data is then used to determine approximate values for these ratios: $B/Y \simeq .35$, $M/Y \simeq .15$, $I/Y \simeq .15$, $M/B \simeq .43$.[1]

A problem arises in the measurement of I' which reflects the change in investment with respect to a change in the real rate of return differential between capital and bonds. The real rate of return differential is written as $[f'(K)-(\dot{R}-X)]=\Omega$.

[1] M1 is used as a measurement of M. Outstanding gross debt measures B, and I is measured as gross private domestic investment. All values are derived from the U.S. Statistical Abstract.

TABLE 3

SLOPES OF IS-LM CURVES

IS: (GT1) (**)

$(dR/dP) = -s(M+B)/I'P^2 < 0$

IS: (GT2)

$(dR/dP) = -\{[s(M+B)/P^2]+\beta(I'-V)R(1-\theta)B/MP\}/\{I'-\beta(I'-V)(1-\theta)B/M\} \lessgtr 0$

LM: (GT1 or GT2) (**)

$(dR/dP) = -\{L_2(f/P)(1-(K/W))+(MK/WP^2)\} / WL_1(1-\theta) > 0$

** Refers to slopes identical to those derived by Tobin and Buiter.

The elasticity of investment with respect to Ω is given by $(dI/d\Omega)(\Omega/I) = I'(\Omega/I)$. An elasticity equal to 0.5 implies that I' equals to $0.5(I/\Omega)$. Dividing through by Y and assuming =0.03 provides an estimate of $(I'/Y)=2.5$. Using this value in conjunction with those given above implies that β cannot exceed 0.60 (if $\theta=.2$) or 0.69 (if $\theta=.3$). Recall Jerome Stein's regression of the growth rate in the nominal money supply on the HEDMS ratio. The coefficient on the HEDMS ratio variable can be interpreted as β. The value found by Stein is equal to 0.35; therefore, for United States data, it appears as if a negatively sloped IS curve can be assumed.

TABLE 4

SHORT-RUN MULTIPLIERS

	GT1	GT2
$dR/dK=R_K=$	$\{A1(A22)+C1(A12)\}\ /DET1 \lessgtr 0$	$\{A1(A22)+C1(A12^*)\}\ /DET2 \lessgtr 0$
$dP/dK=P_K=$	$\{-C1(A11)-A1(A21)\}/DET1 \lessgtr 0$	$\{C1(A11^*)-A1(A21)\}/DET2 \gtrless 0$
$dR/dM=R_M=$	$\{A2(A22)-C3(A12)\}\ /DET1<0$	$\{A2(A22)-C3(A12^*)\}\ /DET2<0$
$dP/dM=P_M=$	$\{C3(A11)-A2(A21)\}\ /DET1>0$	$\{C3(A11^*)-A2(A21)\}\ /DET2>0$
$dR/dB=R_B=$	$\{A2(A22)+C2(A12)\}\ /DET1>0$	$\{A4(A22)+C2(A12^*)\}\ /DET2>0$
$dP/dB=P_B=$	$\{-C2(A11)-A2(A21)\}/DET1 \lessgtr 0$	$\{-C2(A11^*)-A4(A21)\}/DET2 \gtrless 0$
$dR/dG=R_G=$	$\{A3(A22)\}\ /DET1 > 0$	$\{A3(A22)\}\ /DET2 >0$
$dP/dG=P_G=$	$\{-A21(A3)\}/DET1 > 0$	$\{-A3(A21)\}/DET2 >0.$

$DET1 = A11(A22)-A21(A12) > 0$

$DET2 = (A11^*)A22-(A12^*)A21 \gtrless 0$

where:

$A1=I'f''-s(uf'-1)-\beta\theta f'(I'-V)P/M \gtrless 0$

$A2=s/P >0,\quad A3=(I'-V)(P\beta/M) > 0$

$A4=(s/P)+\beta(I'-V)(1-\theta)R/M >0$

$C1=L_2f'-(L_2f/W)+L > 0$

$C2=(1/WP)[(M/P)-(L_2f)] > 0$

$C3=(1/P)[1-(M/PW)+(L_2f/W)] > 0$

$V=\{(M+B)/P\} - (M/\beta P) \gtrless 0$

$A11=I'>0,\quad (A11^*)=I'-\beta(I'-V)(1-\theta)B/M \gtrless 0$

$A22=(L_2f/P)(1-(K/W))+(MK/WP^2) >0$

$A12=s(M+B)/P^2>0,\quad (A12^*)=s(M+B)/P^2)+\beta(I'-V)R(1-\theta)B/MP >0$

$A21=L_1(1-\theta)W <0.$

At this stage the assumption of a 0.5 elasticity of investment demand with respect to the real rate of return differential needs to be justified. The subjective value 0.5 was chosen for the following reasons. C.W. Bischoff (Branson p.236) found a long-run elasticity of investment demand with respect to interest rate changes equal to about -0.5. In the narrowest sense, an investment elasticity with respect to changes in the real rate of return differential may be approximated by the negative of this value. A more recent study by Feldstein, reprinted in his book (17), indicates that the elasticity of the investment/income ratio with respect to the real net rate of return that ultimate suppliers of capital can obtain on non-residential fixed investment is approximately 0.5.

By using Feldstein's data, real investment (I) is regressed on the net real rate of return, denoted by RN, in order to derive the elasticity in terms of investment rather than the investment/ income ratio. The real net rate of return is not a precise definition of the real rate of return differential but it seems unlikely that the elasticities under the two approaches would be significantly different. After correcting for first order autocorrelation by using a Cochrane-Orcutt procedure offered in the [illegible]azam 5 statistical package, the following results are obtained. The bracketed terms are t-ratios.

$I_t = 13.59 + 394.79 RN_t$

(2.4) (4.1)

RHO=0.81

R^2=0.76

D.W.= 1.69

The elasticity at the mean equals 0.55. Therefore, a 0.5 elasticity assumption appears to be a reasonable approximation.

The signs on the multipliers are based on the assumption that the IS curve is negatively sloped. Notice, that even if a positively sloped IS curve exists, as long as the LM curve slope is steeper than the IS curve slope, all signs on the derivatives are reversed. There are however exceptions: dP/dK and dP/dB remain ambiguous, while the sign on dP/dM becomes ambiguous.

The derived multipliers yield standard IS-LM solutions. Expansion of either G or G' shifts the IS curve north-east, thereby increasing both the interest rate and the price level. An increase in the outstanding stock of bonds has two effects. First, by increasing wealth, consumption demand increases; the rightward IS curve shift increases R and P. Secondly, the increase in wealth raises the demand for money. This creates a leftward movement in the LM curve; R increases, P decreases. The combined effect results in a higher interest rate but ambiguous price level. It can be shown that as L_1 or s approaches infinity, the price level increases. As the interest sensitivity of the demand for money, (L_1) approaches infinity, the LM curve becomes flatter. For a given IS curve shift, the effect on P is more pronounced for a flat LM curve as opposed to a steep LM curve. Also the consumption effect is larger, the faster is the speed of adjustment between actual and desired levels of wealth, s. A large s implies a steeper IS curve and a larger horizontal shift of the IS curve.

An expansion of the money supply creates the same type of wealth effects as an increase in bonds. However, a third effect must also be considered. Consider a position of equilibrium in which the demand for money equals the supply of money. An increase in the stock of money, ceteris paribus, implies a rightward shift of the LM curve and therefore, a lower interest rate. It turns out that this rightward movement dominates the wealth effect movements; the price level increases and the interest rate unambiguously decreases.

An expansion in capital stock leads to ambiguous price and interest rate multipliers. Such a rise creates wealth effects qualitatively identical to those previously discussed. A quantitative difference does arise since the capital stock change affects desired as well as actual levels of wealth. Since desired wealth is positively related to long-run output, Eq.(2-2), and therefore the stock of capital, an increase in either M or B will not alter the desired level of wealth.

Changes in capital stock also have investment demand consequences. By lowering the marginal product of capital and inflationary expectations, (the HEDMS ratio also decreases as tax revenues increase), both of which lower the attractiveness of investing in new capital, a capital stock increase has the effect of decreasing investment demand. In other words, the IS curve shifts towards the left, lowering both R and P. This shift directly opposes that of the positive consumption effect created by additional wealth.

The effect which a capital stock increase has on the short-run interest rate and price level, R_K and P_K, is of great importance in the dynamic adjustment analysis. Unfortunately, both the partial derivatives are ambiguous in sign (see page 63). Consider diagrams 3A and 3B on page 68. Diagram 3A is drawn on the assumption that the negative investment effect dominates the positive consumption effect of a capital stock increase; the overall shift of the IS curve is to the left. As the higher level of wealth increases the demand for money, the LM curve also shifts to the left. In this case, P_K is unambiguously negative but R_K, since it depends on the relative slopes of the IS and LM curves and the horizontal movement of the two curves, is ambiguous.

In diagram 3B, the IS curve movement is drawn on the assumption that the consumption effect dominates the investment effect. Here, R_k is unambiguously positive but P_K now becomes ambiguous. It should be kept in mind that the consumption effect dominates only if the speed of adjustment, s, is sufficiently large. We turn now to a discussion of the long-run stability properties when the GBC is specified as GT1.

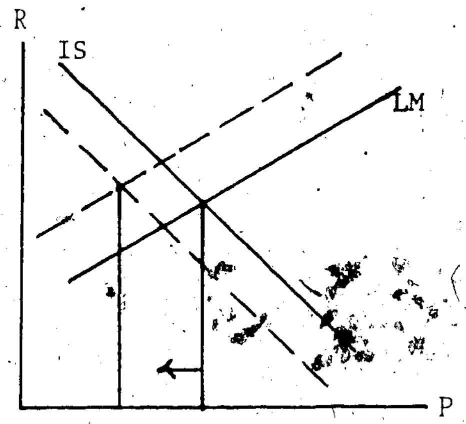

Diagram 3A

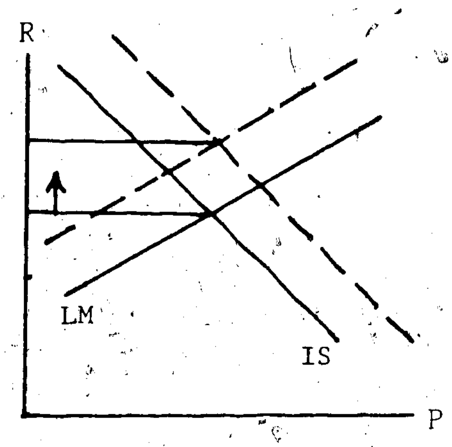

Diagram 3B

3.2.1 ON LONG-RUN STABILITY, (GT1)

Two policies are considered in this section, a MFEXP policy and a BFEXP policy. Only their effects on the interest rate need to be considered; price effects will not disturb the stability of the system. This becomes clear upon inspection of the appropriate dynamic equations. When a MFEXP policy is pursued, DB equals zero and when a BFEXP policy is undertaken, DM equals zero.

(2-9) $DK=I[f'(K)-(R-X)]$ where: $X=\beta(G'-\theta f)P/M$

(2-10) $DB=(G'-\theta f)P$

(2-11) $DM=(G'-\theta f)P$

The price level has no role in the stability of the system because it does not disturb the deficit; $G'-\theta f(K)$ remains constant for a given change in the price level. However, a change in R directly affects the rate of investment as evidenced in equation (2-9).

The immediate effect of a BFEXP policy is to raise the interest rate. This occurs through two channels. One, an increase in G' by stimulating aggregate demand raises both R and P. Two, the expansion in G' creates a government deficit which is financed by new bond issuance. As the stock of outstanding bonds rises, wealth effects continue to place upwards pressure on the interest rate. In turn, the interest rate effect, by decreasing the attractiveness of new capital stock investment vis-a-vis bonds, lowers the stock of capital. This further increases the deficit by lowering the tax revenues of the government; thus, bond expansion continues. Only one possible stabilizing factor exists. If the interest rate effect of a capital stock expansion is positive ($R_K>0$), then the decrease in K will dampen the initial destabilizing interest rate effect. This dampening effect is not a sufficient condition to reverse the initial destabilizing process. The interest rate effect of deficit financing must also be negative ($R_B<0$). In fact, R_B is unambiguously positive. These arguments are verified by a formal analysis of the stability requirements.

By linearizing around the equilibrium values K* and B* of Eqs. (2-9) and (2-10), the following matrix is derived.

$$\begin{bmatrix} 0 & -P\theta f' \\ -I'R_B & I'(f''-R_K-\theta f'P\ /M) \end{bmatrix}$$

Stability for a BFEXP policy requires that the trace of this matrix be negative and the determinant be positive, where

trace $=I'(f''-R_K-\theta f'P\beta/M)<0$

DET $= -I'P\theta f'R_B <0.$

The trace condition is satisfied if R_K is weakly negative or positive. It is immediately apparent that the necessary conditions for stability are not obtained, as the determinant condition is not satisfied. The determinant is positive if and only if R_B is negative. On the other hand, the trace condition is satisfied if R_K is greater than zero. This is a sufficient but not a necessary condition. Notice that if $R_K<0$, its absolute value cannot exceed the absolute value of $f''-\theta f'P\beta/M$; otherwise, the trace condition is also broken.

The same basic line of argument can be used when analyzing a MFEXP policy. Linearization of Eqs.(2-9) and (2-11) around the equilibrium values K* and M*, leads to the following trace and determinant expressions.

trace$=I'(f''-R_K-\theta f'\beta P/M)<0$

DET $=-I'P\theta f'R_M >0$

Again, if R_K is weakly negative or positive, the trace condition is satisfied. Unlike a BFEXP policy where a higher stock of bonds raises the interest rate, with a MFEXP policy, a larger stock of money lowers the interest rate. As a consequence, investment demand is increased, the stock of capital rises and lowers the deficit by increasing the tax revenues of the government.

The trace condition is identical to that found under bond-financed deficits. Again, if R_K is strongly negative, the trace condition will not hold, since the rate of investment demand will not approach zero. The lowering of R as K expands will place further stimulative pressure on investment which cannot be curtailed.

The determinant condition is also the same as before except that R_M [illegible] substituted for R_B. Because R_M is negative, the determinant $(-I'P\delta f'R_M)$ is unambiguously positive. We observe that a MFEXP policy will lead to a stable solution provided that R_K is not strongly negative. R_K could be strongly negative, if the following two conditions are satisfied. First, if the negative investment effect strongly outweighs the positive consumption effect of a rise in K, and second, if the LM curve slope is sufficiently steep, R_K is strongly negative and potentially destabilizing. A stronger condition for stability is $R_K>0$.

3.2.2 ON LONG-RUN STABILITY, (GT2)

The analysis is complicated considerably by the replacement of GT2 for GT1 since changes in the price level now have an important effect on the deficit. Consider the three dynamic equations as given by (2-12) through (2-14).

$$(2\text{-}12)\qquad DK=I[f'-R+\beta P(G+R(1-\beta)B/P-\theta f)/M]$$

$$(2\text{-}13)\qquad DB=\lambda(G-\theta f+R(1-\theta)B/P)P$$

$$(2\text{-}14)\qquad DM=(G-\theta f+R(1-\theta)B/P)P$$

An increase in the price level implies an immediate negative rate of growth for all assets. Starting from a position of equilibrium, (DK=DB=DM=0), an increase in the price level creates a budget surplus by reducing real after tax interest payments. Given the two policies of concern, MFEXP's and BFEXP's, the growth rate of either the stock of outstanding bonds or the stock of money becomes negative. The budget surplus implies the existence of a negative HEDMS ratio. This reduces inflationary expectations, thereby increasing the real rate of return on bonds. As the relative attractiveness of bonds against new capital increases, a process of disinvestment is undertaken; i.e., the rate of growth of capital also becomes negative, causing the budget surplus to decline.

Interest rate movements alter the analysis of section 3.2.1 in two fundamental ways. First, any change in the interest rate will affect the budget deficit directly through real after-tax interest payments; Eqs.(2-13) and (2-14) are increasing functions of the nominal interest rate. Second, the investment effect is lower than previously found. Under the GT1 specification, ceteris paribus, a rise in the rate of interest lowers investment demand by $-I'dR$. Under a GT2 specification, ceteris paribus, a rise in the rate of interest lowers investment demand by

$$-I'dR+I'[\beta(1-\theta)B/M]dR=I'\{[\beta(1-\theta)B/M]-1]\}dR.$$

This disinvestment effect is obviously lower when the GBC is modelled as GT2 rather than GT1. In fact, it is conceivable that the investment effect of a rise in R is positive rather than negative. If $(1-\beta(1-\theta)B/M)<0$, a rise in the interest rate increases investment demand because the increase in expected inflation, caused by larger deficits, more than outweighs the increase in the interest rate. Algebraically, (R-X) decreases and thus stimulates investment demand.

Recall the restriction placed on β in order to derive a downwards sloping IS curve; β must not exceed a value greater than 0.69 assuming a marginal tax rate equal to 0.3. The restriction here is a little stronger. Given a marginal tax rate of 30%, if [illegible] greater than 0.60, a rise in the interest rate will increase investment demand. As before, it is assumed that β remains within the bounds compatible with a negatively sloped IS curve.

We are now in a position to more formally analyze the stability conditions. By linearizing Eqs.(2-12),(2-13) for a BFEXP policy and Eqs. (2-12) and (2-14) for a MFEXP policy around their equilibrium values, we again define appropriate matrices from which the following trace and determinant expressions can be derived.

Bond Financing.

trace: $(1-\theta)B\{R_B-(R/P)P_B\}+I'(1-\theta)\beta(B/M)\{R_K-(R/P)P_K\}$
$+I'\{f''-R_K-(\beta\theta f'P/M)\}+R(1-\theta) \lessgtr 0$

DET: $I'R(1-\theta)R_K\{(B/P)P_B-1\}-I'R_B\{P_KR(1-\theta)(B/P)+P\theta f'\}$
$+I'f''(1-\theta)B\{(R/B)+(R_B-(R/P)P_B)\} \lessgtr 0$

Money-Financing.

trace: $(1-\theta)B\{R_M-(R/P)P_M\}+I'(1-\theta)\beta(B/M)\{R_K-(R/P)P_K\}$
$+I'\{f''-R_K-(\beta\theta f'P/M)\} \lessgtr 0$

DET: $I'R(1-\theta)R_K\{(B/P)P_M-1\}-I'R_M\{P_KR(1-\theta)(B/P)+P\theta f'\}$
$+I'f''(1-\theta)B\{R_M-(R/P)P_M\} \lessgtr 0$

For the system to be stable, the trace must be negative and the determinant, DET, must be positive. As can be seen, the signs are ambiguous in all four expressions.

A casual inspection of the above expressions reveals a high degree of parallelism between the bond-financing and money-financing cases. When analyzing a MFEXP policy the terms R_M and P_M replace R_B and P_B respectively. But, the expressions $R(1-\theta)$ and R/B do not appear in the stability conditions of money financing. Although $R(1-\theta)$ is small in magnitude and can safely be ignored in the present discussion, R/B has an important role in the determination of one of the stability conditions for a BFEXP policy.

Unfortunately, there is a great deal of ambiguity involved for these stability conditions. A wide range of possible scenarios exist, making it very difficult to derive any necessary and sufficient conditions.

The first term in the respective trace conditions shows how real after-tax interest payments react to changes in R and P caused by deficit financing. If the deficit is money-financed, this term is unambiguously negative, given $R_M<0$ and $P_M>0$ according to Table 4, page 63. For a BFEXP policy, if P_B is negative, interest payments increase, thereby creating a destabilizing element. Even if P_B is positive, this first term may still be positive and destabilizing since R_B is positive. Interest payments decrease under bond-financing if and only if $R_B/P_B < R/P$. That is, the price elasticity of an increase in bonds must outweigh the interest elasticity of an increase in bonds.

Notice that the term $(R_B - P_B R/P)$ also appears in the third term of the determinant expression, in conjunction with R/B. The condition under which this term becomes positive is stricter than that found above. The price elasticity minus the interest elasticity of an increase in bonds must be greater than unity. If this condition is satisfied, the likelihood of the trace and determinant conditions being satisfied is enhanced. Under money-financing, the third expression of the determinant is unambiguously positive.

The second term in the trace conditions are identical but are also the most difficult to analyze. It shows the effect on investment demand due to changes in inflationary expectations caused by real, after-tax interest payment changes. After-tax real interest payments are altered as R and P react to capital stock changes. Recall from Section 3.2 the ambiguous sign of both R_K and P_K. This is caused by three different movements in the IS and LM curves. Since an increase in capital stock raises wealth, the LM curve shifts leftward because the demand for money is higher, while the IS curve shifts rightward as consumption demand increases. Also, by lowering the marginal product of capital, the capital stock increase raises the attractiveness of bonds relative to capital; hence the IS curve shifts leftward.

If the overall IS curve shift is to the left, the investment effect outweighs the consumption effect, P_K is less than zero but R_K is ambiguous. The steeper is the LM curve and the flatter is the IS curve, the greater is the probability that $R_K<0$. On the other hand, if the overall shift in the IS curve is to the right, R_K is greater than zero, while P_K is ambiguous. The flatter is the LM curve and the steeper is the IS curve the greater the likelihood that $P_K>0$. Three possible combinations exist, two of which require different conditions so that the second term of the trace expressions is to be negative.

(1) $R_K>0$, $P_K<0$.

(2) $R_K>0$, $P_K>0$.

(3) $R_K<0$, $P_K<0$.

If case (1) prevails, the second term in the trace expressions, $(R_K-P_K R/P)$, is unambiguously positive and therefore destabilizing. In case (2) the price elasticity of an increase in capital must outweigh the interest elasticity of an increase in capital. In case (3), for a given change in the capital stock, the interest rate must be more elastic than the price level.

No matter what signs R_K and P_K take, both have further implications for stability, especially in the determinant expressions. Recall the trace condition from section 3.2.1 which is identical to the third term of the trace condition found in this section. A negative R_K is possible but if its absolute value is larger than the absolute value of $f''-(P\beta/M)\theta f'$, then this third term also becomes positive. Also, a negative R_K has a significant impact on the first term found in the determinant conditions. This term is positive, given a negative R_K, if the price elasticity of an increase in bonds is less than one. Recall, however, that the third term of the determinant is positive only if this elasticity exceeds not only unity but the interest elasticity of an increase in bonds. Therefore, if R_K is negative, either the first or third term of the determinant expression is destabilizing for a BFEXP policy. We can show that under a MFEXP policy, if the price elasticity of an increase in money is greater than M/B, the first term in the determinant is negative and therefore destabilizing.

The final term to be discussed is the second term of the determinant expressions. If P_K is positive this term is negative under bond-financing but negative under money-financing. A negative P_K is sufficient to reverse these findings if $P_K R(1-\theta)B/P < -P\theta f'$. This expression can be re-written in terms of the price elasticity of an increase in capital. Using the knowledege that $f'(K)=R$ in equilibrium, we can show that if the absolute value of this elasticity is greater than $P\theta K/(1-\theta)B$, stability is enhanced under a BFEXP policy but retarded under a MFEXP policy, given a negative P_K.

The above discussion allows us to form some general conditions which appear to enhance the overall stability of the system; namely,

(1) $R_K > 0$

(2) $\{P_B B/P\}-\{R_B B/R\}>1$

(3) $\{P_K K/P\}-\{R_K K/R\}>0$

(4) $\{P_M M/P\}-\{M/B\}>0$.

Conditions (1) and (3) are sufficient to satisfy the trace condition for a MFEXP policy. These two conditions imply that P_K is greater than zero; therefore, conditions (1), (3) and (4) are sufficient to satisfy the determinant condition for a MFEXP policy. For a BFEXP policy, the first three conditions are sufficient to satisfy the trace condition. The second condition also increases the probability that the determinant condition is fulfilled for a BFEXP policy as the first and third terms become unambiguously positive. However, since the second term is negative, it is possible that the determinant condition will not be satisfied.

We conclude this section by briefly comparing the T-B stability conditions, under the assumption of static expectations, with the extended model stability conditions assuming GT1. Recall from Chapter I, Section 4, the following conditions.

	Bond-Financing:	Money-Financing:
trace	$I'(f''-R_K) < 0$	$I'(f''-R_K) < 0$
DET	$-I'P\theta f'R_B > 0$	$-I'P\theta f'R_M > 0$

Bond-financing is unstable since the determinant condition is not satisfied; however, money-financing is stable given the sufficient condition, $R_K > 0$. These conditions are virtually identical to those found in the extended model, section 3.2.1, rewritten below.

	Bond-Financing:	Money-Financing:
trace	$I'(f''-R_K-\theta f'\beta P/M < 0$	$I'(f''-R_K-\theta f'\beta P/M) < 0$
DET	$-I'P\theta f'R_B > 0$	$-I'P\theta f'R_M > 0$

Comparing the trace conditions of the two models, we find the term $I'\theta f\beta P/M$ appears in the extended version. This term marginally increases the lower bound value which R_K can take; if R_K is negative the bounds allow it to be more negative in the extended model than in the T-B model.

As can be seen,, the determinant conditions are qualitatively equivalent. Some quantitative difference exists because the IS slope in the extended model is not the same as found by T-B; in terms of stability, this difference is inconsequential. Combining the above observation with the fact that the long-run comparative static results are identical in the two versions, (assuming GT1), suggests that there is little distinction to be made between models which form inflationary expectations as a function of the HEDMS ratio as opposed to statically formed expectations; i.e., $\beta = 0$. This is quite an interesting result since expectations are continually revised in the extended model as the HEDMS ratio changes. Only in the long-run, when equilibrium is reached, will inflationary expectations cease to alter.

If T-B had also analysed the flexible price version of their model, with GT2 replacing GT1 and assuming static expectations, X=0, the same long-run multipliers of P,R and K would have been derived as in our extended model. This is again due to the fact that the formation of expectations do not in any way influence the long-run analysis. The only difference again would be in terms of the stability analysis. Under a GT1 specification, strong similarities exist between static and inflationary expectations which are formed as a function of the HEDMS ratio. Hence, it would not be surprising if the stability analysis discussed under a GT2 specification is also similar to static expectations.

4. SUMMARY

Our intent in this chapter was to extend the model developed by Tobin and Buiter (1976) by forming inflationary expectations in terms of the high-employment deficit/ money stock ratio. The major findings in this chapter are as follows.

A MFEXP policy decreases R but increases P and K in the long-run. A BFEXP policy lowers R and P while increasing the stock of capital if the slope of the GT2 curve is flatter than the slope of the LLMB curve. On the other hand, if the slope of the LLMB curve is flatter than the GT2 curve, a BFEXP policy increase R and P, while lowering the stock of capital. The relative slopes depend crucially on the interest sensitivity of the demand for money.

The above findings assume constant full-employment. If we relax this assumption, it is found that Stein's conclusion of a lower steady state stock of capital and higher rate of inflation, given a MFEXP policy, cannot prevail in the extended model. If global stability exists in the extended model, a MFEXP policy leads to a higher stock of capital. To achieve stock equilibrium the rate of inflation must equal zero.

An interesting finding occurs in the short-run analysis when inflationary expectations are formed as a function of the HEDMS ratio. If β, the proportion of the deficit individuals expect to be monetized, is greater than approximately 0.6 the IS curve will be upwards sloping. As well, an increase in R will raise, not lower, investment demand. Due to the already complex nature of the stability analysis, it was assumed that the IS curve is downwrds sloping; i.e., $\beta < 0.6$.

When forming inflationary expectations as a function of the HEDMS ratio, the stability conditions under a GT1 specification of the GBC, is virtually identical to those found when inflationary expectations are assumed to be statically formed. This implies that there is little if any difference in the short-run dynamic adjustment process between models which form inflationary expectations as a function of the HEDMS ratio rather than statically. The previous statement does not apply to the traditional adaptive expectations formulation, which happens to be very different than the adaptive mechanism used in this chapter, studied by Cagan. It is also found, that a BFEXP policy is unstable, while a MFEXP policy is stable if R_K is not a large negative number. R_K is the partial derivative of the interest rate whith respect to an increase in the stock of capital.

Unfortunately, the stability of the system, given a GT2 specification of the GBC is an empirical question. Certain conditions which are sufficient to satisfy the trace conditions for both a BFEXP and MFEXP policy were derived. These conditions are also sufficient to ensure stability for a MFEXP policy.

Appendix

In this appendix, we derive the matrix from which the short-run comparative static results for a GT2 specification emerge. The GT1 comparative static results are derived in a similar fashion.

Consider the IS equation, from page 39, rewritten below:

$$\text{(IS)}\quad I[f'(K)-R+X]-s[uf(K)-K-((M+B)/P)]-X(M+B)/P$$
$$+G-\theta f(K)+R(1-\theta)B/P=0$$

Since $X=\beta\{G-\theta f(K)+R(1-\theta)B/P\}/(M/P)$ or $(MX/P\beta)=G-\theta f(K)+R(1-\theta)B/P$, we can write the final two terms of the IS equation as $-XV$, where $V=\{(M+B)/P\}-(M/P\beta)$.

Differentiating the IS equation totally, we find:

$$I'\{f''dK-dR+dX\}-s(uf'-1)dK+s(dB+dM)/P-s\{(M+B)/P^2\}dP-VdX=0$$

where

$$dX=(\beta P/M)\{dG-\theta f'dK+dR(1-\theta)B/P+((1-\theta)R/P)dB-((1-\theta)RB/P^2)dP\}.$$

The only difference between this derivation and the GT1 derivation is in the expression dX. For a GT1 specification, $dX=(\beta P/M)\{dG'-\theta f'dK\}$. Substitution of the more complicated dX expression into the differentiated IS expression, upon rearranging, leads to

Eq.(A-1) $\{I'-\beta(I'-V)(1-\theta)B/M\}dR+\{[s(M+B)/P^2]+\beta(I'-V)(1-\theta)RB/MP\}dP$

$=(s/P)dM+\{(s/P)+\beta(I'-V)(1-\theta)R/M\}dB$

$+\{I'f''-s(uf'-1)-\beta(I'-V)P\theta f'/M\}dK+\{\beta(I'-V)P/M\}dG$

Consider now the LM curve, which is written as:

$M/P=L\{R(1-\theta),\ Pf(K)/(M+B+PK)\}\{K+(M+B)/P\}.$

Upon totally differentiating and rearranging, we write

Eq.(A-2) $L_1(1-\theta)WdR+\{(L_2f/P)((W-K)/W)+MK/WP^2\}dP=$

$-\{L_2f'+L-L_2f/W\}dK-\{[(M/P)-L_2f)]/(1/WP)\}dB$

$+(1/P)\{1-(M/PW)+(L_2f/W)\}dM.$

Finally, we write Eqs.(A-1) and (A-2) in a simplified version as:

$(A11^*)dR+(A12^*)dP=A1dK+A2dM+A3dG+A4dB$

$A21dR+A22dP=-C1dK-C2dB+C3dM.$

Use of Cramer's rule allows us to derive the short-run comparitive static results found in Table 4 from the following matrix:

$$\begin{bmatrix} A11^* & A12^* \\ A21 & A22 \end{bmatrix} \begin{bmatrix} dR \\ dP \end{bmatrix} = \begin{bmatrix} A1dK + A2dM + A3dG + A4dB \\ -C1dK - C2dB + C3dM \end{bmatrix}$$

As a further elaboration to the discussion of R_K and P_K, found on page 67, consider the dK expression of Eq.(A-1), which is comprised of four terms. The terms $I'f''-\beta I'P\theta f'/M$ represent the investment effect of an increase in capital stock. The terms $-s(uf'-1)+\beta VP\theta f'/M$ is the consumption effect of an increase in capital stock.

$I'f''dK$ is the decrease in investment caused by a decreasing marginal product of capital, while $-(\beta I'P\theta f'/M)dK$ is the decrease in investment caused by a lower expected inflation. The expected rate of inflation decreases since the HEDMS ratio is lowered as K rises. The lower are inflationary expectations, the higher is the rate of return earned on bonds. This necessarily implies a decrease in investment demand. Diagramatically, the lower investment implies a leftward IS curve shift.

There are four factors which must be considered when analysing the consumption effect. An increase in capital stock raises the long-run desired level of wealth; therefore, individuals save more and consume less by $(-suf'dK)$. However, the actual level of wealth has also risen, thus the need to save diminishes, thereby increasing consumption by sdK. Since $uf'<1$, the latter effect dominates. The term $\beta VP\theta f'/M$ may be positive or negative since the sign of V is ambiguous. As β approaches zero, V becomes unambiguously negative. An increase in K affects this term in two ways. By lowering expected inflation, an increase in K lowers the expected capital losses of financial assets; therefore, consumption demand increases. However, the increase in K lowers the deficit by creating a larger tax base. This implies that the government is lowering its dissaving (the government dissaves if a deficit exists). In other words the government's consumption demand is decreasing. It is possible that the government's increased saving dominates the combined effect of the three other factors but this is possible only if V is negative and if s approaches zero. We assume throughout the analysis that an increase in K increases consumption demand. The result is a rightward IS curve shift.

III. A SIMPLE EMPIRICAL INVESTIGATION

INTRODUCTION

The purpose of this chapter is to provide estimates, as a first approximation, of the impact which deficits have on the rate of inflation. Any impact flows indirectly via the money supply. A small simultaneous model is developed to achieve this task. Equations explaining inflation, money supply growth and output are presented. Inflationary expectations are assumed to depend on an augmented adaptive mechanism which takes into consideration the long-run relationship between the growth rate of money and the inflation rate.

Our major findings can be summarized as follow: ordinary least squares (OLS) estimates show that the lagged high-employment deficit/ money stock ratio (HEDMS) and lagged money growth variable explain approximately two-thirds of the variation in the growth rate of the money supply. Both reduced form equation estimates and instrumental variable estimates, assuming either an exogenous or endogenous output-gap, yield some strong results. First, dummy variables which take into account the after effects of the oil price shock of 1973-74, are jointly significant in the most efficiently estimated inflation rate equations. Second, the output-gap and inflation rate have a significant systematic relationship in the opposite direction of what was originally anticipated. Third, the lagged inflation rate is insignificant. When the lagged output-gap is substituted for the current output-gap in the inflation rate equation, the lagged inflation rate becomes a significant variable, while the money growth variable becomes insignificant. The HEDMS ratio has had a relatively small impact on the rate of inflation over the time period considered. Reduced form estimates indicate that the HEDMS ratio is an insignificant variable, while instrumental variable (IV) estimates show that a 1% point rise in this ratio raises the inflation rate by 0.05 to 0.21% points.

1. ON THE FORMATION OF INFLATIONARY EXPECTATIONS

In Chapter II, the dynamics of the inflationary process, given a positive rate of unemployment, are explained by the following two equations:

(3-1) $(DP/P) = z(Y-f(K)) + X$ $\quad 0 < z < 1$

(3-2) $X = \beta(G'-\theta f(K))/(M/P)$ $\quad 0 < \beta < 1$

Within this framework, inflation is a function of two variables, the output-gap and expected inflation. The output-gap (the difference between actual and potential output) is a measure of the relative demand pressure which exists in the economy. The closer the actual output is to potential, the greater is the demand pressure and the greater is the likelihood that prices will rise. However, prices are not fully flexible thus the term z is included to provide a measure of the responsiveness of prices to demand pressure.

Eq.(3-2) states that the expected inflation rate is a function of the HEDMS ratio, where β is the proportion of the deficit individuals expect to be monetized. There are three basic reasons why such a formulation is valid in a theoretical setting such as Chapter II. First, a consolidated government sector comprised of the Treasury and Central Bank is assumed to exist. This implies that high-powered money is created for the sole purpose of deficit financing. Second, by assuming a money multiplier equal to unity, any increase in the growth rate of high-powered money implies an equal increase in the growth rate of the money supply (M1). Third, it is assumed that money supply growth has a direct, immediate impact on inflation through its effect on expectations. These three reasons together imply that as long as the government sector finances its deficits in a consistent manner; i.e., a certain proportion of the deficit is money-financed, large errors in formulating inflation expectations are less likely to occur.

In reality, the above assumptions are too restrictive. The Central Bank is an independent agency and the money multiplier is not equal to unity. Therefore, even if the Central Bank maintains a steady policy in terms of deficit monetization, there is still a random component attached to the rate of growth in the money supply. By itself this is not a serious deficiency. Random shocks by definition are unpredictable and as such, uncontrollable. Forming expectations of money supply growth and thereby inflation, in terms of the HEDMS ratio, might be the best individuals can do. However, two important considerations suggest that this may not be the case.

In the first place, it takes time for money supply growth to affect inflation. A five percentage point increase in the growth rate of M1 does not imply an immediate five percentage point increase in the rate of inflation. The hypothesis that inflation is purely a "monetary phenomena" refers to a long-run rather than short-run relationship. This implies that individuals attempting to form expectations of inflation, for time period t at time period t-1, may make major errors if expectations are entirely based on expected money supply growth.

In the second place, it is unlikely that individuals have even heard about the HEDMS ratio. This variable may be a part of the monetary authority's reaction function; however, individuals are unlikely to know this. Also, if individuals believe that money supply growth information is important but that its effect is slow to build, current information about the money supply should be used. This type of information is readily available. In this way, individuals, from past experience, can attempt to determine how much the current growth rate in the money supply will affect inflation for the time horizon being considered. Efficiency, in the sense of individuals trying to predict the inflation rate, may be better served by using this information rather than trying to determine an elaborate reaction fuction.

How might individuals proceed to form expectations about inflation? They might proceed by using an adaptive expectations framework in which past observations are used to correct for any deviations between observed and expected inflation. This simple mechanism, although it is a legitimate alternative to models incorporating rational expectations, is deficient in one key respect; it does not take into account the long-run relationship between money growth and inflation. In order to correct this deficiency, an augmenting factor is added to the usual adaptive expectations formulation. Consider the following continuous time adjustment equation used by Scarfe in certain theoretical papers (40, 41, 42).

(3-3) $DX=n_1((DP/P)-X) + n_2((DM/M)-X) \quad n_1,n_2>0$

The first term in this equation is the usual continuous time version of Cagan's expectations formula. Individuals adjust their expectations to take into account any errors made in previous forecasts. The adjustment is made at a speed equal to n_1.

The second term is the augmenting factor which illustrates how expectations are adjusted when any deviation between actual money supply growth and inflationary expectations prevails. As the rate of growth in the money supply increases relative to expected inflation, expectations of inflation are revised upwards. The increased growth rate does not change expectations by a one-for-one factor. Expectations change depending on the speed of adjustment (n_2) between the current rate of money supply growth and current expectations of inflation. In the long-run, expectations are realized; i.e., the expected rate of inflation equals the inflation rate which in turn equals the growth rate of the nominal money supply.

Notice that if the growth rate of the money supply is a function of the HEDMS ratio, inflationary expectations indirectly depend on this variable. By increasing the growth rate of money, a rise in the HEDMS ratio leads to an upward revision of expected inflation as individuals anticipate a higher, long-run level of inflation to prevail. Individuals do NOT need information about the HEDMS ratio. As long as the monetary authority reacts to the HEDMS ratio, inflationary expectations will be revised to reflect changes in the growth rate of money.

2. MODEL SPECIFICATIONS

The full model comprising four equations and one identity is written as Eqs.(3-4) through (3-8). The rate of inflation and the growth rate of the money supply are rewritten in discrete time as CP and CM, respectively. These variables are defined as follows:

$$CP_t=\{(GNPD_t-GNPD_{t-1})/GNPD_{t-1}\}*100$$

$$CM_t=\{(M_t-M_{t-1})/M_{t-1}\}*100$$

where GNPD is the gross national product price deflator.

(3-4) $CP_t = a_0 + a_1 GAP_t + a_2 X_t + \mu_{1t}$

(3-5) $X_t - X_{t-1} = n_1(CP_t - X_{t-1}) + n_2(CM_t - X_{t-1}) + \epsilon_t$

(3-6) $CM_t = b_0 + b_1 CM_{t-1} + b_2 HEDMS_{t-1} + \mu_{2t}$

(3-7) $Y_t = g_0 + g_1 (M/P)_t + g_2 X_t + g_3 G_t + \mu_{3t}$

(3-8) $GAP_t = YF_t - Y_t$

(3-9) $a_1 < 0,\ b_1, b_2, g_1, g_2, g_3 > 0$ and $0 < n_1, n_2, a_2 < 1$.

Eq.(3-4) shows that inflation at time period t is inversely related to the output-gap and directly related to expected inflation.[1] A rise in the output-gap, by lowering demand pressures should lead to a fall in the rate of inflation. For notational convenience, the output gap of Eq.(3-1) has been rewritten as Eq.(3-8), where YF is the full-employment level of output.

Eq.(3-5) is the discrete time version of the augmented adaptive expectations formulation presented in Section 1. Notice that the lagged expected inflation rate is subtracted from the current period inflation rate and the current period money supply growth rate. Due to lags which may exist in acquiring information, it may be argued that the two current values should be replaced by their lagged values. Information

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[1] Given the assumption that the unemployment rate is a linear function of the output-gap, Eq.(3-4) can be thought of as an augmented Phillips curve relationship.

is available weekly with regards to the money supply; however, only monthly statistics on the inflation rate are available. Nonetheless, it is felt that this lag is sufficiently short to justify the use of current period data. Kajal Lahiri (25) has estimated adaptive expectations forms in which this assumption is implicitly made.

The growth rate of the nominal money supply is hypothesized to take the form of Eq.(3-6). This simple reaction function of the monetary authority depends upon a lagged money growth variable and the lagged value of the HEDMS ratio. This equation is very similar to the form used by Barro and others. Allen and Smith, who are the most thorough of the authors estimating the Barro equation, found the unemployment rate and real government expenditures divided by real trend GNP to be insignificant. These variables were, therefore, not considered for the purposes of this study.

A complication arises with respect to the output-gap identity given by equation (3-8). Because of the endogeneity of output, the output-gap should be considered as endogenous rather than predetermined. Two models are estimated. Model I treats the gap as an exogenous variable and Model II treats the gap as an endogenous variable. Model II necessitates the derivation of an equation explaining output. Eq.(3-7), which explains real output as a function of government expenditures, real money balances and expected inflation, is derived below. However, before proceeding, a brief comment about the endogeneity of YF is in order.

Although YF is theoretically dependent on the stock of capital, to our knowledge, no one has tried to construct a series which adjusts capital stock to its potential level. YF is derived solely by adjusting the labour force to its full-employment level, given the stock of capital prevailing during the time period considered. Therefore, YF is treated as an exogenous component of the gap.

Eq.(3-7), which can be thought of as a quasi-reduced form equation, is derived from a simple IS-LM framework in the following way.

The LM equation, which describes equilibrium in the money market, is most often written in the form

$(M/P)=L(Y,R,X)$ where $L_Y>0$, $L_R<0$ and $L_X<0$.

This in turn can be rewritten in terms of the nominal interest rate as

$R=R(Y,(M/P),X)$ where $R_Y>0$, $R_m<0$ and $R_X>0$.

This equation is then substituted into the IS equation, derived below, in order to eliminate R. In other words, Eq.(3-7) represents the trace of equilibrium points for both the commodity market and money market simultaneously.

Typically, the IS curve relationship illustrates equilibrium in the goods market. This means that the variable Y refers to aggregate demand as well as real output. Given this proposition, we write Y as a

function of the nominal rate of interest, the expected rate of inflation and real government expenditures to give
$Y=Y(R,X,G)$ where $Y_R<0$, $Y_X>0$ and $Y_G>0$.

Although consumption and investment demand do not explicitly appear in the above formulation, they are implicit within the model. Consumption is a function of real disposable income and therefore Y, while investment demand is a function of the real rate of interest and therefore R and X. Furthermore, the demand for consumer durables is likely to be a function of R and X.[2]

Substitution of R from the LM equation into the IS equation leads to, upon linearization, a compact equation in the form of Eq.(3-7).

Because inflationary expectations are unobservable, equation (3-5) cannot be estimated. The use of lag operator notation helps us to rewrite this equation as

(3-10) $X_t(1-\lambda L)=\{n_1 CP_t+n_2 CM_t\}+\epsilon_t$

where $\lambda=1-n_1-n_2$ and L is the lag operator.

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[2] Because estimation is carried out with the use of United States data, the trade deficit is excluded from this formulation. Given the size of the American economy in relation to its international sector, it is believed that this omission is not too serious.

Dividing Eq.(3-10) by $(1-\lambda L)$ leads to:

(3-11) $X_t=(1-\lambda L)^{-1}\{n_1CP_t+n_2CM_t+\epsilon_t\}$.

By substituting Eq.(3-11) into Eq.(3-4) the inflation rate equation can be written as:

$$CP_t=a_0+a_1GAP_t+a_2(1-\lambda L)^{-1}\{n_1CP_t+n_2CM_t+\epsilon_t\}+\mu_{1t}.$$

Multiplying through by $(1-\lambda L)$ allows us to write the above equation as

(3-12) $CP_t(1-\lambda L)=a_0(1-\lambda L)+a_1(1-\lambda L)GAP_t+a_2n_1CP_t$
$+a_2n_2CM_t+a_2\ {}_t+a_2(1-\lambda L)\mu_{1t}$.

Dividing Eq.(3-12) through by $(1-\lambda L)$ and collecting terms leads to

(3-13) $CP_t=p_0+p_1GAP_t+p_2GAP_{t-1}+p_3CP_{t-1}+p_4CM_t+v_{1t}$

where $p_0=a_0(n_1+n_2)/(1-a_2n_1)\ \gtrless 0$

$p_1=a_1/(1-a_2n_1)<0$

$p_2=-a_1(1-n_1-n_2)/(1-a_2n_1)>0$

$p_3=a_2n_2/(1-a_2n_1)>0$

$p_4=(1-n_1-n_2)/(1-a_2n_1)>0$

$v_{1t}=\{a_2/(1-a_2n_1)\}\epsilon_t+\{1/(1-a_2n_1)\}\mu_{1t}-\{\lambda/(1-a_2n_1)\}\mu_{1t-1}$.

The error term has two components: a component which exhibits a first-order moving average process (MA1) and a component, ϵ_t, which is assumed to be normally distributed with zero mean and variance σ^2.

Because ε_t and μ_{1t} cannot, a priori, be assumed to be independent, the structure of v_{1t} is unknown. The unknown error structure does not in any way affect the estimation procedure which is used.

In a similar fashion Eq.(3-11) can be substituted into Eq.(3-7). An equation of the following form is derived .

$$(3\text{-}14)\quad Y_t=c_0+c_1Y_{t-1}+c_2(M/P)_t+c_3(M/P)_{t-1}+c_4G_t+c_5G_{t-1}+c_6CP_t+c_7CM_t+v_{2t}$$

where $c_0=g_0(n_1+n_2) \gtrless 0$

$c_1=(1-n_1-n_2) >0$

$c_2=g_1 >0$

$c_3=-g_1(1-n_1-n_2) <0$

$c_4=g_3 >0$

$c_5=-g_3(1-n_1-n_2) <0$

$c_6=g_2n_1 >0$

$c_7=g_2n_2 >0$

$v_{2t}=g_2\varepsilon_t+\mu_{3t}-\lambda\mu_{3t-1}$.

The reader should note that to derive the signs of Eqs.(3-13) and (3-14), it is necessary to impose the restriction that $0<n_1+n_2<1$.

There is one further hypothesis to be tested and that is whether or not the oil price shock of 1973 created a structural shift in the trend rate of inflation and output. This task is accomplished by adding a dummy variable (DV) to the inflation rate and output equations, where

DV=0 for years prior to 1974 and DV=1 from 1974 onwards.

In subsequent regressions, a slope dummy variable for the current output-gap is also included. This variable is defined as GAPD=DV*RGAP. The inclusion of this variable arose from the need to explain the positive relationship between the output-gap and inflation.

In summary, we rewrite the four equations which form the basis of our estimations.

(3-6) $CM_t=b_0+b_1CM_{t-1}+b_2HEDMS_{t-1}+\mu_{2t}$

(3-8) $GAP_t= YF_t-Y_t$

(3-13) $CP_t=p_0+p_1GAP_t+p_2GAP_{t-1}+p_3CP_{t-1}+p_4CM_t+p_5DV_t+v_{1t}$

(3-14) $Y_t=c_0+c_1Y_{t-1}+c_2(M/P)_t+c_3(M/P)_{t-1}+c_4G_t$

$c_5G_{t-1}+c_6CP_t+c_7CM_t+c_8DV_t+v_{2t}$

These four equations comprise a simultaneous system the nature of which can best be seen by considering the effect of an increase in the HEDMS ratio. As this ratio increases, the rate of growth in the money supply rises, which through Eq.(3-13) pushes the rate of inflation to a higher level. Also, by increasing output, the increased money supply growth is expected to raise the inflation rate indirectly through its impact on the output-gap which has been lowered. The rise in the inflation rate then feeds back positively onto output; the output-gap is further decreased, thereby pushing the inflation rate even higher.

3. ON ESTIMATION AND DATA

Estimation of Model I, in which the output-gap is assumed to be exogenous, reduces to the question of whether or not the system is recursive. If it is recursive, the money growth variable of Eq.(3-13) can be treated as an exogenous variable rather than an endogenous variable.

Two conditions must be met for a system to be recursive. First, the matrix of coefficients of the endogenous variables must be lower triangular. Second, each equation's error term must be independent of the error terms in the other equations (Stewart and Wallis, 46). In Model II, the triangular condition is satisfied; however, a priori, the error terms cannot be assumed to be independent. In fact, the usual procedure is to assume dependent error terms.[3] Since recursiveness cannot be assumed, the money growth variable must be considered endogenous for purposes of estimation.

A final difficulty arises because of the moving average error structure generated by the assumption of adaptive expectations. This implies that the lagged endogenous variable is contemporaneously correlated with the error term. Therefore, a two-stage least squares (2SLS) procedure will

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[3] We are grateful to Professor A.Buse for confirming this point.

produce inconsistent estimates. To obtain consistent estimates, instrumental variable estimation is employed. In other words, the lagged endogenous variable and the right-hand side endogenous variables are replaced by a set of instruments. The instruments must be highly correlated with the variables they replace and they must be uncorrelated with the error term.

As an example of how the list of instruments have been chosen, consider a typical reduced form equation which we write in matrix notation as

$$(3\text{-}15) \quad Y^*_1 = (Y^*_{-1} \mid X^*)\begin{pmatrix}\Pi_y \\ \hline \Pi_{x^*}\end{pmatrix} + v_1$$

Y^*_{-1} is the lagged endogenous variable, X^* is a matrix of all predetermined variables other than the lagged endogenous variable and v_1 is the error term which has an unknown structure but is autocorrelated. The first set of instruments includes X^* which is typically included in two-stage least squares estimation procedures. If Eq.(3-15) is lagged one period, then clearly Y^*_{-1} should be highly correlated to Y^*_{-2} and X^*_{-1}. These two sets of instruments are typically included in the estimation of structural equations in which the error term exhibits autocorrelation (46, pp 295-99). Therefore, we include as the second set of instruments Y^*_{-2} and X^*_{-1}. To summarize, the full list of instruments are $(Y^*_{-2}, X^*, X^*_{-1})$. The IV approach leads to consistent but inefficient estimates.

In Model II, where the gap is assumed to be endogenous, the full set of instruments is very large; 19 instruments are required which is greater than the number of observations. To maintain consistency but at the cost of lower efficiency, the list of instruments is modified to omit, as few as possible, the predetermined variables excluded from the structural equation in question.

There have been some attempts to deal with simultaneous equation models with moving average error processess. For example, in the context of limited information maximum likelihood estimators see Hall and Pagan (19). Unfortunately, the techniques involved provide consistent but inefficient estimates. Algorithms which would provide both consistent and efficient estimates do not appear in standard software packages.

For the purpose of comparison, the reduced form inflation rate equation of Model I is estimated by ordinary least squares (OLS) and by an iterative maximum likelihood procedure developed by Beach and Mackinnon to correct for first-order autocorrelation. Due to the large number of variables which would appear in the reduced form inflation rate equation of Model II, these procedures are applied only to Model I.

The data covers the period between 1961 to 1981 for the U.S. economy (Table 12, located at the end of this chapter). However, due to the construction of the inflation rate and money supply growth variables, the data set is useful only for the period 1963-1981. The current M1 definition of the money supply is used to calculate real money balances and the rate of money growth. Inflation is calculated from the GNP price deflator. The GNP deflator and money supply data are obtained from Business Statistics (49). All other data are available in an article by Frank de Leeuw and Thomas Holloway (28).

4. EMPIRICAL RESULTS

The empirical results are summarized in Tables 5 through 11, found at the end of this chapter. We begin with a discussion of the estimated money supply growth equations. This is then followed by a review of the major findings of each of the two models under the assumption of dependent error terms. An attempt to quantify the impact of deficits on the rate of inflation is made in Section 5 with the aid of reduced form coefficients from Table 11.

4.1 MONEY SUPPLY GROWTH

Recall Jerome Stein's finding that the HEDMS ratio explains roughly two-thirds of the growth rate in the money supply for the period from 1957 to 1979. The high-employment deficit data series used in that study does not correct for the distortionary effects of inflation. In other words, the HEDMS ratio used by Stein is in part a function of the inflation rate. The series used in the current study has been corrected for automatic inflation effects. If the correction was not made, a further equation explaining the HEDMS ratio would have to be added.[4]

Stein also makes the claim that the ADEFMS ratio should not significantly affect the growth rate of the money supply because of two opposing forces which tend to cancel each other out. The argument is as follows. If economic activity is below capacity, two things will occur. First, due to its procyclical nature the money multiplier will decrease, which in turn causes CM_t to decrease. Second, the actual deficit will exceed the high-employment level, which causes CM_t to increase. These two effects tend to offset one another; thus the insignificance of the ADEFM ratio. This result is confirmed in Cols. 3 and 4. of Table 5.

If we include a lagged money growth variable along with the two deficit variables, a very interesting result emerges; both deficit variables become insignificant at a 5% level of significance (Col.2). Is there any way in which we can reconcile the apparent contradictions of this result with those of Cols.3 and 4? Every indication is that the lagged money growth term should be included in the specification. This would be consistent with the findings of other studies reviewed in Chapter I. The problem then is reduced to the relevance of the two deficit variables.

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[4] The adjustment is made to Federal expenditure programs and to tax receipts which respond automatically to changes in the price level.

There are various reasons why the validity of estimating equations including both deficit variables is questionable. First, the ADEFMS ratio is not corrected for the effects of inflation or the output-gap, thereby creating a bias in the estimates. Second, since the two variables have a fairly high level of correlation, (0.77), they both convey roughly the same information with regards to the trend in the government's fiscal position. However, the ADEFMS ratio is unable to distinguish whether or not the trends are due to structural shifts in policy or cyclical shifts in the economy. These reasons combined with the argument presented by Stein imply that the ADEFMS ratio should not be included in the specified equations.

Omission of the ADEFMS ratio leads to an equation such as estimated in Col.1. Both the lagged money growth and HEDMS ratio variables are significant and take the correct sign. First-order autocorrelation can be safely rejected given the low Durbin H statistic. Almost two-thirds of the variation in the growth rate of the money supply can be explained by the two variables. Also, the constant term indicates a high trend rate of growth equal to 3%.

4.2 INFLATION RATE EQUATIONS

In Model I, the output-gap is assumed to be exogenous while in Model II, the output-gap is assumed to be endogenous. The system is reproduced below for convenience.

(3-6) $CM_t = b0 + b1CM_{t-1} + b2HEDMS_{t-1} + \mu_{2t}$

(3-8) $GAP_t = YF_t - Y_t$

(3-13) $\dot{C}P_t = p0 + p1GAP_t + p2GAP_{t-1} + p3CP_{t-1} + p4CM_t + DV_t + v1_t$

(3-14) $Y_t = c0 + c1Y_{t-1} + c2(M/P)_t + c3(M/P)_{t-1} + c4G_t$
$+ c5G_{t-1} + c6CP_t + c7CM_t + c8DV_t + v2_t$

The estimated inflation rate equations for Models I and II are summarized in Tables 6 and 7, respectively.[5] A number of observations can be made. First, the lagged inflation rate is in general insignificant. Second, the two output-gap variables, although significant in each equation, carry the wrong sign. Third, the dummy variable included to test the hypothesis of a structural shift in the trend rate of inflation since 1973 is significant in a positive direction (Col.2 of Tables 6 and 7). When the slope dummy for the current output-gap is included, its individual t-statistic is insignificant (Col.3, Tables 6 and 7); however, the joint hypothesis that both dummy variables should not be included in the estimations is rejected. To test this hypothesis, a Wald chi-square test statistic, given in the Shazam 5 output, is compared to $\chi^2(2)$ at a 5% level of significance.

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5 The Breusch-Godfrey test, as presented in Johnson (22 pp 319-21), was applied to test the hypothesis of time dependent error terms. This Lagrange multiplier test (LM) is usually applied to determine the

As seen in Table 8, Cols. 3 and 4, the above results are unaltered when estimating the reduced form equation of Model I. One striking result found in these two columns is the insignificance of the HEDMS variable.

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existence of an autocorrelated or a moving average error process. However, due to the unknown structure of the error terms in Eqs.(3-13) and (3-14), the Breusch-Godfrey test provides, only as an approximation, evidence of some form of time dependence. The test statistic is $LM=nR^2$, where n equals the number of observations in a regression of the residuals on its lagged value and all other variables appearing in the originally estimated structural equation. We compare LM to a chi-square statistic with one degree of freedom. Interestingly enough, the null hypothesis of normally distributed error terms with variance σ^2 cannot be rejected at a 5% level of significance in Col.3 of Table 6, and Cols.2 and 3 of Table 7. It appears as if the inclusion of the dummy variables produces some sort of transformation which provides clean error terms. It is also possible that the error term exhibits first-order autocorrelation. If this is the case and if $\lambda=\rho$, the error term will be transformed into two components, both of which are normally distributed with mean equal to zero and variance σ^2. This result, however, is not found for the two output equations, Cols.3 and 4 of Table 7.

Although the reduced form estimates are consistent but inefficient, this finding suggests that the HEDMS ratio may have an even smaller impact on the rate of inflation than indicated by the reduced form coefficients derived from the IV estimates, presented in Table 11.[6]

The coefficient on the money growth variable is significant and has the correct sign. Its magnitude, however, varies substantially in the different equations. The range of this coefficient is between 0.64 (Table 7, Col.2) to 1.73 (Table 6, Col.1). It would appear that the lower values are more appropriate since the dummy variables are found to be significant.

Only brief comments are offered with regards to the estimated output equations, Table 7, Cols.4 and 5. It is apparent that the performance of this equation is very poor. Only the lagged output variable and the constant coefficient dummy variable are significant. This is likely a result of multicollinearity. The two government expenditure variables have an especially high correlation with the lagged output term. The only interesting observation which can be offered is that the findings

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6 The reduced form equation coefficients of Cols.3 and 4, show a larger value for the lagged money growth variable but smaller values for the constant coefficient and the lagged HEDMS ratio than the reduced form estimates of Model I, found in Col.1 of Table 11.

of Model I, in which the output-gap is exogenous, are altered little by assuming an endogenous output-gap.

We now turn our attention to an explanation of some of the more surprising results found in Tables 6 and 7. The insignificance of the lagged inflation rate variable may be caused by a multicollinearity problem. An in depth examination of this possibility was not attempted; however, a few simple observations indicate that this may be the case. First, it should be observed that the lagged inflation rate has been found to be a significant variable in studies attempting to explain the inflation adjustment process. As an example, see Mullineaux (34) or Stein (45). Second, the lagged inflation rate and current period GAP variable have a fairly high level of correlation. The simple correlation coefficient between the current GAP variable and the lagged rate of inflation is equal to 0.80. The two pieces of evidence suggest that the GAP variable may be picking up the effects of the lagged inflation rate as well as its own. Another possibility, not related to the muticollinearity problem, is that the adaptive expectations hypothesis from which the lagged inflation rate is derived is not valid.

The incorrect sign on the two output-gap variables is much more difficult to explain. As seen in Tables 6 and 7, both variables are consistently significant but the wrong sign. This result holds even when the slope dummy variable is added to test whether or not the current output-gap slope has changed since the 1973-74 oil price shock. A stronger positive relationship has existed since 1973 but this still does not preclude the fact that the relationship between inflation and the current output-gap, before the oil price shock, is positive. Thus, the stagflation period which arose as a result of the oil price shock cannot explain why the relationship is positive in the 1960's and early 70's.

Another possibility is that the theoretical model developed in this chapter is not an accurate description of the inflation adjustment process. There are two points that can be made. The insignificance of the lagged inflation rate variable, possibly due to multicollinearity, indicates that the augmented adaptive expectations hypothesis may be invalid. The second piece of evidence is somewhat stronger. When calculating the structural coefficients of the model, the values found for the speed of adjustment coefficients are well outside their hypothesized bounds. In fact, n_1 is a negative as opposed to positive value.

One final argument can be offered. This concerns the logic of including the contemporaneous values of variables in a model such as the one developed in this chapter. Clearly it will take time for a variable such as the output-gap to influence the rate of inflation, especially in a downwards direction. We could assume that it is the lagged output-gap which should have the inverse relationship with the inflation rate and not the current output-gap. In the estimated equations, the lagged GAP variable would therefore possess the correct sign. What we may in fact be witnessing with regards to the current period output-gap is a statistical relationship and nothing more. It is quite possible that the current period output-gap should not be included in the model specification. Theoretical models using this variable implicitly assume that price flexibility exists and occurs immediately. In fact, it may be more appropriate to model some form of initial price rigidity which can be relaxed after a certain period of time has elapsed.

To test this hypothesis, a third model in which the lagged output-gap is substituted for the current output-gap of Eq.(3-4), is estimated. Eq.(3-10) is then substituted into Eq.(3-4) in order to derive the inflation rate equation.

(3-16) $CP_t = d_0 + d_1 GAP_{t-1} + d_2 GAP_{t-2} + d_3 CP_{t-1} + d_4 CM_t + \mu_{4t}$

where $d_0 = a_0(1-n_1-n_2)/(1-a_2n_1) \gtrless 0$

$d_1 = a_1/(1-a_2n_1) < 0$

$d_2 = -a_1(1-n_1-n_2)/(1-a_2n_1) > 0$

$d_3 = (1-n_1-n_2)/(1-a_2n_1) > 0$

$d_4 = a_2n_2/(1-a_2n_1) > 0$

$\mu_{4t} = \{a_2/(1-a_2n_1)\}\varepsilon_t + \{1/(1-a_2n_1)\}\mu_{4t} - \{\lambda/(1-a_2n_1)\}\mu_{4t-1}$

Notice that the endogeneity of the output-gap is no longer a factor since only the lagged values of this variable are included in the estimation.

Our findings are summarized in Tables 9 and 10. The IV estimates of Table 9 provide an interesting contrast to those of Models I and II. Unlike previous results, the lagged inflation rate variable is the correct sign and significant while the current money growth variable is insignificant. The insignificance of the money growth variable raises the question of whether or not the HEDMS ratio is a significant variable. We also find that the output-gap variables carry the correct sign, although the two period lagged output-gap is insignificant, with the exception of the Col.1 estimates which exclude the dummy variables.[7]

The intercept dummy variable and the slope dummy variable for the lagged output-gap are jointly as well as individually significant. When the slope dummy is excluded from the estimated equation, the intercept dummy is significant.

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[7] The LM test indicates some form of time dependence for the estimates of Cols.1 and 3 of Table 9.

Derivation of the structural coefficients proves to be a more fruitful endeavour for Model III than in Model I. All structural coefficients have the correct sign and lie within their assumed bounds. For example, n_1 equals 0.8 for the 1964-73 period and 0.9 for the 1974-81 period. As well, n_2 takes on values between 0.05 and 0.02 for the two periods.[8]. These estimates provide much more credence to the augmented adaptive expectations hypothesis than previously found.

The reduced form coefficients, shown in Col.6 of Table 11, vary substantially to those found from the reduced form equation estimates of Table 10, especially the coefficient on the lagged inflation rate which is much larger than the reduced form equation estimates. Notice also, the differences in significance between the variables in Tables 9 and 10. From Table 10, we see that the one period lagged inflation rate, output-gap and HEDMS ratio are all insignificant, while the lagged money growth variable is significant. In contrast, the IV

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[8] The coefficient a_2, which measures the relationship between the current period inflation rate and the current period expected rate of inflation, is equal to unity. This value indicates that there is no long-run trade-off between the lagged output-gap and inflation. It is questionable whether or not such an interpretation has any theoretical basis in economics, since any trade-off is usually assumed to exist between the contemporaneous output-gap and inflation.

estimates show a significant lagged inflation rate but an insignificant money growth variable. The lack of significance of many of the variables in conjunction with high R^2 values provide some evidence that a multicollinearity problem exists.

5. INFLATION, DEFICITS AND THE OUTPUT-GAP

Table 11 is a summary of the various reduced form coefficients of the three models calculated from the IV estimates of Tables 6, 7 and 9. It is important to remember that the estimated coefficients are only asymptotically consistent rather than unbiased.

These coefficients can be used to provide some insight about the degree to which the inflation rate is affected by certain variables. Cols. 1, 2 and 3 show that as the HEDMS ratio increases the rate of inflation by 0.16 to 0.21% points for a 1% point rise in its value. However, Col.6, which is derived from the IV estimates of Model III, shows that a 1% point increase in the HEDMS ratio leads to, at most, a 0.5% point increase in the rate of inflation. At first glance these estimates may not seem to be of a significant magnitude. The largest increase in any one year in the HEDMS ratio occurred from 1974 to 1975 when it rose 10% points. This amounts to a 0.5 to 2.1% point increase in the rate of inflation.

However, it is also true that in the time period studied deficits were relatively small compared to recent years. It is well known that the United States has experienced 150 to 200 billion dollar deficits since 1983. Apparently a very high proportion of this deficit is now structural. Comparing this with any one year in the current study, we find an exponentially increasing growth rate in the HEDMS ratio. Although it is beyond the scope of this work to suggest that the relationship between inflation and the HEDMS ratio holds outside of the time period considered, given the evidence, large current deficits may pose a potential threat in the fight against inflation. It is not advocated that deficits cause high levels of inflation but rather that a certain amount of inflation is generated indirectly through the effect deficits have on the money supply. The reason that the effect in the 1960's and 1970's was weak is because the HEDMS ratio was really quite small.

Brief remarks should be offered on the impact which the output-gap has had on inflation. Whether we examine the GAP coefficients of Col.1 or the individual coefficients comprising the GAP variable in Model II, it is apparent that the magnitude of this coefficient is relatively small, approximately 0.03 to 0.05. The numbers are approximately the same for the lagged output-gap in Col.7, except that the signs are reversed.

It should be noted that the scale on the output-gap variables is in billions of dollars. Therefore, if we consider Model I or Model II, a 1 billion dollar increase in the current period output-gap causes the inflation rate to rise by approximately 0.03 to 0.05% points. When this coefficient is multiplied by 30 or 40 billion dollars, which is not uncommon, this typically involves a 1.2 to 2.0% point hike in the inflation rate.

When Model I or II is considered, it appears that for the time period studied, there is no single variable which stands out as the major instigator of inflation. The lagged money supply growth term has also, in general, had a relatively mild impact on the inflation rate. What is apparent is that there is a large constant term in the estimated equations, which indicates a high trend rate of inflation. The values range from between 1.5 to 4.4% points before 1974, and between 3.1 to 4.8% points since.

On the other hand, the Model III estimates indicate clearly that the lagged inflation rate has had a strong significant impact on the rate of inflation. A 1% point rise in the lagged inflation rate raises the the current inflation rate by 0.76% points.

6. SUMMARY

Both Models I and II provide relatively reasonable estimates of the inflation rate equation, although quite unsatisfactory estimates of the output equation. However, the structural coefficient estimates, along with the positive output-gap/ inflation rate relationship do not support the augmented adaptive expectations hypothesis. When the lagged output-gap is substituted in place of the current period output-gap in the original inflation rate equation, this finding is reversed.

In Model I and II, the lagged inflation rate is, in general, insignificant while the current period output-gap varies positively with the rate of inflation. This positive relationship cannot be explained by the stagflation period which occurred after the 1973-74 oil price shock. Therefore, a third model is estimated in which the lagged output-gap is substituted for the current output-gap. In this model, the lagged inflation rate becomes significant. In fact, it has the most powerful effect of all the variables on the rate of inflation.

IV estimates of Models I and II show that the HEDMS ratio has had a minor impact on inflation, while Model III estimates indicate that the HEDMS ratio may be an altogether insignificant variable. Finally, reduced form equation estimates of the inflation rate for Model I also cast some doubt on the significance of the HEDMS ratio. Although more work in this area is required before any firm conclusion can be reached, the evidence in this chapter indicates that the HEDMS ratio has a small and possibly negligible impact on the rate of inflation.

Table 5

Estimation of Money Supply Growth Equations

	(1)	(2)	(3)	(4)
	OLS.	OLS	OLS	AUTO
	CM_t	CM_t	CM_t	CM_t
Const	3.06 (3.85)*	2.80 (3.62)*	4.71 (13.29)*	4.86 (11.43)*
CM_{t-1}	0.39 (2.68)*	0.39 (2.84)*	- -	- -
$HEDMS_{t-1}$	0.21 (3.37)*	0.12 (1.43)	0.24 (2.73)*	0.23 (2.28)*
$ADEFMS_{t-1}$	- -	0.07 (1.68)	0.06 (1.11)	0.03 (0.46)
R^2	0.63	0.69	0.58	0.63
σ^2	1.01	0.91	1.33	1.04
D.W.	2.13	2.30	1.39	1.90
D.H.	-0.50	-0.94	-	-
OBS.	20	20	21	20
RHO.	- -	- -	- -	0.34 (1.63)

Notes 1 to 4, below, refer to Tables 5 to 10, while notes 5, 6 and 7 refer to Tables 6, 7 and 9.

1) t-ratios are given in parenthesis.
2) t-ratios which appear with a star(*) are significant at a 5% level assuming a one-tailed test.
3) All aymptotic t-ratios are compared to normally distributed Z statistics.
4) OBS is the number of observations used in the estimation.
5) When IV estimation is used, the t-ratio is aymptotic.
6) LM refers to the Lagrange multiplier test statistic, as developed by Breusch-Godfrey, which is used to test for the presence of time dependent error terms.
7) CHI-SQ is the Wald chi-square statistic used when testing the joint hypothesis that the coefficients on the two dummy variables is equal to zero.

TABLE 6

Model I (IV Estimates)

	(1)	(2)	(3)
	IV	IV	IV
	CP_t	CP_t	CP_t
Const	-3.861	-1.417	-0.921
	(-2.129)*	(-1.486)	(-1.367)
DV_t	-	1.764	1.596
	-	(3.271)*	(3.050)*
$GAPD_t$	-	-	0.016
	-	-	(1.323)
CP_{t-1}	-0.284	0.168	0.237
	(-0.552)	(0.810)	(1.567)
GAP_t	0.109	0.052	0.034
	(2.642)*	(2.720)*	(2.707)*
GAP_{t-1}	-0.082	-0.061	-0.057
	(-3.744)*	(-6.122)*	(-7.849)*
CM_t	1.729	0.927	0.783
	(2.873)*	(3.340)*	(4.618)*
R^2	0.788	0.946	0.957
σ^2	1.276	0.329	0.261
D.W.	1.731	1.936	1.946
OBS	18	18	18
LM	9.347	4.134	0.422
CHI-SQ	-	-	19.335

Table 7

Model II (IV Estimates)

	(1)	(2)	(3)		(4)	(5)
	IV	IV	IV		IV	IV
	CP_t	CP_t	CP_t		Y_t	Y_t
Const	-3.299	-0.522	-0.586	Const	-130.07	-131.60
	(-2.373)*	(-0.747)	(-0.955)		(-1.03)	(-1.28)
DV_t	-	2.029	1.737	DV_t	-	38.46
	-	(4.192)*	(3.403)*		-	(1.94)*
$GAPD_t$	-	-	0.014	$GAPD_{t-1}$	-	-
	-	-	(1.170)		-	-
CP_{t-1}	0.017	0.346	0.280	Y_{t-1}	1.05	1.09
	(0.049)	(1.968)*	(1.884)*		(8.66)*	(11.16)*
GAP_t	0.079	0.034	0.030	$(M/P)_{t-1}$	-33.11	30.23
	(2.874)*	(2.173)*	(2.308)*		(-1.14)	(0.69)
GAP_{t-1}	-0.067	-0.052	-0.054	$(M/P)_{t-1}$	33.43	-29.64
	(-4.473)*	(-6.467)*	(-7.739)*		(1.14)	(-0.68)
CM_t	1.430	0.637	0.691	G_t	0.11	-0.49
	(3.426)*	(3.338)*	(4.624)*		(0.20)	(-0.92)
				G_{t-1}	0.21	0.30
					(0.43)	(0.72)
				CP_t	-90.07	46.62
					(-1.40)	(0.49)
				CM_t	82.26	-54.50
					(1.27)	(-0.57)
R^2	0.852	0.953	0.956	R^2	0.996	0.998
σ^2	0.892	0.292	0.262	σ^2	128.62	85.117
D.W.	1.372	2.063	1.972	D.W	1.998	2.535
OBS	18	18	18	OBS	18	18
LM	6.232	0.714	0.371	LM	4.034	4.384
CHI-SQ	-	-	20.843	CHI-SQ	-	-

TABLE 8

Model I (Reduced Form Equation Estimates)

	OLS (1) CP_t	OLS (2) CP_t	AUTO (3) CP_t	AUTO (4) CP_t
Const	0.431 (0.468)	0.419 (0.446)	-0.005 (-0.014)	-0.072 (-0.190)
DV_t	1.961 (1.962)*	1.574 (1.380)	1.520 (3.697)*	1.300 (2.465)*
$GAPD_t$	- -	0.017 (0.750)	-	0.007 (0.671)
CP_{t-1}	0.462 (1.362)	0.351 (1.052)	0.199 (1.091)	0.169 (0.888)
GAP_t	0.014 (0.527)	0.009 (0.303)	0.034 (2.236)*	0.032 (2.090)*
GAP_{t-1}	-0.029 (-2.460)*	-0.027 (-2.305)*	-0.034 (-6.470)*	-0.033 (-6.147)*
CM_{t-1}	0.417 (1.708)	0.473 (1.822)	0.647 (5.470)*	0.682 (5.168)*
$HEDMS_{t-1}$	0.003 (0.028)	-0.001 (-0.006)	0.058 (0.992)	0.055 (0.949)
R^2	0.920	0.923	0.958	0.959
σ^2	1.02	0.84	0.48	0.28
D.W	2.885	2.859	2.06	2.50
OBS	19	19	19	19
RHO	- -	- -	-0.733 (-4.243)*	-0.724 (-4.072)*

Table 9

Model III (IV Estimates)

	(1)	(2)	(3)
	IV	IV	IV
	CP_t	CP_t	CP_t
Const	0.525 (0.323)	0.233 (0.248)	0.045 (0.057)
DV_t	– –	1.941 (1.843)*	2.723 (3.097)*
$GAPD_{t-1}$	– –	– –	-0.030 (-1.980)*
CP_{t-1}	1.299 (6.481)*	0.801 (3.175)*	0.759 (3.824)*
GAP_{t-1}	-0.056 (-3.822)*	-0.043 (-3.952)*	-0.024 (-1.929)*
GAP'_{t-1}	0.040 (2.293)*	0.009 (0.561)	0.004 (0.339)
CM_t	-0.215 (-0.553)	0.178 (0.804)	0.234 (1.480)
R^2	0.806	0.905	0.927
σ^2	1.168	0.573	0.442
D.W.	2.750	2.314	2.453
OBS	18	18	18
LM	6.322	3.395	5.168
CHI-SQ	–	–	11.126

Table 10

Model III (Reduced Form Equation Estimates)

	OLS	OLS	AUTO	AUTO
	(1)	(2)	(3)	(4)
	CP_t	CP_t	CP_t	CP_t
Const	0.437 (0.522)	0.455 (0.539)	-0.163 (-0.435)	0.045 (0.132)
DV_t	3.652 (3.365)*	3.954 (3.452)*	3.363 (3.826)*	3.719 (5.913)*
$GAPD_{t-1}$	- -	-0.020 (-0.894)	- -	-0.022 (-2.126)*
CP_{t-1}	-0.046 (-0.119)	0.021 (0.053)	0.017 (0.048)	0.119 (0.466)
GAP_{t-1}	0.013 (0.541)	0.020 (0.771)	0.009 (0.402)	0.012 (0.806)
GAP_{t-2}	-0.041 (-1.700)	-0.039 (-1.573)	-0.037 (-1.639)	-0.032 (-1.980)*
CM_{t-1}	0.802 (2.484)*	0.726 (2.158)*	0.869 (3.153)*	0.724 (3.428)*
$HEDMS_{t-1}$	-0.055 (-0.958)	-0.200 (-0.276)	-0.057 (-2.036)*	-0.022 (-0.245)
R^2	0.934	0.938	0.958	0.966
σ^2	0.693	0.704	0.277	0.223
D.W	2.781	2.847	2.150	18
OBS	19	19	18	2.333
RHO	- -	- -	-0.665 (-2.532)*	-0.748 (-4.069)*

Table 11

REDUCED FORM COEFFICIENTS

(Derived from IV Estimates)

	(1)	(2)	(3)	(4)	(5)	(6)
	Model 1	Model 2 (64-73)	Model 2 (74-81)	Model 2 (64-73)	Model 2 (74-81)	Model 3
	CP_t	CP_t	CP_t	Y_t	Y_t	CP_t
Const(0)	1.476	4.369	4.804	-94.692	-74.438	0.761
Const(1)	3.071	4.612	4.818	-44.896	-35.298	3.484
$GAP(0)_t$	0.034	-	-	-	-	-
$GAP(1)_t$	0.050	-	-	-	-	-
GAP_{t-1}	-0.057	-	-	-	-	-
CP_{t-1}	0.230	0.117	0.092	5.442	4.278	0.759
CM_{t-1}	0.305	0.378	0.395	-3.624	-2.849	0.091
$HEDMS_{t-1}$	0.164	0.204	0.213	-1.951	-1.534	0.049
$GAP(0)_{t-1}$	-	-	-	-	-	-0.024
$GAP(1)_{t-1}$	-	-	-	-	-	-0.054
GAP_{t-2}	-	-	-	-	-	0.004
$YF(0)_t$	-	0.013	-	0.583	-	-
$YF(1)_t$	-	-	0.01	-	0.672	-
YF_{t-1}	-	-0.023	-0.018	-1.050	-0.825	-
$(M/P)_t$	-	-0.378	-0.436	12.604	9.908	-
$(M/P)_{t-1}$	-	[illegible]71	0.427	-12.357	-9.714	-
G_t	-	[illegible]06	0.007	-0.204	-0.161	-
G_{t-1}	-	-[illegible]004	-0.004	0.123	0.097	-
Y_{t-1}	-	0.009	0.002	1.506	1.184	-

Bracketed terms which appear with a zero refer to the time period 1964-73, while terms which appear with the number one refer to the time period 1974-1981.

Table 12

Data File

YEAR	GNPD	Y	YF	G	M1
1961.000	69.33000	756.6710	801.0962	146.9782	144.3000
1962.000	70.61000	800.1699	828.9194	156.3518	147.9000
1963.000	71.67000	832.5659	859.7740	159.3414	152.4000
1964.000	72.77000	876.3227	893.9123	162.4296	158.4000
1965.000	74.36000	929.3975	927.6493	166.4874	165.1000
1966.000	76.76000	984.8880	963.5227	187.0766	172.7000
1967.000	79.06000	1011.384	999.1146	207.0579	179.5000
1968.000	82.54000	1058.154	1034.892	218.6819	192.1000
1969.000	86.79000	1087.683	1072.128	217.0757	203.5000
1970.000	91.45000	1085.511	1110.443	223.4008	211.2000
1971.000	96.01000	1122.383	1150.297	229.7677	225.5000
1972.000	100.0000	1185.900	1191.600	244.3000	241.7000
1973.000	105.7500	1254.279	1234.326	249.8345	259.3000
1974.000	115.0800	1246.263	1276.330	260.0799	272.3000
1975.000	125.7900	1231.576	1317.990	283.4884	284.9000
1976.000	132.3400	1298.171	1362.778	290.7662	301.1000
1977.000	140.0500	1369.511	1409.068	300.9639	324.2000
1978.000	150.4200	1433.387	1455.458	306.2758	350.7000
1979.000	163.4200	1477.114	1498.776	311.5898	377.7000
1980.000	178.4200	1471.864	1539.738	337.4061	401.4000
1981.000	195.1400	1499.180	1582.095	352.7724	430.0000
1982.000	206.8800	0.0	0.0	0.0	458.0000

YEAR	CM	CP	GAP	HEDMS	DV
1961.000	0.0	0.0	44.42521	-4.920305	0.0
1962.000	2.494802	1.846243	28.74947	-2.028398	0.0
1963.000	3.042596	1.501204	27.20804	-4.855643	0.0
1964.000	3.937008	1.534812	17.58967	-0.6944444	0.0
1965.000	4.229798	2.184966	-1.748252	-0.5451242	0.0
1966.000	4.603271	3.227542	-21.36529	3.2426170	0.0
1967.000	3.937464	2.996352	-12.26916	8.412256	0.0
1968.000	7.019499	4.401720	-23.26145	5.726184	0.0
1969.000	5.934409	5.149019	-15.55479	-2.407862	0.0
1970.000	3.783784	5.369282	24.93166	2.178030	0.0
1971.000	6.770833	4.986331	27.91376	5.011086	0.0
1972.000	7.184035	4.155817	5.700000	5.006206	0.0
1973.000	7.281754	5.750000	-19.95272	3.663710	0.0
1974.000	5.013498	8.822695	30.06604	0.1101726	1.0
1975.000	4.627249	9.306569	86.41386	10.21411	1.0
1976.000	5.686206	5.207091	64.60632	5.778811	1.0
1977.000	7.671870	5.825903	39.55730	6.693399	1.0
1978.000	8.173967	7.404498	22.07153	4.305674	1.0
1979.000	7.698888	8.642468	21.66198	0.5559968	1.0
1980.000	6.274821	9.178803	67.87356	5.032387	1.0
1981.000	7.125062	9.371147	82.91483	0.6046512	1.0
1982.000	6.511628	6.016194	0.0	0.0	1.0

CONCLUSION

The objective of this thesis was to examine both a theoretical and empirical model in which the rate of inflation is assumed to be influenced by the high-employment deficit. Tobin and Buiter's model (1976) was extended by replacing the usual inflationary expectation mechanisms (static, myopic perfect foresight and adaptive expectations) with one which focuses on the expected growth rate of the nominal money supply. In turn, the expected growth rate of the money supply depends on the high-employment deficit/ money stock ratio. This formulation was originally developed by Jerome Stein by recognizing the financing constraint of the government.

Two government budget constraint specifications were considered. The GT1 constraint defined government expenditures to include real after-tax interest payments while the GT2 constraint defined government expenditures separatley from real after-tax interest payments.

It was found that a money-financed expenditure policy will, in the long-run, lead to a lower interest rate but higher price level and a higher stock of capital under either a GT1 or GT2 constraint. On the other hand, a bond-financed expenditure policy leads to a lower long-run interest rate and price level, and a higher stock of capital under a GT1 specification. If a GT2 specification is adopted and the slope of the GT2 curve is steeper than the slope of the long-run LM curve, a bond-financed expenditure policy leads to a higher interest rate and price level but lower stock of capital in the long-run.

The above findings are conditional on the stability of the system. Given a GT1 budget constraint, a bond-financed expenditure policy is unstable, while a money-financed expenditure policy is stable as long as the interest sensitivity of an increase in capital stock is not strongly negative. The most interesting finding here is that the trace and determinant expressions are very similar to those found by Tobin and Buiter under the assumption of static inflationary expectations. As a result, there appears to be little difference in the short-run adjustment process when modelling inflationary expectations either statically or as a function of the high-employment deficit/ money stock ratio. Intuitively, this result should carry over when modelling the government's financing constraint as GT2; this, however, was not proven formally.

Unfortunately, stability for either a money-financed or a bond-financed expenditure policy becomes an empirical question, given a GT2 government budget constraint. Stability is enhanced considerably for both policies if an increase in the capital stock raises the nominal rate of interest and if the price elasticity of an increase in capital outweighs the interest rate elasticity of an increase in capital. Further, the probability of stability for a bond-financed expenditure policy will increase if the price elasticity of an increase in bonds minus the interest rate elasticity of an increase in bonds is greater than unity. In the case of a money-financed expenditure policy, the likelihood of stability is greater if the price elasticity of an increase in money is greater than the money stock to bond stock ratio.

In chapter III, a small simultaneous equation model was developed in an attempt to quantify the impact of the high-employment deficit/ money stock ratio (HEDMS) on the rate of inflation. The system is comprised of four equations and an identity explaining the output-gap. The growth rate of the money-supply is assumed to be a function of its lagged value and the lagged value of the high-employment/ money stock ratio. Inflation is assumed to be a function of the output-gap and inflationary expectations. We model inflationary expectations adaptively but add an augmenting factor to recognize the long-run relationship between the growth rate of the money supply and the inflation rate. In this particular framework individuals do not require information about the HEDMS ratio. As long as the monetary authority reacts to this variable, an indirect link between the inflation rate and the HEDMS ratio will exist through the growth rate of the nominal money supply.

Initially, two versions of the model are estimated. In the first model, the output-gap is assumed to be exogenous while in the second model the output-gap is assumed to be endogenous. Both procedures lead to very similar results. The reduced form coefficients of the two models show that the high-employment deficit/ money stock ratio has had a mild impact on the rate of inflation in the United States for the period 1963-1981. A 1% point increase in the HEDMS ratio increases the inflation rate in the following year by 0.16 to 0.21% points. However, certain difficulties were encountered with these two models. For instance, the lagged inflation rate is consistently insignificant, while the relationship between the contemporaneous output-gap and inflation is positive rather than negative.

Two dummy variables were added to the model in recognition of the oil price shock of 1973-74. These include a intercept dummy variable and a slope dummy variable for the contemporaneous output-gap. The two dummy variables are jointly significant. However, a positive inflation rate/ output-gap relationship still prevails for both time periods, 1964-73 and 1974-81. This is an indication that the stagflation years of the 1970's cannot by themselves explain the positive relationship. Therefore, a third model was estimated in which the contemporaneous output-gap is replaced by the lagged output-gap.

Instrumental variable estimates of the third model yield structural coefficients which support the augmented adaptive expectations hypothesis. As well, the output-gap coefficients have the correct sign and the lagged inflation rate becomes significant. In fact, the lagged inflation rate clearly has the greatest impact on the current inflation rate.

The one sour note which can be attached to the Model III estimates is the insignificance of the money supply growth variable. Due to the insignificance of the growth rate in the nominal money supply, it is questionable if the HEDMS ratio has any impact on the rate of inflation. Reduced form estimates show that if the HEDMS ratio is a significant variable, its impact on the rate of inflation is quite small; i.e., a 1% point increase in the lagged HEDMS ratio leads to a 0.05 point increase in the rate of inflation.

There are a number of extensions which could be made to improve this study. Development of an open economy model, although greatly complicating the analysis, would likely prove useful. The assumption of perfect substitutability between bonds and capital is extreme and should be relaxed. Consolidation of the monetary authority with the Treasury is also an extreme assumption. By assuming an independent monetary authority, a more realistic model could be developed which would allow a much broader scope of analysis, especially with regards to the monetization of deficits. Finally, in terms of the estimated models of Chapter III, it would be useful to introduce a more rigorous money supply growth equation. The equation used in this study is the simplest version found in the literature.

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