Structural Engineering Report No. 59

THE DESTABILIZING FORCES CAUSED BY GRAVITY LOADS ACTING ON INITIALLY OUT-OF-PLUMB MEMBERS IN STRUCTURES

by

D. Beaulieu

and

P.F. Adams

February, 1977

THE DESTABILIZING FORCES CAUSED BY GRAVITY LOADS ACTING ON INITIALLY OUT-OF-PLUMB MEMBERS IN STRUCTURES

BY

D. BEAULIEU

AND

P.F. ADAMS

DEPARTMENT OF CIVIL ENGINEERING

THE UNIVERSITY OF ALBERTA

EDMONTON, ALBERTA

ABSTRACT

This investigation attempts to determine rational design rules related to the overall stability of a structure and to the stability of individual members. The effects of the vertical loads acting on the deformed structure and on initially out-of-plumb members are investigated. The statistical characteristics obtained from measurements taken on out-of-plumb columns and walls in three structures are used in the derivations of several expressions for the different out-of-plumb effects. Statistical methods are used in the development of appropriate design procedures to account for out-of-plumbs; this constitutes the major contribution of the study. The results of the study indicate that many of the procedures used to account for stability in present design standards are generally inadequate.

ACKNOWLEDGEMENTS

This study was performed in the Department of Civil Engineering at the University of Alberta. Financial assistance from the Canadian Steel Industries Construction Council and the National Research Council of Canada is acknowledged.

The authors wish to express their sincere appreciation to D. Kelker, J.G. MacGregor, D. Nixon, and G. Taylor for their professional assistance and constructive discussions and to Poole Construction Limited and Dominion Bridge Company Limited for granting permission to take the measurements.

TABLE OF CONTENTS

Page

Page

LIST OF SYMBOLS

A = column tributary area

A_0 = area of fictitious bracing member

A_1 = column influence area

b = building width

E = modulus of elasticity

e = eccentricity

$\bar{e}$ = mean value of the eccentricity e

F = random horizontal shear in joints and floor systems

F_d = design shear F (absolute value)

f_i = frequency

F_{st} = standardized horizontal shear

F_t = equivalent horizontal load on entire structure

H = factored applied lateral load

h = storey height

h_{avg} = average of two storey heights

H_c = lateral load due to column out-of-plumbs

H_e = equivalent uniformly distributed load

h_i = height of storey i

H'_i = fictitious sway force at floor level i

h_t = total height of building

H_w = fictitious lateral load due to wall out-of-plumbs

i = storey index

I_b = beam moment of inertia

I_c = column moment of inertia

j = column or wall index

K = spring stiffness

k	=	storey index ($i + 1 \le k \le m$)
L	=	length of a wall
ℓ	=	distance from a wall segment to the center of the wall
L_x	=	lever arm parallel to the x axis
L_y	=	lever arm parallel to the y axis
L_0	=	length of fictitious bracing member
M	=	random moment in a section of a floor
m	=	total number of storeys
M_d	=	design moment M (absolute value)
M_{di}	=	design moment M_i (absolute value)
M_i	=	random moment in the core at floor level i
m_i	=	total number of storeys above floor level i
M''	=	second order column moment
M''_1	=	second order moment at the bottom of a column
M''_2	=	second order moment at the top of a column
n	=	number of columns or walls
n_i	=	total number of columns at storey i
P	=	maximum factored axial load
p	=	load contribution in a vertical load-bearing element from the floor above
ΣP_i	=	sum of factored axial loads at storey i
P_j	=	factored axial load in column j
P_0	=	non-deterministic axial load
q	=	prescribed wind pressure
s	=	number of segments in a wall
S_{di}	=	design shear S_i (absolute value)
S_i	=	random shear in the core at storey i
T_c	=	torque due to column out-of-plumbs

T_{di}	=	design torque T_i (absolute value)
T_i	=	random torque in the core at floor level i
T_w	=	torque due to wall out-of-plumbs
t_1, t_2	=	wall thicknesses
v	=	coefficient of variation
ΣV_i	=	total first order shear at storey i
V''	=	second order shear in column
V'_i	=	artificial shear at storey i due to sway forces
V'_j	=	share of fictitious horizontal shear in column j
V''_j	=	actual (exact) second order shear in column j
V^a_j	=	shear in column j obtained from an approximate second-order analysis (value in error)
W	=	weight per cubic meter of a structure
w	=	class width of a histogram
x,y,z	=	Cartesian coordinates
α	=	angle of inclination of bracing member
β	=	safety index
γ	=	factor accounting for tolerance requirements and degree of control
Δ	=	first-order lateral deflection
Δ_d	=	design out-of-plumb
Δ_i	=	first-order lateral deflection of the i'th floor relative to the floor below or to the base of the structure
Δ''_i	=	second-order lateral deflection of the i'th floor relative to the floor below
δ_t	=	deviation from the mean wall thickness at a section
Δ_x	=	random initial out-of-plumb in the direction of the x axis
Δ_y	=	random initial out-of-plumb in the direction of the y axis
Δ_0	=	random initial out-of-plumb

Δ_0' = growth in initial out-of-plumb

$\bar{\Delta}_0$ = mean out-of-plumb

ε = total change in slope between two columns at their intersection

ε_n = ε for n sets of two columns

ε_1 = ε for one set of two columns

λ = safety index

μ = expected mean

μ_c = mean of column out-of-plumb population

μ_d = mean of dead load population

μ_f = mean of distributed horizontal shear

μ_ℓ = mean of live load population

μ_p = mean of gravity load population

ρ = coefficient of correlation

σ = standard deviation

σ_c = standard deviation of column out-of-plumb population

σ_d = standard deviation of dead load population

σ_e = standard deviation of e/L population

σ_f = standard deviation of distributed horizontal shear

σ_ℓ = standard deviation of live load population

σ_p = standard deviation of gravity load population

$\sigma_{x,y}$ = covariance of variables x and y

σ_w = standard deviation of wall out-of-plumb population

ψ = safety index

CHAPTER I

INTRODUCTION

Several factors are known to impair the strength and stability of a structure. Among the most important is the so-called P-Δ effect. The axial loads acting on the columns and load-carrying walls produce additional moments and forces when acting through the lateral displacement of the structure. The effect is particularly important in tall buildings. When these effects are taken into account in the structural analysis of the building, the analysis is referred to as a "second-order analysis". A great deal of time and effort has been spent on developing methods to account for the P-Δ effects. The most recent studies have been oriented towards the development of simple and practical design techniques and an in depth examination of the forces involved.

A second factor which affects the stability of structures but has attracted very little attention until recently is the initial out-of-plumbness of columns and load carrying walls. In a manner similar to the P-Δ effect, the gravity loads acting through initially inclined members generate extra horizontal forces within the structure. The out-of-plumb forces, so far, are either totally neglected in design or improperly accounted for by using over-simplified techniques. A thorough examination of the out-of-plumb effects and the development of appropriate modifications of existing design procedures, as well as new methods, therefore are warranted.

The main objective of this study is to propose rational clauses for design standards which are related to the stability of complete structures and individual members. In this respect, the thesis is divided into two distinct sections: first, a short section which deals with some limited aspects of the P-Δ effects and then a section on out-of-plumbs which constitutes by far the essence of the study. Important measurements taken on steel columns and concrete walls are presented and are used in statistical calculations. Simple methods based on statistics are developed to determine the nature and the significance of the extra horizontal forces due to column and wall out-of-plumbs. The transfer of these forces among the resisting elements of the structure is given special attention.

Although the procedures used for the determination of the out-of-plumb forces should be applicable to many different framing schemes, the investigation uses the core-braced structure as a model for the computations. Basically, the structure consists of a concrete core enclosed by an orthogonal steel framing system as shown in Fig. 1.1. All connections are assumed to be pinned. The frame is designed to carry a share of the vertical loads while the core has the double function of supporting a portion of the vertical loads and stabilizing the structure against lateral forces.

The core-braced structure has been selected because of its relatively common use in medium rise buildings and because the requirements for the transfer of horizontal forces are relatively severe. Other types of structures, with and without rigid connections, are discussed briefly.

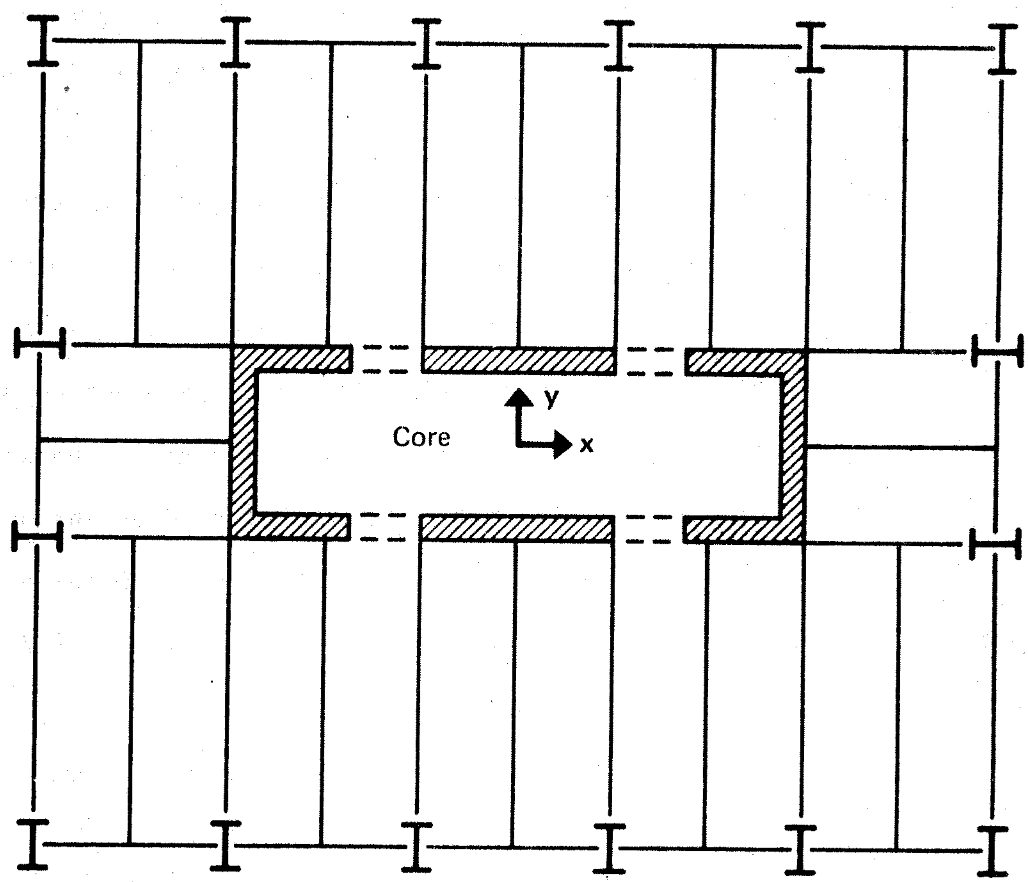


Figure 1. Core-braced structure (plan view)

The different horizontal forces present in structures are classified and defined in Chapter II. Chapter III contains a review of those methods of analysis for core-braced buildings which present a reasonable estimate of the forces defined in the preceding section. The survey is limited to elastic first order and second order methods. The distribution of the horizontal forces resulting from the approximate second order analyses presented in Chapter III is discussed in Chapter IV.

Chapter V reviews the existing literature and clauses in design standards related to column and wall out-of-plumbs. Chapter VI presents the distributions and statistical characteristics of a series of out-of-plumb measurements taken on buildings under construction. Chapter VII is devoted to the derivation of design equations based on statistics for each resisting element affected by the out-of-plumb forces. This is followed in Chapter VIII by examples of applications which attempt to demonstrate the applicability of the equations derived in the previous chapter.

Each individual out-of-plumb effect is calibrated in Chapter IX against the corresponding wind load effect, in order to determine its significance in design. Where appropriate, the recommendations listed in Chapter V are compared with the results of the techniques developed in Chapter VII.

Chapter X contains a general discussion while Chapter XI consists of a brief summary and finally concludes by listing the proposed clauses for design standards related to structural stability.

CHAPTER II

HORIZONTAL FORCES

In a core-braced structure, horizontal forces must be resisted at each beam-to-column connection and portions of these forces must be transferred to the core if the integrity of the structure is to be maintained. The magnitude and distribution of the forces depend upon the relative stiffnesses of the columns and the core, the stiffness of the connections, the characteristics of the floor system, and the general arrangement of the structure.

Several types of horizontal forces must be resisted to ensure the stability of a structure. For convenience, the forces may be classified under three separate headings: first order forces, P-Δ forces, and forces due to column and wall out-of-plumbs.

2.1 First Order Forces

The forces normally considered in the design of the individual members are those resulting from a first order analysis of the structure. The structure is assumed to be subjected to gravity loads alone or in combination with wind or earthquake loads. In the analysis the response is assumed to be elastic and the equilibrium equations are formulated on the undeformed structure[1]. Several methods which are applicable to core-braced systems and which make use of elastic first order principles will be briefly described in the next chapter.

The first order forces are more or less adequately distributed among the stiffening elements depending on the type of analysis used and the simplifying assumptions made.

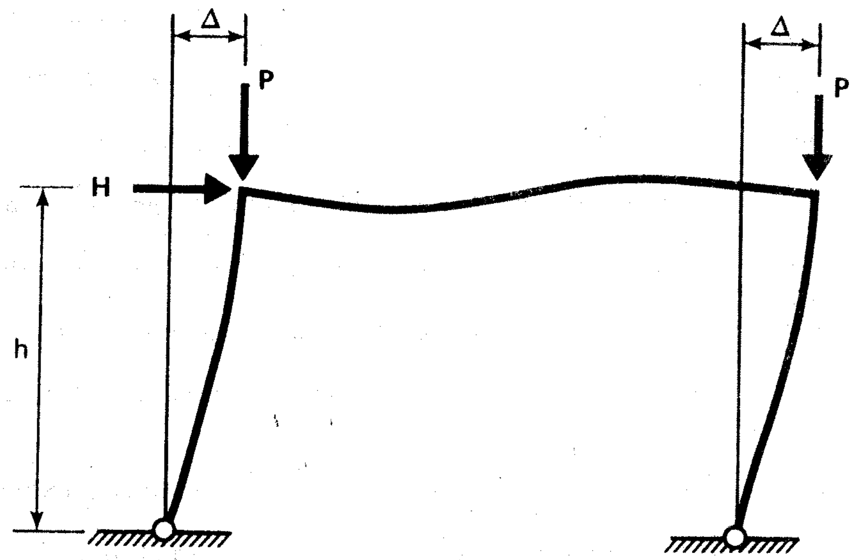
2.2 P-Δ Forces

The assessment of the overall stability of a structure under the action of applied loads is a major problem. In a first order analysis, the stability effects are not considered since the equilibrium equations are formulated on the undeflected structure[1,2]. The designer generally neglects these effects in the analysis to simplify the calculations. The stability effects must then be indirectly accounted for in design as will be discussed below.

The vertical loads, acting through the lateral displacements of a structure, produce additional forces and moments. Fig. 2.1 shows a simple frame subjected to a lateral load, H, and vertical loads, P, applied at the tops of the columns. The extra storey moment, $2P\Delta$, the corresponding fictitious shear, $2P\Delta/h$, the extra sway, and the distribution of the additional moments and forces among the resisting elements of the structure are called P-Δ effects[1,5]. They are especially significant in tall buildings where stability considerations very often control the design[6]. An analysis in which equilibrium is formulated on the deformed structure, thus accounting for these effects, is called a second order analysis.

2.3 Forces Due to Column Out-of-Plumbs

The unavoidable out-of-plumbs of the columns are among the geometric imperfections which most significantly affect the stability



H = Lateral load
P = Column axial load
h = Column height
Δ = Relative lateral deflexion of a column

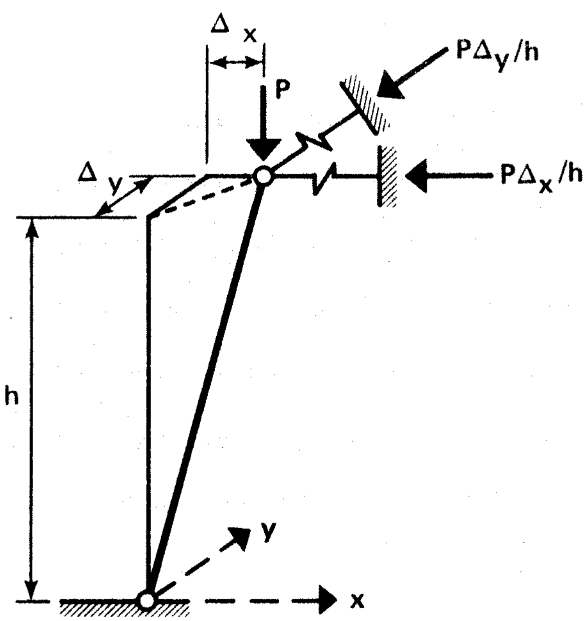
Figure 2.1 Second-order effects on sway permitted frames

of a structure. Horizontal forces of value $P\Delta_x/h$ and $P\Delta_y/h$ in the direction of the x and y axes are generated by the axial load P acting on the inclined column shown in Fig. 2.2. These additional forces may affect the integrity of the structure and provisions should be made for their transfer. The extra moments and shears must be accounted for in the design of connections, floor diaphragms, and vertical bracing systems. This is particularly important in tall core-braced buildings where the vertical loads are high and the horizontal forces from each column must generally be transferred to the core.

When the members of a frame are rigidly jointed, extra moments are developed in the columns and girders by the initial out-of-plumbs with the result that the force transfer requirement is reduced. This particular case is not investigated in this report. If, however, the columns are pin-connected, larger lateral forces are induced. These forces must be transferred by the beam-to-column connections through the floor systems to the central core.

While the direction and intensity of the extra P-Δ forces are calculated directly from an analysis of the structure, the same parameters related to the out-of-plumb forces are practically undefined. Referring to Fig. 2.2, a column may be inclined along both the x and y axes and the magnitude of the initial inclinations may vary between zero and a value greater than the maximum prescribed erection tolerance[7].

The estimation of the out-of-plumb forces and their distribution in a structure is complex and requires special consideration. Similar to the P-Δ forces, the out-of-plumb forces affect the strength and stability of tall buildings. For these reasons they should be accounted for in design.



P = Column axial load
h = Column height
$\Delta_{x,\,y}$ = Initial out-of-plumbs in the direction of the x and y axes

Figure 2.2 Out-of-plumb column

CHAPTER III

ANALYSIS AND DESIGN

This chapter is divided into three parts. The first portion contains a review of the available methods of analysis applicable to core-braced systems. Although the frame is generally simply connected and depends entirely on the core for its stability, methods are also presented which apply to moment resisting frames. The application of approximate second order analyses is discussed in the second part. The last section presents an outline of the current design procedures.

3.1 First Order Analysis

Practical analyses are generally characterized by a number of appropriate structural and loading simplifications. It is common practice to assume that the floor diaphragms are infinitely rigid in their own planes, that the applied lateral loads are concentrated at floor levels, that the joints are ideally rigid or pinned (as appropriate), and that the core acts as a reinforced concrete flexural element having a finite width.

Depending on whether the frame systems offers significant stiffness (rigid connections), a first order analysis is performed on the whole structure or on the core alone.

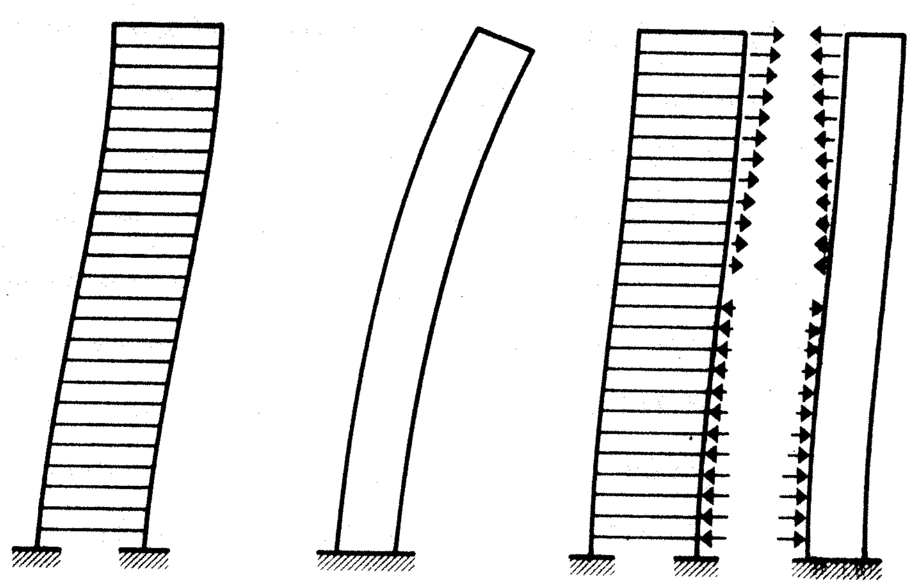
3.1.1 Two-Dimensional Analysis

In the case of structures symmetric in plan and subjected to symmetrically applied horizontal loads, a two-dimensional analysis is appropriate.

A framing system with negligible stiffness is assumed simply connected and is designed to carry the appropriate share of the vertical loads. The core, in turn, has the double function of supporting vertical loads and resisting the horizontal forces by cantilever action. If the frame possesses a significant stiffness, the above assumptions may lead to an unconservative frame design[(8)]. In this case, the interaction between the walls and the frame results in a significant redistribution of the forces in the structure[(9,10)]. As shown in Fig. 3.1, the frame tends to restrain the core in the top storeys and the opposite occurs in the lower region. As a result, forces are created that cannot be predicted by isolating the two systems.

When the frame plays a significant role in resisting the lateral loads, all parallel bents in the structure may be placed side by side and linked together for analysis[(9,10)]. The frame members are modelled as linear elements while the core is simulated by a plane shear-wall of equivalent strength and stiffness[(12)]. The horizontal loads acting on the resulting plane structure are then distributed among the elements according to their relative stiffnesses[(11)]. Only one-half or one-quarter of a symmetric structure need be analyzed.

In many cases, simplification can be achieved by replacing the actual frame by an "idealized frame" as shown in Fig. 3.2(a). The areas and stiffnesses of the members in the idealized structure are obtained by a "lumping" technique as described in Ref. 9. The model



(a) Free Frame (b) Free Core (c) Combined Frame & Core

Figure 3.1 Force redistribution

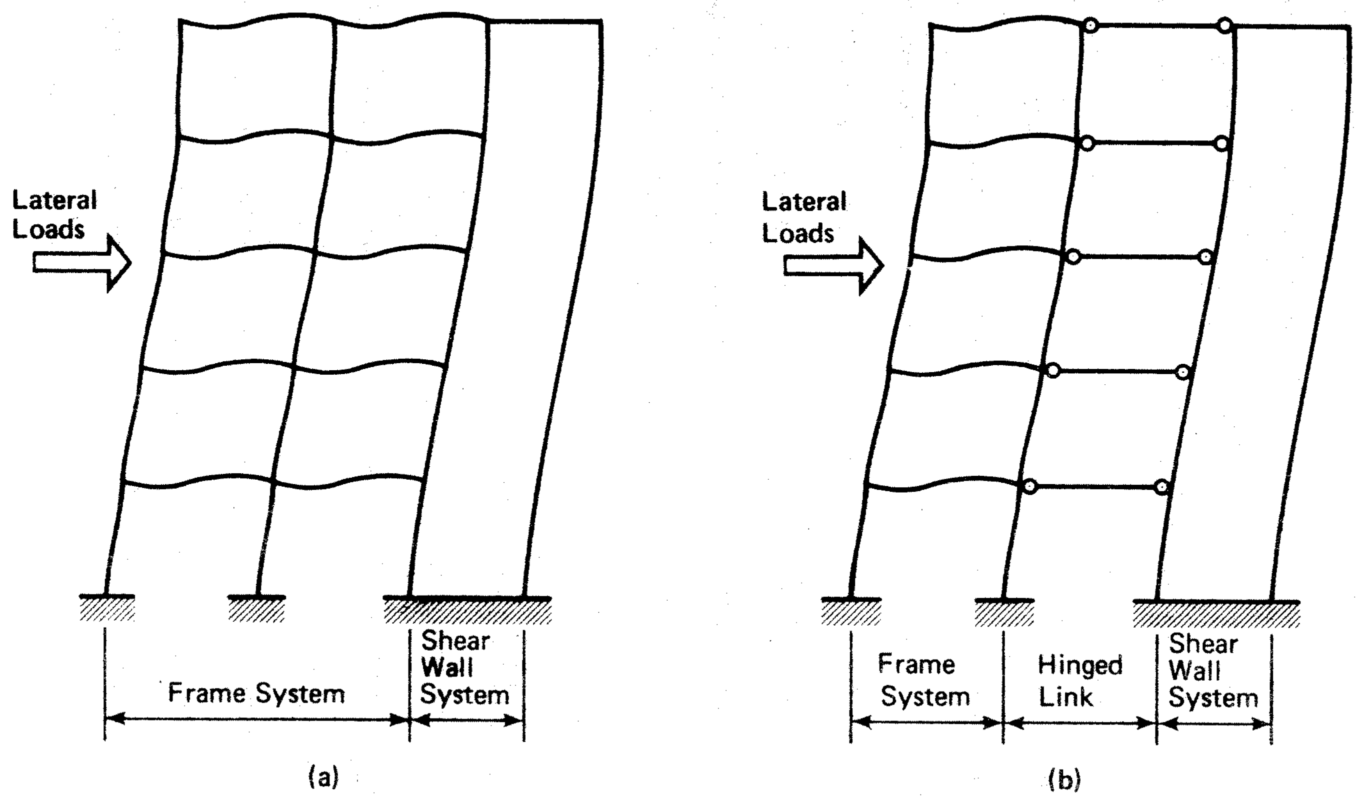


Figure 3.2 Idealized structural model

may be further simplified by using fictitious link bars between the frame and the core as shown in Fig. 3.2(b). These members of infinite area are pinned and simulate the action of the floors in enforcing equal lateral deformations on each of the load resisting elements[11]. For structures having uniform properties over the height of the building, various techniques which model the structure as a continuum are appropriate[12]. For most practical structures, however, the member properties change with height and a discrete approach is used which generally necessitates the use of a large computer. Computer programs using stiffness matrices have been extended to include interacting shear-walls and frames[13,14]. Standard plane frame programs may also be used to analyze mixed structures if the core or shear-wall (with appropriate coupling beams) is replaced by an "equivalent wide column"[15,16]. This simulation is particularly useful in cases where the frame-core connections are rigid so that the width of the core must be considered.

3.1.2 Three-Dimensional Analysis

A torsional analysis is required for symmetric structures subjected to eccentric lateral loads caused by wind or earthquake as well as for structures asymmetric in plan.

If the torsional and lateral resistance of a structure depends primarily on the central core, an analysis of the core acting alone may be appropriate[17,18,19]. Studies of several existing core-braced systems have indicated that a core, with properly interconnected walls, is approximately 10 to 40 times as stiff in torsion as the perimeter steel frame, assuming that the frame is rigidly connected[20].

If, in a special case, the frame system plays a definite role in resisting the eccentric lateral loads, it should be included in the analysis[14,21,22].

3.2 Second Order Analysis

Stability effects have not been considered in the various techniques described in the previous sections[1]. The most important stability or second order effect to consider in an elastic analysis is the P-Δ effect described in section 2.2.

The extra P-Δ moments and shears can be determined by the use of approximate or rigorous second order analysis procedures. A rigorous second order analysis requires the use of a large and expensive computer program which may include wide and stiff elements such as cores[14].

Approximate techniques, however, are generally more desirable than a rigorous analysis. They are of three types:

1. Amplification Factor

The moments and deflections determined by a first order analysis are multiplied by an amplification factor to simulate the effects of the vertical loads acting on the laterally deflected structure[24]. The factor has the form $1/(1-P\Delta/Hh)$, where the terms are defined in Fig. 2.1. The amplification factor technique is appropriate for hand calculations and can sometimes be very useful. However, due to its simplicity, the technique is subject to some restrictions[24] and does not allow directly for the transfer of second order forces; which is of prime concern in this thesis.

2. Fictitious Horizontal Load

The action of a vertical load on a laterally deflected column can be simulated by a horizontal force $P\Delta/h$ applied at the top of the column. This concept has been developed and applied to multi-bay, multi-storey frames for dynamic[25] and static analyses[23,26].

The technique presented in references 23 and 26 is a versatile method which could be used in the analysis of core-braced systems. Since the procedure is recommended by the Canadian Standard S16.1-1974[36] and is the subject of a discussion in the following chapter, the procedure is detailed below:

(a) Perform a first order analysis of the structure under the factored loads to determine the horizontal deflection, Δ_i, at each floor level i.

(b) As shown in Fig. 3.3, compute the artificial storey shears which would produce column end moments equivalent to those caused by the vertical loads:

$$V'_i = \frac{\Sigma P_i}{h_i} (\Delta_{i+1} - \Delta_i) \tag{3.1}$$

where

V'_i = artificial shear at storey i due to sway forces,

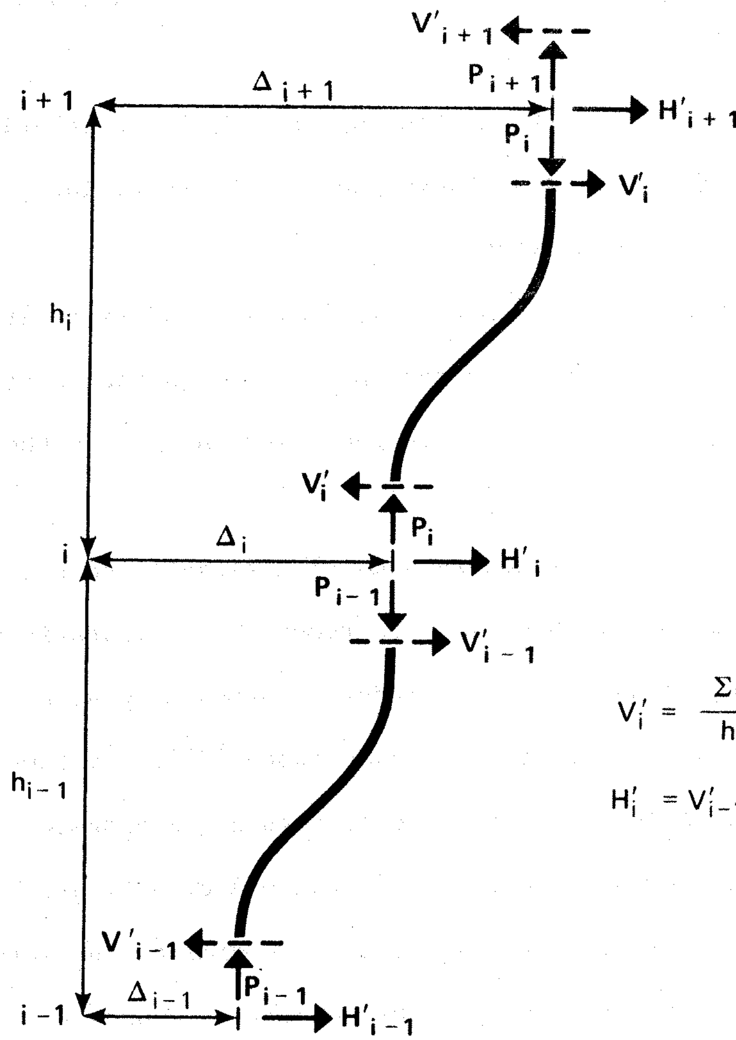
ΣP_i = sum of factored column axial loads at storey i,

h_i = height of storey i,

Δ_i = horizontal displacement of level i with respect to the base of the structure.

(c) Compute the resulting sway forces H'_i at each floor level:

$$H'_i = V'_{i-1} - V'_i \tag{3.2}$$



$$V_i' = \frac{\Sigma P_i}{h_i} \quad (\Delta_{i+1} - \Delta_i)$$

$$H_i' = V_{i-1}' - V_i'$$

Figure 3.3 Sway forces due to vertical loads

(d) Add the sway forces H_i' to the applied lateral loads and reanalyse the structure.

(e) When the deflected shape at the end of a cycle is relatively unchanged, the method has converged and the resulting forces now include the P-Δ effect.

For practical structures, the convergence is fast and the first iteration produces acceptable results. Slow convergence is a sign that a structure is excessively flexible and lack of convergence indicates that the structure is unstable[27].

Since the iterative procedure adds significantly to the cost of the analysis, attempts have been made to reduce the computational effort involved. A suitable deflection index is often used as a basis for the first trial calculation of sway forces[27]. If the resulting deflections of the structure are less than the assumed deflection index, a conservative estimate of the P-Δ effects has been obtained and the results may be used in design. A similar but more rational approach consists of computing the converged second order deflections at each storey from the corresponding first order values by using the amplification factor of method 1[28], thus:

$$\Delta_i'' = \frac{1}{1 - \frac{\Sigma P_i(\Delta_{i+1} - \Delta_i)}{\Sigma V_i h_i}} (\Delta_{i+1} - \Delta_i) \qquad (3.3)$$

where

Δ_i'' = relative second order lateral deflection of storey i (top of storey with respect to bottom),

ΣP_i = sum of factored column axial loads at storey i,

ΣV_i = total first order shear at storey i.

Once Δ''_i is obtained, the fictitious storey shears equivalent to the converged V'_i of Eq. (3.1) are calculated by

$$V'_i = \frac{\Sigma P_i \Delta''_i}{h_i} \tag{3.4}$$

Equation (3.4) may be expressed in a more convenient form so that the fictitious storey shears may be calculated directly from the first order results:

$$V'_i = \frac{1}{\dfrac{1}{\dfrac{\Sigma P_i(\Delta_{i+1} - \Delta_i)}{h_i}} - \dfrac{1}{\Sigma V_i}} \tag{3.5}$$

The sway forces are then evaluated by Eq. (3.2) and applied at each storey to give the converged second order forces and moments directly.

It has been suggested in Ref. 23 that where the vertical loads act alone, the initial forces be computed on the basis of sway deflections equal to 0.002 times the storey height. The initial deflections are equal to the maximum out-of-plumbs permitted during erection of the structure according to Ref. 7. This implies that the structure would be erected with initial imperfections and that the vertical loads acting through the corresponding lateral deflections would produce initial P-Δ shears and forces. This will be discussed in detail in Chapter V.

3. Simplified P-Δ Method

A method which allows a second order analysis to be performed by using a first order computer program (without iteration) has been presented recently[29].

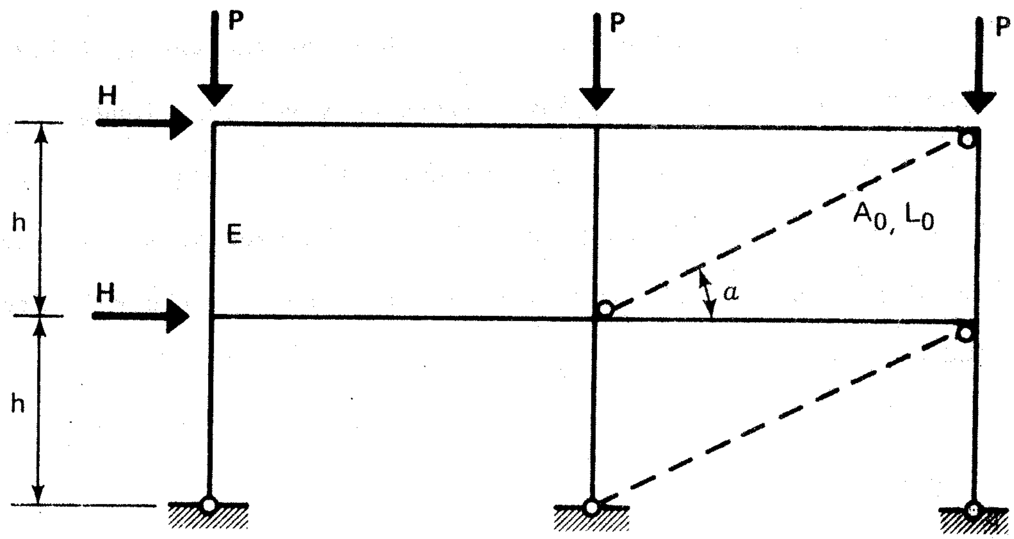
A fictitious "negative bracing" member having an area equal to

$$A_0 = -\frac{\Sigma P}{h}\frac{L_0}{E\cos^2\alpha} \tag{3.6}$$

is added to the frame at each storey as shown in Fig. 3.4. In this equation, ΣP is the sum of the factored column axial loads at a specified storey, E is the modulus of elasticity of the columns, and h, L_0, and α are defined in Fig. 3.4. The "braced" frame is analysed by a standard first order program and the results obtained represent the second order deflections, moments, and forces in the members. The number of equations remains the same as for a first order solution and the computer input is unchanged except for the addition of the extra bracing members. The horizontal component of the force in the bracing member of level i is the converged sway shear V_i' of the previous technique.

3.2.1 Two-Dimensional Analysis

If the frame system lacks significant stiffness, the P-Δ shears corresponding to the total gravity loads on the structures, as well as those caused by wind or earthquake, must be resisted by the core alone. The methods described in section 3.1 along with the second approximate technique of section 3.2 can be used in a second order analysis. If the frame does possess significant rigidity, the core-frame interaction must be considered. Again, an appropriate method outlined in section 3.1 can be used jointly with an approximate technique (no. 2 or 3) in a second order analysis.



$$A_0 = \frac{-\Sigma P}{h} \quad \frac{L_0}{E \cos^2 a}$$

Figure 3.4 Negative bracing

3.2.2 Three-Dimensional Analysis

The assessment of stability effects in three-dimensional structures is complex[30]. The stability aspects of three-dimensional building frames are discussed in Ref. 31, but among the analytical methods presented, only a few apply directly to core-braced structures. A rigorous technique for a stability analysis is presented in Ref. 32 for structures similar to that of Fig. 1.1. It is assumed that the core resists torsion, bending, and its share of the vertical loads, and that the frame is pinned and resists only vertical loads.

As for the plane case, the deflections computed from first order three-dimensional analyses[14,22] may be used to assess the deflections, moments, and forces in a structure with the P-Δ effects included[31].

3.3 Design

One option in designing for stability is the traditional effective length or "K factor" method[7,33].

In this technique, the bending moments obtained from a first order analysis are used directly in the design of the girders and the moments and forces in the columns are adjusted through the interaction equations to arrive at suitable column sections. In regular moment resisting frames the P-Δ effects in the columns can be accommodated indirectly by basing the member selection procedure on the sway permitted condition[26].

As applied to a core-braced structure, a blind application of this procedure is inappropriate. The presence of a stiff concrete core is used to justify the assumption that the columns are sway prevented. The effective length factor for the columns is then equal to or less than unity[23]. In order to be consistent with this assumption, the core must be designed to resist all of the P-Δ effects. The second order effects in a core-braced building are properly accounted for by applying the P-Δ forces to the whole structure (if the frame is moment resistant) or to the core alone (if the frame is simply connected) at each floor level[23,26,34]. This procedure amounts to a second-order analysis and may be performed using the techniques described above.

When the P-Δ forces have been included in the analysis, the columns and the core may be designed for the sway prevented case[23,26,34] in which the effective length factor is equal to or smaller than unity[11].

CHAPTER IV

SECOND ORDER FORCE DISTRIBUTION

The use of an approximate second order analysis, as described in section 3.2, results in an improper distribution of the horizontal forces among the resisting elements.

It is intended in this chapter to formulate a simple technique for the redistribution of second order horizontal forces to be used in the design of connections and floor diaphragms.

4.1 Basic Force Transfer

Three situations are encountered in a typical framed structure. They are schematically represented in Fig. 4.1:

1. Columns and beams in a bent develop their own resistance to lateral loads.

   In the case of a regular unbraced frame with moment resisting connections as shown in Fig. 4.1(a), no extra shears are to be transferred by diaphragm action at the beam level and extra forces need not be transferred through base connections unless the columns have different stiffnesses or carry different axial loads. The beams and columns in this type of structure are subjected to extra P-Δ moments. Any column splice should be designed for an extra shear since there is an increase in the shear within the columns.

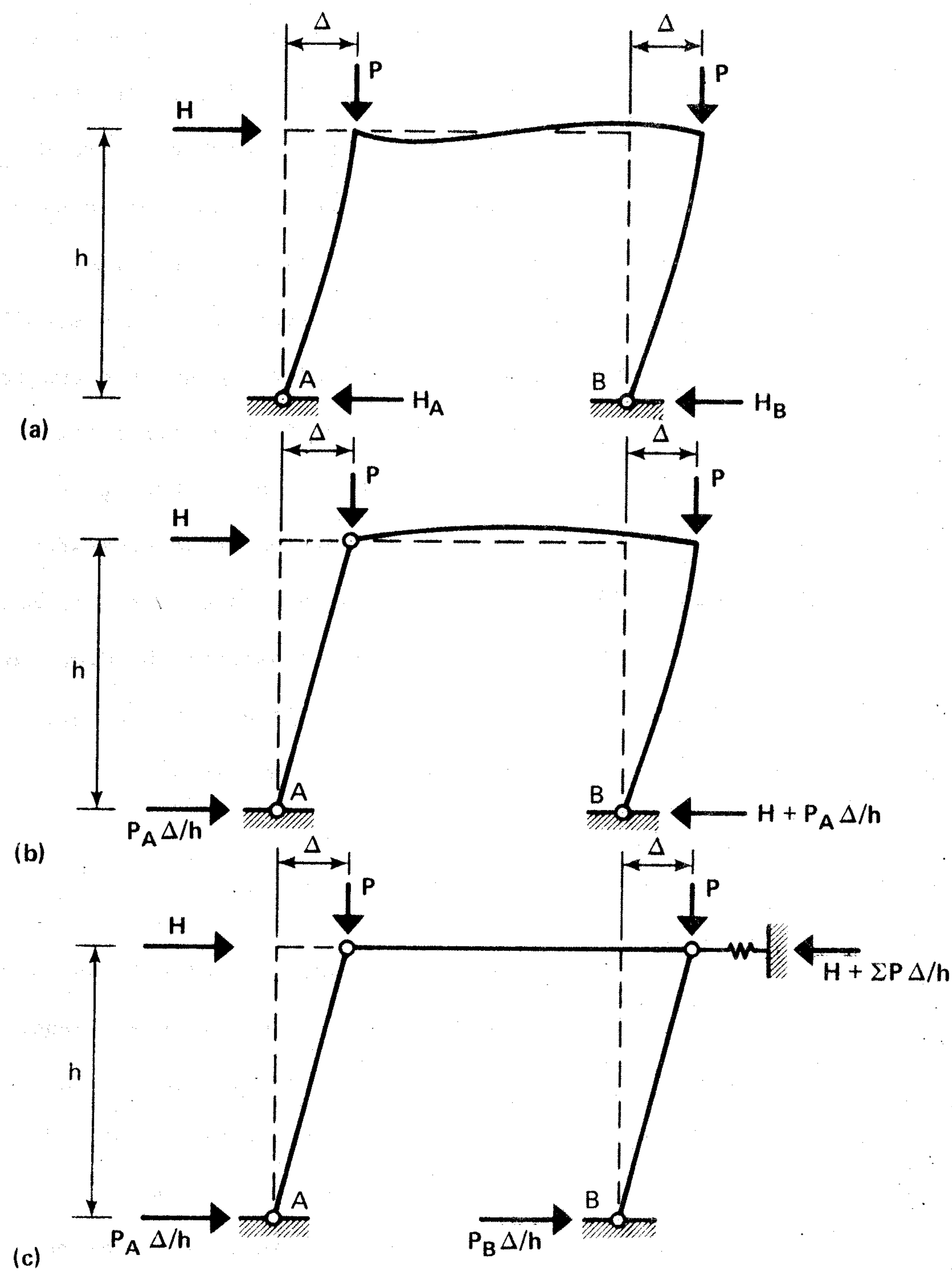


Figure 4.1 Basic second order force transfer

2. A few columns only develop resistance to lateral loads. Connection A for the simple frame of Fig. 4.1(b) must now be designed for a shear of value $P_A\Delta/h$. The three other connections, however, have to resist a force equal to $H+P_A\Delta/h$. Since the shear resistance is assumed to be provided within the bent, no additional diaphragm action is required.

3. Pinned frame relying on a stiffer structure for its stability. When the columns in a bent are all simply connected and rely on a bracing system in the plane of the bent for lateral support, the forces are transmitted directly through the beams and the connections to the vertical bracing system. If the supporting system is not in the plane of the frame, a horizontal force $H+\Sigma P\Delta/h$ must be transmitted by diaphragm action as shown in Fig. 4.1(c). The joints at the base must be designed for the appropriate $P\Delta/h$ shears.

4.2 Problems in Approximate Second Order Techniques

All horizontal forces and shears, such as axial loads in beams, shears in columns, and horizontal forces in joints are generally incorrect if generated by fictitious storey forces applied at one face of the structure. This is due to the transfer of the second order forces through a particular bent. Errors in the analysis could be avoided by applying the extra shears for the individual columns at the top of each column but this is an impractical solution. Both the fictitious horizontal load technique and the simplified P-Δ method of section 3.2 have this limitation. The problem is clearly demonstrated in Fig. 4.2 where the results of an exact and an

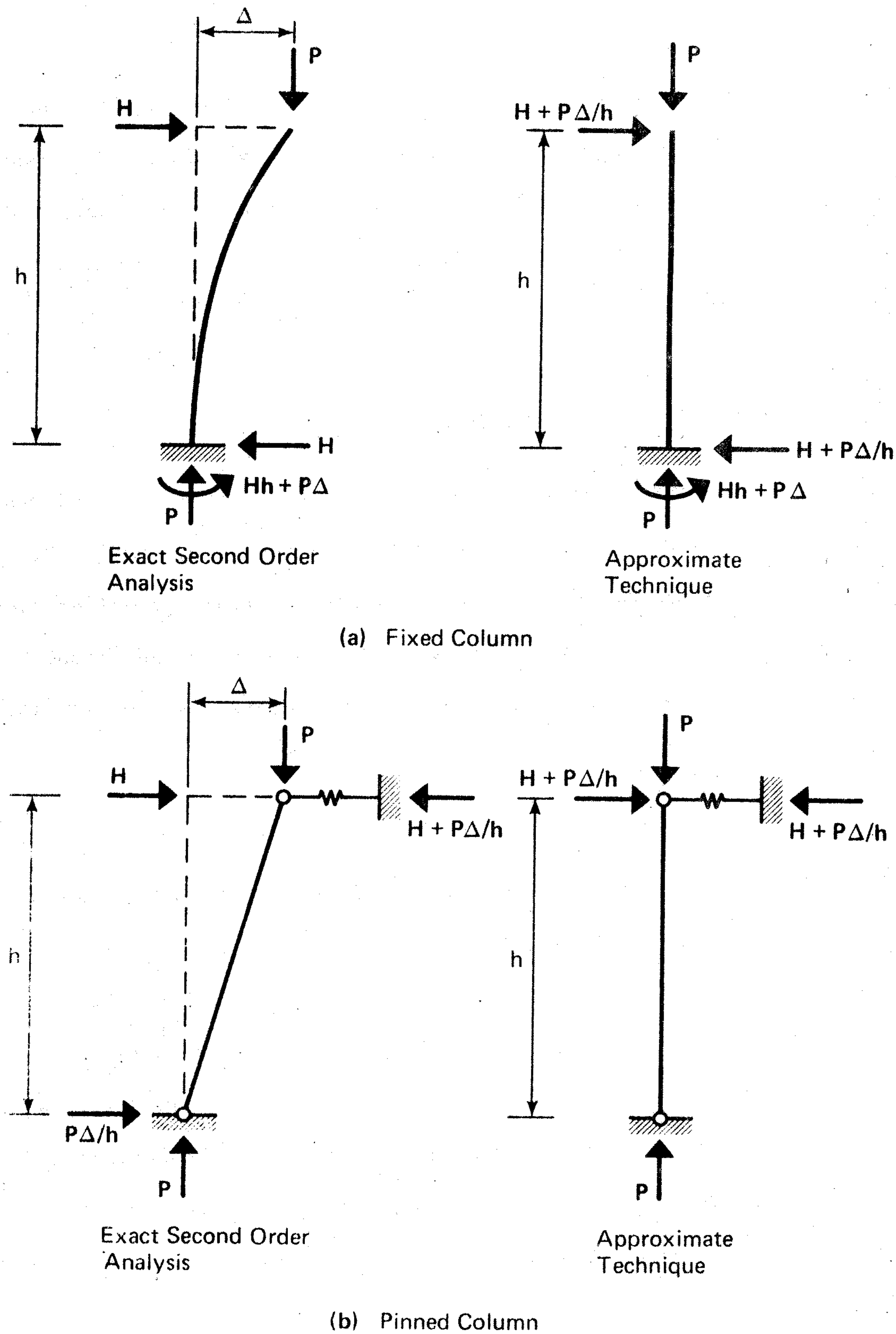


Figure 4.2 Exact vs. approximate second order analyses

approximate second order analysis are compared for two different column end conditions.

In the case of a moment resisting frame, the shear at the base of a column is larger than the actual shear by a factor $P\Delta/h$ when a simplified technique is used (Fig. 4.2(a)). In a simple frame, the actual shear $P\Delta/h$ at the column base is not predicted by the approximate method (Fig. 4.2(b)).

4.3 Force Distribution by Free-Body Diagrams

Since correct second order moments, deflections, and column axial loads are obtained from an approximate second order analysis, the corresponding column shears can be determined from a consideration of column free-body diagrams. The contribution of each individual column to the second order storey shear is calculated by equation (4.1).

$$V'' = \frac{\Sigma M'' - P\Delta}{h} \tag{4.1}$$

where $\Sigma M''$ = the sum of the column end moments M_1'' and M_2'' shown in Fig. 4.3.

Once the shears in the columns above and below a specific floor in a plane frame are known, equilibrium of all the horizontal forces acting on the floor, including the externally applied lateral loads, can be ensured. The horizontal shears in the connections and axial loads in the beams, calculated in this manner, are correctly distributed. An example illustrating this procedure will be presented in section 4.5.2.

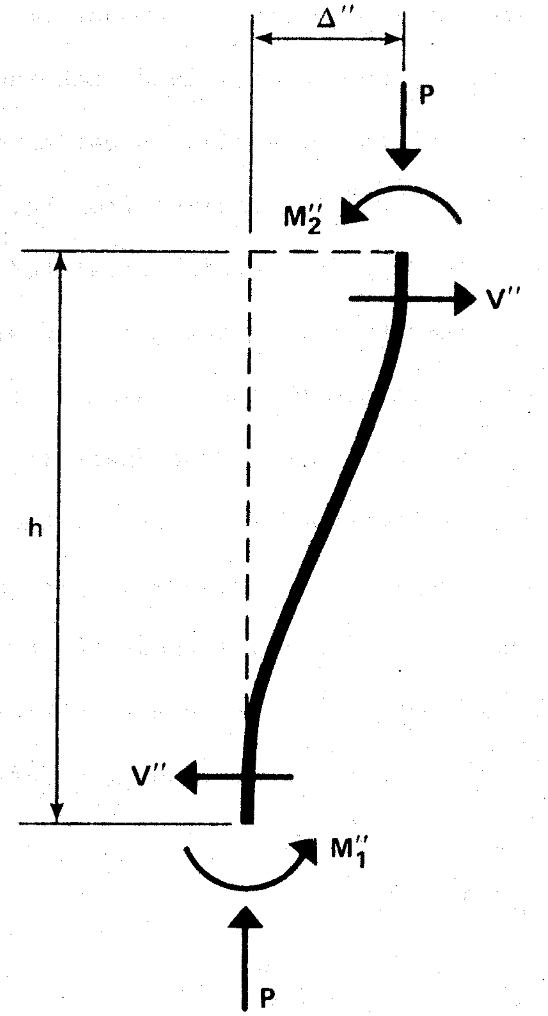


Figure 4.3 Column free-body diagram (positive directions shown)

4.4 Alternate Force Distribution Technique

This section presents an alternate method for the redistribution of horizontal forces which gives nearly the same results as the exact method but requires a reduced amount of calculation.

When the fictitious lateral load technique or the simplified P-Δ method has been used, the artificial shear at each floor level is a known quantity. This shear is obtained from Eq. (3.1) in the fictitious lateral load technique or is calculated as the horizontal component of the fictitious bracing force in the simplified P-Δ method. Also known is the sum of the column axial loads at each storey, since this quantity has been used in the analysis.

Regardless of the type of structure, the extra P-Δ shears in the columns, at a specific storey, are proportional to the axial loads in the columns. The artificial shear V'_i at storey i (the sum of the individual column P-Δ shears for this storey) is then redistributed among the columns according to the following equation:

$$V'_j = \frac{P_j}{\sum_{j=1}^{n_i} P_j} V'_i \qquad (4.2)$$

where

V'_j = share of fictitious shear in column j

j = column index

i = storey index

n_i = total number of columns at storey i

P_j = factored axial load in column j

The exact second order shear corresponding to the shear of Eq. (4.1) is obtained by taking the algebraic difference between the shear (in error) obtained in the approximate analysis, V^a, and the corresponding shear, V', from Eq. (4.2).

$$V''_j = V^a_j - V'_j \tag{4.3}$$

As before, after the actual shears are calculated in the columns above and below a specific floor level, the horizontal forces at each beam-to-column connection at that level are calculated from the equilibrium of the joint. An example of the application of this technique is given in the next section.

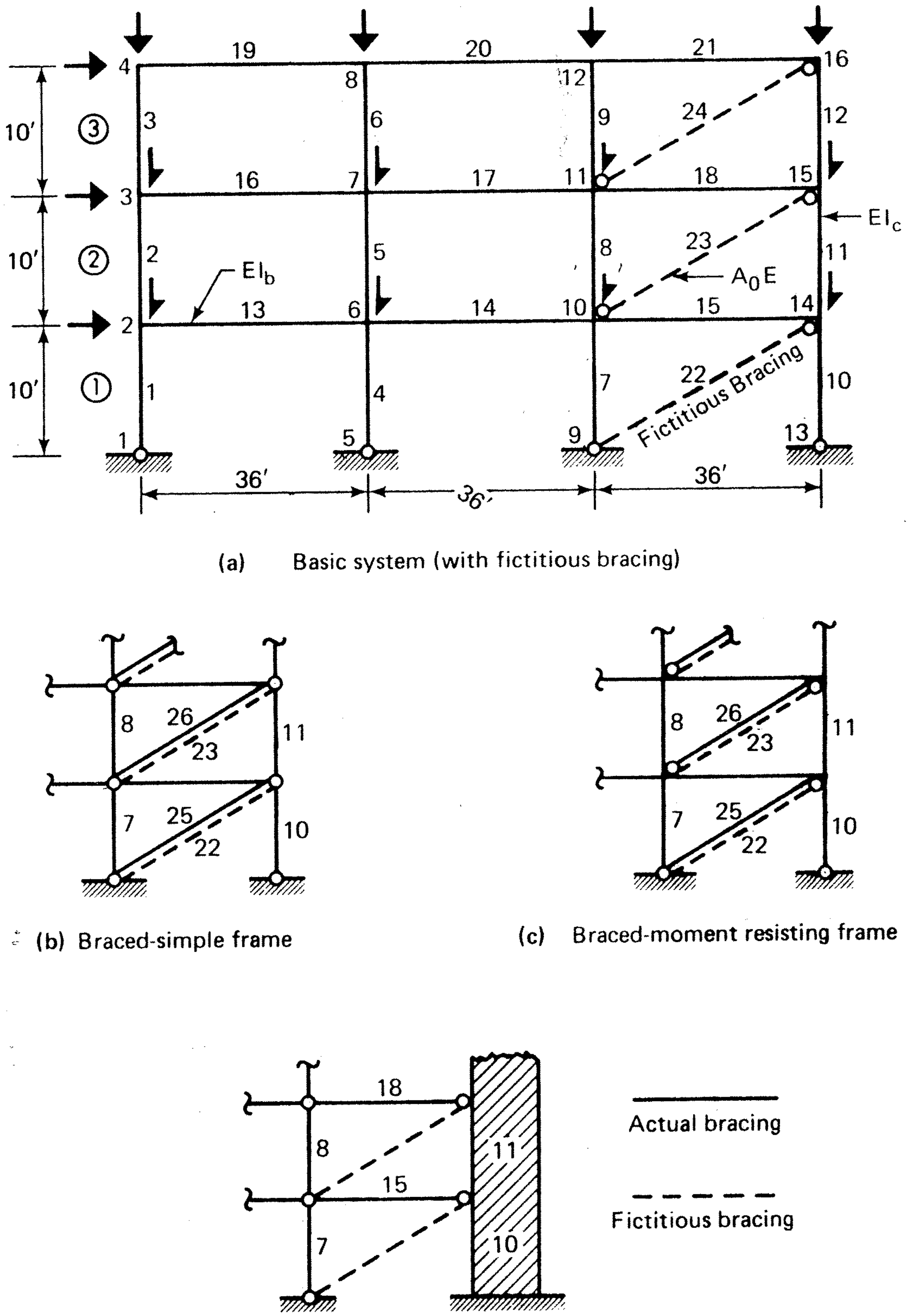
The type of redistribution described above is only useful in the design of beam-to-column connections, column splices, and floor diaphragms for the extra forces produced by the sway of the structure.

4.5 Analysis of Framing Systems

In order to justify the application of the alternate distribution technique described above, different structural systems have been studied using the simplified second order technique of section 3.2(3). The results of the distributions have been compared with the exact values.

4.5.1 Structural Systems

The simple three-bay, three-storey plane frame shown in Fig. 4.4(a) was selected as the basic structure in this study. The cross-sectional area of the structural members (except the bracing members) were increased so that the effects of axial deformations were



(a) Basic system (with fictitious bracing)

(b) Braced-simple frame

(c) Braced-moment resisting frame

(d) Pinned frame and shear-wall

Figure 4.4 Framing systems

eliminated. The gravity and lateral forces were slightly increased from the design values for the frame in order to emphasize the P-Δ effect. All loads were applied at the joints to simplify the study. The different types of structures analysed were:

1. An unbraced frame with moment resisting connections (Fig. 4.4(a)). The gravity loading was unsymmetrical and the column stiffnesses varied.
2. A pin-connected frame braced by a steel truss (Fig. 4.4(b)).
3. A moment resisting frame with one braced bay (Fig. 4.4(c)).
4. A pin-connected frame braced by a shear-wall (core) (Fig. 4.4(d)).

4.5.2 Example

The distribution of the P-Δ forces calculated by the above technique, as applied to the first two storeys of the braced frame shown in Fig. 4.4(d), is given in detail in Fig. 4.5. The first order forces and the second order forces (before and after correction) are given for comparison. The horizontal components of the axial loads in the fictitious bracing members are respectively 7.92 kips and 11.49 kips for the first and second storeys. The force in the first storey is lower since the base of the shear-wall is fixed. The column shears calculated according to Eqs. (4.2) and (4.3) are summarized in Table 4.1. The last column of the table lists the column shears calculated according to Eq. (4.1), based on the results of the second order analysis. The comparison shows that the alternate distribution technique applies properly in this case.

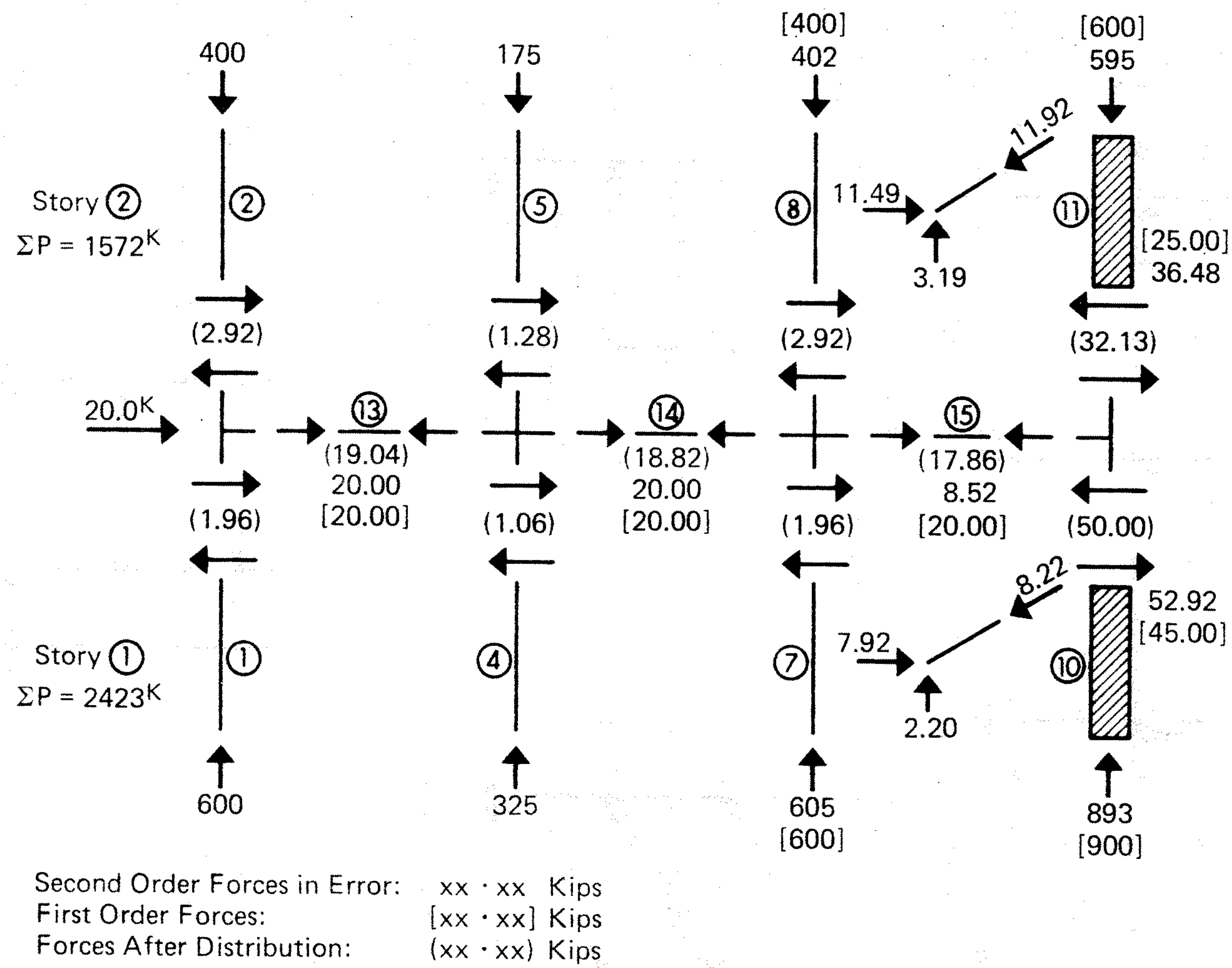


Figure 4.5 **Force distribution in a pinned frame braced by a shear-wall**

Column	$P_j/\Sigma P_j$	Fictitious Shear (Eqn. 4.2) kips	Exact 2nd Order Shear (Eqn. 4.3) kips	Exact 2nd Order Shear (Eqn. 4.1) kips
1	0.248	1.96	-1.96	-2.00
4	0.134	1.06	-1.06	-1.08
7	0.248	1.96	-1.96	-2.02
10	0.368	2.92	50.00	49.94
2	0.254	2.92	-2.92	-2.96
5	0.111	1.28	-1.28	-1.29
8	0.254	2.92	-2.92	-2.97
11	0.379	4.35	32.13	32.08

Sign convention given in Fig. 4.3.

TABLE 4.1 SHEAR DISTRIBUTION

4.5.3 Results and Observations

The alternate technique was successfully applied to all the structures described in Section 4.5.1. For all practical purposes, the column shears calculated in this manner were found to be the same as those computed by Eq. (4.1).

It has been observed that in a moment resisting frame of normal proportions, such as that shown in Fig. 4.4(a), the exact second order horizontal forces differ only slightly from the corresponding first order forces. Thus the first order horizontal forces can be safely used for the design of connections and floor diaphragms. The horizontal force distribution for the second order forces can then be avoided. However, if the fictitious storey shears, V_i', appear to be relatively important at some levels and if the column stiffnesses differ significantly, the exact horizontal forces should be determined at these levels.

Some discrepancies between the first and second order axial loads in the columns adjacent to the fictitious bracing members are observed in Fig. 4.5. The difference is explained by the presence of the vertical components of the bracing member axial loads. These forces are 2.2, 3.19, and 1.8 kips respectively for the first, second, and third storeys of the structure. Although this effect has been exaggerated in the example, it is still negligible, since the errors account for less than one percent of the actual axial loads in the columns.

As demonstrated in Fig. 4.2(b), the P-Δ shears in pin-connected columns are not simulated by an approximate second order analysis. They should therefore be distributed as described above whenever the total storey shears are found significant.

The axial loads in the actual bracing members of the trusses shown in Fig. 4.4(b) and (c) are not in error when an approximate second order analysis is used. The exact distribution, in this case, is predicted by the analysis and is a function of the relative stiffnesses of the resisting elements at each storey. In this type of structure, the fictitious bracing member should be placed in parallel with the actual bracing member in order to determine directly the amount of shear to be distributed among the columns.

The frame-shear-wall system of Fig. 4.4(d) exhibits the same force distribution characteristics as the braced frame of Fig. 4.4(b).

CHAPTER V

STABILITY INFLUENCE OF OUT-OF-PLUMBS

Very few studies have been conducted on the effects of column and wall out-of-plumbs. North American data are virtually non-existent but a few studies have been published in Europe (Sweden) and in Russia, where the influence of material and geometric imperfections has been investigated. It seems appropriate to review the existing literature at this stage of the study in order to introduce the many features peculiar to out-of-plumbs. In the first three sections of this chapter, the basic design principles are presented, a survey of column and wall erection tolerances is given, and the degree of accuracy achieved in plumbing is discussed. This information is useful to understand the final section, which reviews the code recommendations of several countries on stability and force transfer related to out-of-plumbs.

5.1 Basic Principles

It has long been recognized that columns and wall elements (prefabricated or cast-in-place) are erected with unintentional deviations from the vertical. Undesirable moments and horizontal forces can result where out-of-plumb elements are subjected to vertical loads.

Very few design codes adequately consider the stability of tall buildings in view of the geometric imperfections of columns and walls. In those design regulations which do consider the problem, the effects of imperfections are considered principally in two different ways[35]:

1. The effect is assumed to be included among other uncertainties and covered by the safety factor.
2. The effect of certain prescribed imperfections, such as eccentricities or out-of-plumbs, is calculated and combined with other actions by means of fictitious loads or added eccentricities.

The two approaches above are generally used in combination so that when a code prescribes a specific value for a geometric imperfection, the value need not be representative of the imperfection itself. It may be that the prescribed value includes other uncertainties. Moreover, it is possible that the effects of geometric imperfections, such as member out-of-plumbs, cannot be correctly described by a simple parameter due to the statistical nature of the imperfection.

Under the above circumstances, a comparison of the different regulations must be performed with care. This is particularly true when, for instance, regulations derived for precast concrete structures are compared with equivalent regulations applied to steel structures. The degrees of accuracy attained in the fabrication and erection of steel and concrete members are remarkably different.

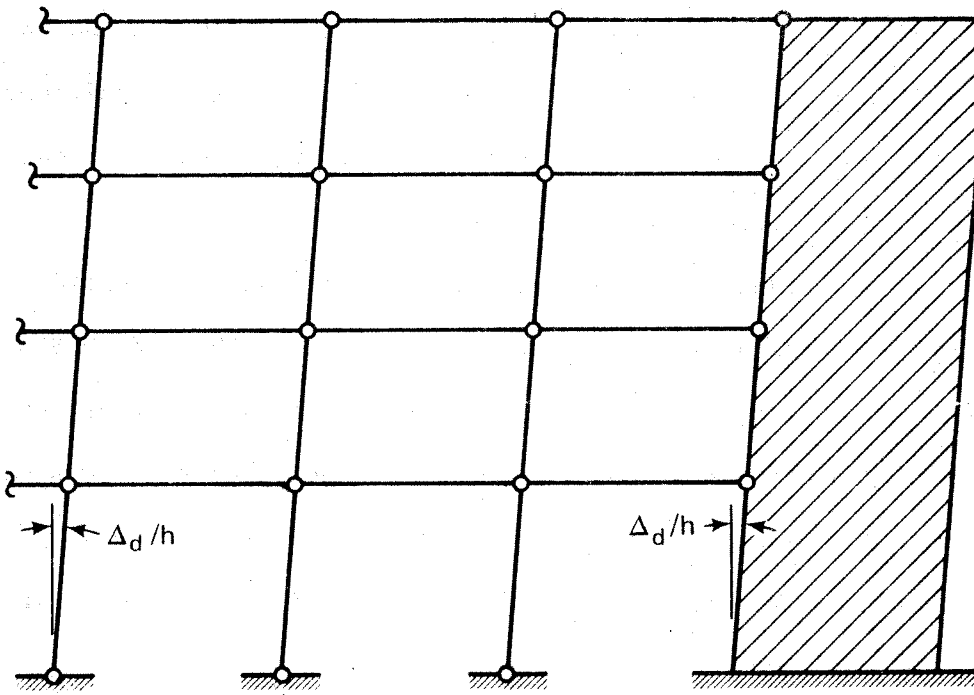
In a design for the overall stability of a structure, all load-bearing columns or walls are generally assumed to lean in the same direction, as shown in Fig. 5.1(a). A similar critical situation can be created for the design of individual structural components if the model assumed in Fig. 5.1(b) is used. The connections and floor system at a specific level must be able to resist the horizontal force resulting from accidental inclinations of the columns above and below the floor. The inclinations in the two storeys are oriented so that the resulting forces add together.

On the basis of the models shown in Fig. 5.1, the calculations required to determine the forces produced by out-of-plumbs are simple and generally conservative. For these reasons, such models have formed the actual basis of several design prescriptions. It is highly unlikely, however, that these idealistic situations will occur in practical structures. The random nature of the variable should therefore be accounted for.

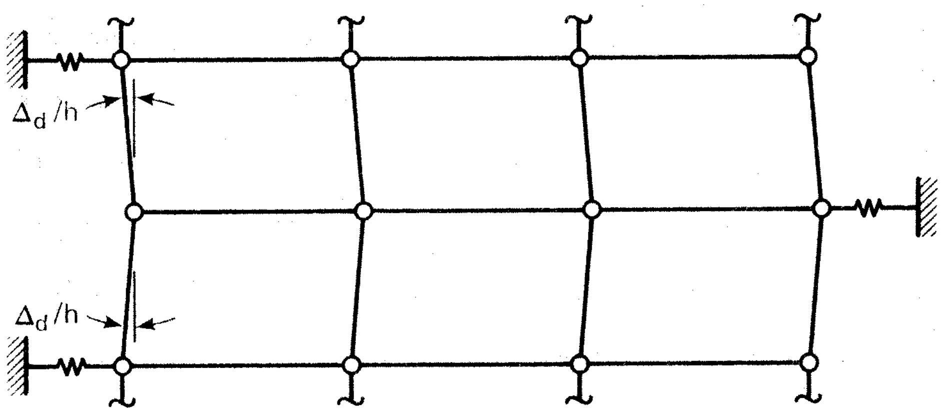
5.2 Erection Tolerances

In order to achieve a certain standard in construction, the builders must comply with a set of specific erection tolerances. These tolerances vary from code to code depending on a number of factors.

The Canadian and American standards for steel construction[7,36,33], for instance, recommend that all exterior columns of multi-storey buildings be erected with an accuracy of 1 to 1000 but not more than 1 inch towards nor 2 inches away from the building line in the first 20 storeys plus 1/16 inch for each additional storey up



(a)



Δ_d/h = Design out-of-plumb (slope)

(b)

Figure 5.1 Assumed deformed shapes

to a maximum of 2 inches towards or 3 inches away from the building line. The columns adjacent to elevator shafts should also satisfy the 1 to 1000 limit but in addition should be out-of-plumb by no more than 1 inch in the first 20 storeys plus 1/32 inch for each additional storey up to a maximum of 2 inches. All other columns are considered plumb when the error does not exceed 1 to 500.

Tolerances for concrete columns and walls taken from four different standards are given in Fig. 5.2[37]. The Swedish SBN-S25:21 (Publ. No. 25) regulations are intended for precast columns, while the Hus AMA-72 values shown are intended for both precast and cast-in situ columns and walls. The ACI (347-68) tolerances shown are valid for cast-in situ work, while the British (BSI PD 6440) values are intended for both cast-in situ and precast structures.

There is usually a difference between the actual dimensions of an element and those specified. This difference is frequently much larger than anticipated[38]. For example, two studies carried out within the past nine years, one in Britain and one in Denmark, showed that with precast concrete units, the actual deviations in dimensions could be as large as two to three times the tolerances specified. It is assumed that the deviations occurred despite serious efforts to minimize inaccuracies. This same observation could be applied to the actual out-of-plumbs of steel columns and concrete walls.

In general, the standard of accuracy thought to be attainable in building construction appears to be much higher than that actually attained in practice[38]. Such a statement implies the need for designers, specification writers, and constructors to be more realistic about the situation. Designers may have to accept the present standard of

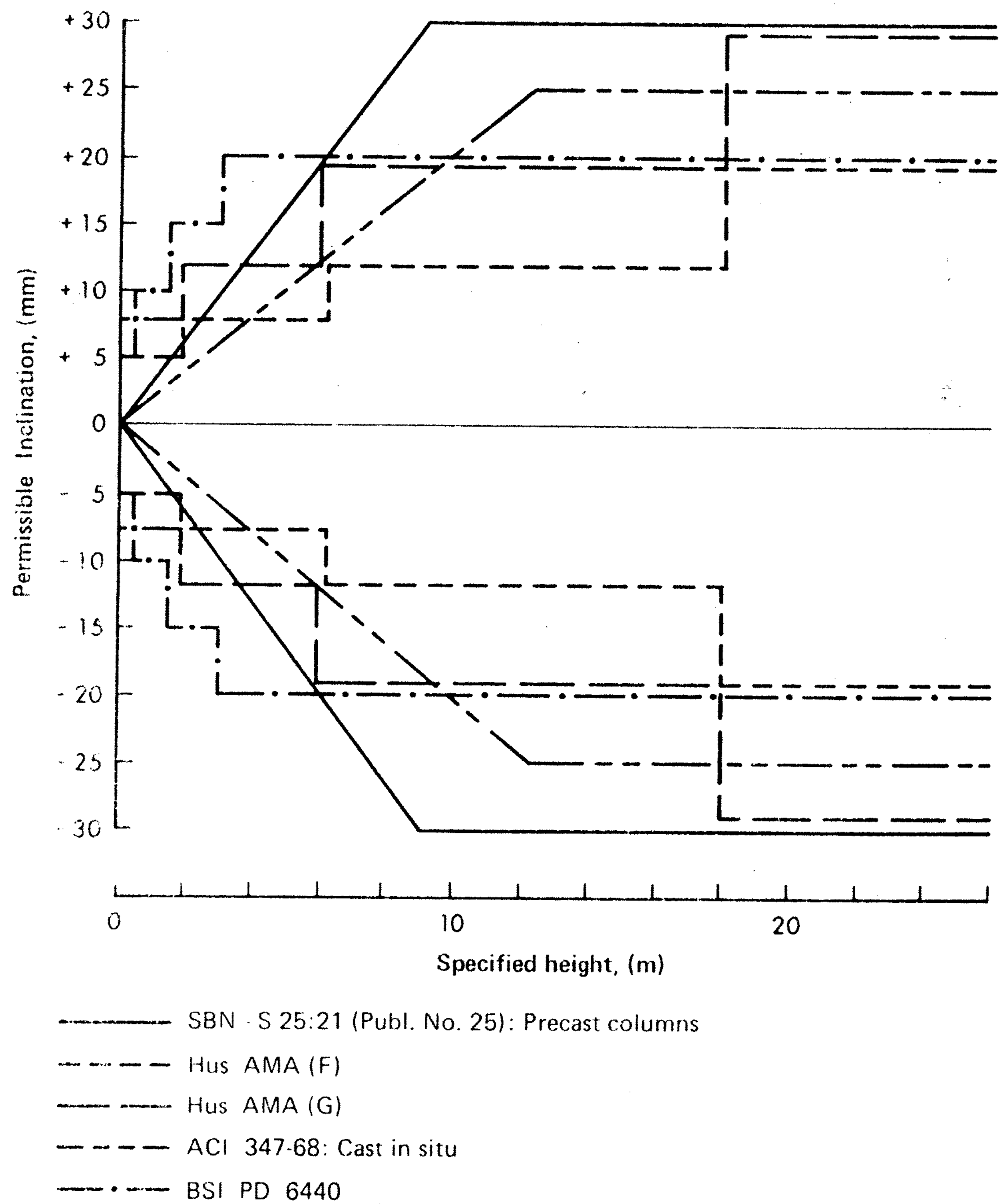


Figure 5.2 Erection tolerances for concrete columns

inaccuracy as a "fact of life" and design all facets of the building in such a way that errors can be accommodated.

Specification writers as well, should not call for unnecessarily tight tolerances. This practice could lead to an excessive number of rejections or to protracted arguments. From the viewpoint of economy, it is often advantageous to specify the largest tolerance which satisfies the structural, construction, functional, and esthetic requirements. If the tolerance limits are too loose, these performance criteria will not be fulfilled. On the other hand, if the tolerance limits are too tight, the cost will increase. In the extreme case it may be physically impossible to satisfy the limits.

Finally constructors should reexamine their present practices with a view to minimizing inaccuracies.

5.3 Accuracy in Plumbing

Ref. 38 contains a review of the actual methods of plumbing the columns in a structure and discusses the degree of accuracy obtainable with each method. The methods depend upon the height involved and the degree of exposure to wind.

Heavy plumb-bobs are used for heights up to 10 ft. Special precautions, such as immersion of the bob in a liquid and protection of the line against wind, are taken for heights up to 30 ft. The accuracy of this method is thought to be ±1/8" in 10 ft. For greater heights, transits are generally used in plumbing by setting up the instrument away from the element to be plumbed and elevating the telescope to sweep the full height of the element. The accuracy attained is ±1/8" in 100 ft. A greater accuracy (±1/6 in 100 ft.)

could be attained with special optical plumbing devices.

In practice, however, the method used and the precision obtained in plumbing determine only partially the final degree of plumbness of the members. Ref. 39 illustrates this fact by attributing the deviations from plumb to four causes:

1. Deviations in the horizontal placement of the columns.
2. Deviations in the lengths of the beams spanning between the columns.
3. Errors in measurement techniques.
4. The assembly procedure, where work at later stages of assembly disturbs columns which have previously been adjusted to their "final" positions.

To this list might be added the initial crookedness of the columns themselves, as well as the adequacy of the hardware used for holding the elements in position during assembly. Any corrections after the erection stage described in item 4 are either impractical or, if applied, could weaken the structure.

A study of differences in the out-of-plumbs of precast wall panels during four stages of construction has been reported in Ref. 53. The dispersion of the measurements was as follows:

Stage	Description	Standard Deviations*
1	Elements placed by crane	0.0050
2	Element position after plumbing	0.0011
3	Elements fixed in position with mortar	0.0014
4	Element position after floor slab above has been placed	0.0016

* Appendix A gives a brief discussion of the standard deviation and other essential statistical concepts.

A similar observation has been reported in Chapter 2 of Ref. 41 as applied to steel structures: "It should be recognized that the displacements of the structural steel frame change during the erection. That is, the displacements of a lower storey are considerably affected by the subsequent erection of storeys above it. Therefore, it is important that the geometrical imperfections should be referred to the finished structure, after including heavy floors, vertical panels, etc."

5.4 Design Recommendations

The design recommendations for stability of structures presented in this section are those directly related to structural out-of-plumbs. Other stability considerations are not mentioned.

5.4.1 European Recommendations (European Convention for Constructional Steelwork)

The ECCS[40,41,42] maintains that for the columns in a multi-storey frame to be considered braced, the bracing system should be adequately designed to resist the direct effect of the factored horizontal loads, plus the full destabilizing (P-Δ) effect of the factored gravity loads acting on the swayed structure, plus the effects of an assumed out-of-plumb equal to 1/200 of the height of each storey or of the structure as a whole. The latter requirement refers to an extra set of horizontal forces derived from a deformed structure such as that shown in Fig. 5.1(a), where all columns lean in the same direction at an angle of 0.005 Rad. The recommendations do not mention any specific requirements for the transfer of forces due to out-of-plumbs among the resisting elements.

5.4.2 Canadian Recommendations

The Canadian Standard S16-1969[7] contains specifications for the stabilization of individual columns but does not consider the overall stability of the structure. It is specified that columns shall be braced in order to develop their full load carrying capacity. The bracing must be proportioned to resist at least 2 percent of the axial load in the column at the brace location unless a suitable analysis is carried out to determine the appropriate strength and stiffness of the bracing members[43]. The rigidity is assumed to be sufficient if the strength requirements are met, but braces must be securely anchored. For example, braces should not be attached to a structure equivalent to that being braced unless the whole assemblage is trussed in order to prevent concurrent buckling[43].

The "2 percent rule" is also widely used in the United States. This provision is based primarily on the results of the investigation described in Ref. 44. The study assumed that the member to be stabilized was an axially loaded column, two storeys in height, having an initial crookedness of h/500 at mid-height, as shown in Fig. 5.3.

Under the assumption that an additional deflection equal to the initial out-of-plumb would occur as the column approaches the buckling load, the stiffness of the supporting structure required to force the column to buckle with a node point at the mid-height support was determined and the equivalent support force computed. The lateral force is approximately 2 percent of the axial force in the column. The North

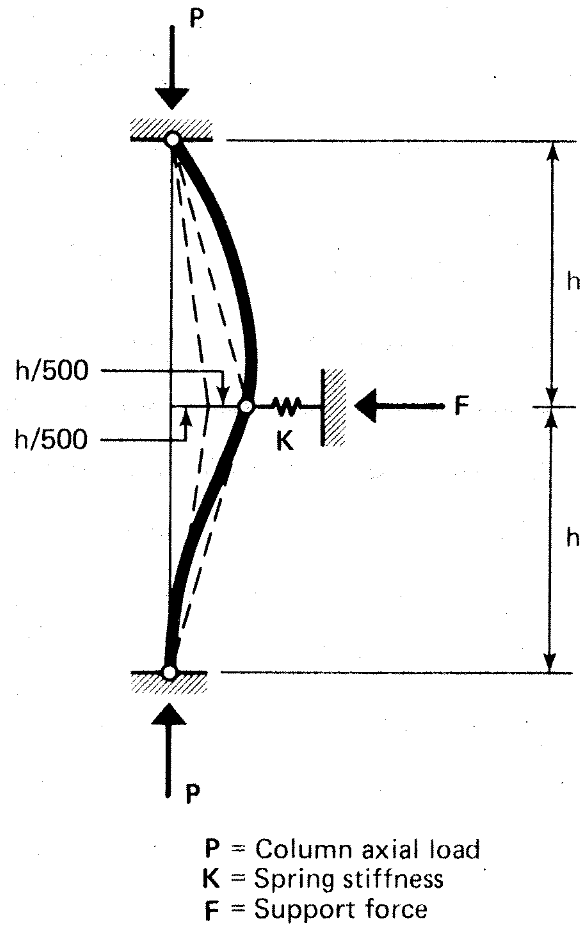


Figure 5.3 Model for computation of bracing force

American specifications for column bracing are then essentially based on buckling considerations.

In view of the severity of the assumptions involved, the requirements have been relaxed somewhat in the recent standard S16.1-1974[36] to provide the designer with a more flexible approach based on the maximum prescribed erection tolerances for columns. The bracing requirement has thus been reduced from 1/50 of the column axial load (2%) to either 2/1000 or 2/500, depending on the erection tolerance.

Ref. 36 also contains recommendations for overall structural stability and design of floor diaphragms. It is stated that the sway effects produced by the vertical loads acting on the displaced structure (second order effects) should not be less than those produced by the vertical loads acting on the structure assumed displaced an amount equal to the maximum out-of-plumbness consistent with the specified erection tolerances[23,34]. In other words, the structure must at least be stabilized against a set of lateral loads computed from the model shown in Fig. 5.1(a) inclined with a constant slope of 0.002 (1/500) Rad.

The deflected configuration of Fig. 5.1(b), in which the column slopes are in opposite directions in adjacent storeys, may produce local effects at a given floor level which govern the design of beam-to-column connections, diaphragms and other elements[23]. The girders and their connections must be capable of transferring the forces due to the column out-of-plumbs, along with the applied lateral loads, to the bracing systems. The provision does not clearly specify whether the second order (P-Δ) forces are considered to be included by this procedure or must be transferred in addition to the two forces mentioned above.

5.4.3 British Recommendations

According to the British Standard Code of Practice[46] (Addendum No. 1 1970 to CP 116) which relates to structures utilizing precast load-bearing wall panels not less than a single-storey in height, both the local and general effects of the out-of-plumbs must be taken into account in design. The code requires that, in the absence of firm data, an allowance of $0.4\sqrt{m}$ (in.) at the top of a building should be made for the calculation of the forces, where m is the number of storeys in the building.

5.4.4 West German Recommendations

In the analysis of buildings according to the 1971 West German code for reinforced concrete, DIN 1045[28,47], the designer must consider the global stability effects caused by horizontal loads, eccentric gravity loads, and unintentional eccentricities of the vertical loads. The latter are computed assuming an unintentional inclination of all vertical elements equal to[34]:

$$\frac{\Delta_d}{h} = \pm \frac{1}{55\sqrt{h_t \quad \text{(in feet)}}} \tag{5.1}$$

where h_t = the total height of the building.

The individual floors must also be able to resist a horizontal force resulting from accidental inclinations of the columns above and below the floor of:

$$\frac{\Delta_d}{h} = \pm \frac{1}{110\sqrt{h_{avg} \quad \text{(in feet)}}} \tag{5.2}$$

where h_{avg} = the average of the two storey heights.

The inclinations in the two storeys are to be oriented so that the resulting forces add rather than subtract (as per Fig. 5.1(b)).

The Germans, in fact, have adopted the concept described in Fig. 5.1 but have introduced a new element in the form of a variable which decreases the magnitude of the deflected slope in a non-linear manner as the height of the structure increases.

5.4.5 Russian Recommendations

Since the Russian Standards were not directly available for examination, some of the concepts in use in that country have been extracted from selected publications[20,48,49].

The Russian designers are conscious of the fact that significant forces from different second order sources are present in structures and should be considered in design. They recommend, for instance, that the horizontal forces due to column out-of-plumbs, column eccentricities, inaccuracies of manufacturing, etc., be computed and combined statistically. The resulting forces should in turn be transmitted by the floor diaphragms to the bracing system to ensure column stability.

Actual measurements of column deviations from the vertical during erection in a number of structures built in Moscow between 1965 and 1970 reveal that forces due to column out-of-plumbs may be determined by[20]:

$$F = P\varepsilon \qquad (5.3)$$

where P = axial load on the column

ε = total change in slope between two columns at their intersection.

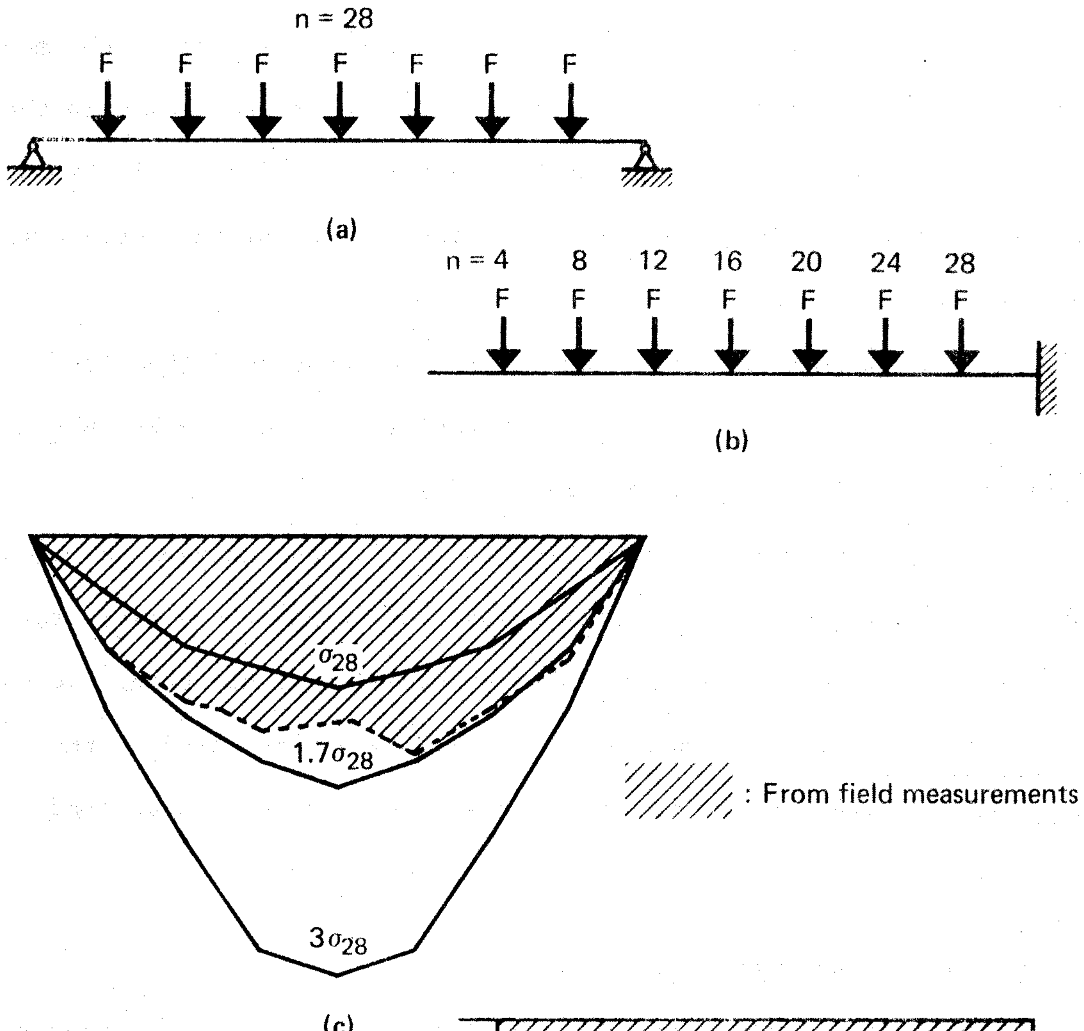
A statistical analysis, based on measurements made on skeletal-type concrete structures, predicts that an equivalent inclination for a group of n connected columns can be obtained by[50]:

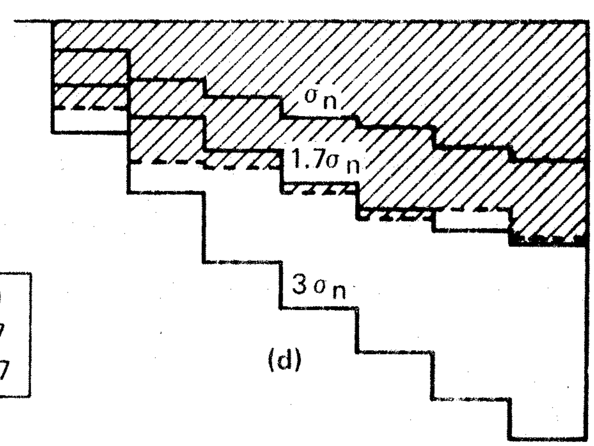
$$\varepsilon_n = \frac{\varepsilon_1}{\sqrt[3]{n}} \tag{5.4}$$

The angle ε_1 in this expression is defined as the maximum inclination of an individual element and is taken as three times the standard deviation obtained from the field data. The prescribed value for ε_1[50] is 0.012 and has been criticized in Ref. 48 as being too large and reflecting an insufficient standard of construction. However, the significance of Eq. (5.4) has been confirmed in Ref. 48 from an observation of 20,000 field measurements on prefabricated multi-storey skeletal buildings.

In addition, an investigation of the forces created by n out-of-plumb columns on the two structural models shown in Fig. 5.4 has shown the existence of a variable safety index[48]. The simple model of Fig. 5.4(a) represents the braced section of a floor diaphragm which is in turn restrained against lateral movement at its extremities. Fig. 5.4(b) shows the same structure restrained at only one end. In both cases, the slabs are loaded by seven concentrated forces at the location of the transverse bents. Each of these bents contains 4 columns for a total of 28 for the entire building.

The bending moment and shear force diagrams obtained from field measurements are indicated by the shaded areas in Figs. 5.4(c) and (d) respectively. These diagrams are compared in the same figures with the corresponding diagrams where the forces are calculated from the expression $F = 4\beta\sigma/\sqrt[3]{n}$ derived from Eqs. (5.3) and (5.4). In this





$$F = \frac{\beta \sigma P}{\sqrt[3]{n}}$$

If, $1 \leqslant n \leqslant 7$, then,		$\beta = 3.0$
If, $7 \leqslant n \leqslant 28$, then,	$3.0 \geqslant \beta \geqslant 1.7$	
If, $n \geqslant 28$, then,		$\beta \leqslant 1.7$

F = Force
β = Safety Index
σ = Standard deviation
P = Axial load in column
n = Number of columns

(e)

Figure 5.4 Variable safety index

expression, the numeral 4 represents the 4 unit loads from the 4 columns, β is a variable safety index, σ is the standard deviation of the measured out-of-plumbs ($\varepsilon_1 = \beta\sigma$), and n is the total number of columns in the structure. For the specific example shown in Fig. 5.4(a), the results obtained from the field data are best approximated by the curve corresponding to $\beta\sigma_{28}$ with $\beta = 1.7$.

From this investigation it has been concluded that for $1 \leq n \leq 7$, $\beta = 3.0$, for $7 \leq n \leq 28$, β varies, and for $n \geq 28$, $\beta \leq 1.7$. These results are tabulated in Fig. 5.4(e).

An observation of the results of 70 such analyses has shown that the use of the variable safety index described above gives for the shears and moments an acceptable probability of not being exceeded in the order of 97 to 99 percent. It will be demonstrated in the following chapters that the safety index can be held constant and serve the same purpose when other variables are considered.

5.4.6 Swedish Recommendations

In the Swedish concrete regulations (B7-1968)[46], hinged columns are assumed to be 0.7 percent (0.007) out-of-plumb. The structural members serving as bracing (e.g. connections and floor diaphragms) must be designed to resist horizontal forces taken as 0.7 percent of the axial forces on the columns.

In the study of the global stability of a structure, however, the total horizontal force acting on the bracing structure is calculated with the columns in one bent inclined at 0.7 percent and the columns in the other bents at 0.35 percent. Thus, for a large number of bents, the average inclination will approach 0.35 percent. This is to account for the probabilistic nature of the forces. The quantities 0.0035 and 0.0070 are, by definition, the initial slopes of the structure of Fig. 5.1(a).

For a design based on the imperfections specified by the regulations, the imperfections should be considered to cause forces which act in the most dangerous manner on the structure. In a laterally braced building, for example, a configuration of interest for the design of the slabs is that in which the floor systems deflect alternately in opposite directions so that the columns form a zigzag line (Fig. 5.1(b)).

The proposed supplement to section 21 of the Swedish Building Regulations[51] contains another set of design provisions written specifically to account for the horizontal forces resulting from the inclination of vertical load bearing elements. The basic content of this supplement is summarized in Fig. 5.5. Provisions are made for:

1. Forces at connections between floor diaphragms and vertical elements (Fig. 5.5(a)).

$$F = 0.017 P \tag{5.6}$$

2. Forces in floor diaphragms at a specified level i (Fig. 5.5(b)).

$$F_i = \gamma\left(\frac{0.024 P}{\sqrt{2n_i}} + 0.003 p\right) \tag{5.7}$$

3. Equivalent horizontal loads on the entire structure (Fig. 5.5(c)).

$$F_{t_i} = 0.015 \gamma p\left(0.2 + \frac{0.8}{\sqrt{m_i n_i}}\right) \tag{5.8}$$

where i = storey index

γ = factor accounting for tolerance requirements and degree of control.

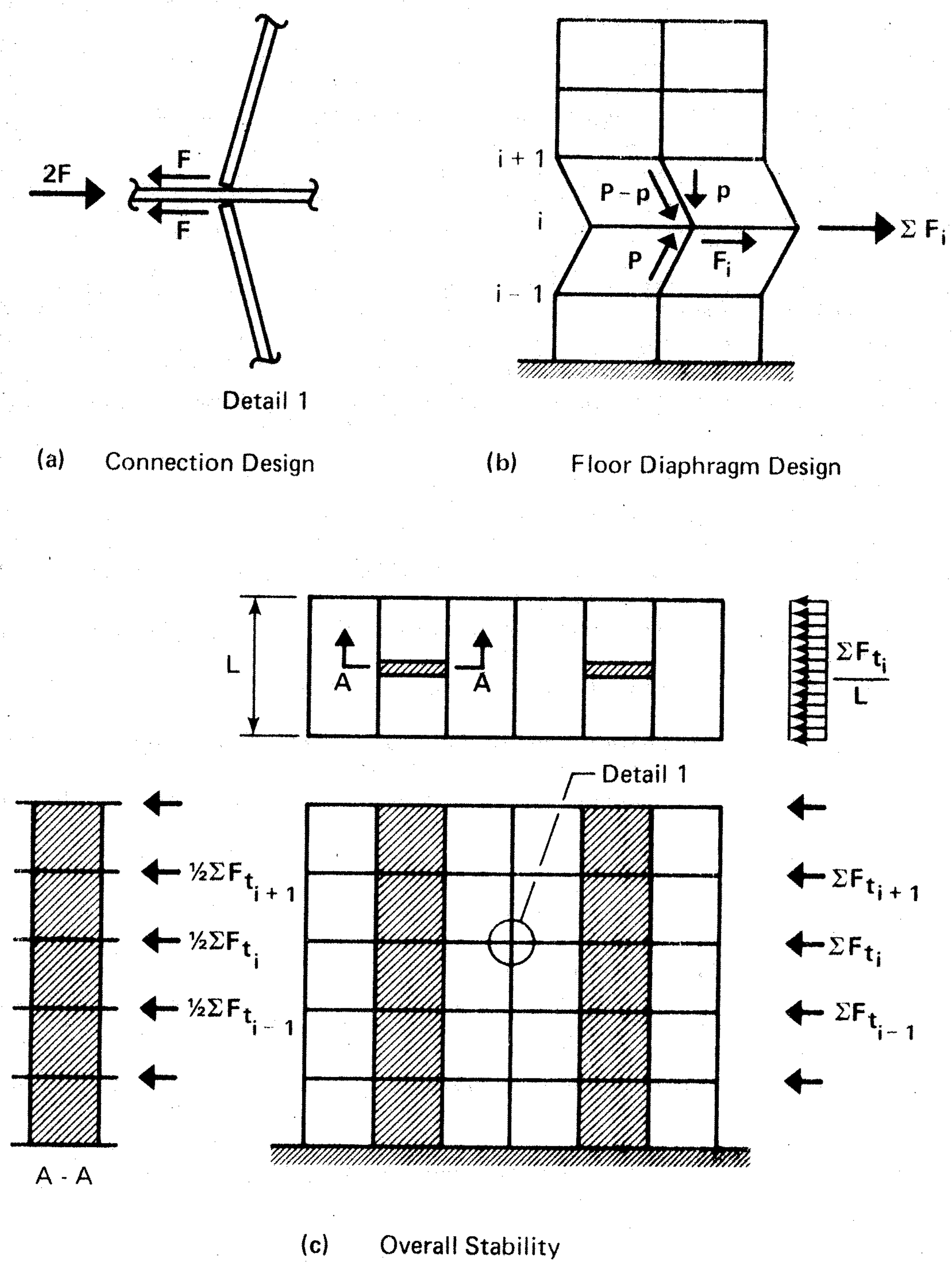


Figure 5.5 Swedish design provisions

P = total load on a vertical load bearing element.

p = load contribution in a vertical load-bearing element from the floor above only

m_i = total number of storeys above floor level i

n_i = total number of load-bearing elements at storey i

It is intended that the loads F, F_i, and F_{ti} should be combined with other anticipated loads, but not with each other.

Based on field measurements, a maximum inclination of 0.015 was assumed for individual elements; 20 percent of this maximum value was taken as a systematic variation and 80 percent was assumed to be random. Statistical results of measurements given in the next chapter indicate that the assumed maximum variation is within the observed values of the mean plus three standard deviations. Assuming a normal distribution for the random variations, the value of the inclination was taken to decrease (with a given probability of being exceeded) with the square root of the total number of load-bearing elements, $\sqrt{m_i n_i}$.

The use of the factor γ recognizes the fact that the specification of tolerances combined with satisfactory control measures can serve to improve workmanship and reduce the variations encountered. For normally specified tolerances, γ is taken as 1.0, but it can be reduced according to the expression:

$$\gamma = \frac{0.0075 + \Delta_d/h}{0.015} \tag{5.9}$$

where Δ_d/h is the maximum permissible deviation over the height of the load-bearing elements. The value of Δ_d/h must be specified in the construction drawings and assembly of the structure must be performed under adequate control, so that the permissible deviation is not exceeded. The expression for γ is based on the assumption that only 50 percent of the maximum deviation can be adjusted during assembly.

Ref. 52 gives the following expression for the horizontal force resulting from the inclination of vertical load-bearing elements in large panel systems:

$$H_e = \frac{b}{450}\left(W + \frac{0.5\,p}{h}\right) \qquad (5.10)$$

where H_e = equivalent uniformly distributed load acting over the entire area of the external walls (kg/m^2)

b = width of the building (m)

W = weight per cubic meter of the structure including partitions

p = superimposed load on a typical floor (kg/m^2)

h = storey height (m)

The derivation of this expression is based on an allowable inclination of 10 mm (0.4 in) per storey height, which is considered valid under average assembly conditions. The value of H_e is combined with the wind load to calculate the horizontal load transferred to the stiffening walls.

5.5 Conclusions

The requirements of several design standards, which attempt to account for instability problems due to column and wall out-of-plumbs, have been presented in this chapter. Most of the requirements are simple in application; others are slightly more involved. Two important conclusions are derived from this study:

1. A statistical analysis is required to correctly represent the actual problem.
2. Only consistently planned field measurements should serve as the basis for the derivation of design equations.

These two observations will guide the course of this research program.

CHAPTER VI

MEASUREMENTS OF OUT-OF-PLUMBS

In the first part of this chapter the results of measurements on column and wall out-of-plumbs available in the published literature are presented. The second part introduces a program initiated for the purpose of this study to record out-of-plumb measurements on buildings under construction. Two tall core-braced buildings and one industrial building were investigated. The results obtained on steel column segments and cast-in situ concrete walls are presented and discussed.

6.1 Reported Measurements

The published measurements refer mainly to precast concrete structures. The results obtained from precast concrete columns can, with a certain reserve, be compared with those obtained from measurements on steel columns. The order of magnitude of the out-of-plumbs and other characteristics differ but several aspects are common to both.

Results of measurements on the state of plumbness of precast column elements and precast walls are presented in Tables 6.1 and 6.2 respectively. Both tables have been extracted from Ref. 37 where the authors' original references are listed. Since the measurements were taken on elements of different heights, the results are shown as two values where possible. The first value represents the total relative lateral displacement while the second (in brackets) expresses the displacement per unit height. The initial displacement is defined as

References	Type of Columns and Locations	Number of Measurements	Arithmetic* Mean	Standard* Deviation
Jacobson and Widmark (Sweden)	Rectangular Columns			
	Ground Storey	20	0.12 [1.2]	0.14 [1.4]
	First Storey	18	0.22 [2.0]	0.15 [1.4]
Van den Berg (Sweden)	Circular Columns			
	Beam Direction	377	-0.02 [0.1]	0.23 [1.9]
	Facade Direction	349	-0.01 [0.1]	0.17 [1.5]
Holmberg, Berner et. al. (Sweden)	Rectangular Facade	34	- [0.1]**	- [0.6]
	Columns	36	- [4.2]**	- [2.7]
	Rect. Interior Col.	15	- [1.2]**	- [3.3]
Klingberg (Sweden)	Square Interior Col.			
	N-S Direction	12	- [0.2]	- [2.3]
	E-W Direction	13	- [2.3]	- [1.9]
Hardwick and Milner (Britain)	Rectangular Columns			
	Ground Storey	30	- -	0.24 -
	First Storey	49	- -	0.28 -

* All values are given in inches except those in brackets which are given in Rad. x 10^3.

** Not reported whether absolute or algebraic values were used.

TABLE 6.1 OUT-OF-PLUMBS MEASURED ON PRECAST CONCRETE COLUMNS

References	Type of Walls and Locations	Number of Measurements	Arithmetic* Mean	Standard* Deviation
Suu** (Sweden)	h = 100"	117	- -	0.25 [2.5]
Van den Berg (Sweden)	Facade Element (2 Measurements per Element) h = 104"	670	0.04 [0.4]	0.14 [1.3]
Klingberg (Sweden)	Apartment Bldg.	-	- -	- [1.6]
	Hospital	-	- -	- [1.5]
Butler (Britain)	Cross Wall Elements h = 96"			
	Block S			
	Ground Storey	24	0.00 [0.0]	0.11 [1.1]
	First Storey	24	0.02 [0.2]	0.15 [1.7]
	Second Storey	24	0.02 [0.2]	0.14 [1.5]
	Third Storey	24	0.04 [0.4]	0.09 [1.0]
	Block T			
	Ground Storey	24	0.02 [0.2]	0.19 [2.0]
	First Storey	24	-0.03 [0.3]	0.19 [2.0]
	Second Storey	24	0.02 [0.2]	0.21 [2.3]
	Third Storey	24	0.03 [0.3]	0.12 [1.3]
	Longitudinal Wall Elements h = 98"			
	Block P			
	Ground Storey	24	0.04 [0.4]	0.13 [1.4]
	First Storey	23	0.04 [0.5]	0.14 [1.5]
	Second Storey	24	0.01 [0.1]	0.22 [2.3]
	Third Storey	24	0.09 [0.9]	0.18 [1.9]
	External Wall Element h = 96"			
	First Storey	19	0.03 [0.3]	0.11 [1.2]
	Second Storey	15	0.08 [0.8]	0.21 [2.2]
	Third Storey	20	-0.07 [0.7]	0.17 [1.8]
	Fourth Storey	15	-0.04 [0.4]	0.13 [1.3]

* All values are given in inches except those in brackets which are given in Rad. x 10^3.

** Absolute values.

TABLE 6.2 OUT-OF-PLUMBS MEASURED ON PRECAST CONCRETE WALLS the horizontal distance between a vertical line passing through the base and the top of the element, as shown in Fig. 2.2.

Suu, in Table 6.2, did not consider the direction of the inclination of the elements, that is, only the absolute values of the out-of-plumbs were recorded. The mean was not given.

Systematic variations in the out-of-plumbness of the elements, that is, mean values different from zero, indicate the tendency of the elements to lean in the same direction. In studying the out-of-plumb variations of various elements it is preferable to specify the direction of the inclinations so that any possible systematic variations can be observed. It is not necessarily exact to assume that the measurements are symmetric about zero. The signs of the mean values in Tables 6.1 and 6.2 are given to indicate that the direction of the inclinations was reported.

A study of initial deviations of precast reinforced concrete columns in fifteen industrial buildings located in southern Sweden is presented in Ref. 56. Some of the buildings were completed while others were under construction. The column lengths varied from 118 to 550 in. The deviations of the top of the columns from a plumb line passing through their base was measured in two perpendicular directions using photogrammetric techniques. The results are summarized in the histogram of Fig. 6.1. All deviations were taken as positive.

Out-of-plumb measurements on a multistorey braced steel frame are given in chapter 2 of Ref. 41. The measurements were presented in the form of histograms for longitudinal and transverse displacements at specific storeys of the building. The calculated standard

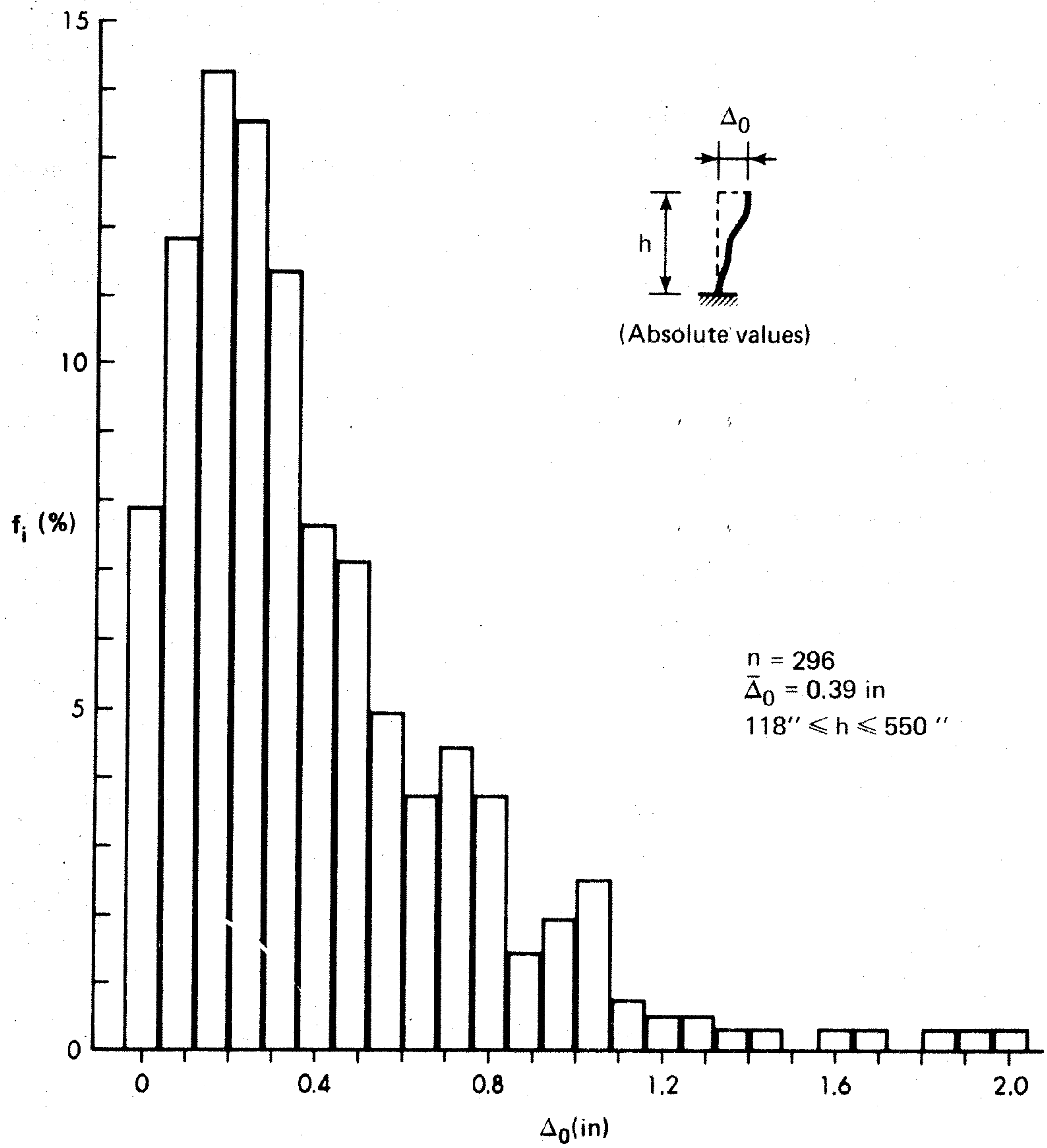


Figure 6.1 Out-of-plumbs observed on precast concrete columns (Ref. 56)

deviations varied from 0.12 in. to 0.43 in., primarily because of the small number of observations recorded. A significant portion of the data was found to exceed the prescribed erection tolerances.

The processing of more than 20,000 field measurements on precast concrete columns in Russia resulted in a reduced angle* of 0.0047 Rad. for column out-of-plumbs[48]. The population was found to be normally distributed and to follow the predictions of Eq. (5.4). Results of measurements on precast concrete columns are also presented in Ref. 49 but the interpretation of the data is confusing.

The columns in three different reinforced concrete buildings in the United States of America were measured to determine the characteristics of the column out-of-plumbs[57]. The purpose of this investigation was to analyze the effect of such variations on the unintentional eccentricity of loading for reinforced concrete columns. An interpretation of the published results was required to derive the values listed in Table 6.3. In the investigation, only the absolute values of the variations were recorded.

It can be concluded from these surveys that the out-of-plumb population generally follows a normal distribution and that the orientation of the deviations is an important factor. When the orientation of the deviations is accounted for, the mean of the population has a tendency to be small and the standard deviation ranges between 0.001 and 0.003 Rad.

---

* The exact nature of this variable is not well defined in the original Russian publication.

Building No.	Number of Measurements	Mean* (Rad.)	Standard* Deviation (Rad.)
I	104	0.00274	0.00277
II	36	0.00216	0.001
III	106	0.00233	0.00152

* Absolute Values

TABLE 6.3 OUT-OF-PLUMBS MEASURED ON REINFORCED CONCRETE COLUMNS (REF. 57)

6.2 Buildings Investigated

A total of three buildings were investigated for the purpose of defining the statistical characteristics associated with the out-of-plumbs of steel columns and concrete walls.

Buildings A and B are tall core-braced buildings while building C is a large industrial building The results obtained in the latter are used to verify if the column out-of-plumb distributions and statistical characteristics observed on core-braced buildings pertain only to this type of structure.

The cross-section and dimensions of building A, a 27-storey core-braced structure, are given in Fig. 6.2. The structure is bisymmetrical and has a central concrete core and 16 steel columns per storey up to level number 14. The core width is reduced and two more columns are added from this level to the roof. The storey heights are uniform at 12 ft. with the exception of the first and last storeys which are 20 ft. high. All connections are simple and are assumed hinged. The composite floor system consists of steel beams, steel floor deck, and concrete topping.

The layout of the 34-storey building B is given in Fig. 6.3. The structure is non-symmetrical and has a rectangular core consisting of nine orthogonal walls. The cross-section of the core is reduced at storey No. 20 where wall No. 3 is removed. Storey heights and column locations are given in Fig. 6.3. The connections are considered to be hinged and the floor system has the same characteristics as the floor system of building A.

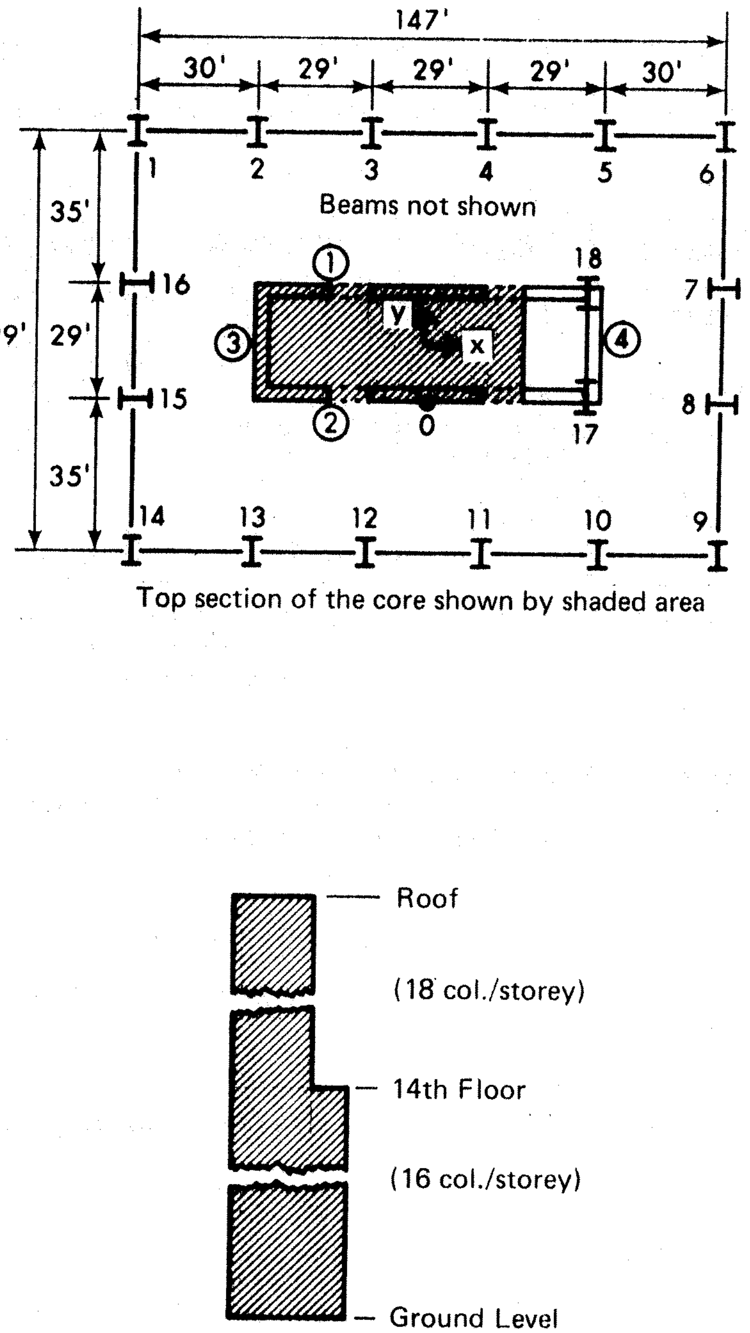
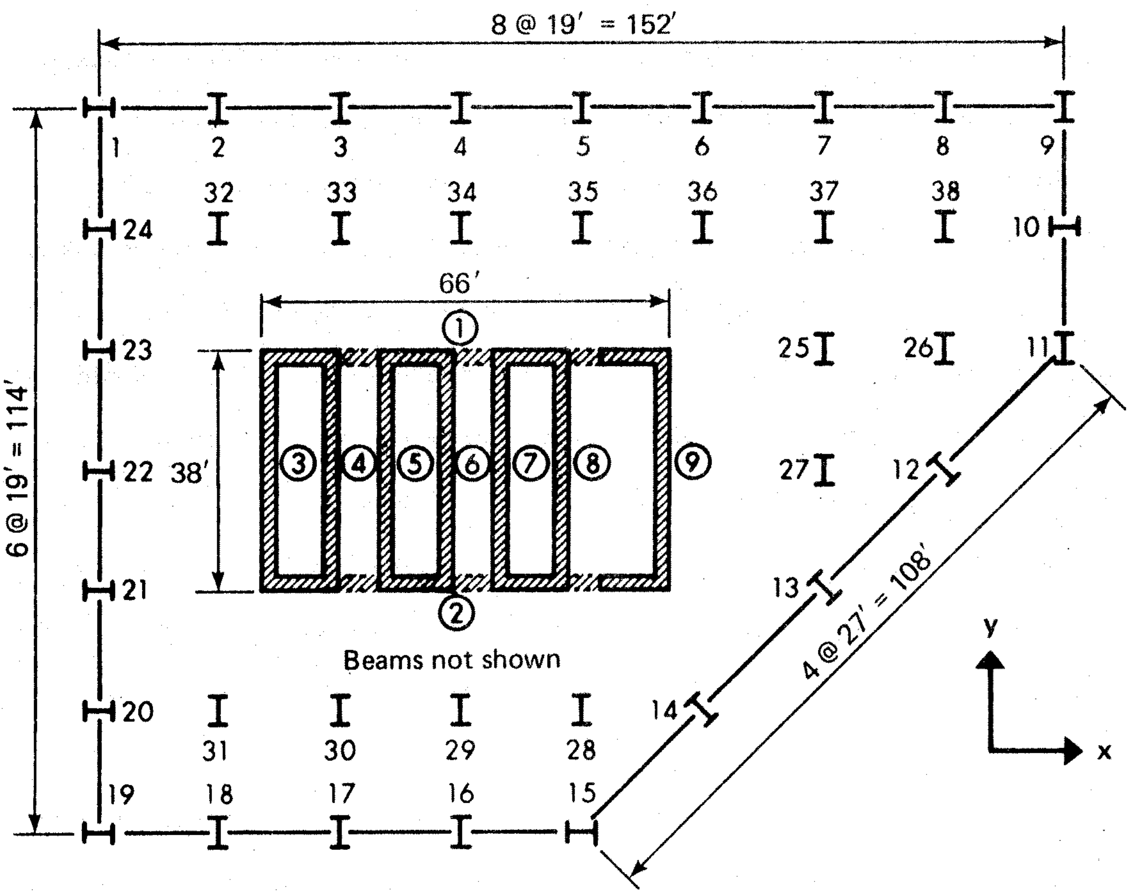


Figure 6.2 Layout of building /\. **(27-storey building)**



Core section reduced at storey 20
(Wall #3 removed)

Storey	Column Locations
1 and 2	1 to 38 inclusive
3	1 to 31 "
4	1 to 26 "
5 to 34	1 to 25 "

Storey	Height
1 and 2	15′
3	16′
4	27′
5	24′
6 to 32	12′
33	14′
34	24′

Figure 6.3 Layout of building B (34-storey building)

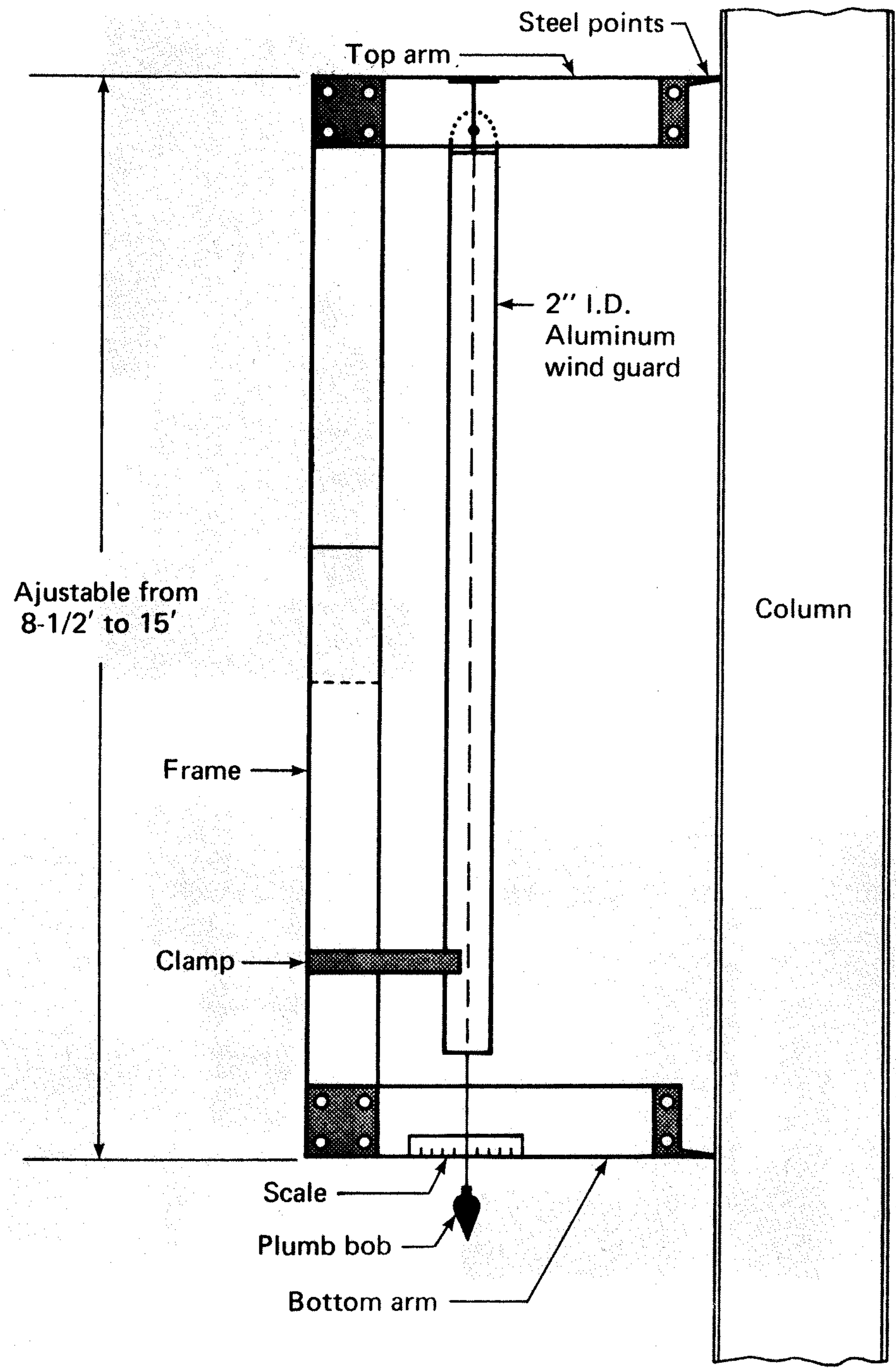
6.3 Method of Measurement

6.3.1 Column Measurements

The values of interest in this study are the deviations of the columns from the vertical, after the structure has been completed. A column does not take its final position until the surrounding structure is definitely fixed in place. A reading taken immediately after plumbing is thus meaningless. Since measurements of the columns in their final positions are not generally recorded by construction companies, additional measurements were taken by the research staff. For the structures under investigation, the out-of-plumbs were measured during building erection after the columns had been plumbed but before fireproofing had been applied. At this stage, the structures were nearly complete and most of the dead load of the building was in place.

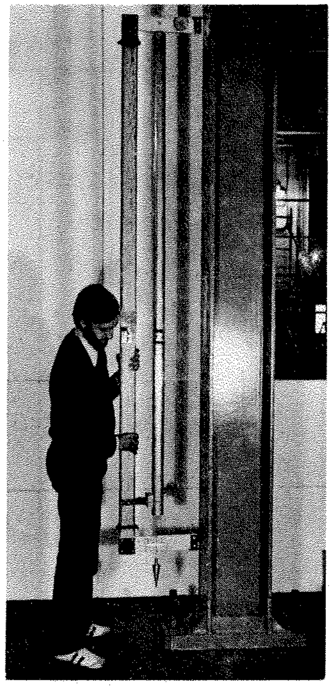
In the case of building A, some measurements taken by the surveyor on the construction site were used since they were taken with a transit after the columns had been bolted into place.

A frame together with a string and a plumb bob was used to measure the column out-of-plumbs. The frame, shown in Fig. 6.4, consists of a main mast with rigid arms at the top and bottom ends. The plumb bob string is attached to the upper arm while the bottom arm supports a scale. The distance from the steel point on the upper arm to the plumb bob string is exactly equal to the distance from the steel point on the lower arm to the center of the scale. Since wind does affect the readings, an aluminium wind guard was added to eliminate this effect.

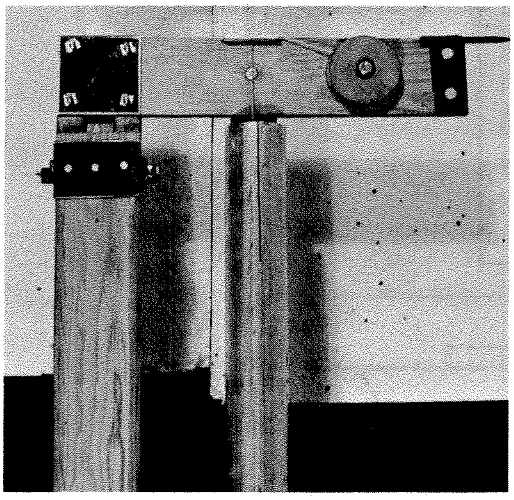


(a)

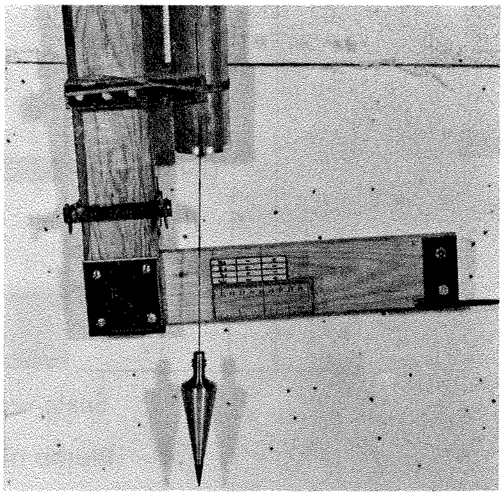
Figure 6.4 Measuring rod



(b) Measuring rod



(c) Top detail



(d) Bottom detail

Figure 6.4 (continued)

The rod is placed against the column and the resulting out-of-plumb is read directly off the scale (Fig. 6.4(b)). For wide flange columns, the rod is held at the web centerline for the reading in one direction and on the outside of the flange-to-web junction for the second reading, when possible. These precautions are taken in order to avoid the measurement of initial twists or other member defects not associated with column out-of-plumbs. To avoid the measurement of initial column curvature, the rod is extended to the full length of the column, if possible.

6.3.2 Core-Wall Measurements

The instrument described above was also used to measure the deviations of core-walls from plumb. Since a wall is not uniformly out-of-plumb at every section, several measurements must be taken at regular intervals along the wall. The out-of-plumb of the wall is then defined by the average of these values.

6.4 Results of Measurements

6.4.1 Column Out-of-Plumbs

Two measurements were taken on each column along the x and y axes, oriented as shown in Figs. 6.2 and 6.3. By definition, a value is positive when the top of a column leans in the positive direction of the axis. For statistical purposes, it is convenient to use the non-dimensional form Δ_0/h to describe the out-of-plumbs. In this manner, column segments of different heights can be considered together.

It is not realistic to present the measurements taken on each column individually. The amount of data is too large and the analysis of individual variables has no significance. As an example, the results of measurements taken on the columns in the top four storeys of building A are given in Table 6.4. The magnitude and sign of the results reflect the random nature of the geomegric imperfection.

The means and standard deviations estimated from Eqs. (A-14) and (A-16) of Appendix A were calculated for buildings A and B in two orthogonal directions at each storey. These quantities, obtained from less than 30 measurements at a time, are meaningless for the present purposes. The absolute values of the means were found to vary between 0.0001 and 0.0018 Rad. and the standard deviations between 0.00065 and 0.0025 Rad.

The probability densities* for the sample population of column out-of-plumbs from building A are given in Table 6.5. The frequency, f_i, defined as the number of results falling in an interval i, is given separately for the measurements in the x and y directions and then for the total population. The resulting histograms can be normalized by dividing the number of elements in a given interval by the product of the total number of elements, n, and the width of the interval, w. The frequency is then expressed as a percentage per unit length and the area of each histogram is 1 or 100 percent.

The results of Table 6.5 for the total 916 measurements are plotted in Fig. 6.5. The distribution has a mean, $\overline{\Delta}_0/h$, of -0.44×10^{-4} Rad. and a standard deviation, σ_c, of 0.162×10^{-2} Rad. In this case, 13 percent of the measured out-of-plumbs exceed the

---

* Defined in Appendix A.

Column Out-of-Plumbs: $\frac{\Delta_0}{h} \times 10^3$ Rad.

Column No.	Storey No. 27		26		25		24	
	x Axis	y Axis	x Axis	y Axis	x Axis	y Axis	x Axis	y Axis
1	0.00	-1.04	0.87	-0.87	4.34	2.60	-4.34	-2.60
2	-0.50	-0.52	-0.92	-0.87	-1.00	0.87	0.00	0.00
3	0.25	-0.52	-0.04	0.87	-0.50	-3.47	0.50	-0.87
4	0.25	0.52	-0.54	-1.74	-1.50	-0.87	-1.00	2.60
5	-0.25	-1.04	0.04	-1.74	0.50	1.74	-1.50	0.00
6	0.52	0.52	2.60	-0.87	-0.87	-1.74	-2.60	3.47
7	0.00	1.00	1.74	1.08	0.87	0.50	-0.87	0.50
8	-1.04	-0.50	0.00	-0.92	-1.74	-1.00	0.00	0.00
9	0.00	1.04	0.87	0.87	-2.60	-0.87	-0.87	3.47
10	-1.50	0.00	-2.00	-1.74	-1.50	2.60	-1.00	0.00
11	-1.00	1.56	-1.33	0.00	-1.00	-1.74	-1.00	1.74
12	-1.00	1.04	-1.33	-0.87	-1.00	-0.87	0.00	-0.87
13	-1.00	-0.52	-1.33	0.00	-1.00	0.87	-0.50	1.74
14	1.04	-0.52	-0.87	-0.87	0.00	0.87	-1.74	1.74
15	0.00	0.00	0.87	0.00	-3.47	0.00	-0.87	2.00
16	2.60	-0.50	0.00	0.58	-2.60	2.00	-0.87	1.50
17	2.00	-1.50	2.67	-0.75	2.00	1.00	-1.50	0.00
18	0.00	-1.00	1.00	-1.33	2.00	-1.00	-2.00	-1.00
Mean	0.02	-0.11	0.13	-0.51	-0.50	0.08	-1.12	0.75
Stand. Dev.*	1.03	0.85	1.31	0.89	1.85	1.60	1.08	1.59

* Defined in Appendix A

TABLE 6.4 MEASURED COLUMN OUT-OF-PLUMBS IN BUILDING A

Class Delimitations (x 10^3 Rad.)	f_i x Axis	f_i y Axis	f_i Total	f_i/nw Total
-7.5 to -6.5	1	1	2	2.2
-6.5 to -5.5	0	1	1	1.1
-5.5 to -4.5	2	1	3	3.3
-4.5 to -3.5	2	3	5	5.5
-3.5 to -2.5	18	24	42	45.9
-2.5 to -1.5	50	40	90	98.3
-1.5 to -0.5	98	100	198	216.2
-0.5 to 0.5	144	144	288	314.4
0.5 to 1.5	77	77	154	168.1
1.5 to 2.5	35	30	65	71.0
2.5 to 3.5	19	31	50	54.6
3.5 to 4.5	8	5	13	14.2
4.5 to 5.5	2	0	2	2.2
5.5 to 6.5	2	1	3	3.3
6.5 to 7.5	0	0	0	0.0
Total	458	458	n=916	

f_i = frequency

w = class width = 0.001 Rad.

n = sample dimension (total number of measurements)

TABLE 6.5 FREQUENCY FUNCTION OF COLUMN OUT-OF-PLUMBS FOR BUILDING A

Column Out-of-Plumbs: $\frac{\Delta_0}{h} \times 10^3$ Rad.

Column No.	Storey No. 27		26		25		24	
	x Axis	y Axis	x Axis	y Axis	x Axis	y Axis	x Axis	y Axis
1	0.00	-1.04	0.87	-0.87	4.34	2.60	-4.34	-2.60
2	-0.50	-0.52	-0.92	-0.87	-1.00	0.87	0.00	0.00
3	0.25	-0.52	-0.04	0.87	-0.50	-3.47	0.50	-0.87
4	0.25	0.52	-0.54	-1.74	-1.50	-0.87	-1.00	2.60
5	-0.25	-1.04	0.04	-1.74	0.50	1.74	-1.50	0.00
6	0.52	0.52	2.60	-0.87	-0.87	-1.74	-2.60	3.47
7	0.00	1.00	1.74	1.08	0.87	0.50	-0.87	0.50
8	-1.04	-0.50	0.00	-0.92	-1.74	-1.00	0.00	0.00
9	0.00	1.04	0.87	0.87	-2.60	-0.87	-0.87	3.47
10	-1.50	0.00	-2.00	-1.74	-1.50	2.60	-1.00	0.00
11	-1.00	1.56	-1.33	0.00	-1.00	-1.74	-1.00	1.74
12	-1.00	1.04	-1.33	-0.87	-1.00	-0.87	0.00	-0.87
13	-1.00	-0.52	-1.33	0.00	-1.00	0.87	-0.50	1.74
14	1.04	-0.52	-0.87	-0.87	0.00	0.87	-1.74	1.74
15	0.00	0.00	0.87	0.00	-3.47	0.00	-0.87	2.00
16	2.60	-0.50	0.00	0.58	-2.60	2.00	-0.87	1.50
17	2.00	-1.50	2.67	-0.75	2.00	1.00	-1.50	0.00
18	0.00	-1.00	1.00	-1.33	2.00	-1.00	-2.00	-1.00
Mean	0.02	-0.11	0.13	-0.51	-0.50	0.08	-1.12	0.75
Stand. Dev.*	1.03	0.85	1.31	0.89	1.85	1.60	1.08	1.59

* Defined in Appendix A

TABLE 6.4 MEASURED COLUMN OUT-OF-PLUMBS IN BUILDING A

Class Delimitations ($x\ 10^3$ Rad.)	f_i x Axis	f_i y Axis	f_i Total	f_i/nw Total
-7.5 to -6.5	1	1	2	2.2
-6.5 to -5.5	0	1	1	1.1
-5.5 to -4.5	2	1	3	3.3
-4.5 to -3.5	2	3	5	5.5
-3.5 to -2.5	18	24	42	45.9
-2.5 to -1.5	50	40	90	98.3
-1.5 to -0.5	98	100	198	216.2
-0.5 to 0.5	144	144	288	314.4
0.5 to 1.5	77	77	154	168.1
1.5 to 2.5	35	30	65	71.0
2.5 to 3.5	19	31	50	54.6
3.5 to 4.5	8	5	13	14.2
4.5 to 5.5	2	0	2	2.2
5.5 to 6.5	2	1	3	3.3
6.5 to 7.5	0	0	0	0.0
Total	458	458	n=916	

f_i = frequency

w = class width = 0.001 Rad.

n = sample dimension (total number of measurements)

TABLE 6.5 FREQUENCY FUNCTION OF COLUMN OUT-OF-PLUMBS FOR BUILDING A

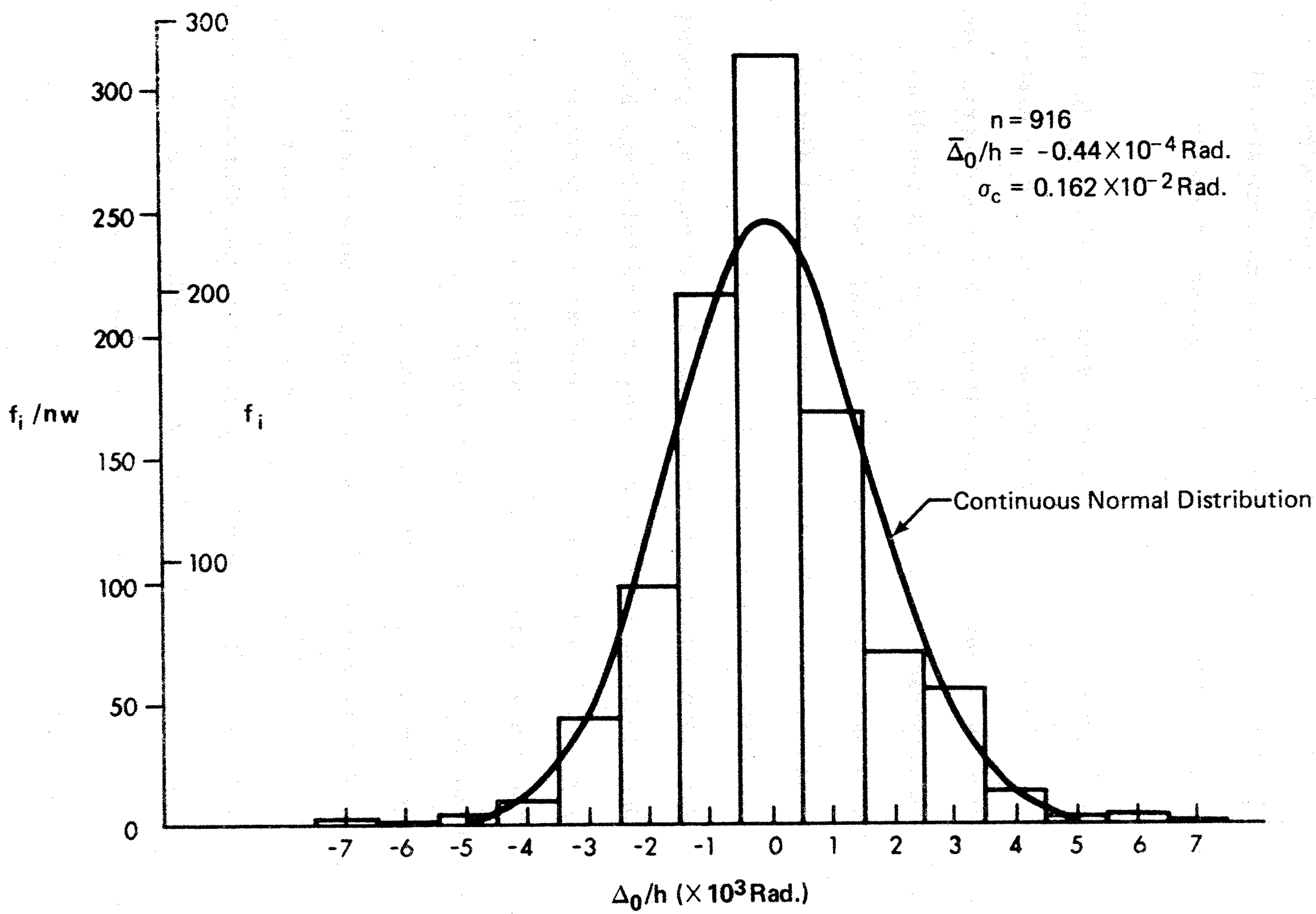


Figure 6.5 Column out-of-plumb distribution for building A

prescribed tolerance of 1/500. The result was expected. Superimposed on the histogram is the normal distribution* calculated from the given mean and standard deviation. The area under both the histogram and the normal curve is unity. A comparison of the two graphs indicates that the out-of-plumb population can effectively be assumed normally distributed.

Two more quantities characterize a distribution. As described in Appendix A, the skewness and the peakedness (kurtosis) of a distribution are defined by non-dimensional quantities. For the distribution of Fig. 6.5, the skewness factor is +0.14 indicating that the distribution is slightly skewed to the right. The kurtosis factor is 4.9, indicating a distribution more peaked than a perfect normal distribution with a kurtosis factor of 3.0.

Similar histograms are given in Figs. 6.6 through 6.8 showing the distributions of column out-of-plumbs measured in buildings B, A and B combined, and C. All populations are approximately normally distributed. The characteristics are quite similar in all cases and are summarized in Table 6.6. The standard deviations given in Table 6.6 are comparable with those of Tables 6.1 and 6.2.

The probability densities of the absolute values of the measurements for buildings A, B, A and B combined, and C are given in Figs. 6.9 through 6.12. The negative part of the normal distributions of Figs. 6.5 to 6.8 is literally folded over and added to the positive part. The distribution that results when the mean is zero or relatively close to zero is called a "half-normal" distribution. A description of the half-normal distribution is given in Appendix A.

---

* Definition given in Appendix A.

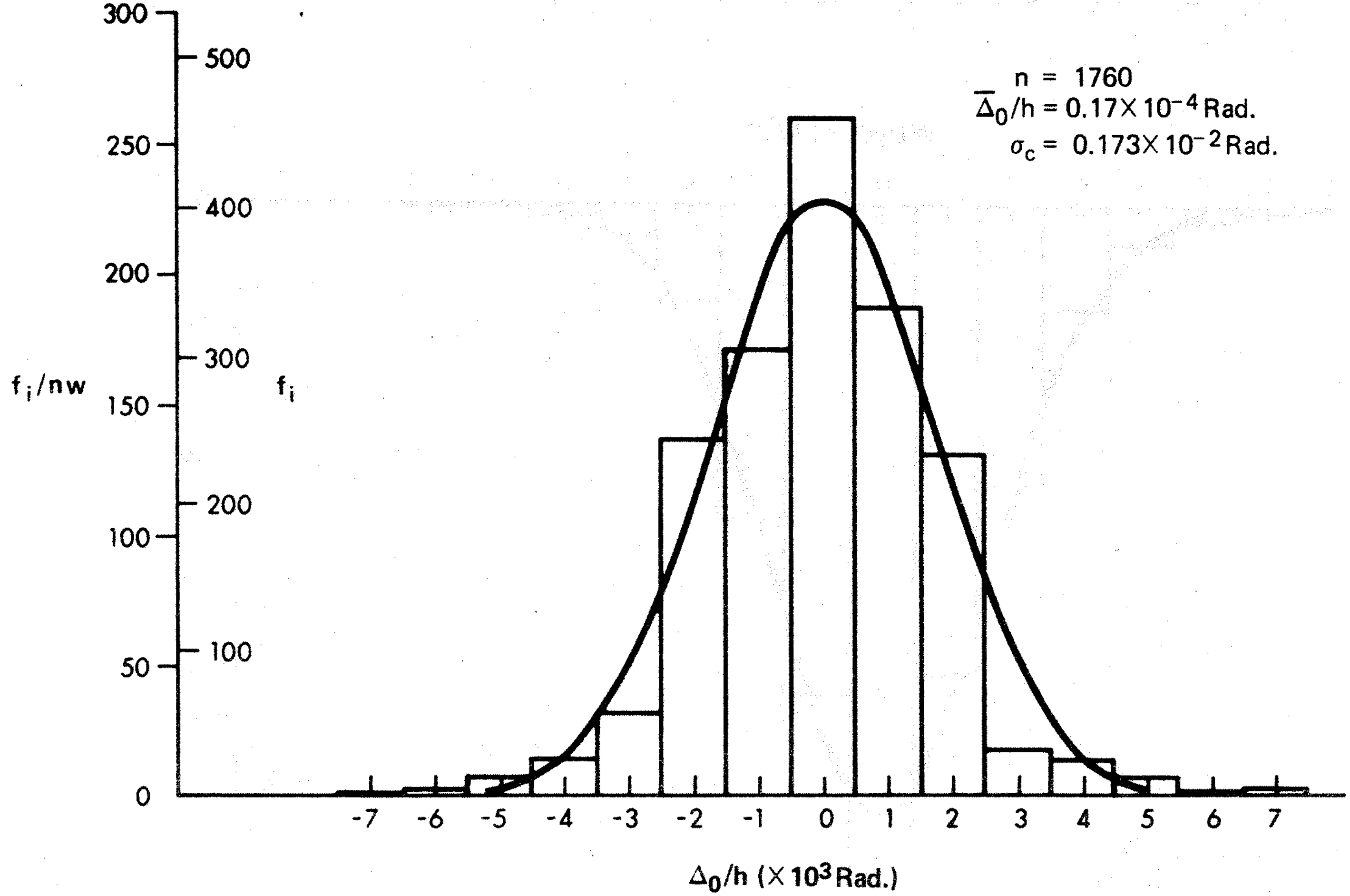


Figure 6.6 Column out-of-plumb distribution for building B

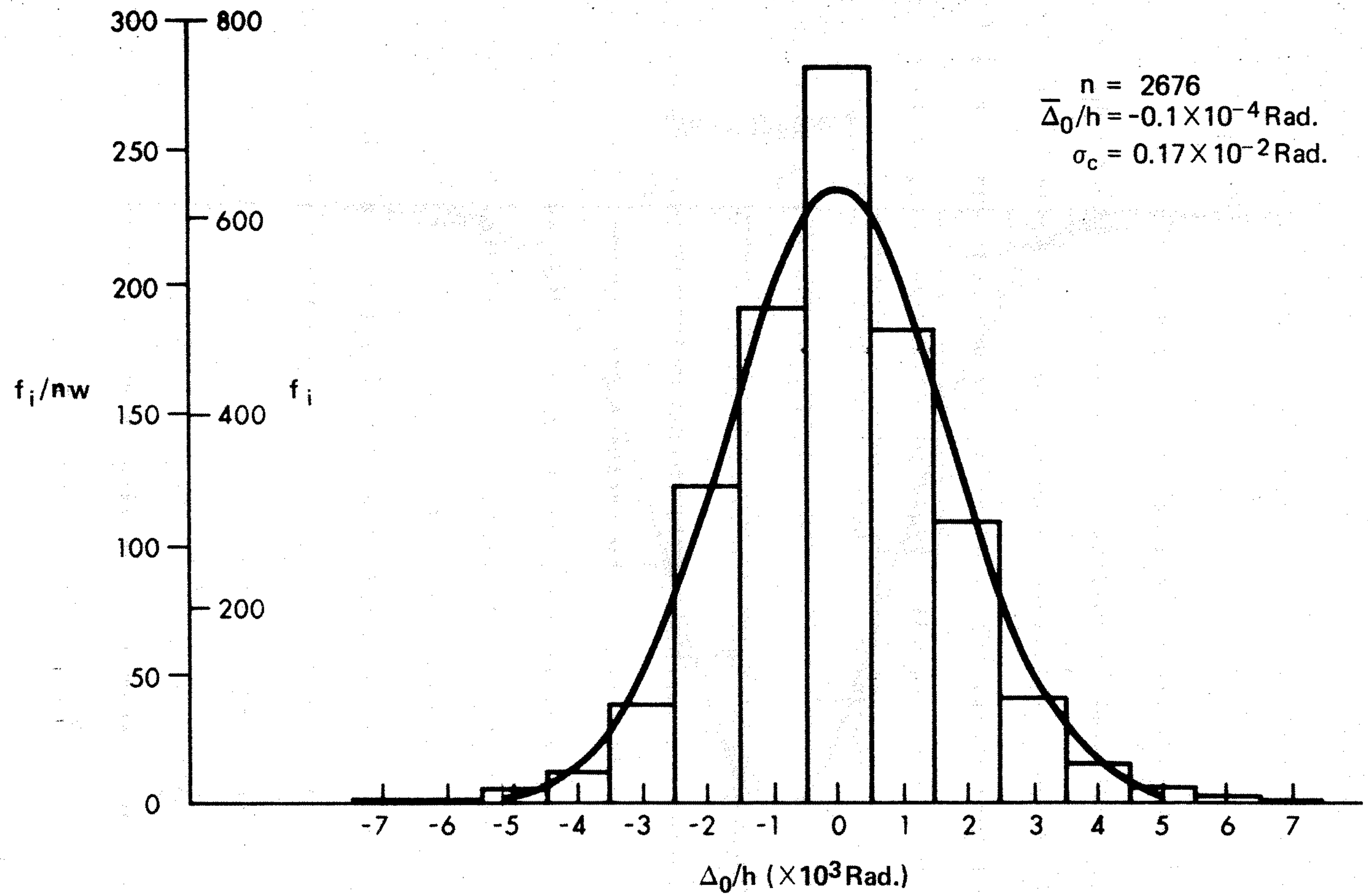


Figure 6.7 Column out-of-plumb distribution for buildings A and B

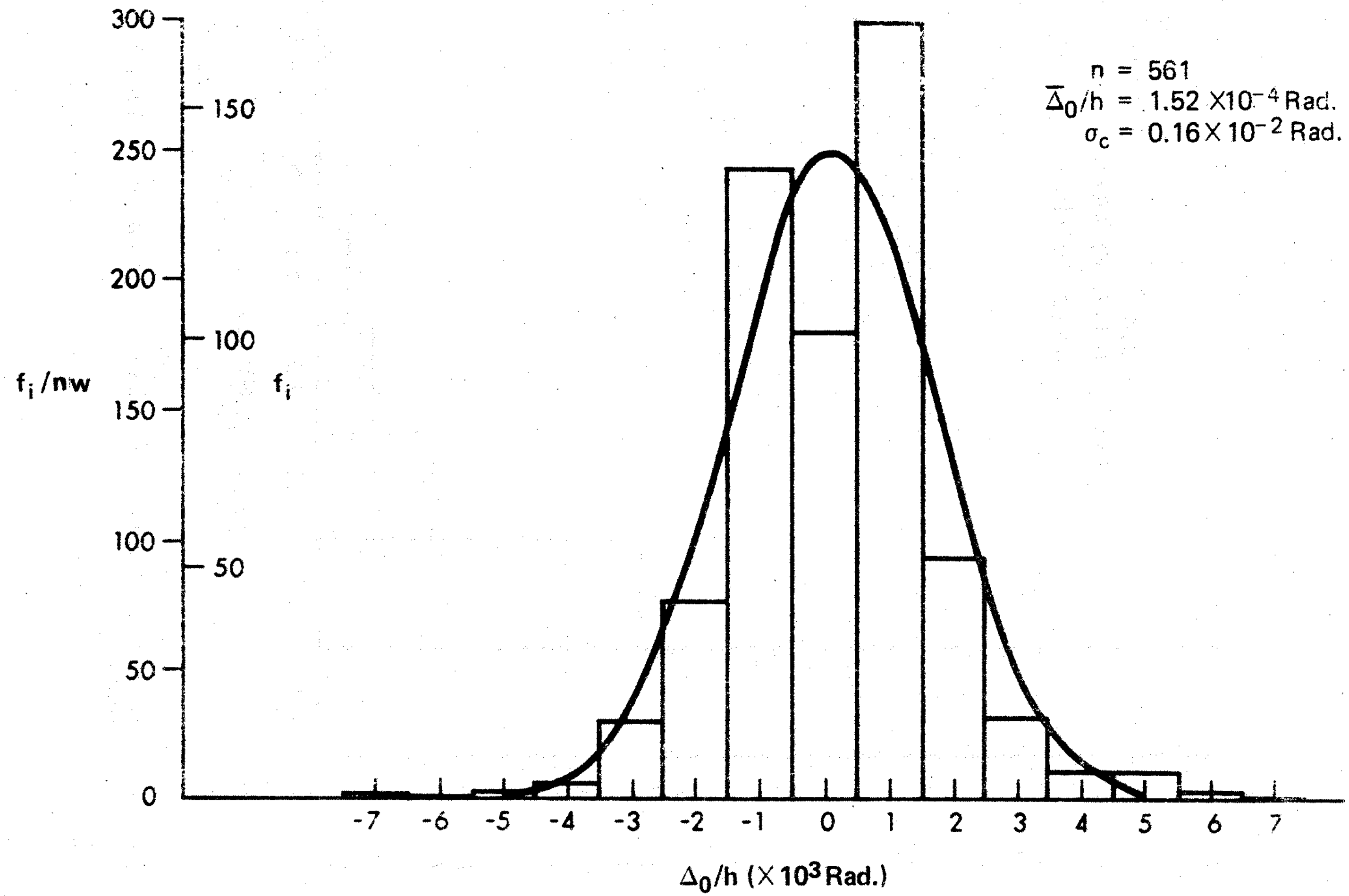


Figure 6.8 Column out-of-plumb distribution for building C

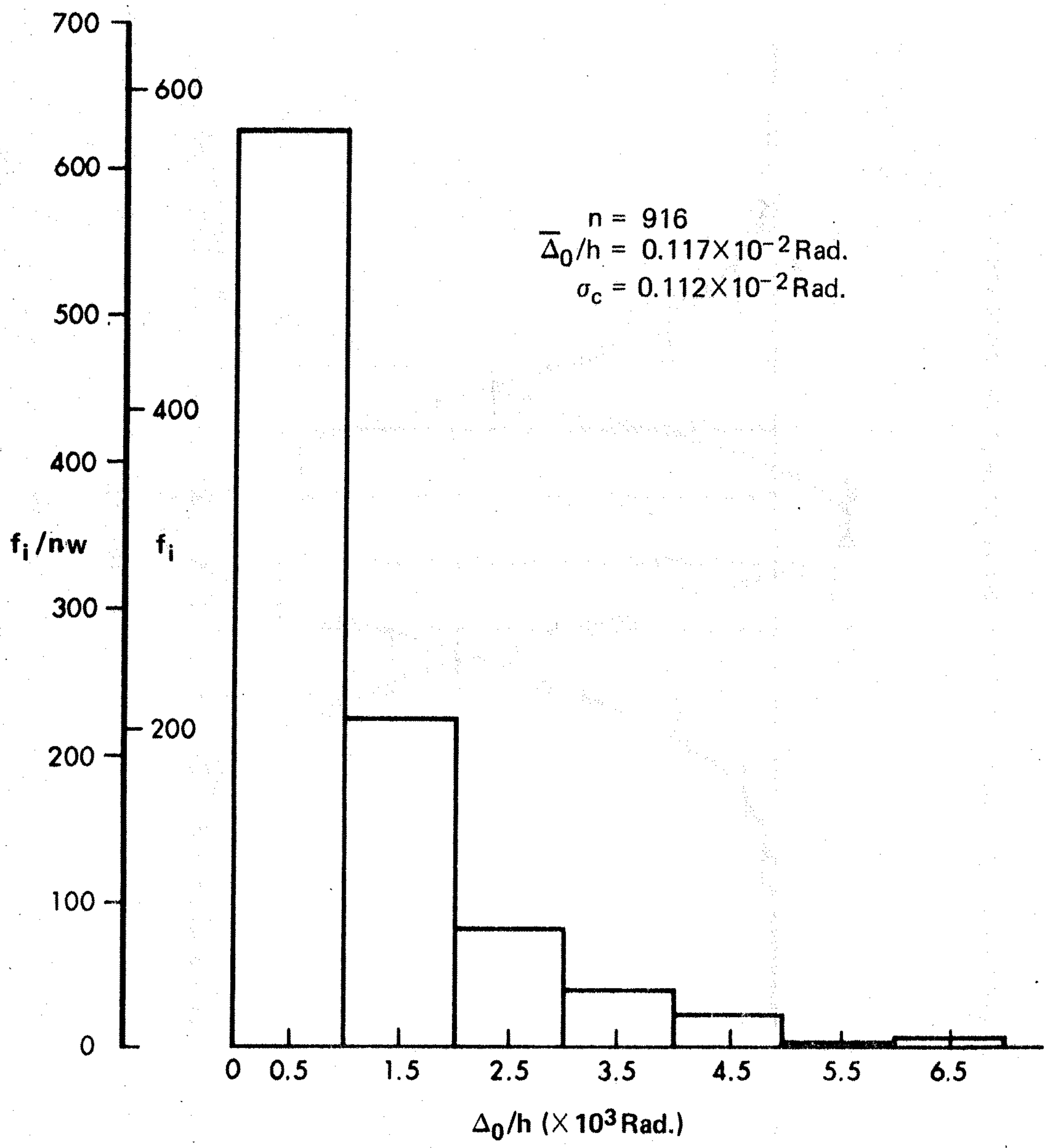


Figure 6.9 Distribution of absolute values of column out-of-plumbs for building A

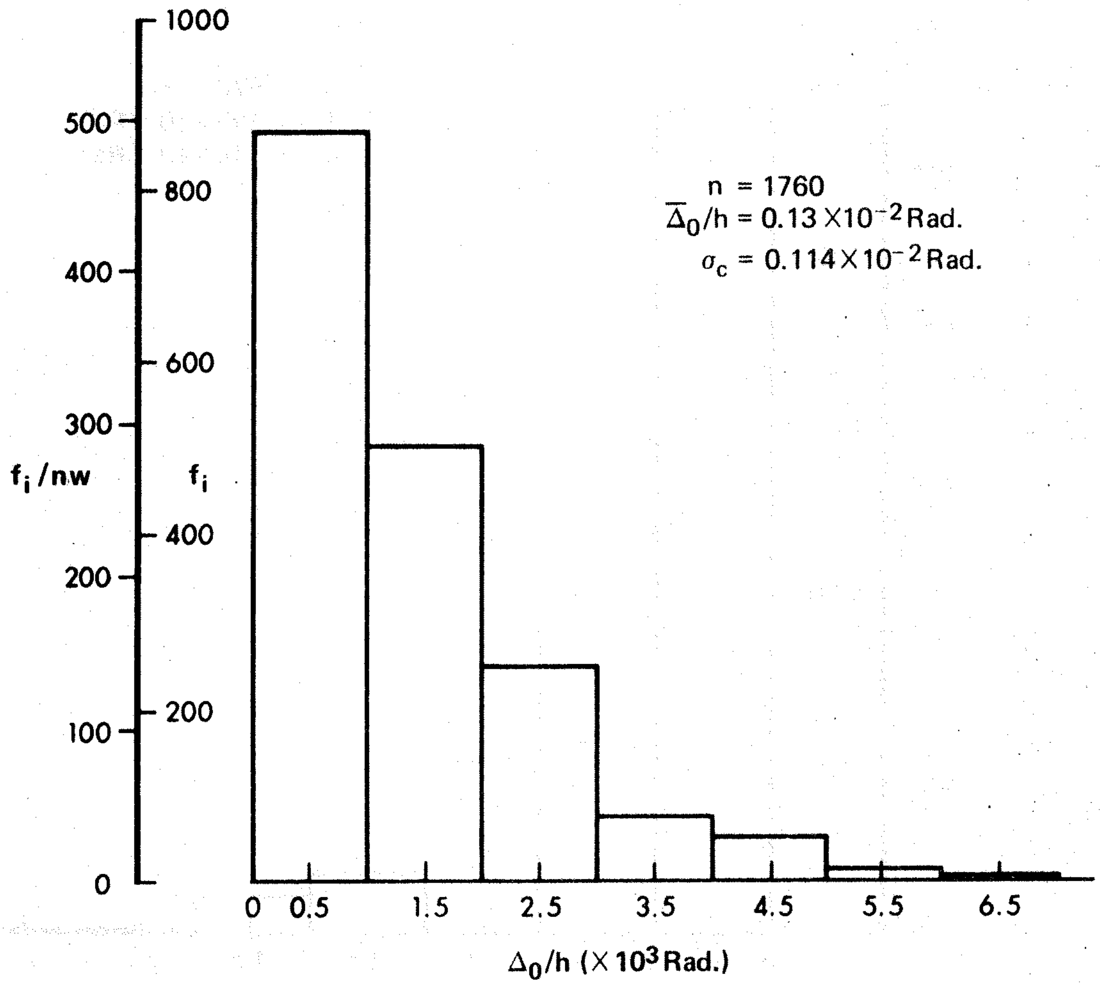


Figure 6.10 Distribution of absolute values of column out-of-plumbs for building B

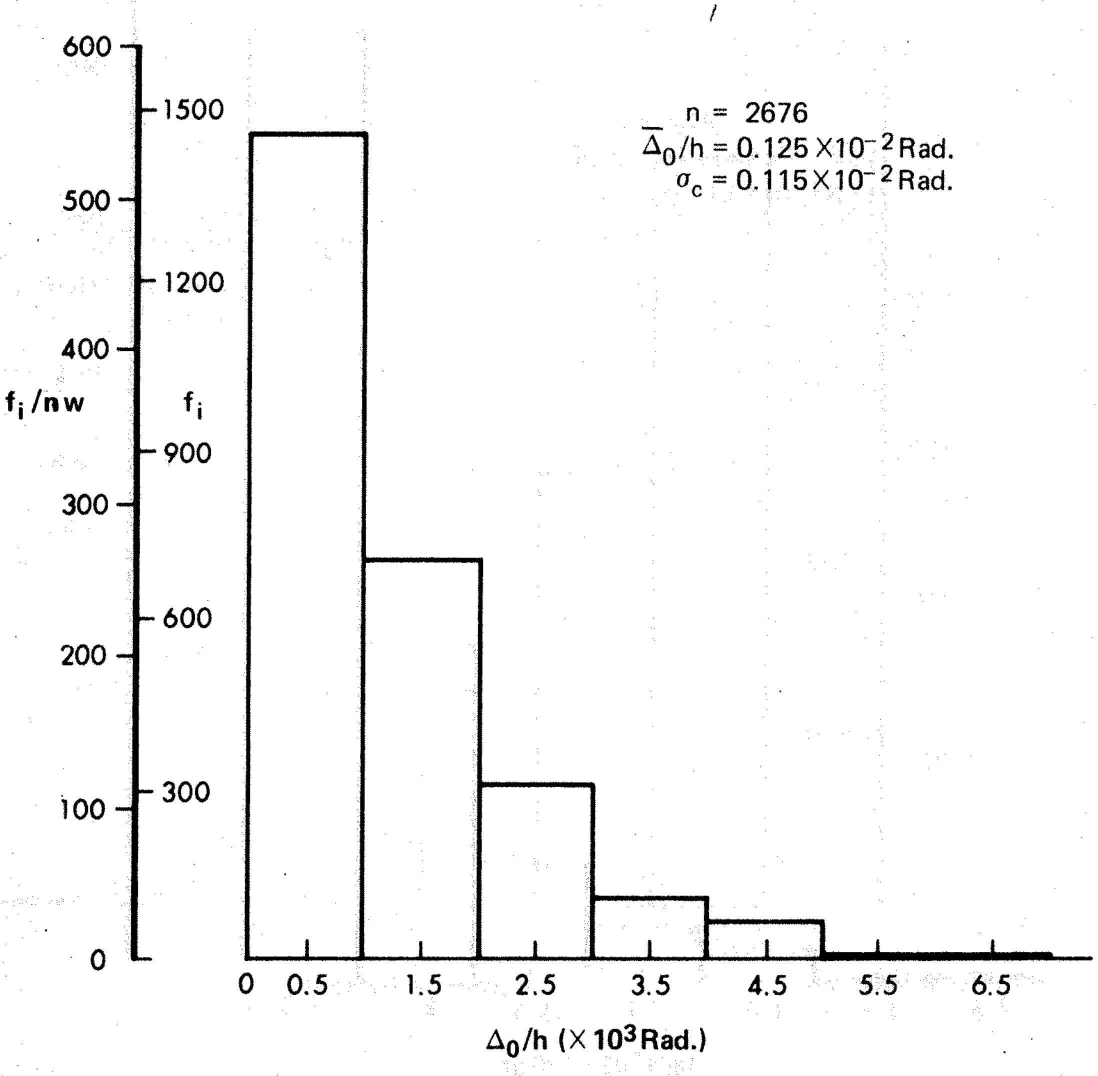


Figure 6.11 Distribution of absolute values of column out-of-plumbs for buildings A and B

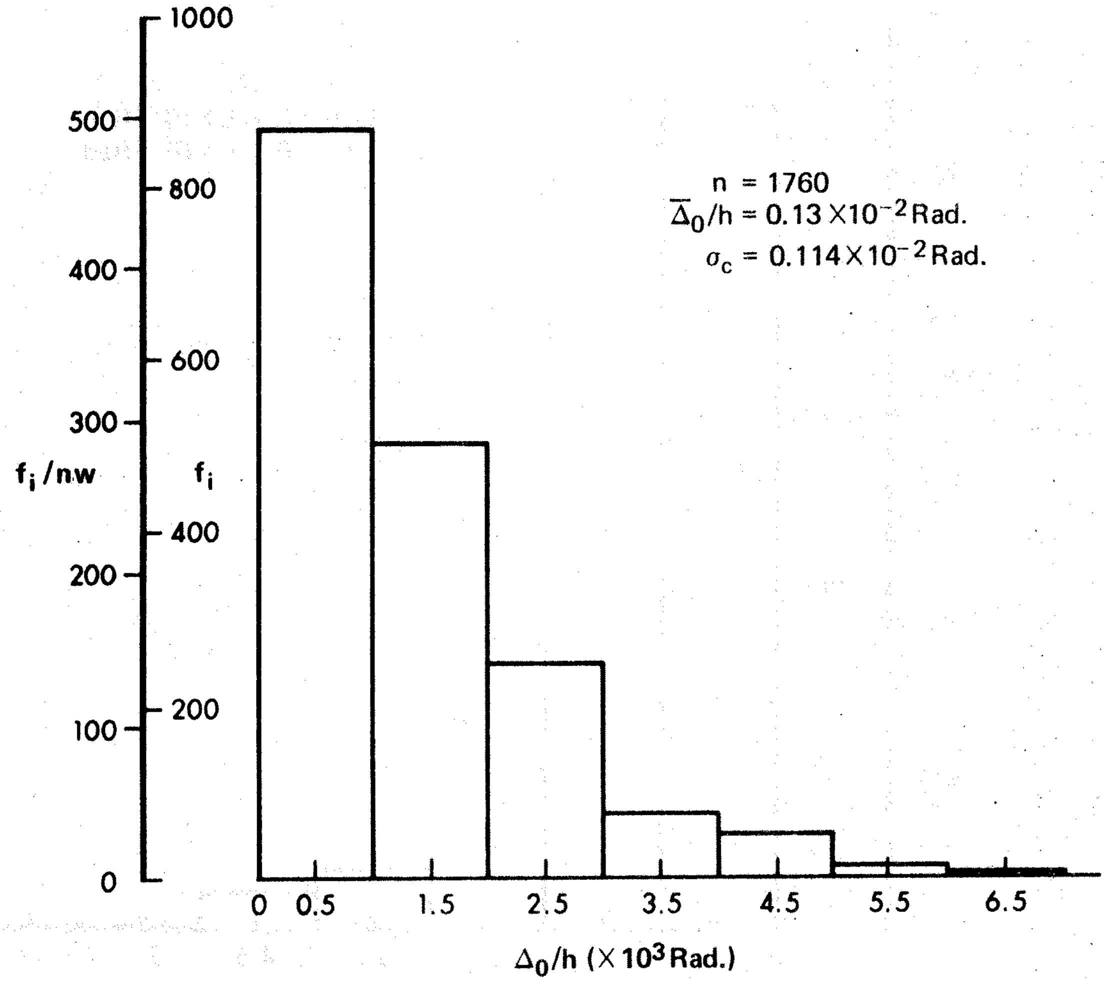


Figure 6.10 Distribution of absolute values of column out-of-plumbs for building B

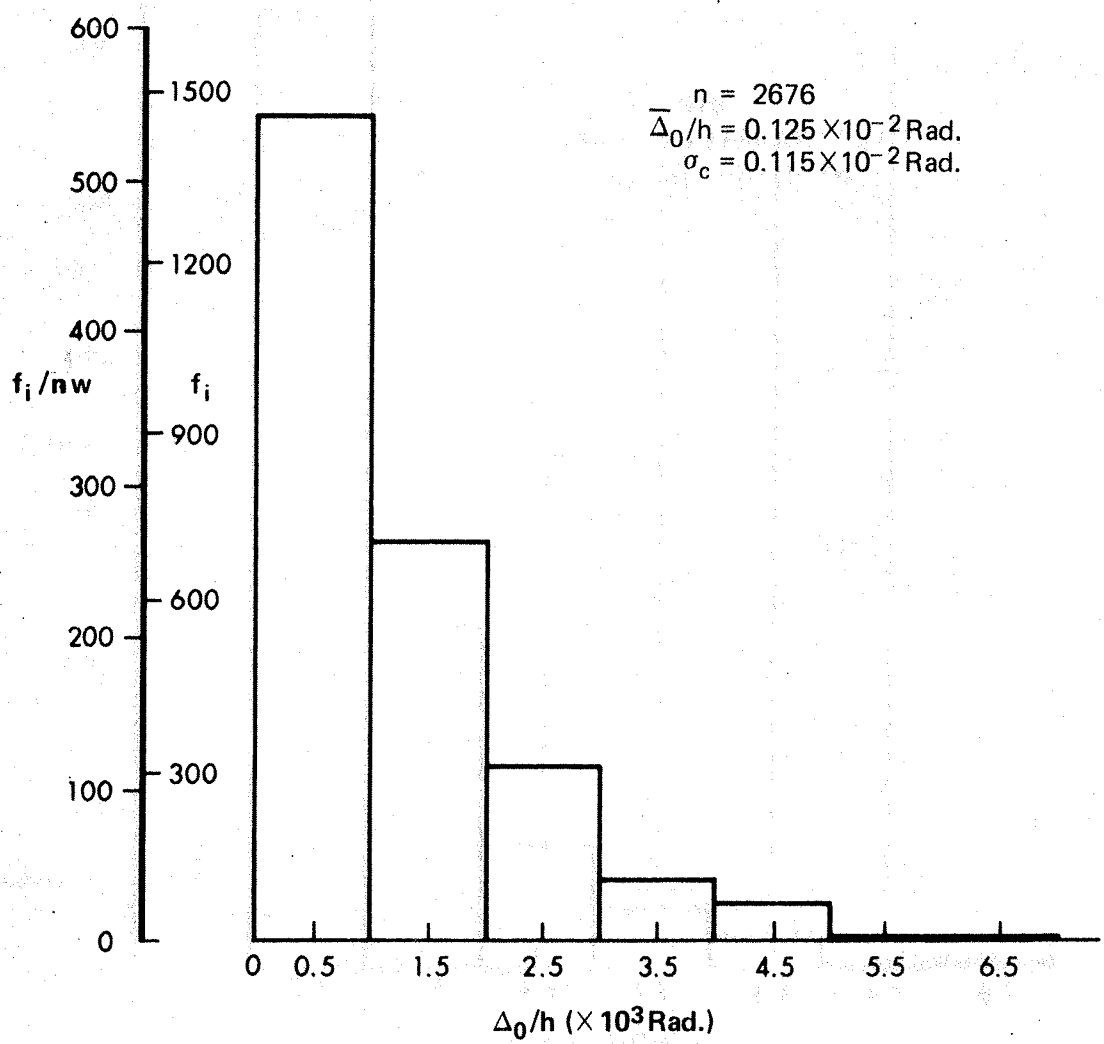


Figure 6.11 Distribution of absolute values of column out-of-plumbs for buildings A and B

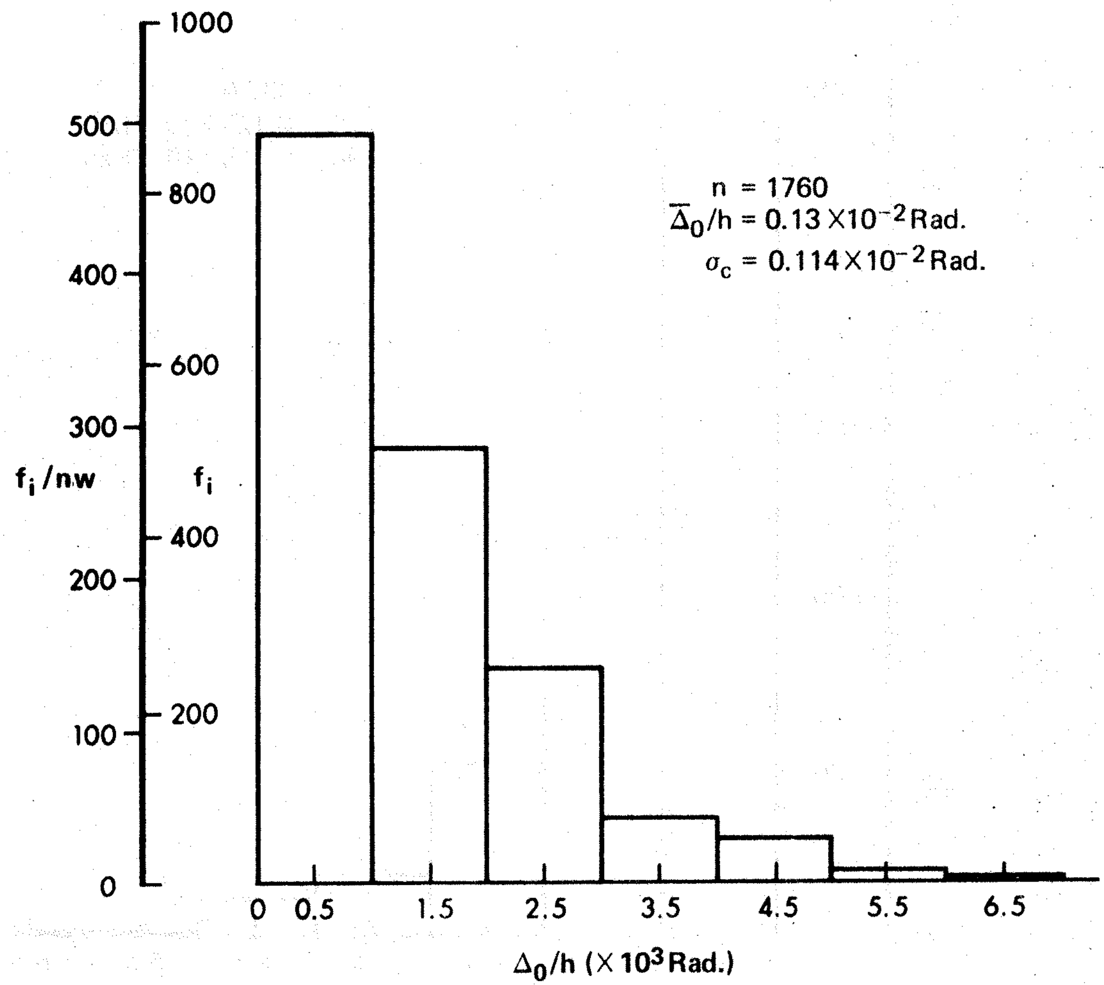


Figure 6.10 Distribution of absolute values of column out-of-plumbs for building B

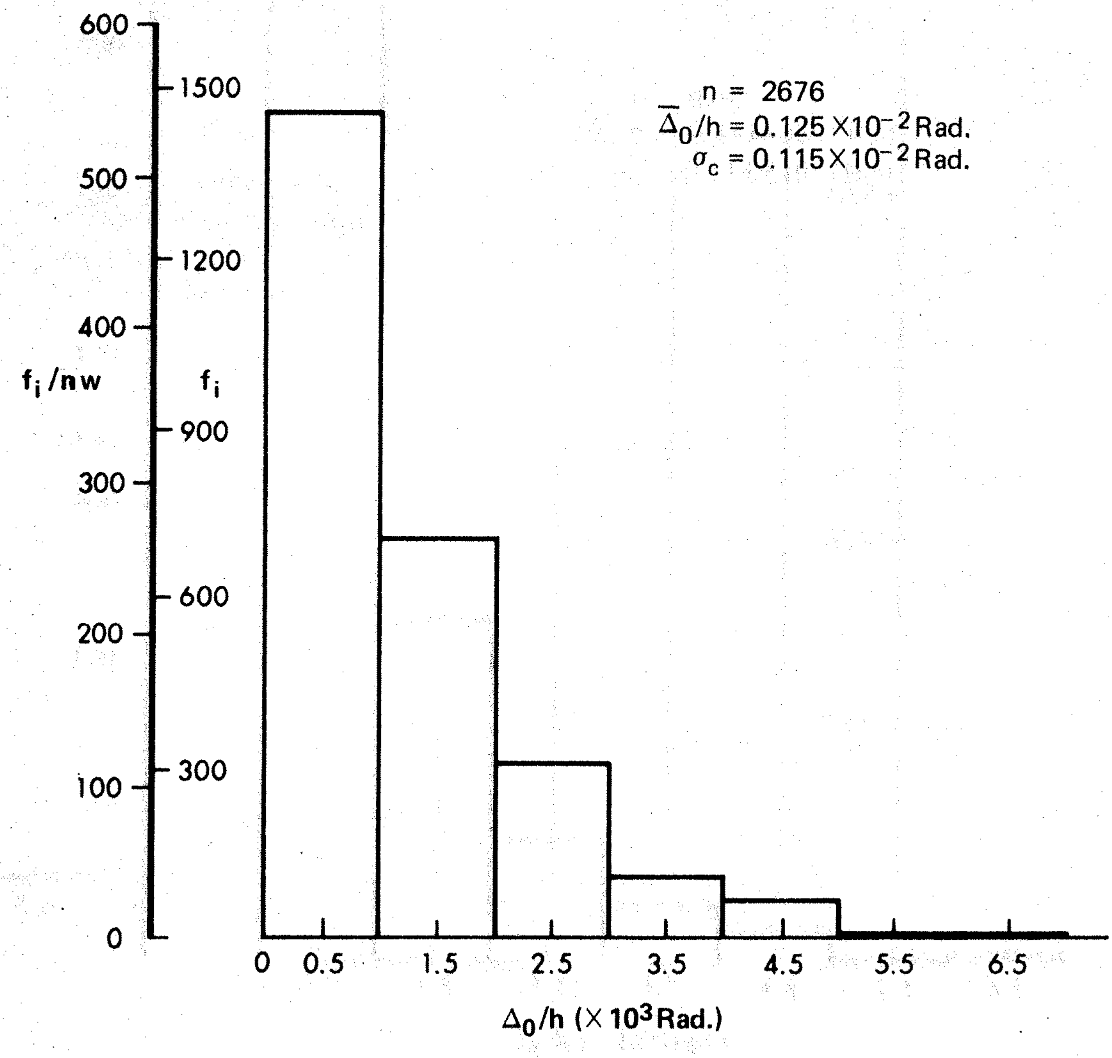


Figure 6.11 Distribution of absolute values of column out-of-plumbs for buildings A and B

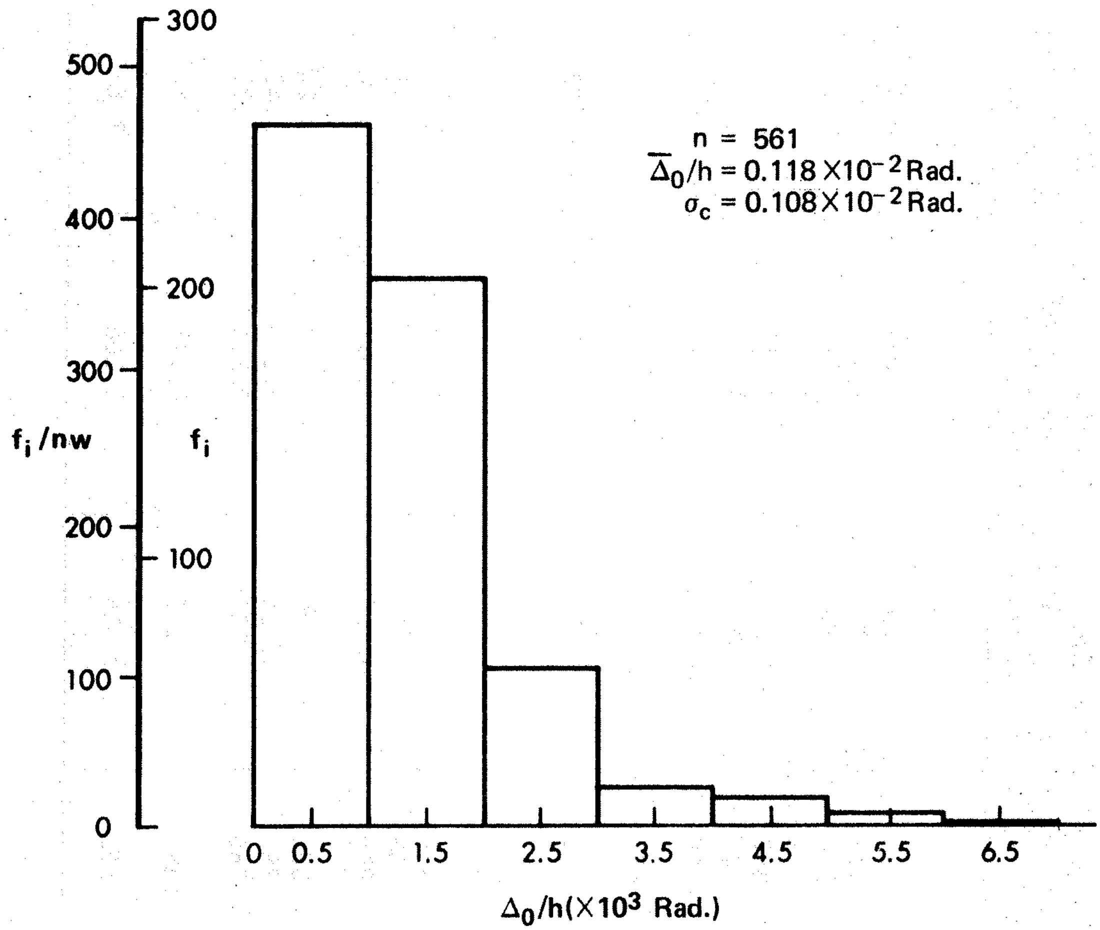


Figure 6.12 Distribution of absolute values of column out-of-plumbs for building C

Building	Type	Sample Dimension n	Mean (Eqn. A-14)* (x 10^2 Rad.)	Stand. Dev. (Eqn. A-16)* (x 10^2 Rad.)	Skewness (Eqn. A-20)*	Kurtosis (Eqn. A-22)*
ALGEBRAIC VALUES						
A	x Axis	458	-0.004	0.164	0.27	5.1
	y Axis	458	-0.005	0.160	0.00	4.7
	Total	916	-0.004	0.162	0.14	4.9
B	x Axis	880	-0.009	0.167	-0.13	3.8
	y Axis	880	0.012	0.178	0.17	4.2
	Total	1760	0.002	0.173	0.05	4.1
A + B	Total	2676	-0.001	0.170	0.09	4.5
C	Total	561	0.015	0.160	0.05	4.7
ABSOLUTE VALUES						
A	x Axis	458	0.119	0.113	1.74	7.5
	y Axis	458	0.115	0.112	1.51	6.1
	Total	916	0.117	0.112	1.62	6.8
B	x Axis	880	0.126	0.110	1.24	5.2
	y Axis	880	0.134	0.119	1.43	6.1
	Total	1760	0.130	0.114	1.35	5.8
A + B	Total	2676	0.125	0.115	1.50	6.4
C	Total	561	0.118	0.108	1.68	6.9

* See Appendix A

TABLE 6.6 STATISTICAL CHARACTERISTICS OF COLUMN OUT-OF-PLUMBS

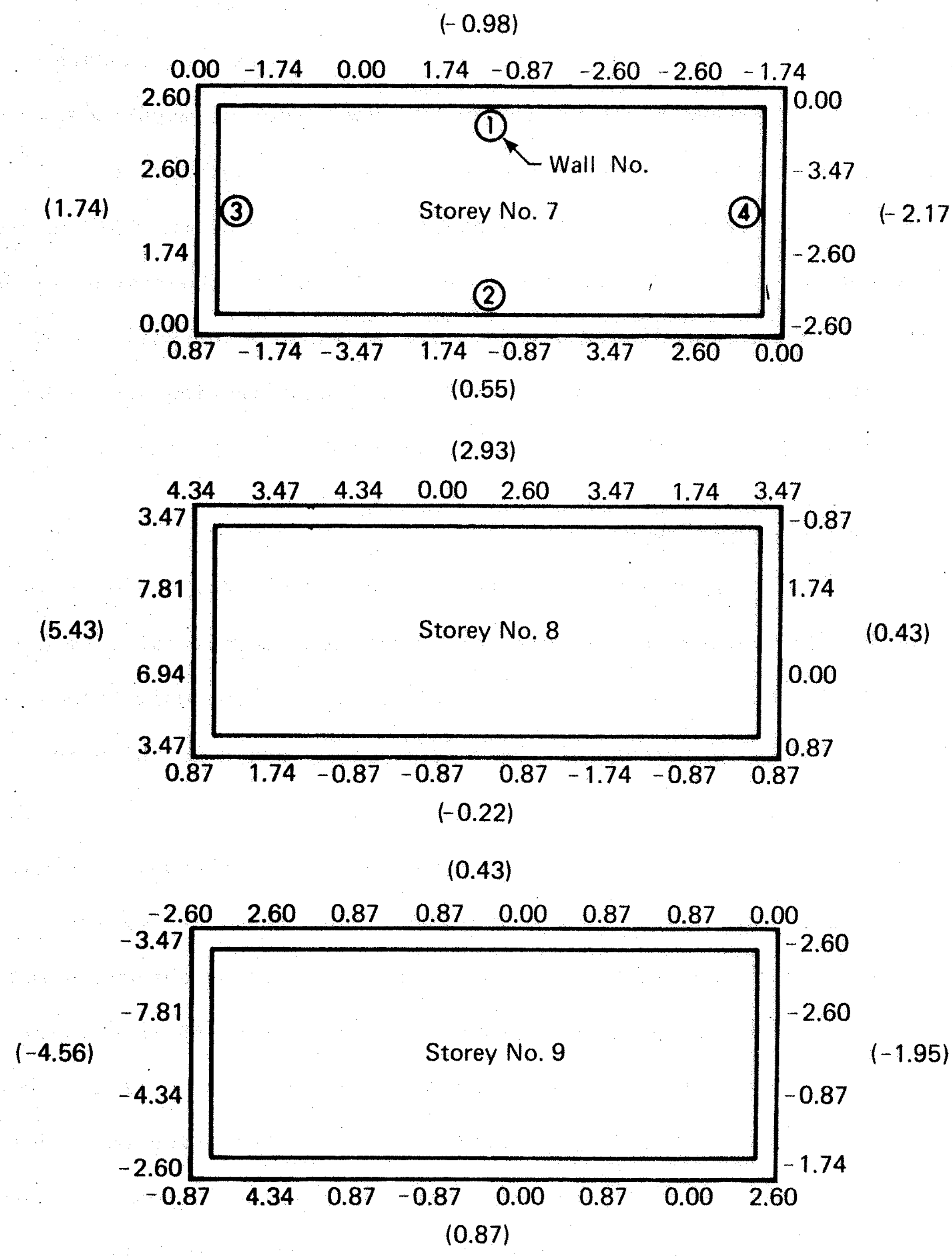
The corresponding statistical characteristics are listed in Table 6.6. In this table, the values of the mean and standard deviation are always positive and of the same order of magnitude, with a resulting coefficient of variation (standard deviation/mean) slightly lower than unity. The standard deviations of the half-normal distributions are 47 percent lower than the standard deviations of the corresponding normal distributions. The measure of kurtosis, in the order of 6.5, indicates that the half-normal distribution approaches the exponential distribution characterized by a factor of 9.0.

6.4.2 Wall Out-of-Plumbs

In a manner similar to the column deviations, the wall out-of-plumbs are conveniently expressed in the non-dimensional form Δ_0/h, where Δ_0 is the horizontal deviation of the top of the wall from a plumb line passing through the base of the wall and h is the height of the wall.

Several measurements are needed to define the out-of-plumb of a wall. A minimum of four measurements were taken at regular intervals along the walls. In some cases, up to 15 measurements were necessary to define the out-of-plumbs of long walls. As an example, the values and locations of measurements taken at three adjacent storeys in building A are given in Fig. 6.13. The sign convention adopted is the same as that used for columns, that is, a value is positive when the top of a wall leans in the positive direction of the axis.

The measurements taken on a cast-in-place reinforced concrete wall are not totally independent of each other. This observation is based on the fact that a wall is being cast in a continuous form and that the chance of measuring large out of plumb variations



Wall Out-of-Plumbs: Δ_0/h ($\times 10^3$ Rad.)

Figure 6.13 Typical core-wall measurements on building A

within a short distance along the wall is remote. The average out-of-plumb value of each individual wall therefore is used in statistical manipulations. This, in turn, implies that the deviations of the individual walls forming a core are independent of each other, which is a reasonable assumption.

Two out-of-plumb values are used to characterize a wall. One is measured in the direction "perpendicular" to the plane of the wall and is the average out-of-plumb discussed above. The second is measured in the direction "parallel" to the plane of the wall and is the average of the measurements taken at the two extremities. According to this definition, the parallel out-of-plumb for wall No. 1 at storey 7 in Fig. 6.13 is (0.0026 + 0)/2 = 0.0013 Rad.

The average perpendicular and parallel wall out-of-plumbs, as defined above, are given in Tables 6.7 through 6.10 for buildings A and B. The measurements for building B were taken by the research team but those for building A were supplied by the surveyor on the job site. The results for the 5 storeys above level 22 in the latter were not available.

The perpendicular and parallel out-of-plumbs are plotted separately on Figs. 6.14 through 6.19 for building A, B, and A and B combined. The type of graph used is the same as for the column data. Once again, each discrete distribution can be fitted by a normal distribution.

The characteristics of each statistical distribution are summarized in Table 6.11. The standard deviation can be taken as 0.0028 Rad. for both parallel and perpendicular out-of-plumbs. For all practical purposes, a common mean value of 0.00028 Rad. can be

Out-of-Plumbs : x 10^3 Rad.

Storey	Wall No.*			
	1	2	3	4
22	1.95	1.02	12.37	-4.20
21	2.86	1.48	1.56	-1.54
20	-0.87	-0.87	0.69	0.72
19	2.60	-1.74	-1.74	5.79
18	0.12	2.11	5.64	0.00
17	-5.82	-6.57	-0.87	-0.93
16	2.93	-0.11	8.25	0.87
15	0.00	-0.22	3.91	-3.69
14	5.32	3.26	-4.12	-1.09
13	-1.20	-0.98	-0.22	1.95
12	-2.50	-1.52	-2.60	-2.60
11	0.87	2.17	3.47	0.87
10	-1.85	-0.98	2.17	0.22
9	0.43	0.87	-4.56	-1.95
8	2.93	-0.22	5.43	0.43
7	-0.98	0.55	1.74	-2.17
6	0.14	-0.43	0.68	-0.65
5	2.92	0.36	-1.76	-0.54
4	-4.17	0.55	1.91	6.95
3	0.31	-0.28	1.74	-2.08
2	3.27	0.40	2.17	-3.76
1	-4.40	1.45	3.76	2.89

* Wall numbering given in Fig. 6.2.

TABLE 6.7 PERPENDICULAR WALL OUT-OF-PLUMBS FOR BUILDING A

Out-of-Plumbs : x 10^3 Rad.

Storey	Wall No.*			
	1	2	3	4
22	-11.28	-10.42	0.00	-1.74
21	-1.30	0.00	0.00	1.74
20	0.00	1.74	0.43	-0.43
19	-0.43	-1.74	-0.43	0.43
18	4.34	4.34	0.87	3.04
17	-3.47	0.87	-1.74	-9.55
16	1.30	-0.87	-2.17	-0.87
15	-3.04	-0.87	1.74	2.60
14	0.43	0.00	1.74	-3.91
13	-2.60	-2.60	2.17	-2.60
12	3.04	3.04	-3.47	2.17
11	-0.87	-0.43	0.43	-1.74
10	-3.04	-2.17	-1.74	1.30
9	1.30	2.17	1.74	2.17
8	1.30	-1.30	0.43	-0.87
7	6.51	0.87	1.30	3.47
6	-1.74	0.00	-0.43	-0.87
5	1.08	-2.15	-0.54	2.15
4	7.29	0.00	0.00	0.00
3	3.13	0.00	2.60	-0.52
2	-0.87	-0.43	-0.87	1.30
1	-1.74	4.34	0.00	-1.74

* Wall numbering given in Fig. 6.2.

TABLE 6.8 PARALLEL WALL OUT-OF-PLUMBS FOR BUILDING A

Out-of-Plumbs : x 10^3 Rad.

Storey	Wall No.*								
	1	2	3	4	5	6	7	8	9
34	-0.97	-0.83	-	2.76	-2.19	1.46	1.98	1.48	-0.42
33	2.19	0.00	-	2.71	-0.83	1.56	-1.67	-2.37	-0.31
32	0.73	2.08	-	5.73	3.33	-3.54	-1.39	-2.37	-3.33
31	-0.26	1.67	-	1.70	1.67	0.42	0.52	-1.22	-0.42
30	1.72	-1.46	-	1.04	-4.04	-0.91	-0.73	-3.31	0.10
29	1.88	1.41	-	-0.31	0.21	-2.50	-0.42	1.47	-0.20
28	2.29	0.52	-	1.17	0.63	-3.44	1.98	-0.59	-1.46
27	0.05	-1.25	-	0.68	-1.15	-2.50	-0.52	-0.26	-0.83
26	1.25	0.10	-	0.21	-0.63	-2.08	-3.44	1.87	1.04
25	0.05	-0.21	-	-2.71	-0.52	1.04	4.17	2.24	-1.67
24	0.89	0.96	-	-0.39	-2.92	1.35	7.29	1.82	6.94
23	0.63	1.82	-	2.08	-3.13	-1.88	0.31	-1.30	-0.21
22	2.08	0.21	-	-0.20	1.04	3.65	2.92	-0.63	1.94
21	0.94	0.28	-	0.76	-2.08	-1.87	1.98	-2.27	2.71
20	0.68	0.26	-	4.69	-1.88	-1.46	1.25	5.00	-1.04
19	-1.72	2.50	5.52	-1.35	-0.83	2.50	1.25	2.08	-0.47
18	0.98	3.52	3.52	1.95	4.04	1.82	5.73	0.00	-1.39
17	1.09	0.47	1.98	3.02	-1.87	0.73	-1.56	0.42	1.15
16	0.37	0.94	3.75	-5.42	-2.92	-1.46	-2.92	8.75	-3.85
15	-0.99	1.20	2.50	-0.73	0.00	1.65	-2.92	-4.17	1.04
14	2.71	1.46	-0.52	-2.78	-2.08	0.83	-2.19	0.00	-0.83
13	2.73	-0.59	0.91	4.43	0.39	-1.25	2.73	0.52	1.04
12	-2.05	0.43	-2.41	0.52	3.52	-0.52	-0.91	-5.21	-2.02
11	1.69	9.33	-1.17	-3.13	-9.51	-2.47	-2.60	-1.82	-4.56
10	1.99	-3.52	0.26	-2.29	2.08	1.30	0.78	-4.69	0.46
9	3.52	1.86	0.26	1.30	0.65	0.76	0.00	-0.52	2.08
8	2.15	-1.60	-0.98	0.91	1.82	2.73	0.13	1.04	2.21
7	0.49	-0.91	0.78	0.70	-1.59	4.77	-4.43	-1.56	2.73
6	-1.63	0.33	2.60	-1.56	0.26	0.00	-1.30	-0.42	3.13
5	2.15	0.52	-0.17	2.99	0.13	-0.15	-3.52	5.28	-1.74
4	-0.62	0.62	2.40	-0.91	-2.21	0.91	1.17	-0.91	2.41
3	-4.86	-0.89	-1.09	-1.04	0.00	0.52	-0.13	0.13	0.70
2	3.99	-0.07	-0.35	-2.74	-0.65	-1.17	-0.52	-0.78	-1.85
1	-1.39	-0.05	-1.27	-1.04	-1.31	-1.56	-0.52	-1.69	1.62

*Wall numbering given in Fig. 6.3

TABLE 6.9 PERPENDICULAR WALL OUT-OF-PLUMBS

FOR BUILDING B

Out-of-Plumbs : x 10^3 Rad.

Storey	Wall No.*								
	1	2	3	4	5	6	7	8	9
34	-2.08	-0.83	-	0.27	-0.83	-0.83	0.42	-3.73	-0.42
33	4.17	-0.20	-	4.17	-2.92	-3.75	-0.42	2.29	3.33
32	-0.42	0.00	-	2.50	0.42	0.83	5.00	3.75	0.21
31	0.79	-0.26	-	2.08	1.25	0.00	-1.67	1.25	0.83
30	-1.04	1.46	-	0.21	2.29	0.21	-0.63	-1.04	1.88
29	-3.33	-2.50	-	5.83	2.08	1.04	0.63	-0.21	0.21
28	-0.73	1.25	-	1.67	1.67	-0.42	2.50	2.08	3.33
27	0.42	-0.42	-	-1.67	-1.25	0.00	-3.13	0.63	2.71
26	-2.50	0.42	-	2.92	3.75	-0.21	0.00	-1.67	0.42
25	-0.83	-0.63	-	0.83	-0.42	0.42	0.00	0.21	0.00
24	-2.50	2.08	-	-0.83	0.00	0.42	2.08	2.50	4.17
23	1.67	-0.42	-	2.08	1.67	-1.46	0.83	2.08	0.83
22	0.00	-0.63	-	0.00	-0.21	6.67	1.04	0.21	1.67
21	0.94	0.28	-	0.76	-2.08	-1.88	1.98	0.42	1.88
20	-4.17	4.17	-	-0.83	1.25	0.42	1.04	1.67	1.46
19	-3.75	-2.50	-1.67	-0.83	2.50	0.00	-3.33	-1.25	-2.29
18	-0.42	2.50	5.42	3.13	1.82	-0.52	1.82	1.82	4.58
17	0.21	0.00	-4.58	1.25	-0.63	1.67	1.25	2.50	1.46
16	-2.92	-5.83	0.63	0.42	3.13	1.46	1.25	-1.67	0.83
15	0.63	2.08	-1.67	2.92	0.00	-2.50	3.75	-3.33	4.38
14	-0.83	-0.21	1.67	2.92	0.42	2.92	2.50	2.92	0.83
13	2.08	-1.56	0.78	-0.52	-0.52	3.80	3.38	0.00	2.08
12	-0.52	-1.95	1.95	-2.47	-0.52	-2.60	-2.21	-0.65	1.43
11	0.00	-2.60	3.65	-0.26	1.04	4.17	4.43	3.65	5.73
10	-1.04	4.43	-1.95	-2.21	-1.56	0.65	1.69	0.26	2.08
9	-2.34	3.91	5.08	1.30	1.04	2.21	1.82	0.52	5.73
8	0.26	-0.13	-0.52	-0.78	0.52	2.08	2.34	1.69	-3.39
7	1.04	2.60	-1.04	-2.08	0.26	0.26	2.60	-1.95	0.78
6	0.26	1.56	-0.26	-1.56	2.08	0.13	0.52	-3.13	-1.56
5	-1.78	-5.73	0.26	4.69	0.78	1.30	-0.78	-1.56	0.78
4	2.08	0.71	2.08	1.41	-0.31	2.19	0.91	-2.40	1.04
3	-4.34	-0.69	2.08	-0.69	0.00	-1.52	0.43	-1.30	-2.60
2	-0.17	-0.35	3.47	0.69	0.17	-2.08	1.39	2.43	2.78
1	-1.56	0.52	-0.28	0.35	-1.39	-1.39	-0.35	0.52	-0.69

*Wall numbering given in Fig. 6.3

TABLE 6.10 PARALLEL WALL OUT-OF-PLUMBS

FOR BUILDING B

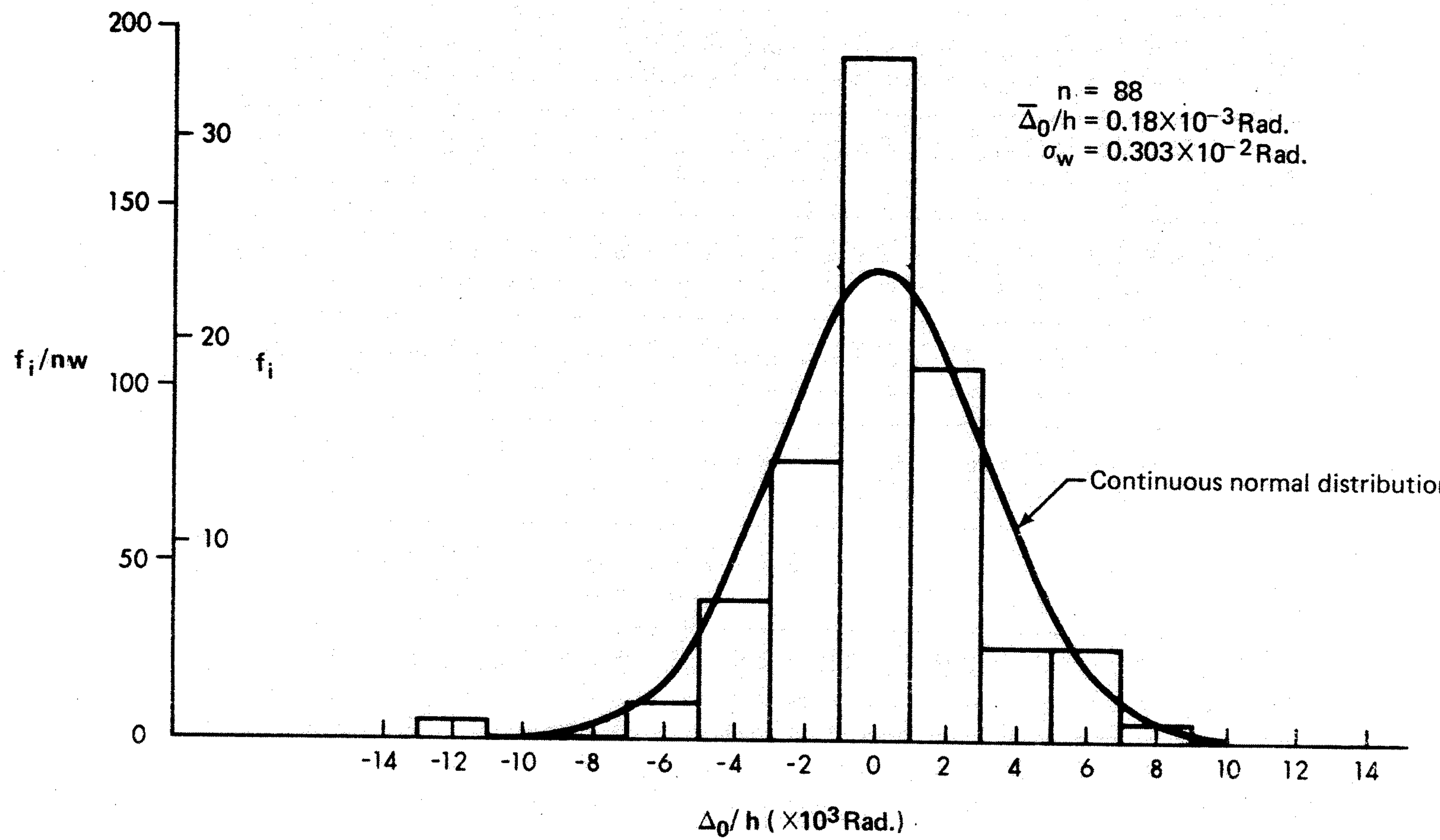


Figure 6.14 Distribution of perpendicular wall out-of-plumbs for building A

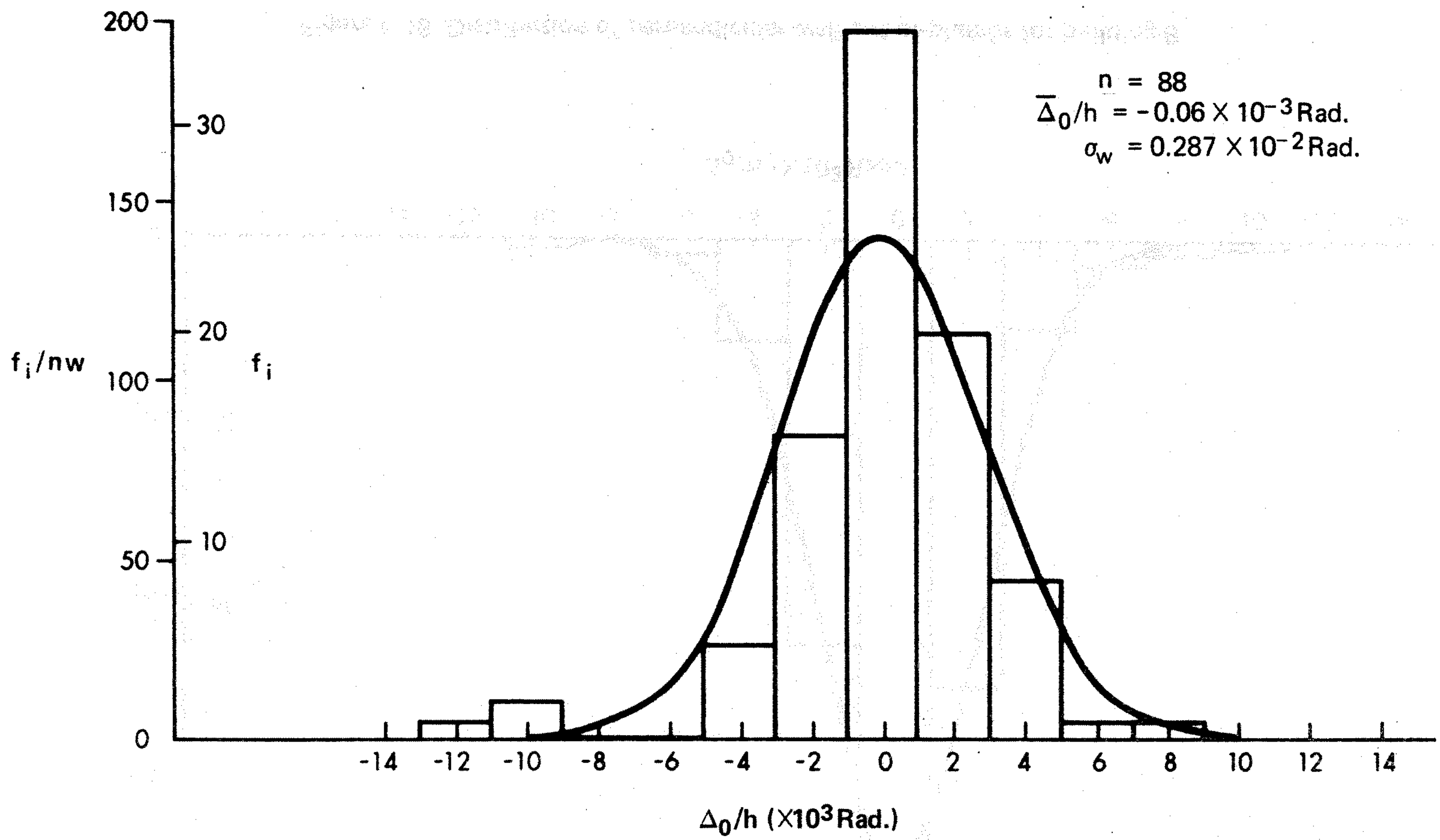


Figure 6.15 Distribution of parallel wall out-of-plumbs for building A

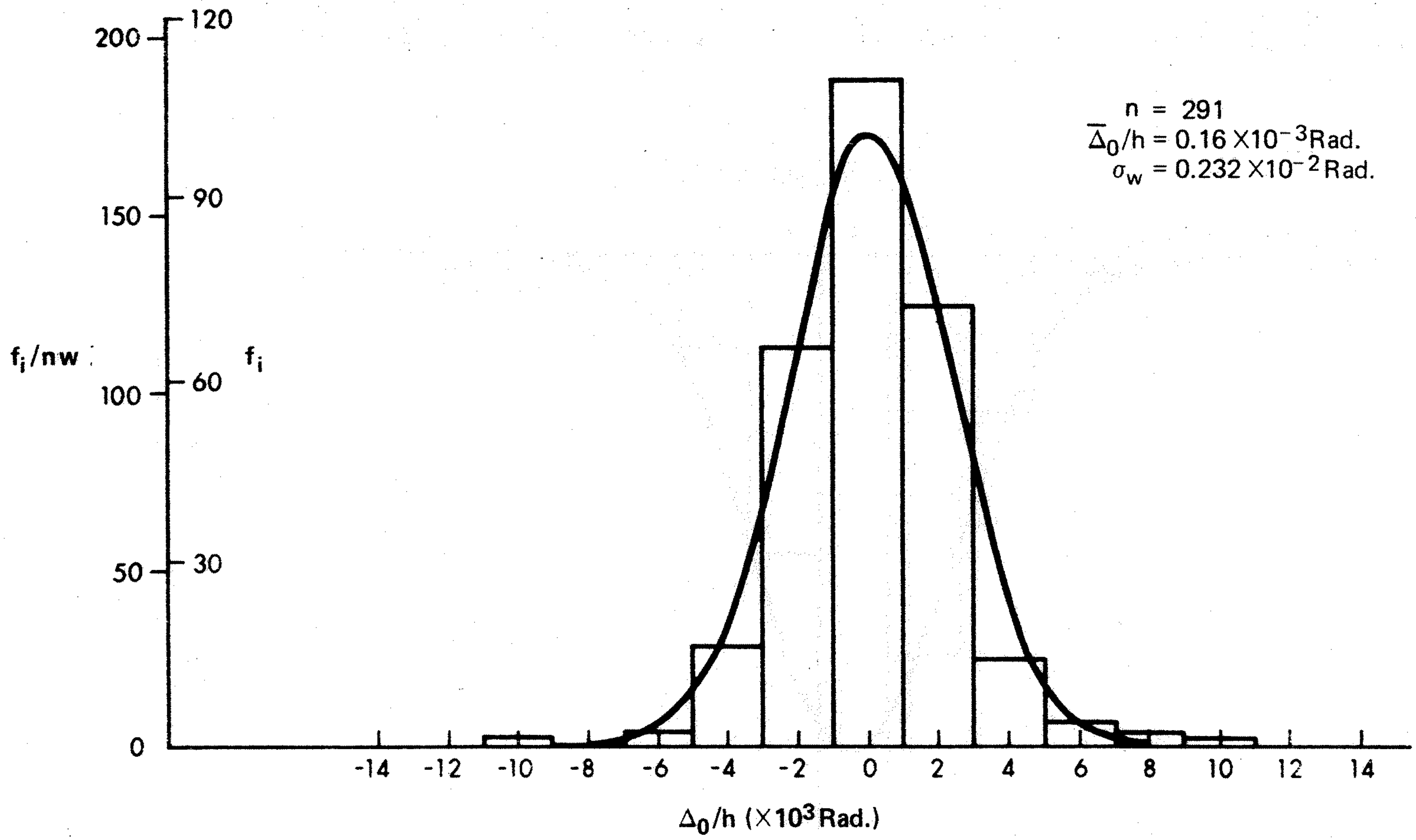


Figure 6.16 Distribution of perpendicular wall out-of-plumbs for building B

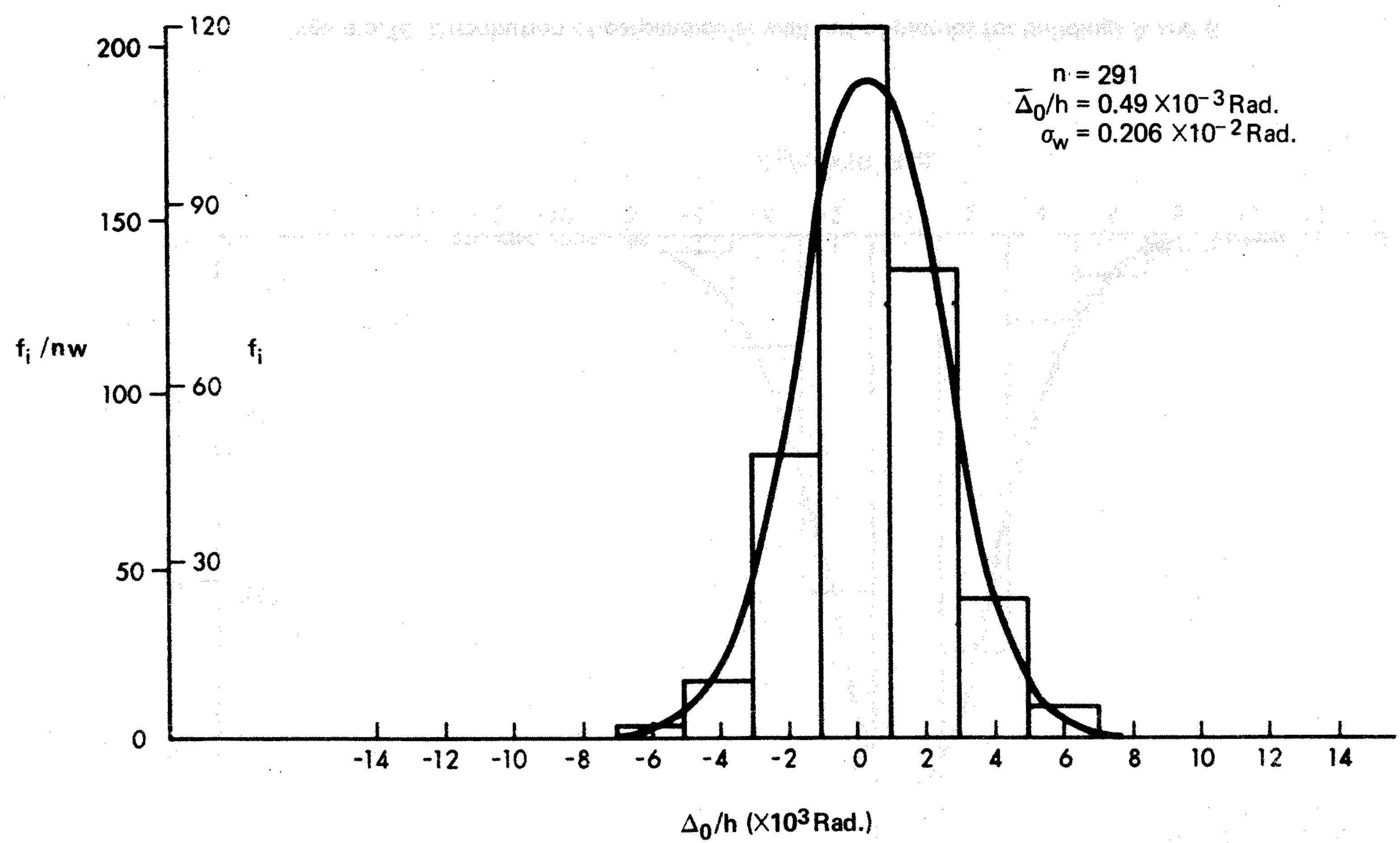


Figure 6.17 Distribution of parallel wall out-of-plumbs for building B

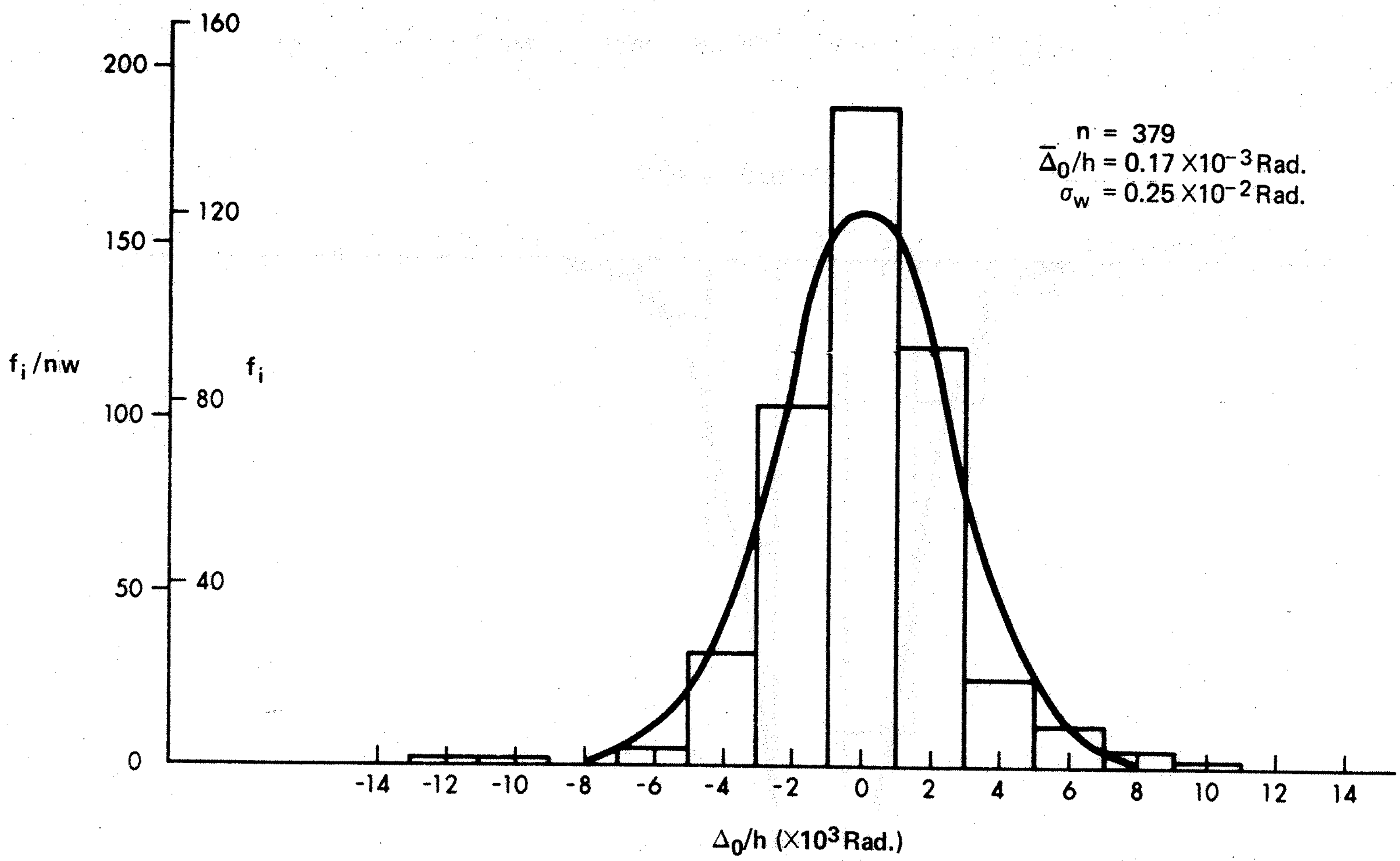


Figure 6.18 Distribution of perpendicular wall out-of-plumbs for buildings A and B

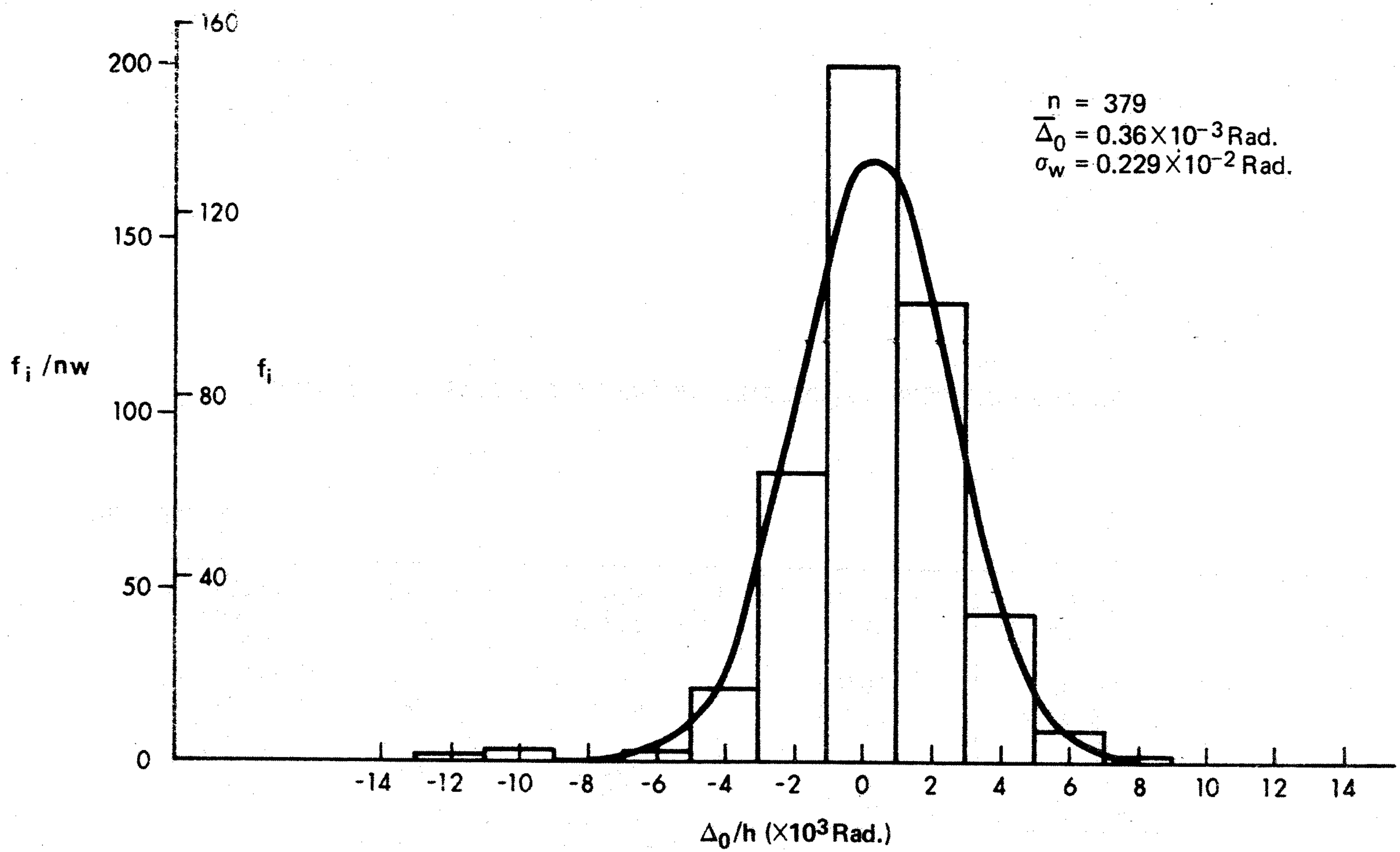


Figure 6.19 Distribution of parallel wall out-of-plumbs for buildings A and B

Building	Type	Sample Dimension n	Mean (Eqn. A-14)* ($x\ 10^2$ Rad.)	Stand. Dev. (Eqn. A-16)* ($x\ 10^2$ Rad.)	Skewness (Eqn. A-20)*	Kurtosis (Eqn. A-22)*
A	Perpen.	88	0.018	0.303	-0.57	5.6
	Parallel	88	-0.006	0.287	-1.18	7.3
B	Perpen.	291	0.016	0.232	0.29	4.9
	Parallel	291	0.049	0.206	0.06	3.3
A + B	Perpen.	379	0.017	0.250	-0.05	5.63
	Parallel	379	0.036	0.229	-0.60	6.37

* See Appendix A

TABLE 6.11 STATISTICAL CHARACTERISTICS OF WALL OUT-OF-PLUMBS assumed for both types of out-of-plumbs, therefore eliminating the need to differentiate between perpendicular and parallel deviations. The values listed in Table 6.11 also demonstrate that the distributions are slightly skewed and somewhat more peaked than a normal distribution.

CHAPTER VII

STATISTICAL ANALYSIS

7.1 Effects of Column Out-of-Plumbs

Since the column out-of-plumb population is normally distributed, the variate Δ_0/h can be standardized as described in Appendix A. A design value for column deviations from plumb is obtained by rearranging the terms in Eq. (A-44).

$$\frac{\Delta_d}{h} = \frac{\overline{\Delta}_0}{h} + \beta\,\sigma_c \tag{7.1}$$

In this expression Δ_d/h is the distributed random variable, β is the standardized Δ_d/h value, $\overline{\Delta}_0/h$ is the arithmetic mean of the discrete population, and σ_c is the standard deviation.

In the present study, the mean, valued at -1.0×10^{-5} Rad. in Fig. 6.7, may clearly be neglected in Eq. (7.1). In other words, the population is assumed normally distributed about a mean of zero. The expression is then reduced to

$$\frac{\Delta_d}{h} = \beta\,\sigma_c \tag{7.2}$$

The quantity β is found from the "Tables of the Standard Cumulative Normal Distribution" (Table A-1) for a prescribed cumulative probability of occurrence. For example, the probability of having a value falling within the limits $\pm\, 2\sigma_c$ is 0.9544. The selection of an appropriate β

in the present case is very arbitrary. A study described in Ref. 58 has shown that the probability of failure of a building under normal conditions should not be higher than 3×10^{-4} during the 30-year life of the structure. This corresponds to a β factor of approximately 3.5. A factor of 3.0 has been used in Ref. 61 in the derivation of design criteria based on limit states. The New Canadian Standard CSA-S16.1, "Steel Structures for Buildings - Limit States Design", has used β factors ranging between 2.9 and 4.0[59]. The selected factor, β, commonly called the "safety index", should fall within these limits. A conservative β of 3.5 corresponding to a probability of being exceeded of 4.6×10^{-4} will be used in this thesis. This choice will be subject to further discussions in Appendices B and C.

7.1.1 Horizontal Force at Connection Point

The horizontal forces shown in Fig. 2.2 result from the fact that the column is out-of-plumb. The force $P\Delta_x/h$, for instance, is transmitted by the connection to the beam or floor diaphragm and then to the core. A safe estimate of this additional force in the connection is:

$$F_d = \beta\sigma_c P = 3.5 \times 0.0017\ P \simeq 0.006\ P \tag{7.3}$$

where F_d is the absolute value of the force, P is the factored axial load in the column obtained for a specific load combination, $\beta = 3.5$, and $\sigma_c = 0.0017$ from Table 6.6. Equation (7.3) indicates that a connection between one column and the adjacent beam should be designed for 0.6 percent of the factored axial load to resist the force created by the out-of-plumb of the column. The force F_d has a probability of not being exceeded, defined by the safety index β, of 99.954 percent or, in other words, a probability of being exceeded of 4.6×10^{-4}, if P is assumed deterministic.*

In the common case of two column segments connected at a floor level and having different axial loads, different heights, and different out-of-plumbs, the extra force, F, at the beam-to-column connection is an algebraic summation of the type shown in Fig. 7.1.

$$F = \left(\frac{P\Delta}{h}\right)_1 + \left(\frac{P\Delta}{h}\right)_2 \qquad (7.4)$$

If $\quad F = P_1 X + P_2 Y$

and $\quad X \sim> N(\mu_x, \sigma_x), \quad Y \sim> N(\mu_y, \sigma_y)$

Then $\quad F \sim> N(P_1 \mu_x + P_2 \mu_y, \sqrt{P_1^2 \sigma_x^2 + P_2^2 \sigma_y^2})$,

if independence is satisfied.†

For $\mu_x = \mu_y = 0$ and $\sigma_x = \sigma_y = \sigma$,

$$F \sim> N(0, \sigma\sqrt{P_1^2 + P_2^2})$$

Thus, in the case of two column segments, the force F is still normally distributed and has a new standard deviation defined as above.

Equations (7.2) and (7.4) are combined to give:

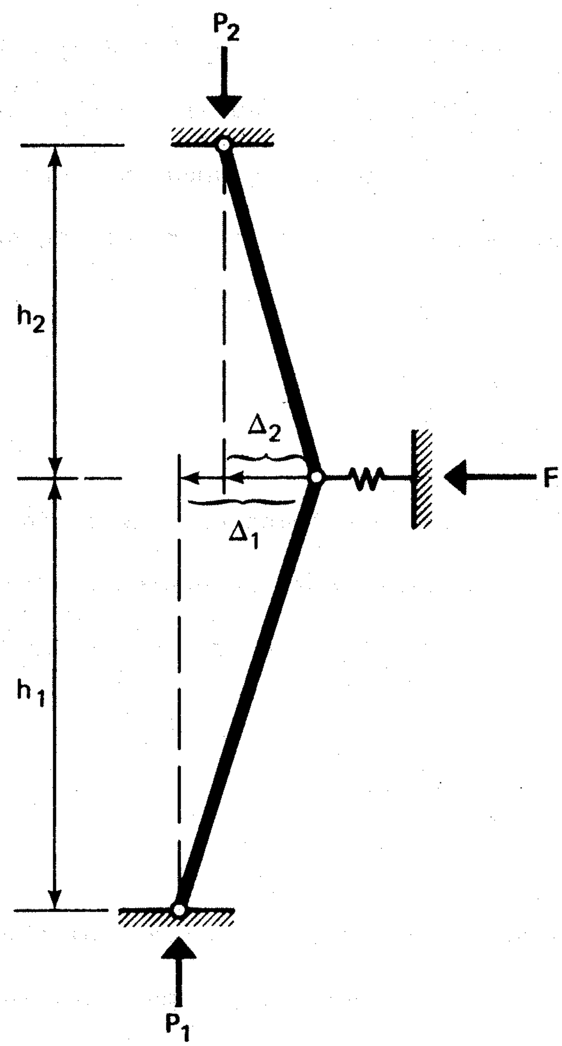
$$F_d = \beta\sigma_c\sqrt{P_1^2 + P_2^2} \qquad (7.5)$$

where F_d is the absolute value of the extra force used in the design of a beam-to-column connection when two columns are present. When $P_1 = P_2$, $F_d = 3.5 \times 0.0017 \times \sqrt{2}\ P = 0.0084\ P$. The extra force to be resisted by the connection as given by expression (7.5) is 0.84 percent of the

---

* See Appendix B

† See Appendix C



$$F = (P\Delta/h)_1 + (P\Delta/h)_2$$

Figure 7.1 Horizontal force required to stabilize two out-of-plumb columns

average axial load in the columns, which is significantly lower than the 2 percent presently used[43].

7.1.2 Horizontal Shear in the Plane of the Floor

The out-of-plumbs of the columns above and below a given floor produce extra shears in the plane of that floor. For example, the floor system of Fig. 6.2 must transmit to the core an extra shear resulting from the 16 out-of-plumb columns on lines 8 to 15. These columns have random inclinations and the resulting shear (for example in the x-direction) is:

$$F = \sum_{j=1}^{n} \left[\frac{P\Delta_0}{h}\right]_j \qquad (7.6)$$

where $n = 16$, the number of columns considered in the example.

As in the preceding case, an expression for F may be obtained from the statistical sum of the standard deviations corresponding to the 16 columns.

$$F_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2} \qquad (7.7)$$

F_d is the absolute value of the extra horizontal shear due to n out-of-plumb columns and P_j is the factored column axial load for the load combination considered. The other terms have been defined previously. Equation (7.7) is general and includes Eqs. (7.3) and (7.5), applied previously to connection design, for n = 1 and 2.

7.1.3 Moment in Floor

Moments in any portion of a floor due to a group of out-of-plumb columns can also be determined. For example, the moment in the plane of the floor at point 0 in Fig. 6.2 is produced by the x and y out-of-plumbs of the columns on lines 8 to 15, thus:

$$M = \sum_{j=1}^{16} [\frac{P\Delta_x}{h} L_y + \frac{P\Delta_y}{h} L_x]_j \qquad (7.8)$$

In Eq. (7.8), L_x and L_y are the lever arms in the x and y directions from the column to the point at which the moment is calculated. Since L_x and L_y are also coefficients (similar to P), the same summation rule applies.

$$M_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n} [P^2(L_x^2 + L_y^2)]_j} \qquad (7.9)$$

M_d is the absolute value of the design moment in the plane of the floor due to a group of n out-of-plumb columns.

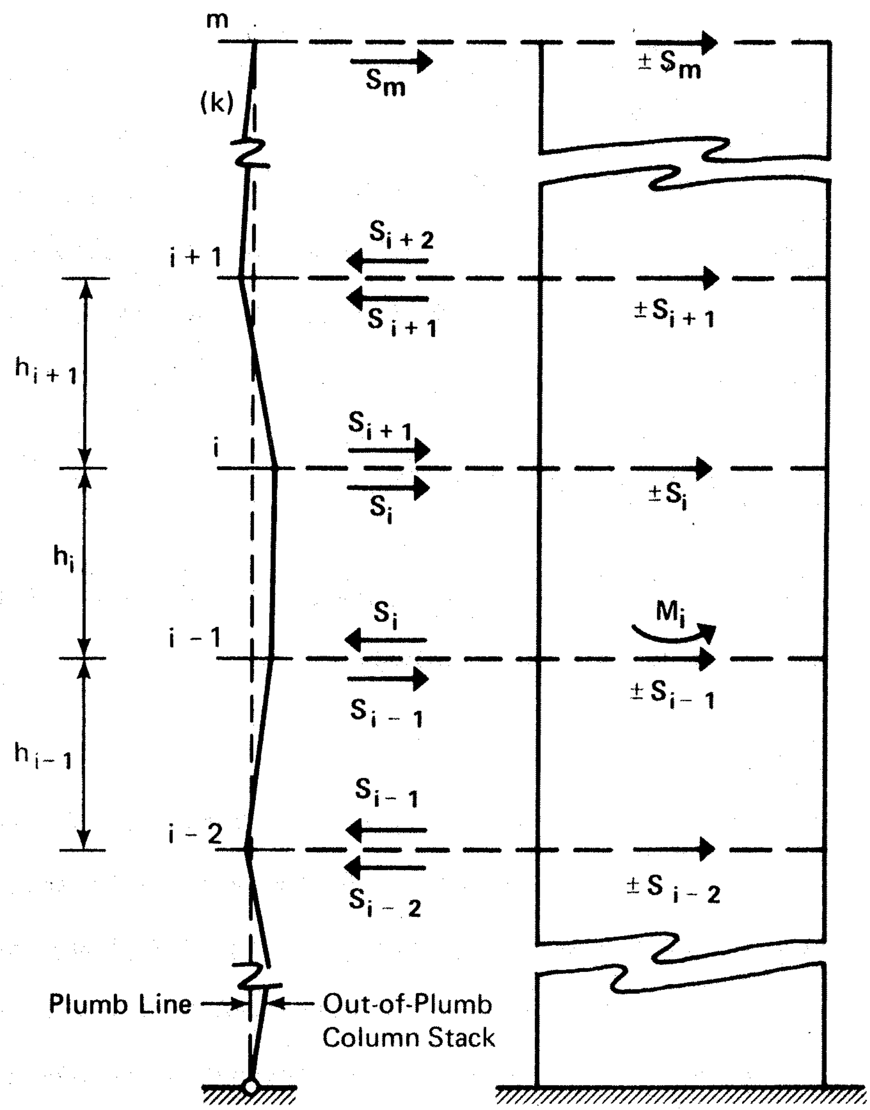
7.1.4 Shear in Core

The core (or any other bracing system) must stabilize the columns by resisting the forces induced by the vertical loads acting on the columns in their deformed positions. Fig. 7.2 shows that the absolute value of the out-of-plumb shear resisted by the core between floors i-1 and i is S_i and depends only on the out-of-plumbs of the columns at storey i (storey below floor level i).

$$S_i = \sum_{j=1}^{n_i} [\frac{P\Delta_0}{h}]_j \qquad (7.10)$$

where n_i is the number of columns at storey i. Then,

$$S_{di} = \beta\sigma_c \sqrt{\sum_{j=1}^{n_i} P_j^2} \qquad (7.11)$$



m = Total number of storeys

i, k = Storey indices $i + 1 \leqslant k \leqslant m$

Figure 7.2 Shears and moments in core due to column out-of-plumbs

where S_{di} is the absolute value of the shear in the core at storey i caused by the out-of-plumbs of the columns and P_j is the factored axial load in column j. The summation extends over the n_i columns in storey i.

7.1.5 Moment in Core

Fig. 7.2 shows that the moment at floor level i due to the out-of-plumb columns is:

$$M_i = \sum_{k=i+1}^{m} S_k h_k \tag{7.12}$$

where k is a storey index used for storeys above level i. Substituting S_k as given by Eq. (7.10) in the expression for the moment, gives:

$$M_i = \sum_{k=i+1}^{m} [\sum_{j=1}^{n_k} (\frac{P\Delta_0}{h})_j]_k h_k$$

Transforming the summation inside the brackets results in:

$$M_i = \sum_{k=i+1}^{m} (\beta\sigma_c \sqrt{\sum_{j=1}^{n_k} P_j^2})_k \, h_k$$

Transforming again for the other summation yields:

$$M_{di} = \beta\sigma_c \sqrt{\sum_{k=i+1}^{m} [\sqrt{\sum_{j=1}^{n_k} P_j^2}]_k^2 \, h_k^2}$$

which can be written as:

$$M_{di} = \sqrt{\sum_{k=i+1}^{m} (S_d h)_k^2} \tag{7.13}$$

where M_{di} is the moment in the core at floor level i caused by the out-of-plumbs of the columns, S_{dk} is the shear at storey k given by Eq. (7.11),

h_k is the height of storey k, and m is the total number of storeys in the building.

7.1.6 Torque in Core

The torque due to out-of-plumb columns, at a specific storey of the core, depends only on the columns at that storey in a manner similar to the shear. The torque at each storey is obtained by combining the expressions developed for the moments in the floors (7.9) and the shears in the core (7.11).

$$T_i = \sum_{j=1}^{n_i} \left[\frac{P\Delta_x}{h} L_y + \frac{P\Delta_y}{h} L_x\right]_j \qquad (7.14)$$

or

$$T_{di} = \beta\sigma_c \sqrt{\sum_{j=1}^{n_i} [P^2(L_x^2 + L_y^2)]_j} \qquad (7.15)$$

where L_x and L_y are the distances (lever arms) along the x and y axes between a particular column and the center of resistance of the core.

7.1.7 Lateral Deflections

An equivalent column inclination, Δ_d/h, constant for a specified number of columns may be obtained from Eqs. (7.6) and (7.7). For $F = F_d$,

$$\frac{\Delta_d}{h} = \frac{\beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2}}{\sum_{j=1}^{n} P_j} \qquad (7.16)$$

This equation would be considerably simplified if expressed only in terms of β, σ_c and n, the total number of columns in the structure. Assuming that

P_j is constant for all the columns gives:

$$\frac{\Delta_d}{h} = \frac{\beta\sigma_c}{\sqrt{n}} \qquad (7.17)$$

Generally, the column axial loads differ greatly in a structure. As demonstrated in Appendix D, the formulation (7.17) is always unconservative with respect to the "exact" expression (7.16). It is also demonstrated in the Appendix that

$$\frac{\Delta_d}{h} = \frac{\beta\sigma_c}{\sqrt[2.2]{n}} \qquad (7.18)$$

gives a safe estimate when n is reasonably large. For structures of one and two storeys, Eq. (7.16) is recommended.

A set of horizontal forces is obtained from the structural configuration shown in Fig. 5.1(a) where the constant slope is defined by either one of the equations above. These forces can be added to the wind forces and used to calculate the lateral deflections of a structure. Applications of Eqs. (7.16) and (7.18) are given in the next chapter.

7.2 Effects of Wall Out-of-Plumbs

The results summarized in Table 6.11 show that the perpendicular and parallel wall out-of-plumb populations can be described by a normal distribution and statistical characteristics common to both. Conservatively, the mean, $\bar{\Delta}_0/h$, is taken as 0.00028 Rad. and the standard deviation, σ_w, as 0.0028 Rad.

Since the core depends entirely on itself for stability (the frame is assumed pinned at each floor level and the core cantilevered from the foundation), only moments, torques, and extra lateral deflections induced in the core by the wall out-of-plumbs must be calculated.

The deviations measured on the walls are affected in some ways by the presence of variations in wall thickness. The problem is treated in Appendix E.

7.2.1 Moment in Core

The expression used to describe a standardized normal variable (A-44) can also be used to describe the wall out-of-plumbs:

$$\frac{\Delta_d}{h} = \frac{\bar{\Delta}_0}{h} + \beta\sigma_w \tag{7.19}$$

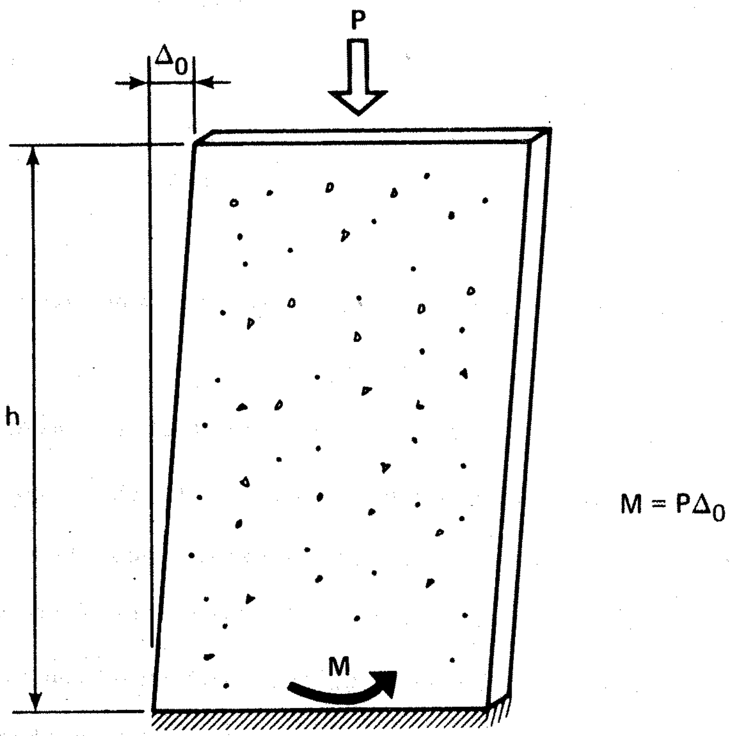
where Δ_d/h is the design wall out-of-plumb and β is the safety index introduced in section 7.1. The moment due to out-of-plumbs at the base of the one-storey wall shown in Fig. 7.3(a) is:

$$M = P\Delta_0 \tag{7.20}$$

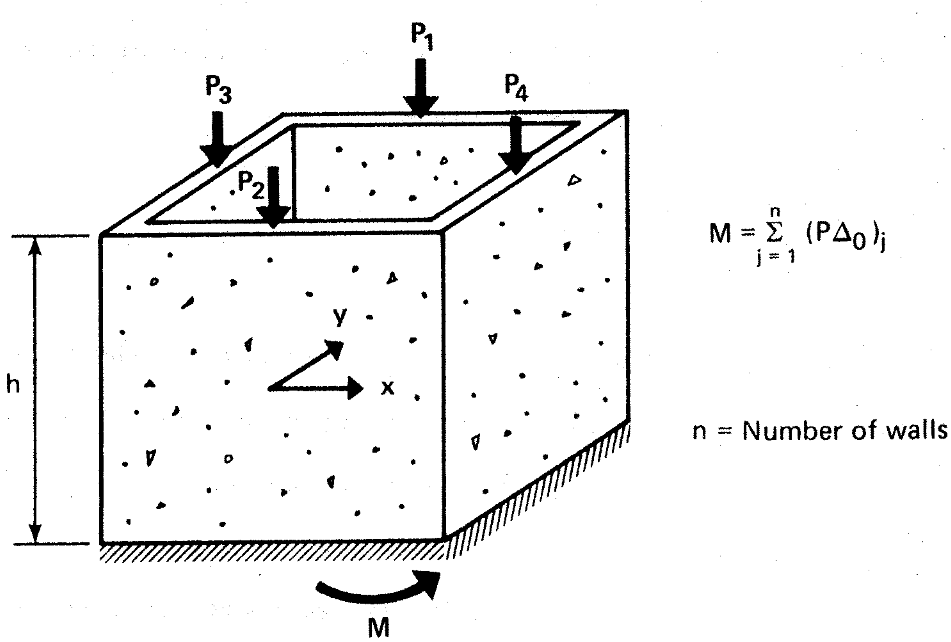
where P is the total factored load carried by the wall and Δ_0 is the actual averaged out-of-plumb of the wall. Similarly, the moment in either the x or y direction at the base of the one-storey core section of Fig. 7.3(b) is:

$$M = \sum_{j=1}^{n} (P\Delta_0)_j \tag{7.21}$$

where P_j is the factored axial load carried by one of the n individual wall segments and Δ_{0j} is the actual perpendicular or parallel out-of-plumb of the wall, depending on the direction considered.



a) One-storey wall



b) One-storey core

Figure 7.3 Moments in one-storey walls

Equations (7.20) and (7.21) together with (7.19) may be adapted for design by the transformation of section 7.1.1.

$$M_d = \frac{\overline{\Delta}_0}{h} \sum_{j=1}^{n} (Ph)_j + \beta\sigma_w \sqrt{\sum_{j=1}^{n} (Ph)_j^2} \qquad (7.22)$$

The terms in the above expression have been defined previously in this chapter.

Equation (7.22) would be simplified if the mean were neglected. The actual mean could be zero, as in the case of column out-of-plumbs, due to the similarities between these two variables. It is possible that the measured mean is different from zero by reason of the relative small sample size (379). The question, at this point, is to find the percentage of the total moment that is contributed by the mean in practical situations.

Assuming constant axial loads, Eq. (7.22) becomes $M_d = 0.00028\ n\ Ph + 3.5 \times 0.0028 \sqrt{n}\ Ph$. The ratio of the first term to the total expression for M_d gives

$$\frac{0.00028\sqrt{n}}{0.00028\sqrt{n} + 0.0098}$$

which is the percentage contribution of the mean to the total moment in terms of n. For n = 1, the mean accounts for less than 3 percent of the total but for n = 5 and 20, the contributions are 6 and 11 percent respectively.

Assuming, for the reasons listed above, that the mean could be negligible, Eq. (7.22) would become:

$$M_d = \beta\sigma_w \sqrt{\sum_{j=1}^{n} (Ph)_j^2} \qquad (7.23)$$

and σ_w is taken as 0.0028. The validity of Eq. (7.23) will be discussed in the next chapter.

The design equation for the moments due to initial wall deviations in a multi-storey core is obtained in a manner similar to Eq. (7.13) for column out-of-plumbs. Using the notation adopted in Fig. 7.2, the moment is calculated as:

$$M_i = \sum_{k=i+1}^{m} M_k \tag{7.24}$$

where M_i, the moment at level i, is the algebraic summation of the individual storey-moments above level i. When the contribution of the mean is accounted for, the corresponding design equation becomes:

$$M_{di} = \frac{\bar{\Delta}_0}{h} \sum_{k=i+1}^{m} \left[\sum_{j=1}^{n_k} (Ph)_j \right]_k + \beta\sigma_w \sqrt{\sum_{k=i+1}^{m} \left[\sum_{j=1}^{n_k} (Ph)_j^2 \right]_k} \tag{7.25}$$

When the contribution of the mean is neglected, this expression is reduced to:

$$M_{di} = \sqrt{\sum_{k=i+1}^{m} (M_d^2)_k} \tag{7.26}$$

where M_{dk} is given by Eq. (7.23) for each level k above level i. The variable n in Eq. (7.23) is then replaced by n_k, the number of walls at storey k.

7.2.2 Torque in Core

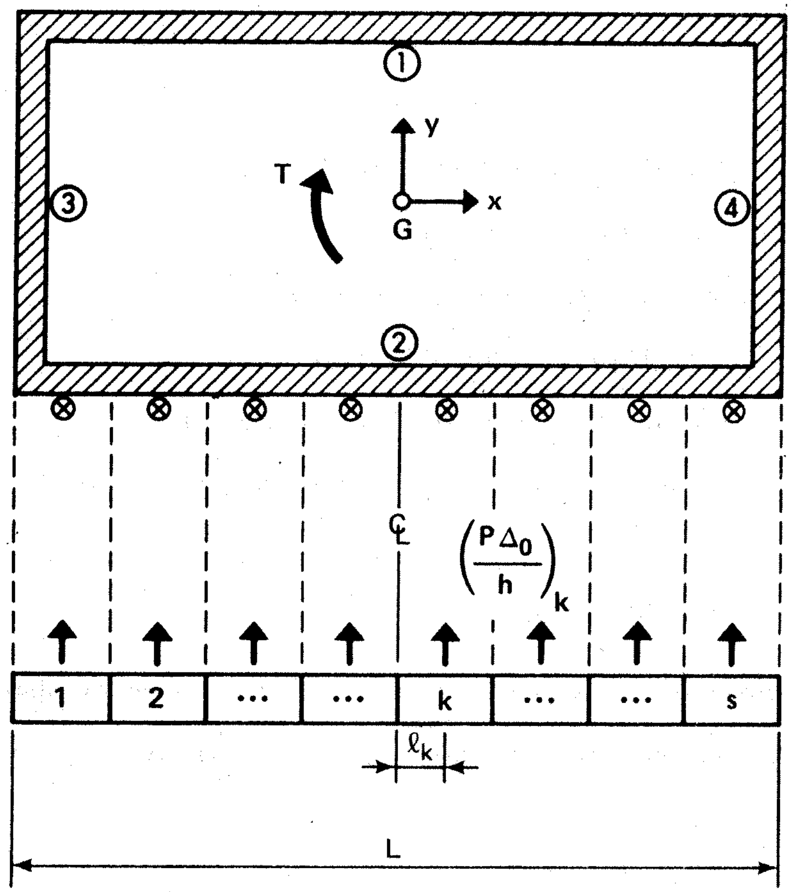
The cantilevered wall shown in Fig. 7.3(a) is stabilized against the in-plane out-of-plumb by a moment at the base. However, due to the relatively small thickness of the wall, the stability against an out-of-plumb in the orthogonal direction must be ensured by some means other than the wall itself.

As an example, the support required to stabilize wall No. 2 in the y-direction, as shown in Fig. 7.3(b), is provided by the adjacent walls, Nos. 3 and 4, spanning at right angles. Assuming conservatively that the base of wall No. 2 is pinned in the y-direction and that the supports of the adjacent walls are only effective at the top of the wall, a total horizontal force of value $P\Delta_y/h$ is induced to stabilize wall No. 2. The out-of-plumb value for wall No. 2 in the y-direction is Δ_y.

This force, however, is not distributed equally to the stabilizing walls (Nos. 3, 4) since the wall to be stabilized (No. 2) has variable out-of-plumbs at different sections. When several walls are assembled orthogonally to form a core, the presence of these unbalanced forces could result in the formation of a significant torque in the core.

A set of unequal horizontal forces on a wall can be visualized as a corresponding force applied eccentrically with respect to the center of the wall. The eccentricity of this force is a function of the actual state of plumbness of the wall and consequently characterizes that wall. The eccentricity can be considered as a variable with a particular distribution and can be evaluated from the out-of-plumb measurements taken on the individual walls by the procedure described below.

As shown in Fig. 7.4, a wall may be subdivided into as many segments as there are measurements taken on that wall. The contribution of the wall to the total torque can be approximated by:



T: Torque (positive clockwise)

G: Center of resistance of the core

ℓ_k: Distance from segment k to the center of the wall

L: Total length of the wall

⊗: Location of measurements

Figure 7.4 Evaluation of ratio e/L

$$T = Pe = \sum_{k=1}^{s} \left(P \frac{\Delta_0}{h} \ell\right)_k$$

where

T = resulting torque

P = total factored axial load on the wall

e = equivalent "eccentricity"

s = number of segments in a wall

P_k = axial load carried by segment k

Δ_{0k} = measured out-of-plumb of segment k

ℓ_k = distance from the center of segment k to the center of the wall

h = height of the wall

Making the assumption that each segment carries an equal share of the total load on the wall and solving for e, gives:

$$e = \sum_{k=1}^{s} \left(\frac{\Delta_0}{h} \ell\right)_k \frac{P_k}{P}$$

The ratio P_k/P is equal to $1/s$. Dividing both sides by the length, L, of the wall gives:

$$\frac{e}{L} = \frac{1}{s} \sum_{k=1}^{s} \left(\frac{\Delta_0}{h} \frac{\ell}{L}\right)_k \qquad (7.27)$$

The measured out-of-plumb, Δ_0/h, of each segment k is multiplied by the ratio ℓ_k/L pertaining to that segment. The sum of the s individual products is then multiplied by a constant, 1/s, to result in a dimensionless equivalent "eccentricity", e/L, which characterizes the wall.

These quantities are listed in Tables 7.1 and 7.2 for the walls of buildings A and B respectively. The values are then plotted separately for each building in Figs. 7.5 and 7.6 and combined in Fig. 7.7. The characteristics of each distribution are listed in Table 7.3.

The distributions are approximately normal. They are reasonably peaked and slightly skewed. The absolute value of the mean $\overline{e}/L$, can be taken as 0.5×10^{-4} and the standard deviation, σ_e, as 4.0×10^{-4}.

The contribution of a wall to the total torque in a core is then calculated as:

$$T = P \frac{e}{L} L \tag{7.28}$$

where e/L, since normally distributed, can be approximated by:

$$\frac{e}{L} = \frac{\overline{e}}{L} + \beta\sigma_e \tag{7.29}$$

The safety index β is 3.5.

The design torque in the core at any storey i is obtained from a statistical formulation combining the two expressions above.

$$T_{di} = \frac{\overline{e}}{L} \sum_{j=1}^{n_i} (PL)_j + \beta\sigma_e \sqrt{\sum_{j=1}^{n_i} (PL)_j^2} \tag{7.30}$$

The torque at storey i depends only on the n_i out-of-plumb walls at that storey. By assuming an eventual mean of zero for a population with a larger sample dimension, Eq. (7.30) would be reduced to:

$e/L \times 10^4$

Storey No.	Wall No.*			
	1	2	3	4
22	1.95	1.24	-0.81	-3.94
21	-0.09	-2.21	0.54	-2.49
20	0.18	3.54	-0.27	5.36
19	-2.30	-4.25	-4.61	4.85
18	-0.09	-3.54	3.80	0.85
17	6.20	9.92	-4.34	4.98
16	-4.68	3.32	1.09	9.22
15	2.44	-3.12	5.97	-1.36
14	-4.68	1.09	-2.44	-1.36
13	4.14	5.49	2.44	-0.81
12	5.49	-2.17	-0.54	-0.54
11	-2.58	-1.09	-2.17	0.54
10	-0.07	0.61	0.00	1.36
9	-0.67	-0.27	-1.90	-1.36
8	1.15	0.14	0.27	-1.09
7	2.10	-2.78	2.71	2.17
6	1.03	1.42	-3.98	-1.36
5	1.89	-2.69	0.74	0.18
4	-5.06	-2.48	3.04	8.68
3	-3.20	8.48	5.21	0.29
2	8.56	-1.53	1.57	-0.97
1	-6.78	1.24	0.49	-3.86

* Wall numbering given in Fig. 6.2

TABLE 7.1 VARIABLE e/L FOR BUILDING A

$e/L \times 10^4$

Storey No.	Wall No.*								
	1	2	3	4	5	6	7	8	9
34	-0.80	2.33	-	-3.92	-5.08	2.87	8.46	0.87	-4.69
33	-8.27	1.17	-	3.91	4.17	-2.21	-8.33	1.11	5.60
32	-7.49	6.06	-	0.91	-3.65	7.55	-1.30	1.42	1.04
31	4.46	-3.65	-	-0.14	-8.33	14.58	-6.38	2.72	0.00
30	0.94	-0.52	-	-4.69	4.40	-6.35	11.26	-1.43	-0.91
29	0.65	8.43	-	0.78	-2.87	2.08	0.13	-1.53	-1.69
28	-3.26	-2.80	-	-5.05	-3.39	4.82	-2.47	2.97	-2.87
27	-0.81	-5.34	-	-0.33	-3.78	1.30	-1.17	-2.35	2.08
26	4.30	3.39	-	-2.34	-1.82	1.04	3.00	-4.45	-4.95
25	0.29	1.04	-	1.95	1.69	-4.18	-0.52	2.23	-2.08
24	5.37	-1.69	-	-4.53	-11.46	3.26	-8.07	-1.87	-1.82
23	0.39	-1.86	-	3.52	12.24	2.87	0.65	-3.14	2.34
22	-2.28	1.95	-	0.39	-3.39	3.52	2.08	1.56	0.78
21	4.23	-3.26	-	1.20	1.04	6.51	-3.00	-5.11	0.78
20	-2.12	-8.04	-	-0.39	-9.64	0.26	2.60	0.80	-2.87
19	-2.51	3.94	-0.39	-6.38	-2.08	-1.04	-9.64	0.93	-4.17
18	3.78	-3.99	-1.79	-6.35	-7.65	-1.95	-7.49	5.97	-1.63
17	-0.29	-4.33	-1.43	-0.91	3.91	-0.91	1.69	5.34	1.95
16	2.31	0.72	0.52	-3.13	-1.04	1.30	2.08	-1.54	3.78
15	-4.07	-1.07	-1.17	5.08	2.08	-1.17	-2.08	-5.38	-1.30
14	0.91	-0.37	-4.82	-5.47	-14.06	3.13	-4.56	3.37	1.04
13	-4.23	3.38	2.12	-1.30	-1.14	-4.75	-8.30	2.18	5.86
12	1.16	-1.57	-3.01	0.65	1.79	-1.89	-7.00	4.46	4.31
11	-6.19	-2.28	1.79	1.63	1.79	-0.10	-7.16	3.01	-0.16
10	1.24	-4.07	-6.51	-2.87	-6.25	-2.93	-11.07	5.41	-6.10
9	-1.99	-1.95	-5.86	-2.93	-8.14	-1.20	-8.14	-1.75	-5.21
8	1.18	0.55	-1.55	-7.32	-11.39	-5.83	-7.98	-7.30	1.47
7	-2.54	0.41	-2.28	-2.85	2.66	1.07	-2.93	5.89	-1.14
6	2.48	1.22	-4.49	0.00	4.88	0.00	1.63	3.93	-3.91
5	4.27	2.22	-0.94	-1.47	-10.25	1.97	-0.49	4.39	1.47
4	-3.00	10.51	0.77	1.14	6.51	-8.63	-6.02	-3.74	6.59
3	-0.52	2.50	-4.50	-6.50	-2.28	-1.30	-1.79	-3.09	0.41
2	-0.75	2.31	-1.30	-1.14	3.74	-0.81	2.60	-0.65	-0.43
1	-1.68	-2.22	-3.36	-3.91	-2.60	-0.65	6.51	-1.14	1.52

* Wall numbering given in Fig. 6.3

TABLE 7.2 VARIABLE e/L FOR BUILDING B

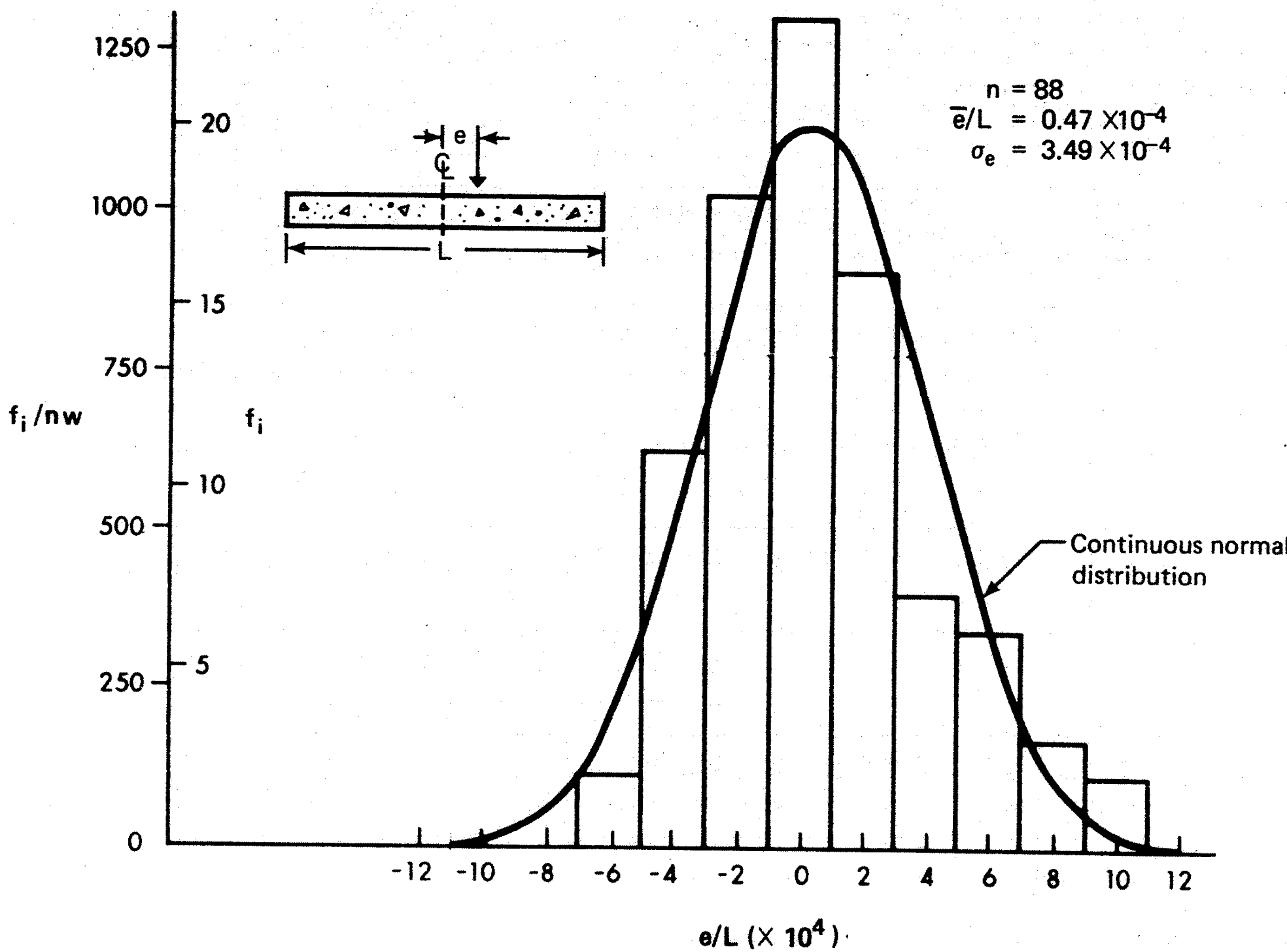


Figure 7.5 Distribution of e/L for building A

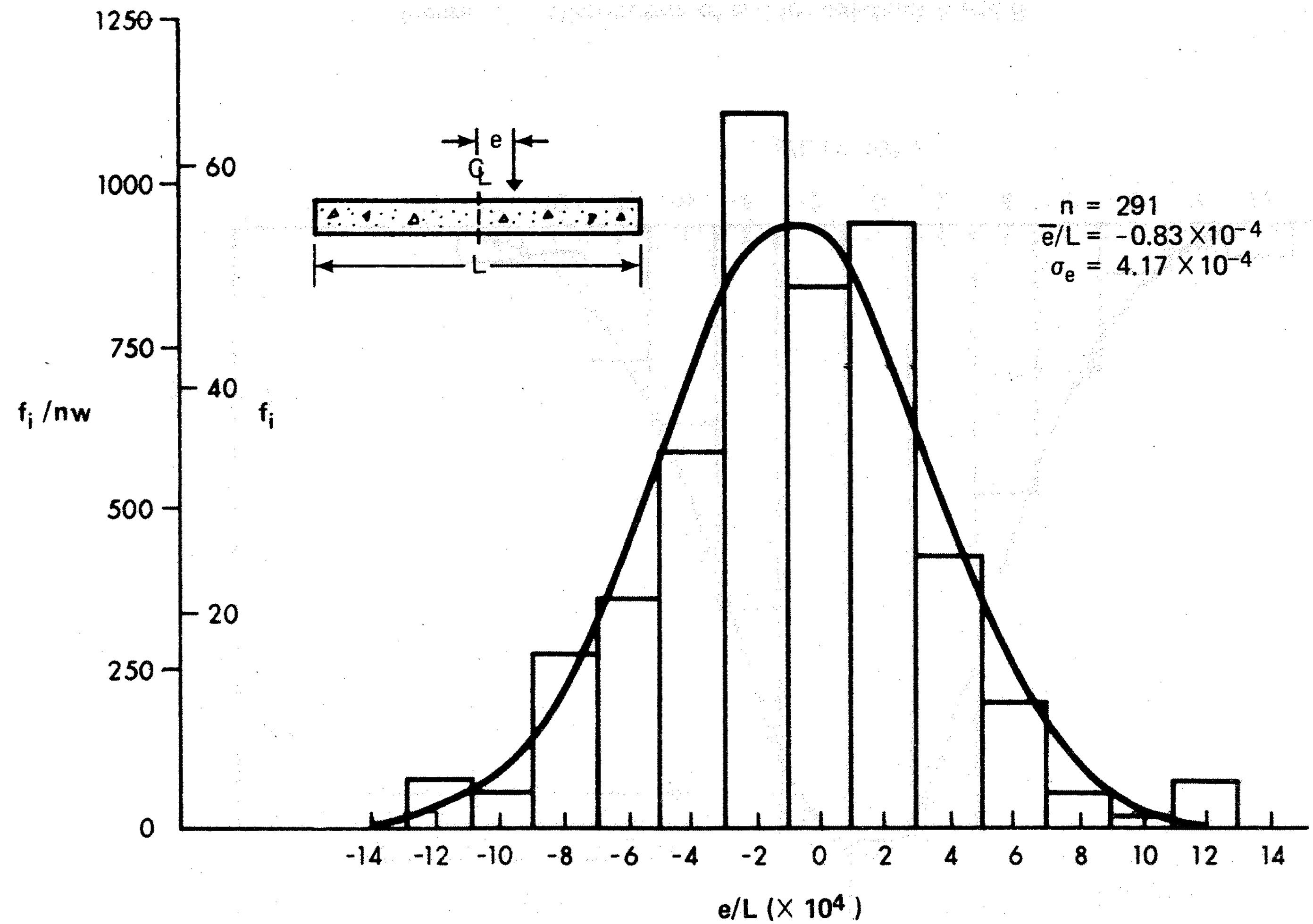


Figure 7.6 Distribution of e/L for building B

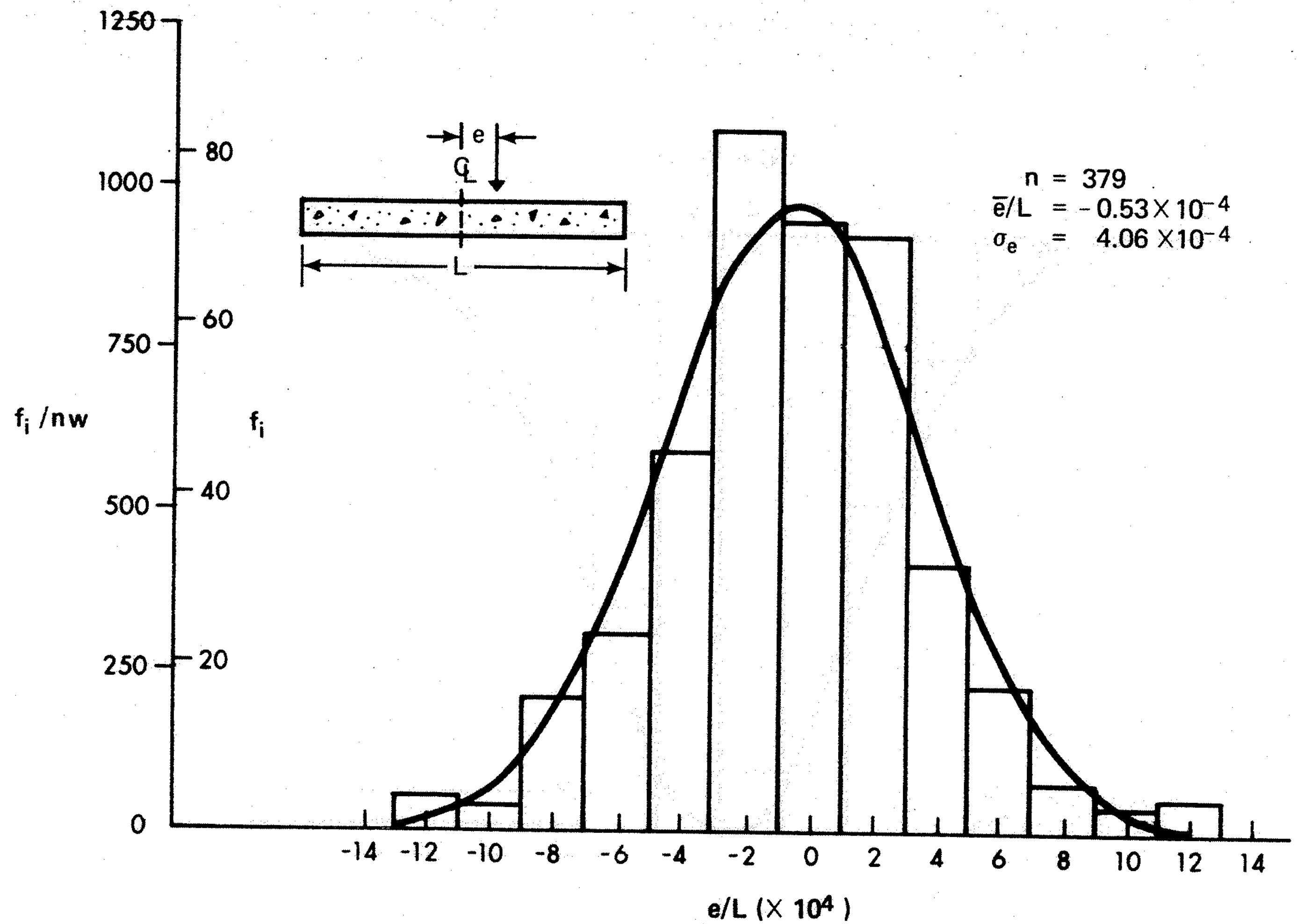


Figure 7.7 Distribution of e/L for buildings A and B

Building	n	Mean ($\times 10^4$)	Stand. Dev. ($\times 10^4$)	Skewness	Kurtosis
A	88	0.47	3.49	0.65	3.21
B	291	-0.83	4.17	0.06	3.79
A + B	379	-0.53	4.06	0.10	3.81

TABLE 7.3 STATISTICAL CHARACTERISTICS FOR e/L

$$T_{di} = \beta\sigma_e \sqrt{\sum_{j=1}^{n_i} (PL)_j^2} \tag{7.31}$$

The two equations above are only applicable to a reinforced concrete structure consisting of an orthogonal assembly of cast-in situ walls.

7.2.3 Lateral Deflections

It has been demonstrated in section 7.1.7 that a structure deflects laterally as the result of the axial loads acting on the out-of-plumb columns. In a similar manner, the axial loads acting on the out-of-plumb walls forming the core force the structure to deflect an additional amount. As shown in Fig. 7.8(a), a vertical load P applied to an out-of-plumb wall section induces an additional lateral deflection Δ_0'. The moment at the base of the one-storey wall is then the sum of the moment $P\Delta_0$ defined in section 7.2.1 and a smaller moment $P\Delta_0'$.

An estimation of the component $P\Delta_0'$ of the total moment can be obtained from the equivalent model shown in Fig. 7.8(b). A fictitious horizontal force $P\Delta_0/h$ is applied at the top of the perfectly vertical wall section. In a manner similar to the iterative procedure described in Chapter III, the structure is analyzed to determine the converged moment $P(\Delta_0 + \Delta_0')$ and the corresponding deflection Δ_0'.

The model shown in Fig. 7.8(b) can be used to evaluate an equivalent wall out-of-plumb value, Δ_d/h, for the general case of a combination of n walls. The derivation is similar to that of section 7.1.7 for columns. The fictitious force at the top of the wall section is:

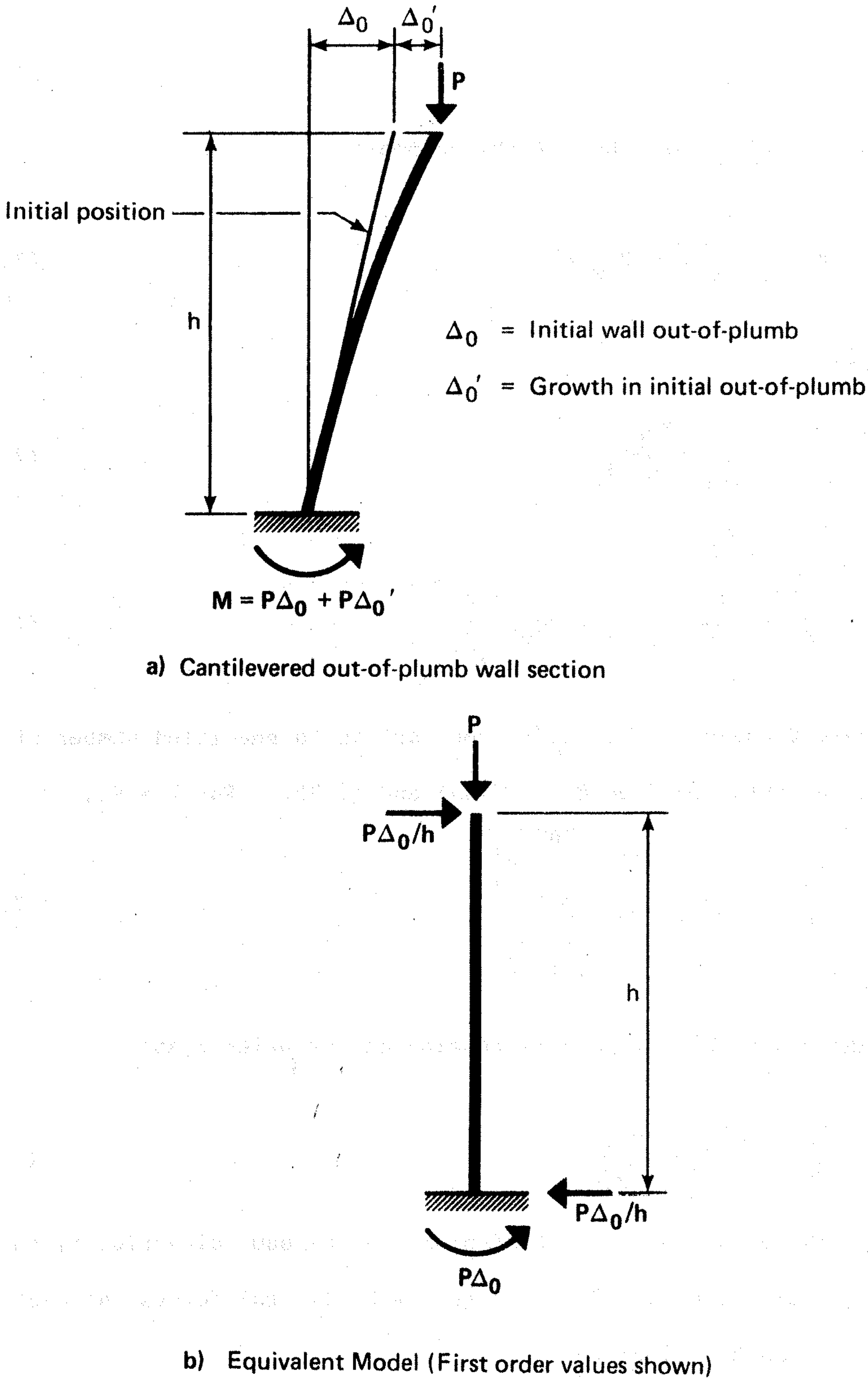


Figure 7.8 Effect of wall out-of-plumbs on the lateral deflection of a core

$$F = \frac{P\Delta_0}{h} \tag{7.32}$$

Eq. (7.32) combined with Eq. (7.19) becomes:

$$F_d = \frac{\overline{\Delta}_0}{h} P + \beta\sigma_w P \tag{7.33}$$

For n walls,

$$F = \sum_{j=1}^{n} \left(\frac{P\Delta_0}{h}\right)_j \tag{7.34}$$

or

$$F_d = \frac{\overline{\Delta}_0}{h} \sum_{j=1}^{n} P_j + \beta\sigma_w \sqrt{\sum_{j=1}^{n} P_j^2} \tag{7.35}$$

An equivalent out-of-plumb, Δ_d/h, constant for a specified number of walls may be obtained from Eqs. (7.34) and (7.35). For $F = F_d$,

$$\frac{\Delta_d}{h} = \frac{\overline{\Delta}_0}{h} + \beta\sigma_w \frac{\sqrt{\sum_{j=1}^{n} P_j^2}}{\sum_{j=1}^{n} P_j} \tag{7.36}$$

According to Appendix D, this expression can be written as:

$$\frac{\Delta_d}{h} = \frac{\overline{\Delta}_0}{h} + \frac{\beta\sigma_w}{2.2\sqrt{n}} \tag{7.37}$$

Assuming that every wall is out-of-plumb in the same direction by the amount Δ_d/h given in Eq. (7.37), a set of horizontal forces, at each floor level can be calculated.

The total sway of a structure is obtained by a second order analysis where the applied forces are those caused by the wind loads together with the lateral forces due to column and wall out-of-plumbs. A statistical combination is required to account for the fact that the walls and the columns may induce deflections in opposite directions. The total lateral load at a specific floor level is then:

$$H = H_{wind} + \sqrt{H_c^2 + H_w^2} \tag{7.38}$$

where H_c and H_w are the lateral loads representing the effects of the column and wall out-of-plumbs respectively.

Eq. (7.38) is not exact if the mean in expression (7.37) is to be included. The exact expression can be easily derived but is more complex. However, the difference in the results is not significant and Eq. (7.38) can be adopted.

7.3 Summary

The various statistical characteristics that have been recommended for use in design in this chapter and in the previous one have been summarized in Table 7.4.

	Function	Mean (Rad.)	Standard Deviation (Rad.)
Column Out-of-Plumbs	All Purposes	$\frac{\overline{\Delta}_0}{h} = 0.0$	$\sigma_c = 0.0017$
Wall Out-of-Plumbs	Moment & Deflection	$\frac{\overline{\Delta}_0}{h} = 0.00028$	$\sigma_w = 0.0028$
	Torque	$\frac{\overline{e}}{L} = 0.00005$	$\sigma_e = 0.0004$

Safety Index	$\beta = 3.5$

TABLE 7.4 DESIGN VALUES

CHAPTER VIII

APPLICATIONS

Several equations serving different purposes have been presented in the previous chapter but no examples of applications have yet been presented. In this chapter the applicability of these equations will be checked against the corresponding results obtained from the measurements taken on buildings A and B.

8.1 Column Out-of-Plumbs

8.1.1 Force at Connection Point

A connection between one column and a beam must be designed to resist the extra horizontal force due to the eventual out-of-plumb of the column. This force was estimated as 0.6 percent of the factored axial load in the column. The force is increased to 0.84 percent of the average axial load in the more common case of two column segments connected at a floor level, as shown in Fig. 7.1.

Two cases must be considered in the transfer of these forces in a braced structure:

1. The bent to be designed is stabilized by a stiffer structure outside the plane of the bent. This could be the case, for instance, for column stacks 1 to 6 in the structure shown in Fig. 6.2. The extra forces originating from each column stack are directly transmitted to the core by the floor diaphragms. The individual connections must be designed for

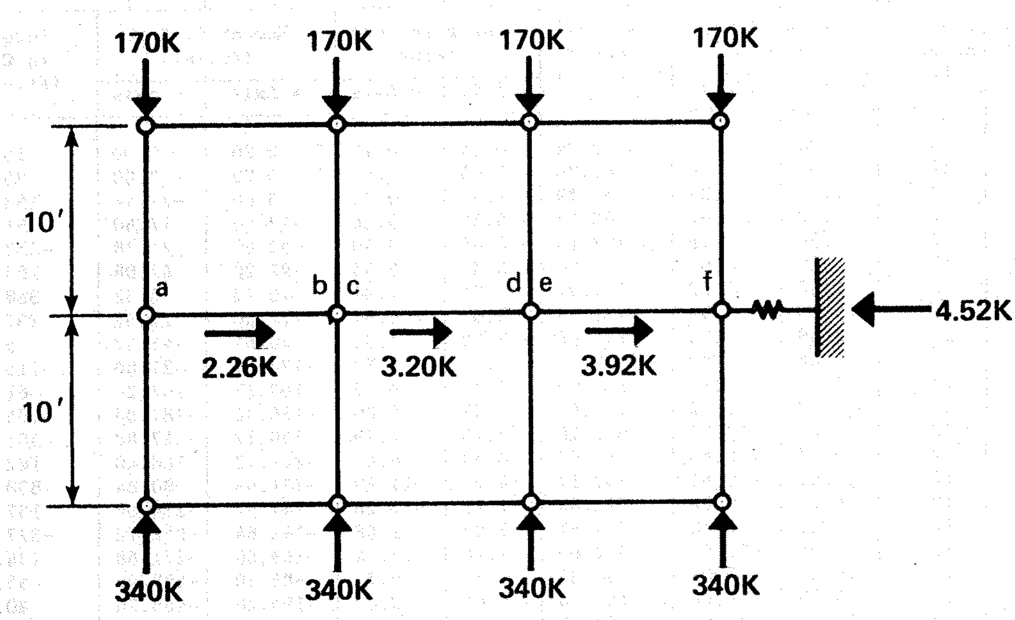
horizontal shears equal to 0.6 or 0.84 percent of the column axial loads, depending on the case (see section 7.1.1). The floor diaphragms, in turn, must be designed to resist the appropriate horizontal shears given by Eq. (7.7).

2. The bent to be designed is braced in the plane of the bent. The extra shears due to column out-of-plumbs are transferred from bay to bay and the connections must be designed accordingly.

An example showing the gradual increase in the horizontal force, when transmitted to the bracing system, is given in Fig. 8.1. The columns in the upper and lower storeys of the frame carry individual axial loads of 170 and 340 kips respectively. According to Eq. (7.7), the force in the girder a-b is 2.26 kips and originates from the two left hand columns. The force in girder c-d is 3.2 kips and is produced by the axial loads acting on four out-of-plumb columns; the shears being transmitted from left to right. The force in girder e-f is 3.92 kips and the bracing system finally resists a total force of 4.52 kips. The gradual increase in shear is non-linear and the connections in the vicinity of the bracing structure have to resist the larger shears.

8.1.2 Shear in the Plane of the Floor

The horizontal shears in the plane of the floor, estimated by Eq. (7.7), are compared to the values calculated from the measured column out-of-plumbs in buildings A and B. The actual forces in the x and y directions, calculated for all the columns at each storey, are listed in columns 3 and 4 of Tables 8.1 and 8.3. The corresponding



$$F_d = \beta \sigma_c \sqrt{\sum_{j=1}^{n} P_j^2} \qquad (7.7)$$

$\beta = 3.5$

$\sigma_c = 0.0017$ Rad.

Figure 8.1 Transfer of shear in beam-to-column connections

1	2	3	4	5	6	7	8	9	10
Floor No.	Storey Height (Ft.)	Storey Force (Kips)		Storey Moment (Ft.-K.)	Shear in Core (Kips)		Moment in Core (Ft.-K.)		Torque in Core (Ft.-K.)
		x Axis	y Axis		x Axis	y Axis	x Axis	y Axis	
27	20	0.04	- 0.16	10.74	0.04	- 0.16	0.00	0.00	10.74
26	12	0.36	- 1.36	85.20	0.40	- 1.52	0.80	- 3.20	95.94
25	12	- 2.35	1.84	57.79	- 1.95	0.32	5.60	-21.44	153.73
24	12	- 3.40	2.92	-205.57	- 5.35	3.24	-17.80	-17.60	- 51.84
23	12	4.50	0.16	-182.13	- 0.85	3.40	-82.00	21.28	-233.97
22	12	7.19	- 8.85	497.56	- 6.34	- 5.45	-92.20	62.08	263.58
21	12	- 6.88	6.77	104.97	- 0.54	1.32	-16.12	- 3.32	368.55
20	12	- 5.13	- 5.99	-131.15	- 5.67	- 4.67	-22.60	12.52	237.40
19	12	2.68	5.98	-157.84	- 2.99	1.31	-90.64	-43.52	5.33
18	12	4.44	- 7.93	-294.82	1.45	- 6.62	-126.52	-27.80	-215.27
17	12	- 3.72	- 0.10	154.05	- 2.27	- 6.72	-109.12	-107.24	-61.22
16	12	2.79	1.72	-123.43	0.52	- 5.00	-136.36	-187.88	-184.65
15	12	- 7.27	12.29	-317.10	- 6.75	7.29	-130.12	-247.88	-501.93
14	12	5.87	- 0.66	684.63	- 0.88	6.63	-211.12	-160.40	182.83
13	12	15.41	-18.61	647.17	14.53	-11.99	-221.68	-80.84	830.05
12	12	-26.74	9.72	-672.88	-12.21	- 2.26	-47.32	-224.60	157.17
11	12	14.23	0.58	-534.81	2.02	- 1.68	-193.84	-251.72	-377.64
10	12	5.08	-25.66	512.65	7.10	-27.34	-169.60	-271.88	134.00
9	12	-16.33	37.07	-190.02	- 9.24	9.73	-84.40	-599.96	-55.01
8	12	4.80	- 9.66	145.26	- 4.44	0.07	-195.28	-483.20	90.17
7	12	- 9.73	23.43	988.50	-14.16	23.50	-248.56	-482.36	1078.75
6	12	15.80	1.47	-830.72	1.64	24.97	-418.48	-200.36	248.03
5	12	5.46	- 8.98	95.87	7.10	15.99	-398.80	99.28	343.89
4	12	14.30	-26.84	140.90	21.40	-10.85	313.60	291.16	484.79
3	12	3.02	- 2.31	-398.58	24.42	-13.16	-56.80	160.96	86.21
2	12	-45.97	24.17	730.38	-21.55	11.01	236.24	3.04	816.59
1	20	- 3.03	-15.38	-1486.87	-24.58	- 4.37	-22.36	135.16	-670.28
							-513.96	47.76	

TABLE 8.1 FORCES IN BUILDING A FROM ACTUAL COLUMN OUT-OF-PLUMBS

1	2	3	4	5	6	7
Floor No.	Storey Height (Ft.)	$\left[\sqrt{\sum_{j=1}^{n_i} P_j^2}\right]_i$ (Kips)	Storey Force (Eq. 7.7) (Kips)	Shear in Core (Eq. 7.11) (Kips)	Moment in Core (Eq. 7.13) (Ft.-K.)	Torque in Core (Eq. 7.15) (Ft.-K.)
27	20	215	1.28	1.28	0.0	79.6
26	12	691	4.31	4.12	25.6	259.6
25	12	1000	7.23	5.95	55.7	377.0
24	12	1265	9.59	7.52	90.5	477.3
23	12	1523	11.77	9.06	127.8	575.1
22	12	1776	13.92	10.56	167.8	670.7
21	12	2029	16.05	12.08	210.3	766.7
20	12	2280	18.16	13.57	255.4	861.4
19	12	2528	20.26	15.05	302.9	955.7
18	12	2777	22.35	16.52	352.6	1049.9
17	12	3024	24.43	17.99	404.6	1143.2
16	12	3271	26.50	19.47	458.5	1236.7
15	12	3517	28.58	20.92	514.6	1329.7
14	12	3337	28.85	19.86	572.6	1393.2
13	12	3556	29.02	21.15	620.2	1439.7
12	12	3773	30.84	22.45	670.1	1527.6
11	12	4003	32.73	23.79	722.2	1615.5
10	12	4207	35.77	25.04	776.7	1703.4
9	12	4422	36.32	26.32	832.8	1786.3
8	12	4640	38.14	27.60	890.7	1878.4
7	12	4855	39.96	28.90	950.3	1965.9
6	12	5071	41.78	30.18	1011.5	2052.9
5	12	5287	43.60	31.46	1074.4	2140.4
4	12	5502	45.41	32.74	1138.8	2227.5
3	12	5718	47.21	34.02	1204.7	2315.0
2	12	5933	49.02	35.29	1272.0	2401.7
1	20	6149	50.84	36.59	1340.7	2489.2
					1527.4	

TABLE 8.2 FORCES IN BUILDING A FROM STATISTICAL CALCULATIONS

1	2	3	4	5	6	7	8	9	10
Floor No.	Storey Height (Ft.)	Storey Force (Kips)		Storey Moment (Ft.-K.)	Shear in Core (Kips)		Moment in Core (Ft.-K.)		Torque in Core (Ft.-K.)
		x Axis	y Axis		x Axis	y Axis	x Axis	y Axis	
34	24	- 2.12	- 0.60	6.31	- 2.12	- 0.60	0.00	0.00	6.31
33	12	0.21	0.96	64.58	- 1.91	0.36	-50.88	-14.40	70.89
32	12	1.48	1.25	-338.27	- 0.43	1.61	-73.80	-10.08	-267.39
31	12	0.68	- 1.70	409.35	0.25	- 0.09	-78.96	9.24	141.96
30	12	- 5.93	0.06	-498.94	- 5.68	- 0.03	-75.96	8.16	-356.98
29	12	7.16	2.83	40.35	1.48	2.81	-144.12	7.80	-316.63
28	12	- 0.60	2.93	125.64	0.88	5.74	-126.36	41.52	-191.00
27	12	-10.56	9.82	321.64	- 9.68	15.56	-115.80	110.40	130.65
26	12	2.01	- 7.48	-398.29	- 7.67	8.08	-231.96	297.12	-267.65
25	12	10.66	- 4.85	709.03	2.99	3.24	-324.00	394.08	441.38
24	12	-15.50	11.14	-828.74	-12.51	14.38	-288.12	432.96	-387.36
23	12	8.17	- 4.72	1528.07	- 4.34	9.66	-438.24	605.52	1140.71
22	12	0.94	-20.54	-726.16	- 3.40	-10.87	-490.32	721.44	414.55
21	12	4.01	22.97	222.34	0.61	12.10	-531.12	591.00	636.88
20	12	1.72	- 2.18	-1987.66	2.34	9.92	-523.80	736.20	-1350.77
19	12	- 9.99	-35.03	1016.84	- 7.65	-25.11	-495.72	855.24	-333.93
18	12	19.37	9.83	841.27	11.71	-15.29	-587.52	553.92	507.33
17	12	- 3.16	30.03	-708.78	8.55	14.74	-447.00	370.44	-201.45
16	12	-17.88	- 6.47	-318.22	- 9.33	8.28	-344.40	547.32	-519.66
15	12	19.00	-14.21	596.11	9.66	- 5.94	-456.36	646.68	76.45
14	12	15.85	4.70	-530.72	25.51	- 1.24	-340.44	575.40	-454.28
13	12	-14.40	2.92	853.66	11.11	1.68	- 34.32	560.52	399.39
12	12	-29.54	-21.74	151.78	-18.43	-20.06	99.00	580.68	551.17
11	12	13.11	28.85	-2289.57	- 5.31	8.79	-122.16	339.96	-1738.40
10	12	5.16	-27.09	1920.94	- 0.15	-18.30	-185.88	445.44	182.54
9	12	- 6.33	11.38	-327.14	- 6.48	- 6.91	-187.68	225.84	-144.60
8	12	7.12	35.16	-585.46	0.64	28.25	-265.44	142.92	-730.05
7	12	-20.56	-36.81	36.68	-19.93	- 8.56	-257.76	481.92	-693.37
6	12	45.88	-12.41	2901.36	25.95	-20.97	-496.92	379.20	2207.99
5	24	- 8.14	21.83	832.05	17.80	0.86	-185.52	127.56	1375.94
4	27	-31.81	15.34	444.57	-14.00	16.20	241.68	148.20	1820.50
3	16	- 3.12	4.27	2528.19	-17.12	20.46	-136.32	585.60	-707.69
2	15	17.49	6.17	994.50	0.36	26.63	-410.24	912.96	286.82
1	15	5.64	3.75	1748.31	6.00	30.38	-404.84	1312.41	-1461.50
							-314.84	1768.11	

TABLE 8.3 FORCES IN BUILDING B FROM ACTUAL COLUMN OUT-OF-PLUMBS

1	2	3	4	5	6	7
Floor No.	Storey Height (Ft.)	$\left[\sqrt{\sum_{j=1}^{n_i} P_j^2}\right]_i$ (Kips)	Storey Force (Eq. 7.7) (Kips)	Shear in Core (Eq. 7.11) (Kips)	Moment in Core (Eq. 7.13) (Ft.-K.)	Torque in Core (Eq. 7.15) (Ft.-K.)
34	24	398	2.37	2.37	0.0	160.6
33	12	689	4.73	4.10	56.9	279.7
32	12	998	7.22	5.94	75.2	407.2
31	12	1303	9.77	7.75	103.6	537.1
30	12	1628	12.41	9.69	139.2	671.5
29	12	1945	15.09	11.57	181.4	801.8
28	12	2263	17.75	13.46	228.4	932.8
27	12	2580	20.42	15.35	279.8	1063.8
26	12	2899	23.09	17.25	335.0	1195.4
25	12	3216	25.76	19.14	393.8	1326.4
24	12	3594	28.70	21.38	455.9	1481.6
23	12	3855	31.36	22.94	523.1	1589.7
22	12	4173	33.80	24.83	591.1	1720.7
21	12	4491	36.48	26.72	662.0	1852.2
20	12	4809	39.15	28.61	735.5	1984.0
19	12	5129	41.83	30.52	811.7	2115.0
18	12	5447	44.52	32.41	890.5	2246.0
17	12	5766	47.20	34.31	971.7	2377.7
16	12	6084	49.87	36.20	1055.4	2508.7
15	12	6404	52.56	38.10	1141.3	2640.3
14	12	6723	55.25	40.00	1229.4	2772.0
13	12	7040	57.92	41.89	1319.8	2903.0
12	12	7361	60.60	43.80	1412.3	3035.3
11	12	7679	63.29	45.69	1506.9	3166.3
10	12	7998	65.97	47.59	1603.6	3298.0
9	12	8316	68.65	49.48	1702.2	3429.0
8	12	8636	71.33	51.38	1802.8	3560.6
7	12	8955	74.02	53.28	1905.3	3692.3
6	12	9274	76.71	55.18	2009.7	3824.0
5	24	9592	79.39	57.07	2116.0	3963.3
4	27	9911	82.07	58.97	2520.6	4095.4
3	16	10234	84.77	60.89	2981.4	4163.0
2	15	10567	87.53	62.87	3136.5	4365.4
1	15	10952	90.55	65.16	3275.2	4516.2
					3418.0	

TABLE 8.4 FORCES IN BUILDING B FROM STATISTICAL CALCULATIONS

storey forces given by Eq. (7.7) are listed in column 4 of Tables 8.2 and 8.4. The absolute values of the measured and predicted forces described above are compared in Figs. 8.2 and 8.3. The forces obtained from the measurements are represented by the solid circles. The direction of the forces is not relevant since the only purpose of these figures is to compare the observed and predicted magnitudes. Equation (7.7), with $\beta = 3.5$ and $\sigma_c = 0.0017$ Rad., appears to be an upper bound on the predicted forces in the floor diaphragms.

If more measurements were available from several similar structures, the computed values shown by the solid circles would eventually fill the area corresponding to the predicted values with a density corresponding to that of a normal distribution. Depending on the probability chosen for design (β factor), a few points may be found outside the limits. In the absence of measurements, such a situation can be artificially created by a Monte Carlo simulation(54). In this method, applied to the present case, out-of-plumbs of known distribution and characteristics are randomly generated by a computer for every column segment in a fictitious structure.

8.1.3 Moment in the Plane of the Floor

The adequacy of Eq. (7.9) in predicting moments in floor diaphragms due to column out-of-plumbs can be checked as in the previous section. The results obtained from Eq. (7.9) are compared in Figs. 8.4 and 8.5 with those taken from column 5 of Tables 8.1 and 8.3. These figures and others to come in this chapter have the characteristics of Figs. 8.2 and 8.3.

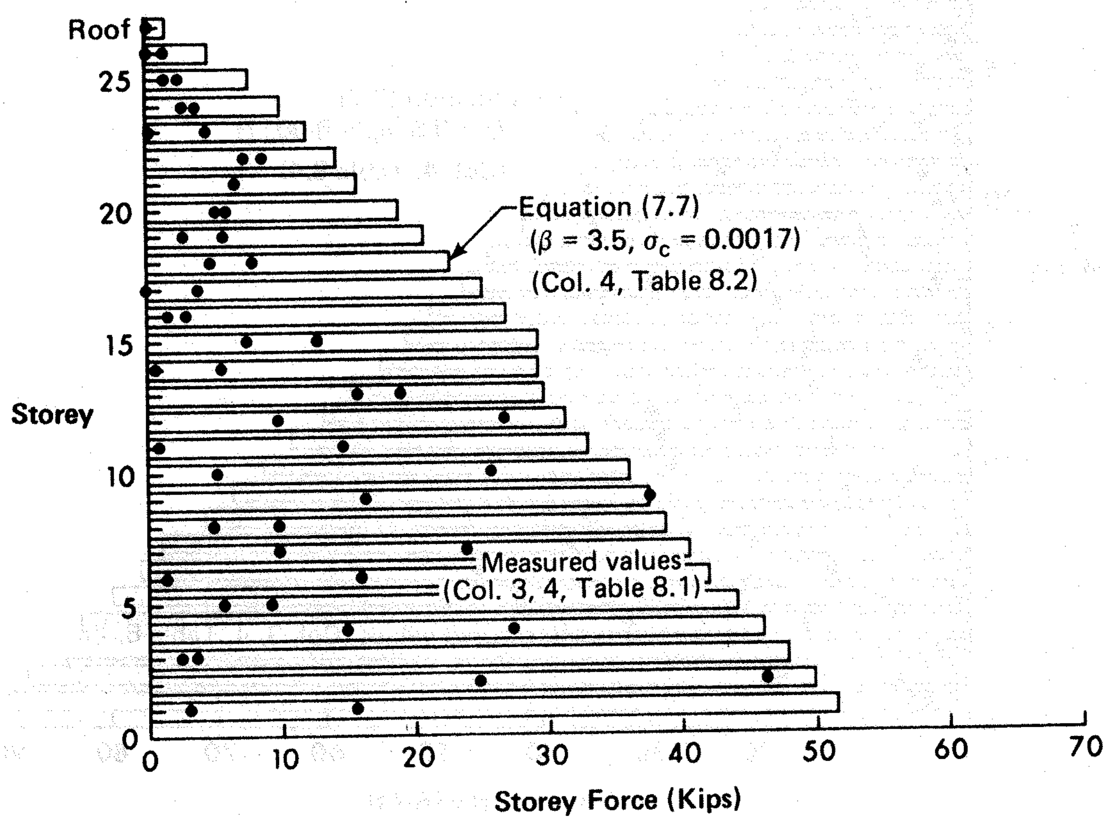


Figure 8.2 Force in the plane of the floor at each level of building A

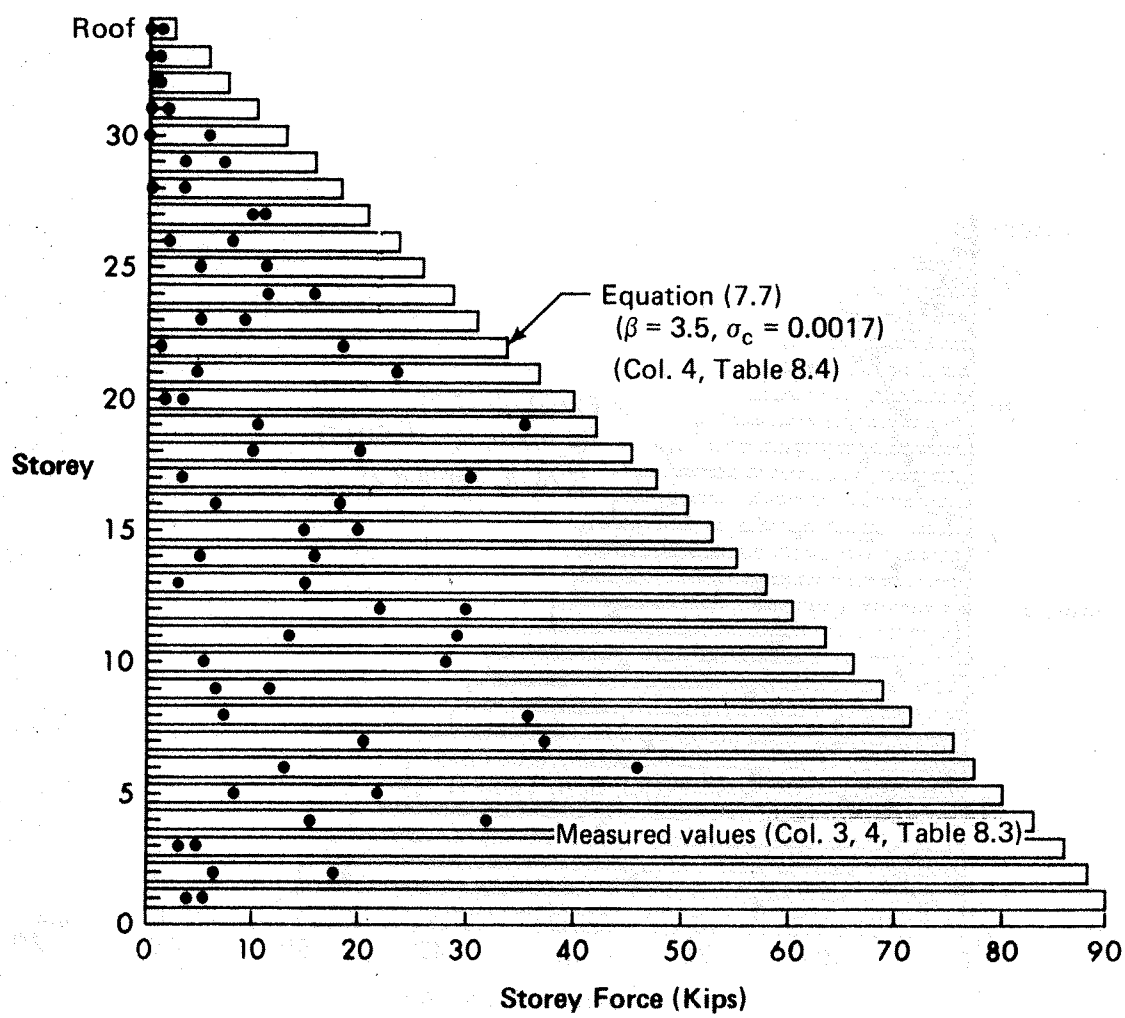


Figure 8.3 Force in the plane of the floor at each level of building B

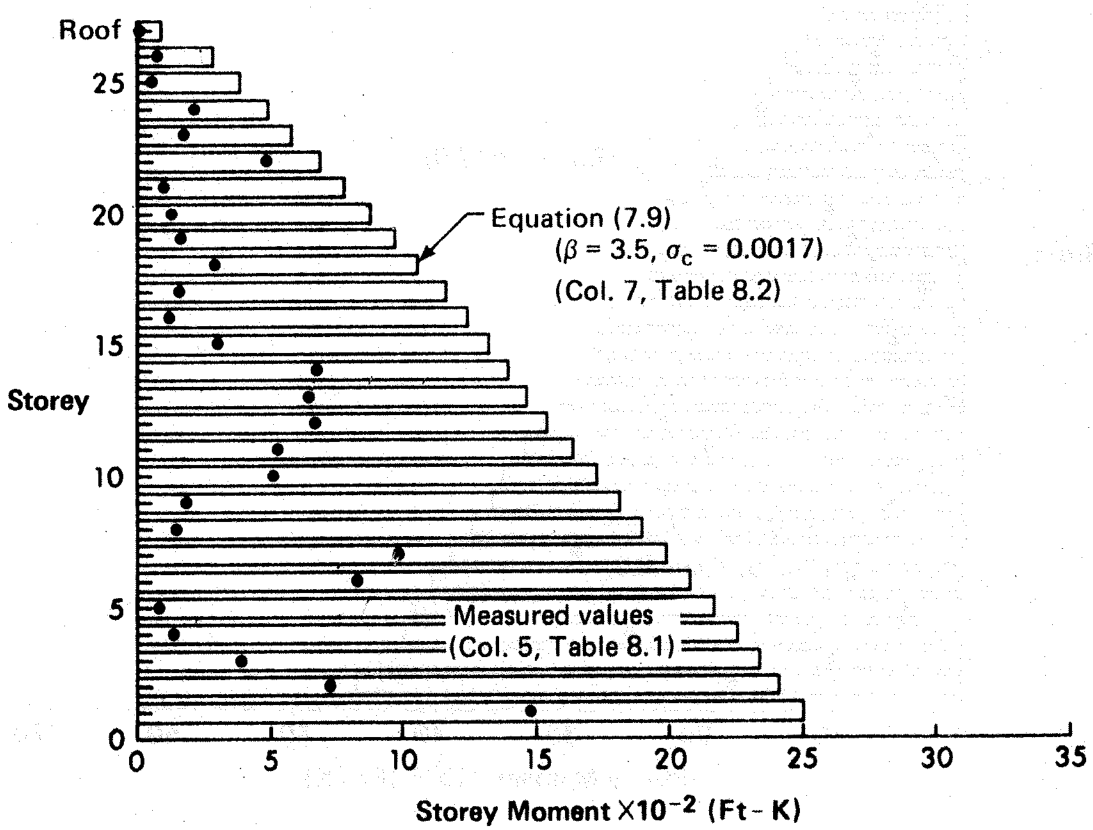


Figure 8.4 Moment in the plane of the floor at each level of building A

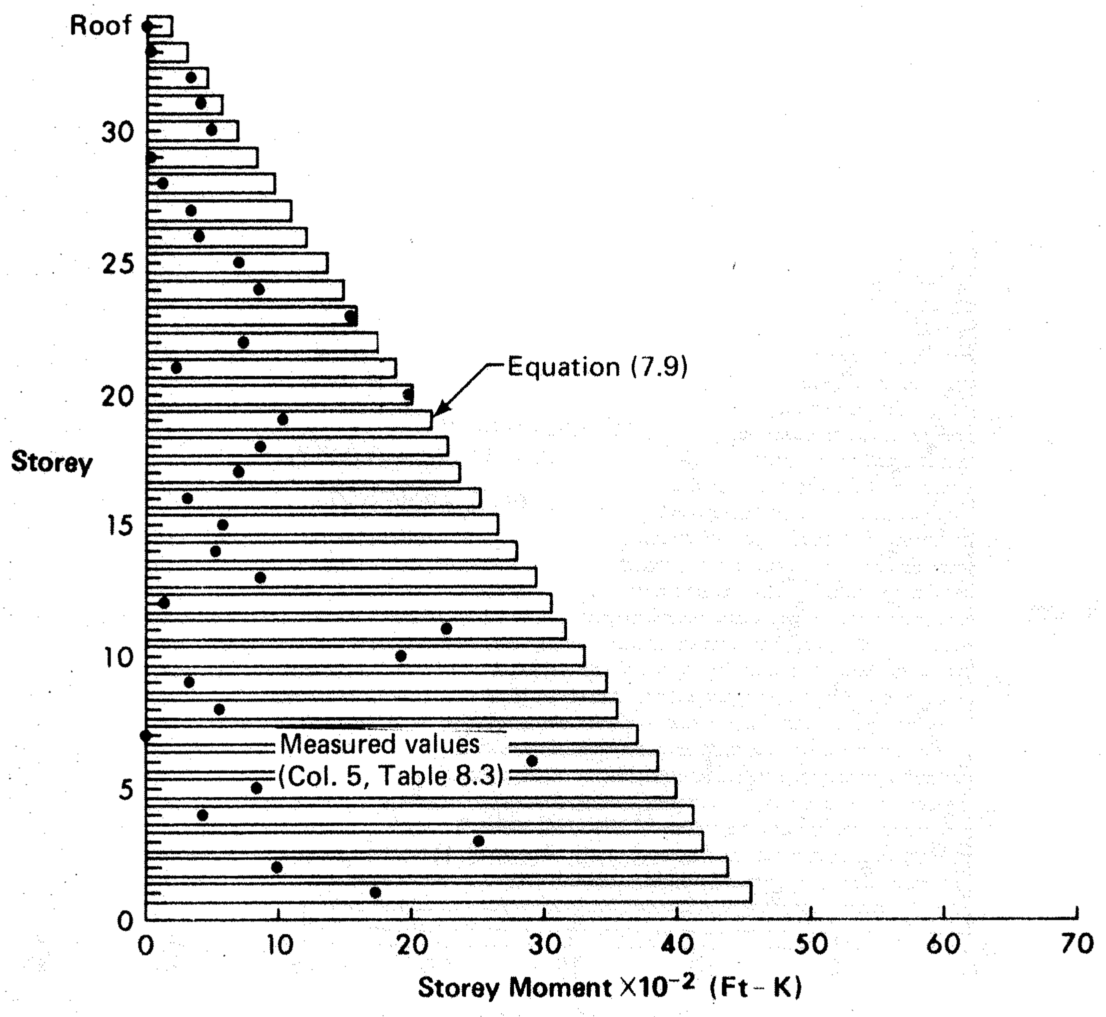


Figure 8.5 Moment in the plane of the floor at each level of building B

The moments shown are calculated, for convenience, by considering every column present in the structure at each storey. In this particular case, n becomes n_i and Eq. (7.9) takes the form of Eq. (7.15) derived to calculate the torques in the core. These results are listed in column 7 of Tables 8.2 and 8.4. The figures show that Eq. (7.9) provides a good estimate of the moments in the floor diaphragms.

8.1.4 Shear, Moment, and Torque in Core

Calculations and figures are provided in this section to verify the application of Eqs. (7.11), (7.13), and (7.15), all three expressions giving estimates of forces to be resisted by the core. The measured and predicted shears resisted by the core at each storey of buildings A and B are compared in Figs. 8.6 and 8.7. Similarly, the moments are compared in Figs. 8.8 and 8.9 and the torques in Figs. 8.10 and 8.11. The plotted quantities are taken from Tables 8.1 to 8.4 and their respective origins are indicated on the figures. In each case, the proposed equation seems adequate.

8.1.5 Lateral Deflections

The presence of out-of-plumb columns in a structure forces the structure to sway laterally. All the columns participate in this action. In the case of building A, the lateral deflection curves obtained from the measurements in the x and y directions have been plotted in Fig. 8.12 against the results given by the equations derived in section 7.1.7. Curves 1 and 2 show the results of Eqs. (7.16) and (7.18) while curves 3 and 4 present the actual deflections obtained from the measurements. The values shown in abcissa have no units since they only serve the purpose of indicating the relative deflections.

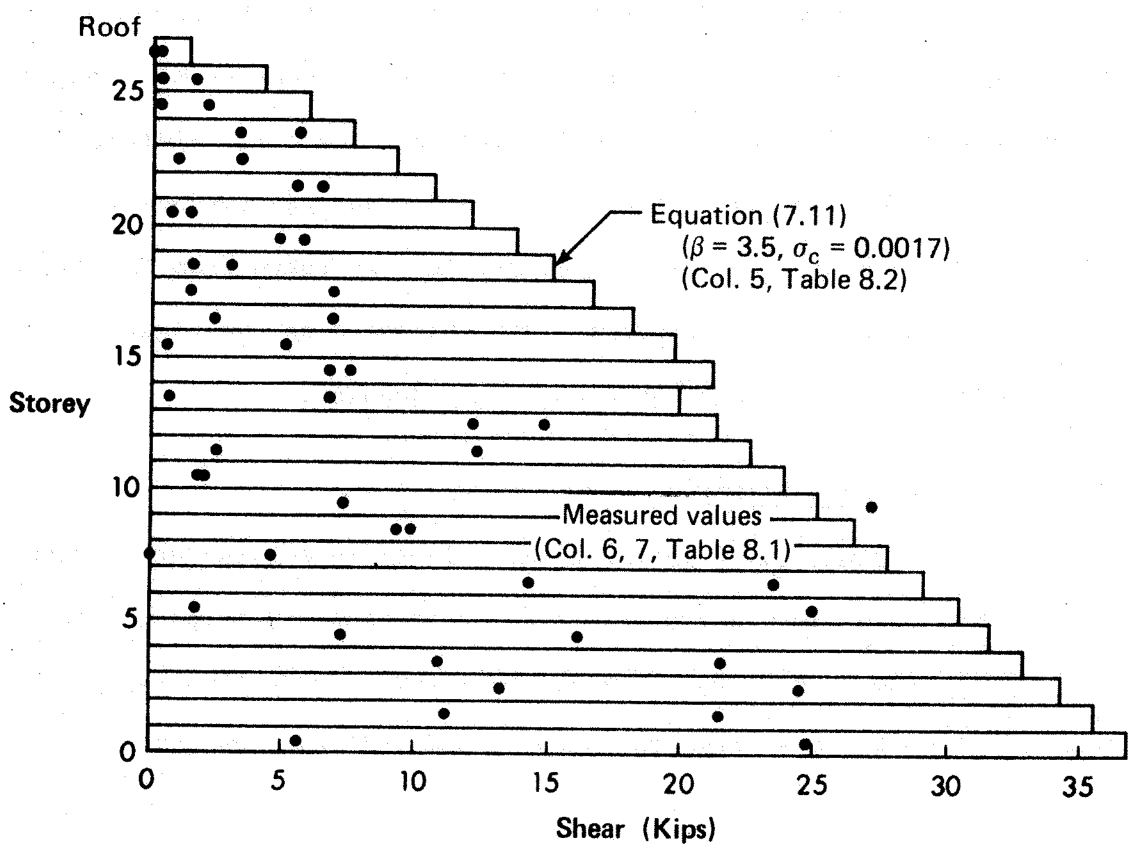


Figure 8.6 Shear due to column out-of-plumbs in the core of building A

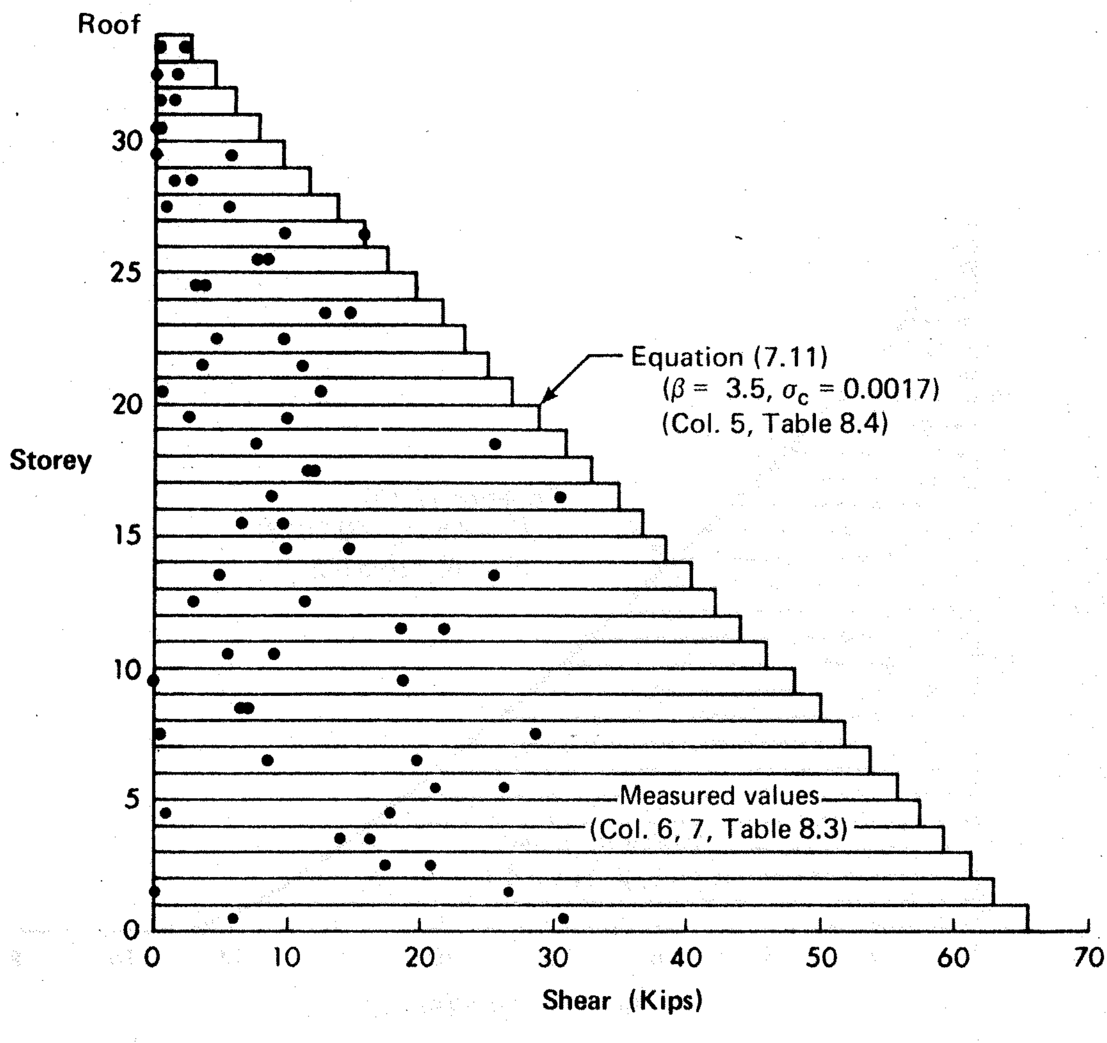


Figure 8.7 Shear due to column out-of-plumbs in the core of building B

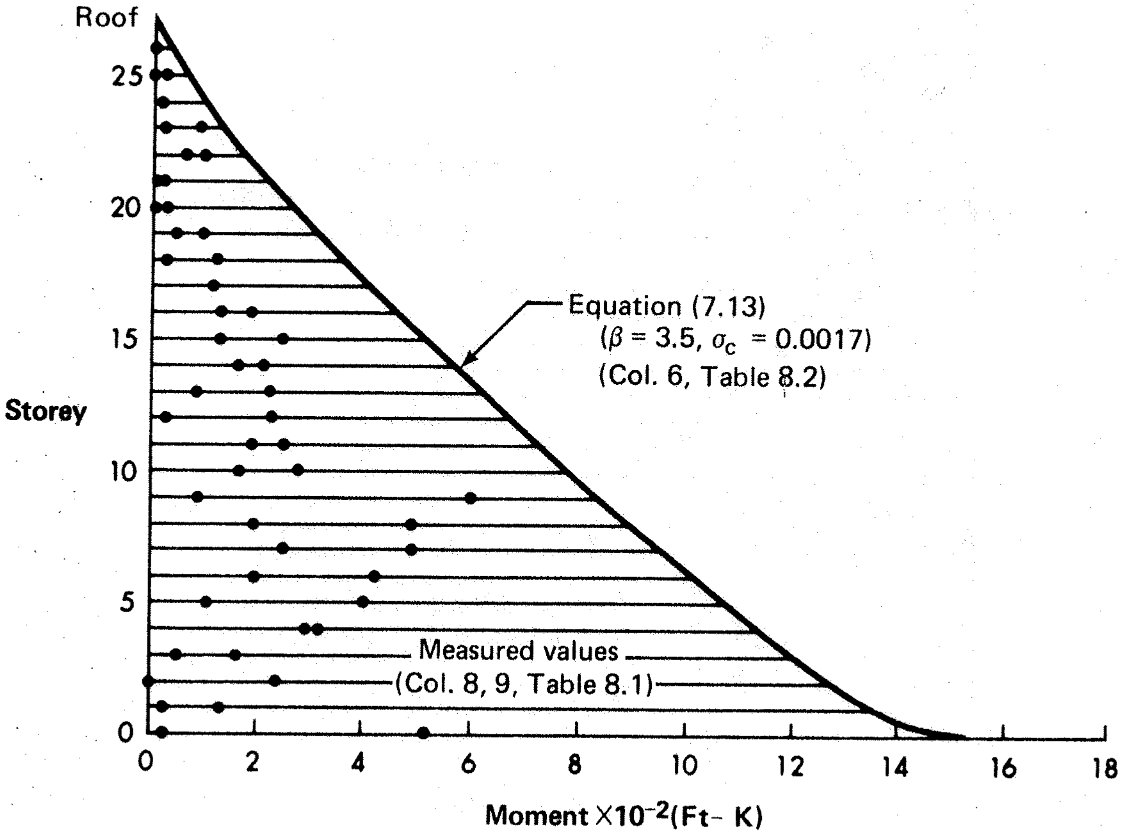


Figure 8.8 Moment due to column out-of-plumbs in the core of building A

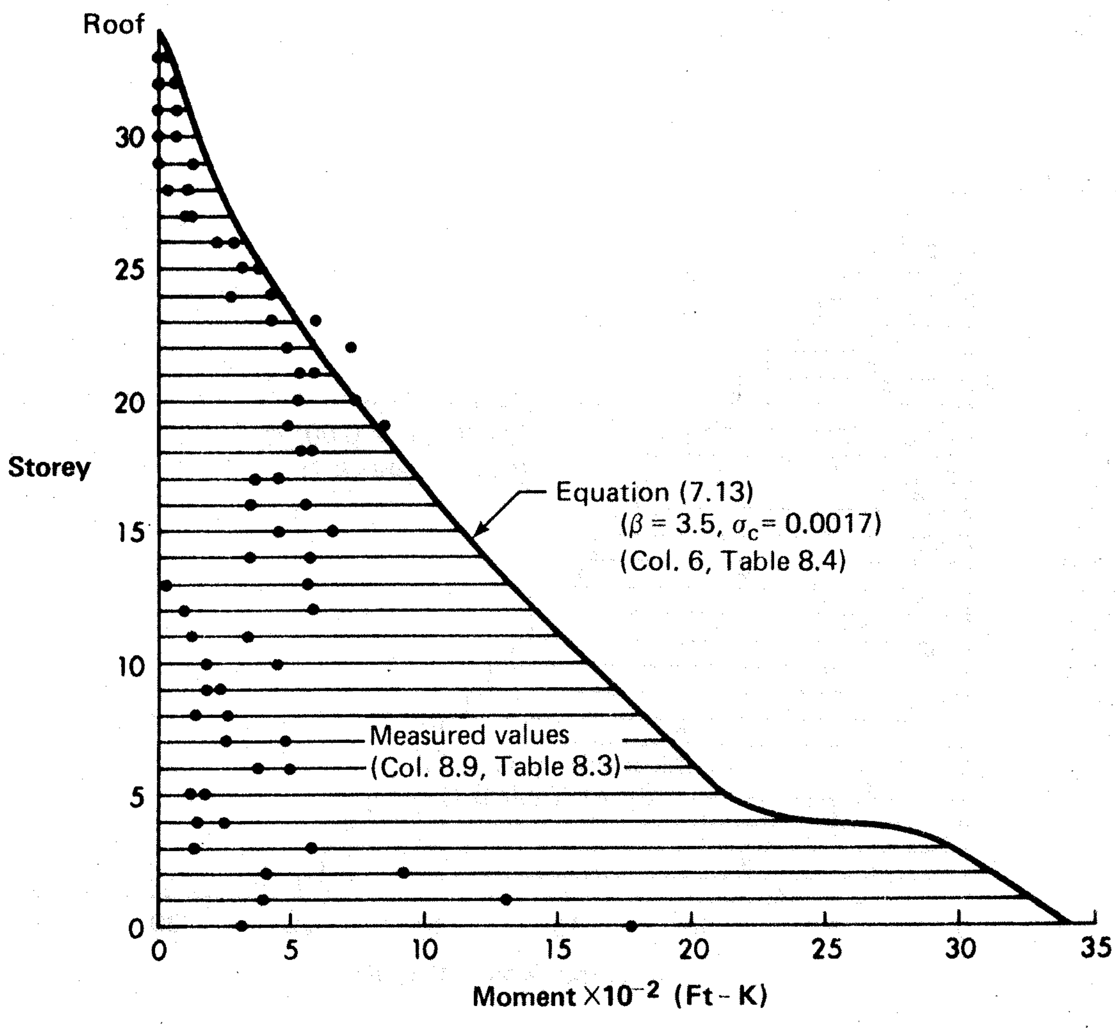


Figure 8.9 Moment due to column out-of-plumbs in the core of building B

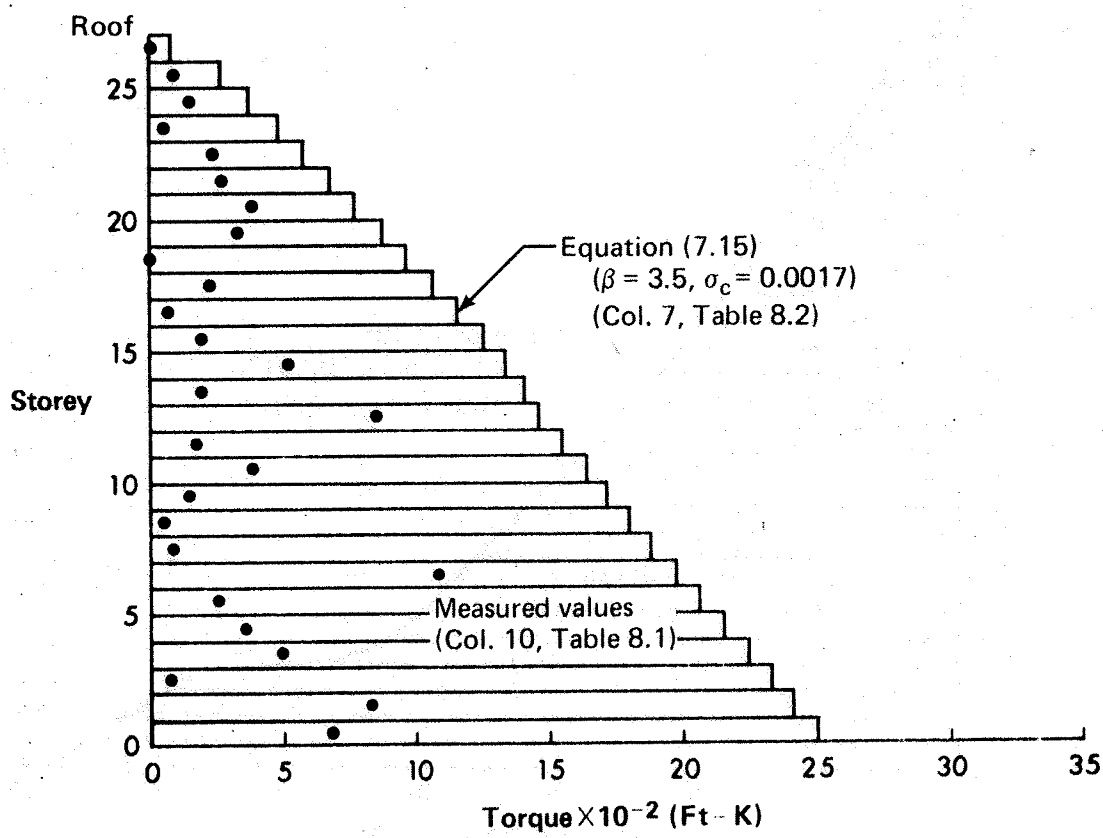


Figure 8.10 Torque due to column out-of-plumbs in the core of building A

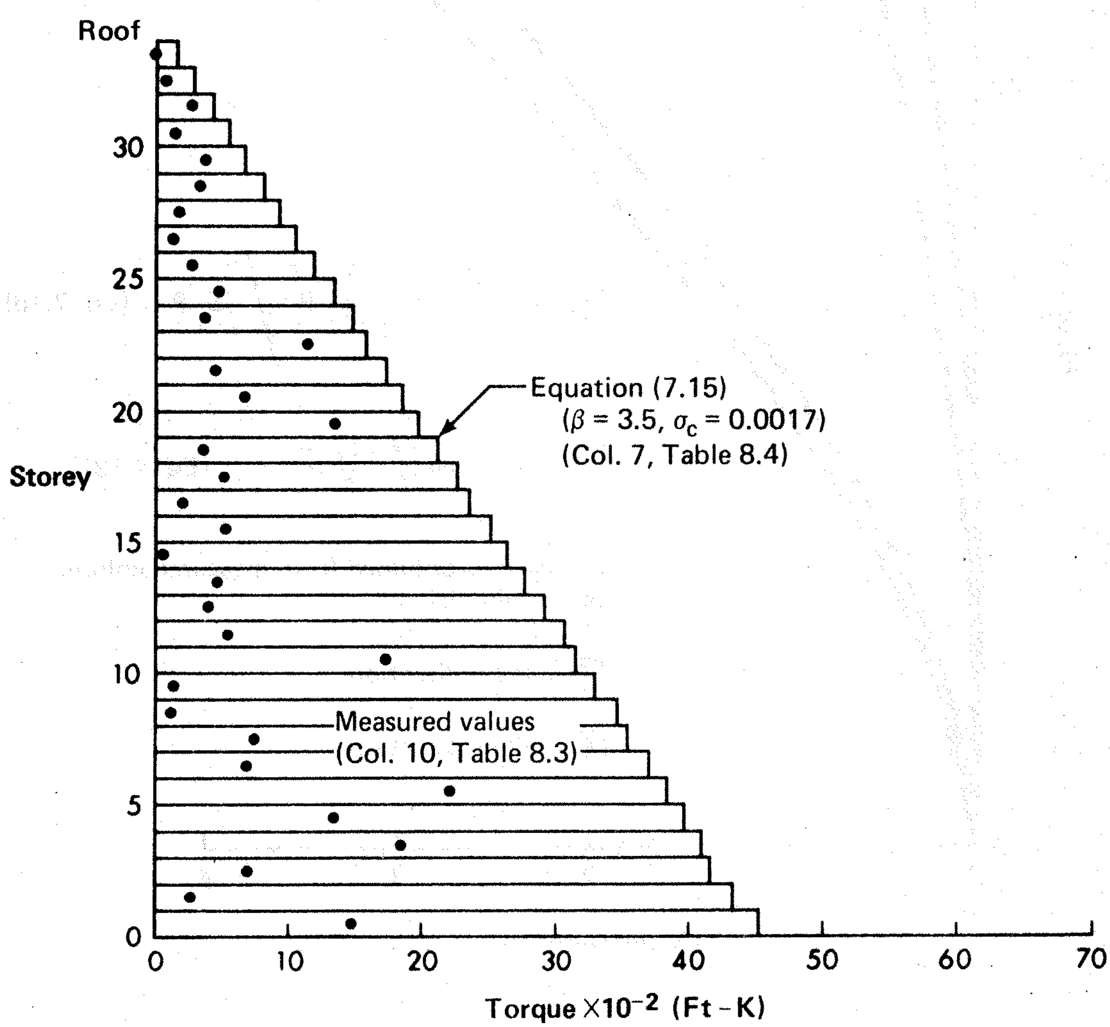


Figure 8.11 Torque due to column out-of-plumbs in the core of building B

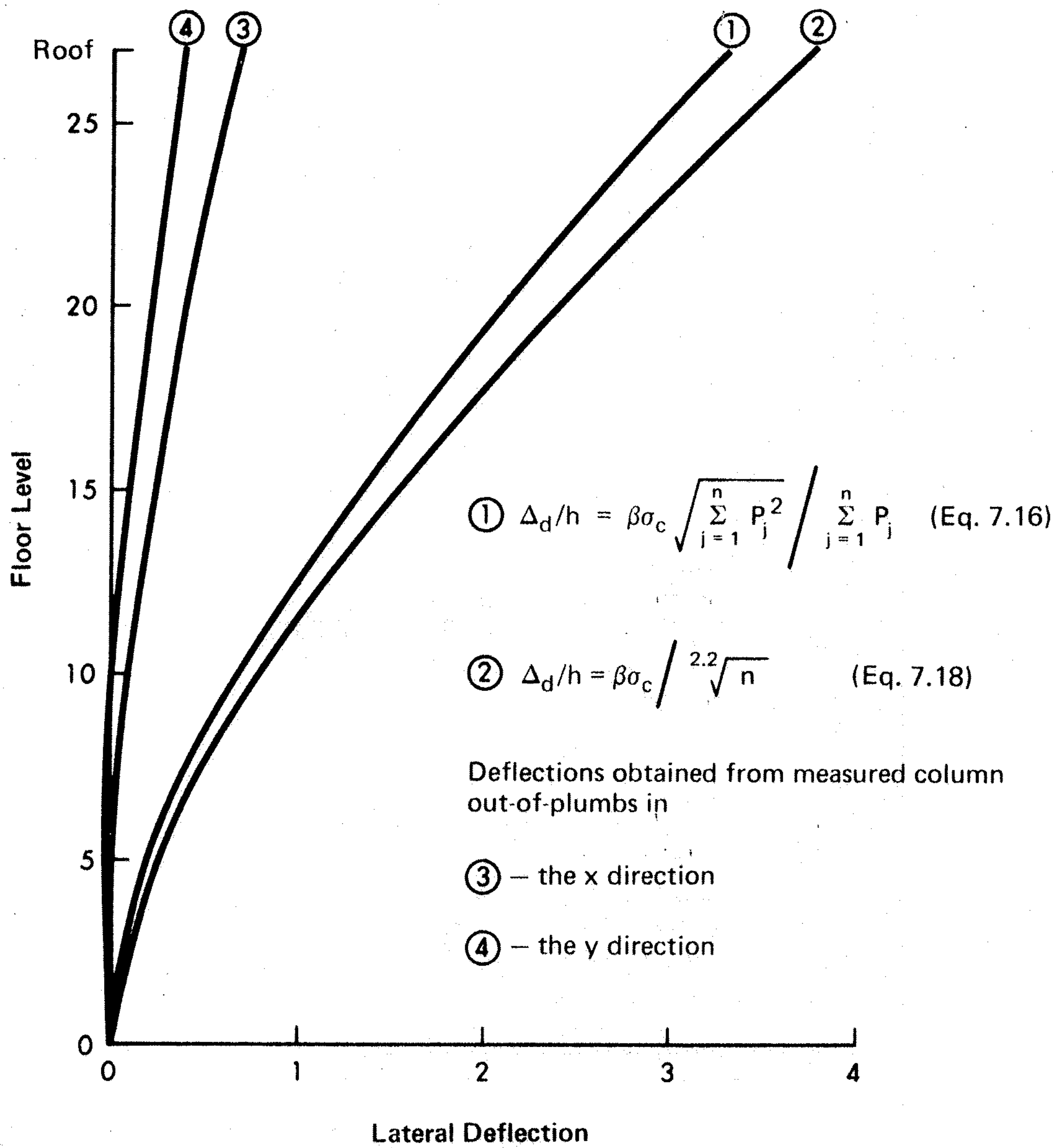


Figure 8.12 Lateral deflections caused by initial column out-of-plumbs for building A

It is seen that Eq. (7.18) gives a conservative estimate of the more exact expression (7.16). The equivalent slope Δ_d/h given by Eq. (7.16) is 3.5 x 0.0017 x 19755/367682 = 0.00032 Rad., compared to 0.00037 Rad. given by Eq. (7.18) with n = 458. The storey forces used in the determination of the sway of building A were obtained by using these values and the actual column axial loads in the structure.

To demonstrate that Eqs. (7.16) and (7.18) are general, the study must be extended to structures of different size. Three simple structures denoted as E.1, E.2, and E.3 have been selected and are shown in the insets to Figs. 8.13 to 8.15. E.1 and E.2 are 5-storey buildings with 20 and 50 columns respectively, while E.3 is a 10-storey building containing 50 columns.

A Monte Carlo simulation has been used to generate random out-of-plumbs for the columns of frame E.1. The population generated was normally distributed and had a mean of zero and a standard deviation of 0.0017 Rad. Every column of the frame has been allocated one of these values. A set of storey forces was obtained and was applied to the shear-wall to calculate the lateral deflections. The process was repeated 50 times with different values and the results were plotted in Fig. 8.13. Also plotted is the curve resulting from Eq. (7.16), where Δ_d/h = 3.5 x 0.0017 x 1173/4500 = 0.00155 Rad. (in this case, the same result is obtained from Eq. (7.18)). It is observed that none of the curves exceeds the limit given by Eq. (7.16) or (7.18).

Similar computations were made for structures E.2 and E.3 and the results are presented in Figs. 8.14 and 8.15. Uniform slopes in the order of 0.001 Rad. were calculated from both Eqs. (7.16) and (7.18). The prescribed curves were again not exceeded. It can be

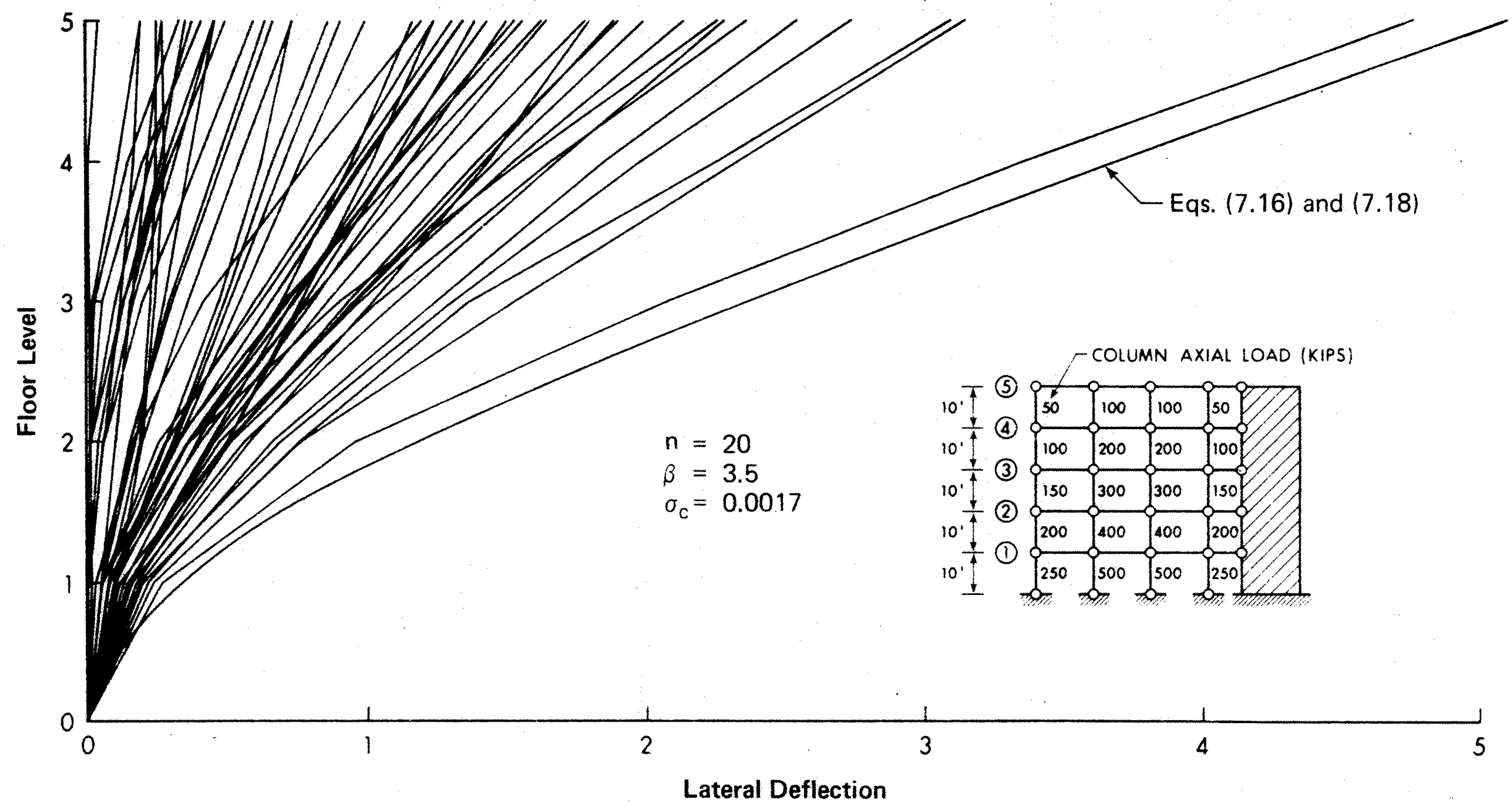


Figure 8.13 Lateral deflections caused by initial column out-of-plumbs for building E.1

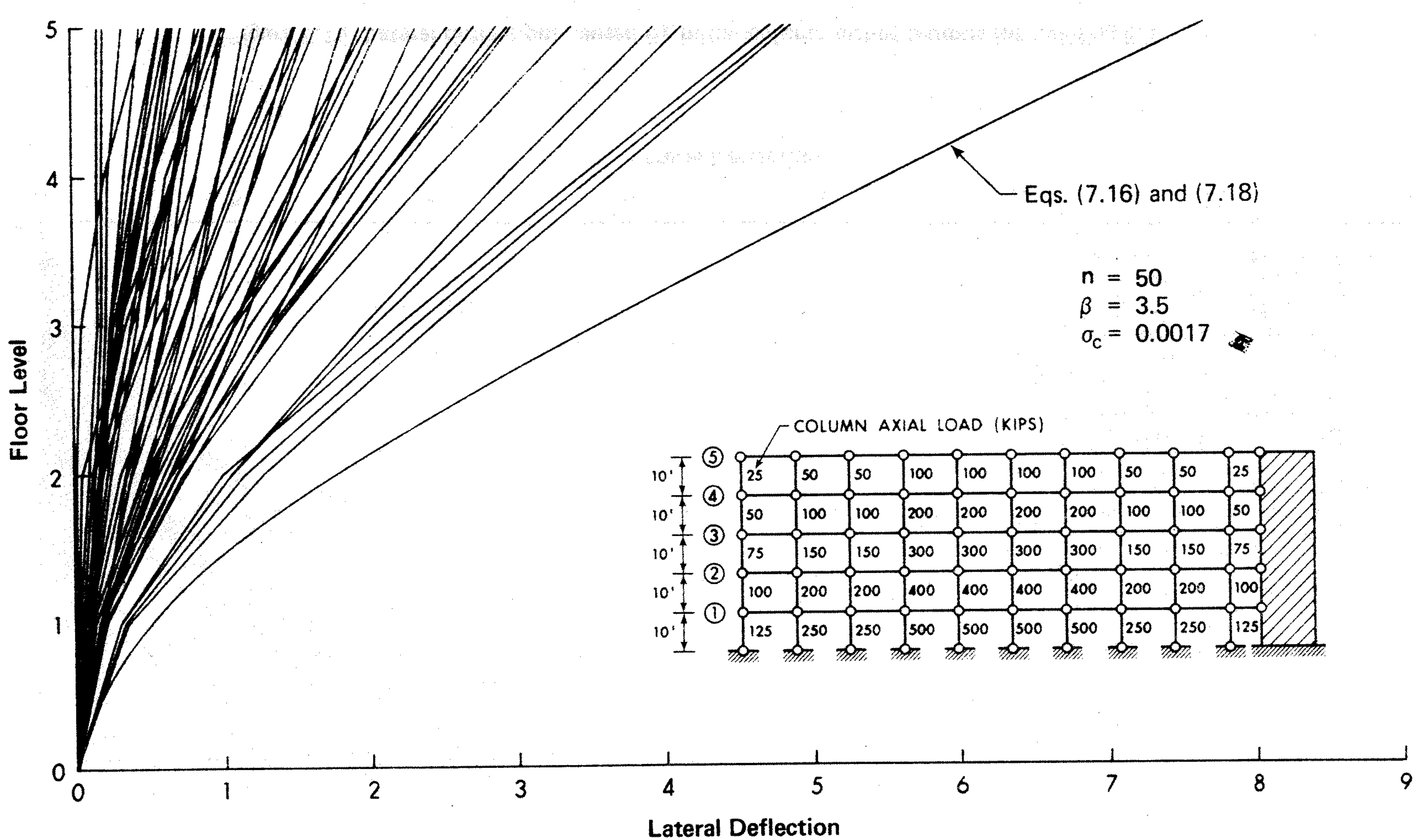


Figure 8.14 Lateral deflections caused by initial column out-of-plumbs for building E.2

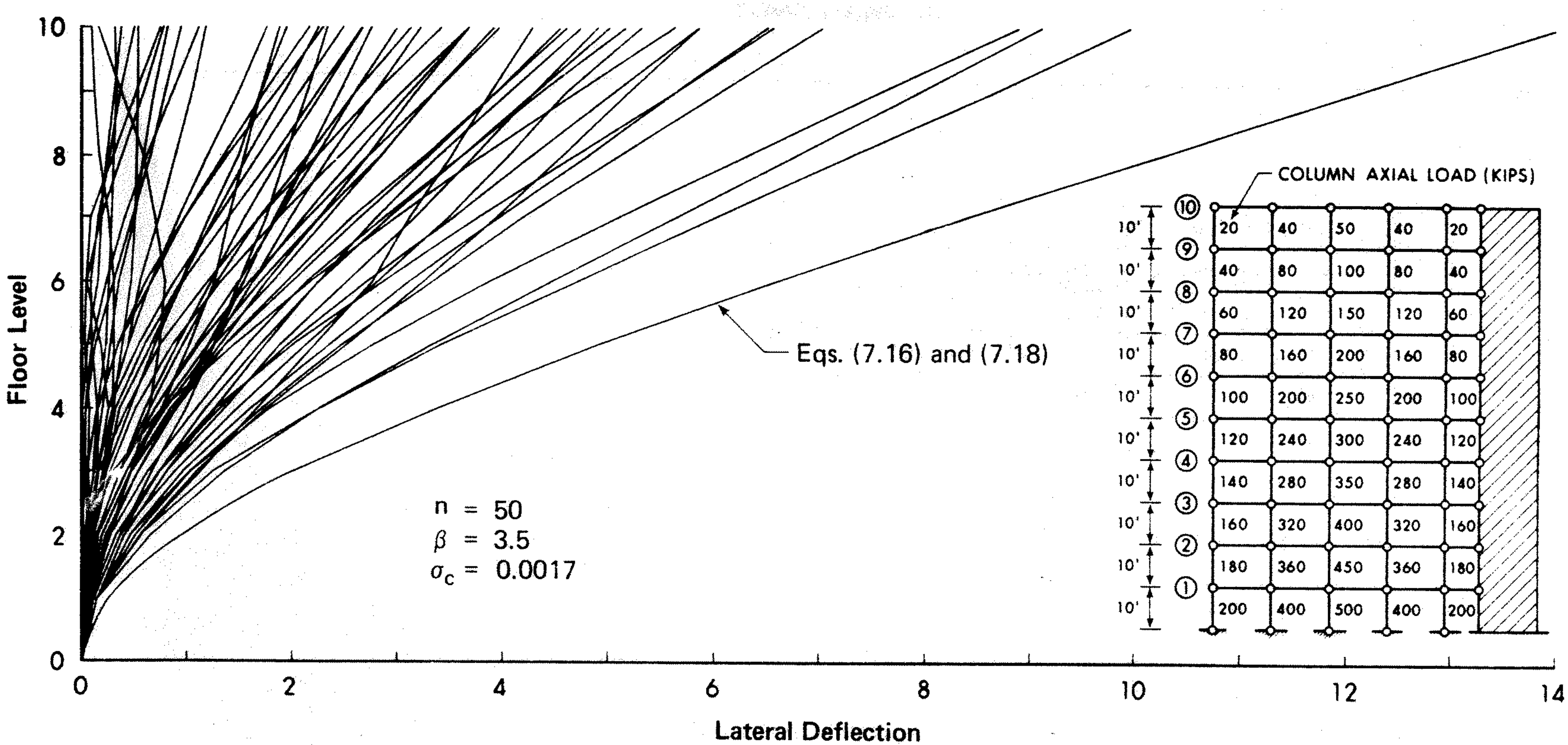


Figure 8.15 Lateral deflections caused by initial column out-of-plumbs for building E.3

concluded that Eq. (7.18) gives a good upper bound for estimating the lateral deflections induced by initial column out-of-plumbs for the types of structures studied.

8.2 Wall Out-of-Plumbs

The actual cores of buildings A and B are formed of eight and nine walls per storey, respectively. However, only the four exterior walls will be considered in the following applications, using the information given by the measurements. It will be assumed that the four walls carry the total axial loads in the actual cores. The moments, torques, and lateral deflections calculated in this manner will be larger than in the actual cases where the same loads are carried by more than four walls. These effects will be discussed in the next chapter.

The wall dimensions used in the computations are given in Figs. 6.2 and 6.3. In building A, the core dimensions are 87' x 29' up to level 14 and are reduced to 67' x 29' in the upper section. In building B, the dimensions are assumed to be 66' x 38' for the total height of the building.

The share of the total axial load carried by the individual walls is assumed proportional to the length of the walls. These values are listed in Tables 8.5 and 8.7. The axial loads in the upper 22 storeys of the 27-storey building A are used in the calculations since results were obtained for only 22 storeys.

8.2.1 Moment in Core

The moments in the cores of buildings A and B have been calculated at each storey from the information given by the measurements on the walls and have been listed in columns 6 and 7 of Tables 8.5 and 8.7. The moments prescribed by Eq. (7.25), which considers the contribution of the mean, and Eq. (7.26), which neglects it, are listed in columns 3 and 4 of Tables 8.6 and 8.8.

The measured and predicted moments described above are plotted in Figs. 8.16 and 8.17 for direct comparison. Fig. 8.17 proves that the apparent conservativeness of the proposed equations in Fig. 8.16 is due to the actual out-of-plumb arrangement in building A. This arrangement resulted in the calculation of comparatively small moments in the core. Moreover, the contribution of the mean, which accounts for 15 and 25 percent of the contribution of the standard deviation at the base of buildings A and B, is apparently a significant factor.

8.2.2 Torque in Core

The torques at each core level, as predicted by Eqs. (7.30) and (7.31), are listed in Tables 8.6 and 8.8 for buildings A and B. The torques calculated from the actual measurements are listed in Tables 8.5 and 8.7 and are plotted against the proposed design values in Figs. 8.18 and 8.19. Since all the measured values lie well within the limits prescribed by Eq. (7.31) and since the contribution of the mean accounts for less than 6 percent of the contribution of the standard deviation in both cases, Eq. (7.31) seems appropriate for use in design.

1	2	3	4	5	6	7	8
Storey No.	Storey Height (Ft.)	Axial Load (Kips)			Moment (Ft.-K.)		Torque (Ft.-K.)
		Total	Walls #1,2*	Walls #3,4*	x Axis	y Axis	
22	12	1382	484	207	0.00	0.00	7.49
21	12	2726	954	409	-167.12	12.93	- 17.01
20	12	4140	1449	621	-181.90	71.14	45.28
19	12	5520	1932	828	-141.17	40.95	84.21
18	12	6894	2413	1034	-151.20	61.08	- 44.74
17	12	8274	2896	1241	170.16	174.15	315.08
16	12	9652	3378	1448	52.90	-424.68	12.51
15	12	11022	3858	1653	288.87	-363.11	4.52
14	12	12406	4652	1551	52.33	-287.06	-162.39
13	12	13782	5168	1723	- 20.38	151.32	441.12
12	12	15154	5683	1894	-307.48	7.22	158.22
11	12	16406	6197	2006	- 11.47	-296.46	-207.35
10	12	17898	6712	2237	- 3.82	-101.87	40.36
9	12	19306	7240	2413	-359.24	-341.45	- 82.02
8	12	20650	7744	2581	-246.09	-115.22	80.77
7	12	22022	8258	2753	- 64.61	123.82	- 9.89
6	12	23398	8774	2925	652.23	238.53	141.72
5	12	24766	9287	3096	288.78	85.89	- 56.38
4	12	26146	9805	3268	123.35	428.35	-532.11
3	12	27526	10322	3441	1127.65	73.94	529.04
2	12	28910	10841	3614	1438.38	148.85	669.33
1	20	30286	11357	3786	1200.10	645.35	-584.38
					1857.10	322.00	

* Wall numbering given in Fig. 6.2.

TABLE 8.5 FORCES IN CORE OF BUILDING A FROM ACTUAL WALL OUT-OF-PLUMBS

1	2	3	4	5	6
Storey No.	Storey Height (ft.)	Moment (Ft.-K.)		Torque (Ft.-K.)	
		Eq. (7.25)*	Eq. (7.26)*	Eq. (7.30)**	Eq. (7.31)**
22	12	0.0	0.0	69.1	65.3
21	12	92.3	87.5	136.3	128.7
20	12	207.9	193.5	207.0	195.5
19	12	354.7	325.9	276.0	260.7
18	12	524.7	477.9	344.7	325.6
17	12	716.9	647.3	413.7	390.7
16	12	930.0	832.8	482.6	455.8
15	12	1162.6	1033.0	551.1	520.5
14	12	1413.5	1246.7	851.2	806.2
13	12	1698.6	1489.8	945.6	895.7
12	12	1999.2	1743.6	1039.9	984.9
11	12	2314.2	2008.2	1133.4	1073.6
10	12	2643.0	2281.8	1228.1	1163.3
9	12	2988.5	2567.3	1324.8	1254.8
8	12	3361.9	2863.9	1417.0	1342.1
7	12	3736.9	3169.3	1511.0	1431.2
6	12	4126.3	3484.3	1605.5	1520.6
5	12	4530.0	3808.8	1699.3	1609.5
4	12	4946.1	4142.1	1794.1	1699.3
3	12	5376.2	4484.6	1888.7	1788.9
2	12	5819.9	4835.9	1983.7	1878.9
1	20	6277.2	5196.0	2078.1	1968.3
		7416.4	6165.2		

* with $\beta = 3.5$, $\overline{\Delta}_0/h = 2.8 \times 10^{-4}$, and $\sigma_w = 2.8 \times 10^{-3}$.

** with $\beta = 3.5$, $\overline{e}/L = 0.5 \times 10^{-4}$, and $\sigma_e = 4.0 \times 10^{-4}$.

TABLE 8.6 FORCES IN CORE OF BUILDING A FROM STATISTICAL CALCULATIONS

1	2	3	4	5	6	7	8
Storey No.	Storey Height (Ft.)	Axial Load (Kips)			Moment (Ft.-K.)		Torque (Ft.-K.)
		Total	Walls #1,2*	Walls #3,9*	x Axis	y Axis	
34	24	312	100	56	0.00	0.00	- 0.82
33	12	1962	628	353	- 3.85	- 4.53	- 16.67
32	12	3208	1027	577	42.79	51.77	- 5.42
31	12	4326	1376	787	54.24	105.18	6.94
30	12	5440	1730	990	75.12	155.94	- 16.27
29	12	6560	2086	1194	97.38	186.10	120.88
28	12	7672	2440	1396	- 81.47	354.80	-139.60
27	12	8790	2795	1600	- 71.02	520.91	-102.81
26	12	9896	3147	1801	- 74.02	500.73	109.83
25	12	11012	3502	2004	-125.68	623.91	29.75
24	12	12114	3852	2205	-292.17	637.38	-232.69
23	12	13226	4206	2407	-138.02	810.64	12.79
22	12	14342	4561	2610	- 20.77	1018.44	1.67
21	12	15456	4915	2813	- 0.60	1196.07	52.63
20	12	16550	5263	3012	220.76	1387.30	-390.23
19	12	17664	5617	3215	352.53	1469.10	- 2.70
18	12	18922	6017	3444	126.16	1369.05	- 53.10
17	12	20186	6419	3674	364.48	2107.00	-188.47
16	12	21446	6820	3903	518.30	2088.91	200.16
15	12	22708	7221	4133	-202.68	2263.77	-283.76
14	12	23938	7612	4357	207.65	2416.14	- 35.45
13	12	25194	8012	4585	41.70	2927.45	94.09
12	12	26456	8413	4815	199.24	3291.20	1.02
11	12	27716	8814	5044	-306.82	3322.93	-461.48
10	12	28974	9214	5273	-929.03	5056.40	-424.77
9	12	30600	9615	5685	-509.12	4895.72	-489.17
8	12	31496	10016	5732	-168.95	6252.96	112.62
7	12	32758	10417	5962	- 68.51	6051.05	-223.92
6	12	34016	10817	6191	638.75	5979.25	66.53
5	24	35548	11304	6470	836.68	5674.81	497.23
4	27	36176	11504	6584	-1496.94	6560.72	754.35
3	16	40886	13002	7441	225.71	7113.85	54.38
2	15	42470	13505	7730	-867.73	5854.57	88.23
1	15	43978	13985	8004	-1228.49	7374.08	-415.94
					-1405.32	6955.80	

* Wall numbering given in Fig. 6.3.

TABLE 8.7 FORCES IN CORE OF BUILDING B FROM ACTUAL WALL OUT-OF-PLUMBS

1	2	3	4	5	6
Storey No.	Storey Height (ft.)	Moment (Ft.-K.)		Torque (Ft.-K.)	
		Eq. (7.25)*	Eq. (7.26)*	Eq. (7.30)**	Eq. (7.31)**
34	24	0.0	0.0	14.6	13.7
33	12	40.2	38.1	91.7	86.3
32	12	134.4	125.7	150.0	141.0
31	12	252.3	232.8	201.4	189.3
30	12	385.7	351.7	253.2	238.0
29	12	535.6	483.3	305.3	287.0
28	12	701.5	627.2	357.1	335.7
27	12	882.4	782.3	409.1	384.6
26	12	1077.7	948.1	460.6	433.0
25	12	1286.5	1123.6	512.5	481.8
24	12	1508.6	1308.7	563.8	530.0
23	12	1743.1	1502.6	615.6	578.7
22	12	1990.1	1705.1	667.5	627.5
21	12	2249.2	1916.0	719.4	676.2
20	12	2520.1	2135.0	770.3	724.1
19	12	2801.9	2361.2	822.1	772.8
18	12	3095.1	2594.9	880.7	827.9
17	12	3403.3	2839.6	939.5	883.2
16	12	3726.1	3094.5	998.2	938.3
15	12	4062.8	3359.2	1056.9	993.5
14	12	4412.9	3633.0	1114.1	1047.3
13	12	4775.2	3914.9	1172.6	1102.3
12	12	5150.1	4205.2	1231.3	1157.5
11	12	5537.5	4503.6	1290.0	1212.7
10	12	5936.9	4809.9	1348.5	1267.7
9	12	6348.0	5123.7	1412.3	1327.2
8	12	6777.2	5450.1	1465.9	1378.0
7	12	7210.8	5777.9	1524.6	1433.2
6	12	7656.0	6113.0	1583.2	1488.3
5	24	8112.1	6454.9	1654.5	1555.3
4	27	9670.1	7773.9	1683.7	1582.8
3	16	11391.1	9221.5	1903.0	1788.9
2	15	12154.4	9801.6	1976.6	1858.1
1	15	12852.8	10321.6	2046.8	1924.1
		13567.5	10851.6		

* with $\beta = 3.5$, $\overline{\Delta}_0/h = 2.8 \times 10^{-4}$, and $\sigma_w = 2.8 \times 10^{-3}$.

** with $\beta = 3.5$, $\overline{e}/L = 0.5 \times 10^{-4}$, and $\sigma_e = 4.0 \times 10^{-4}$.

TABLE 8.8 FORCES IN CORE OF BUILDING B

FROM STATISTICAL CALCULATIONS

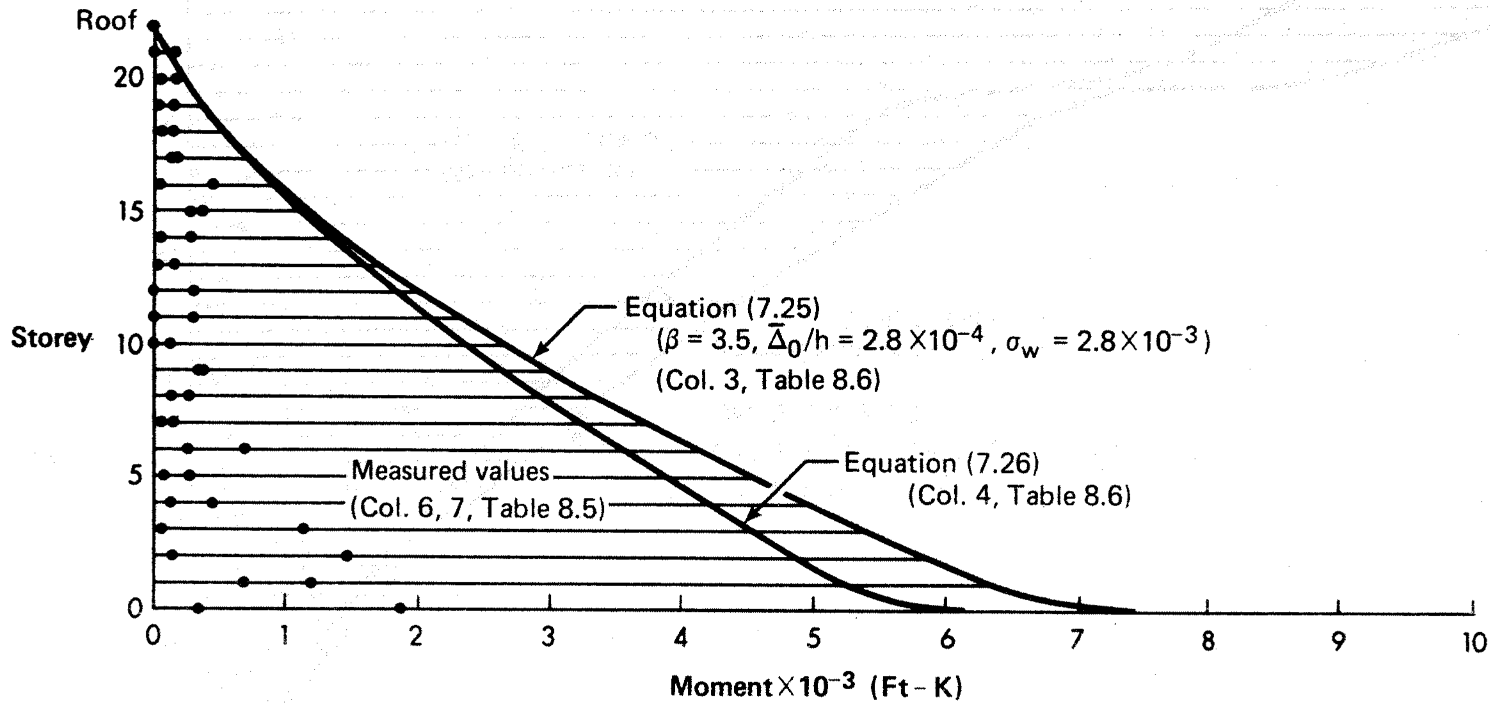


Figure 8.16 Moment due to wall out-of-plumbs in the core of building A

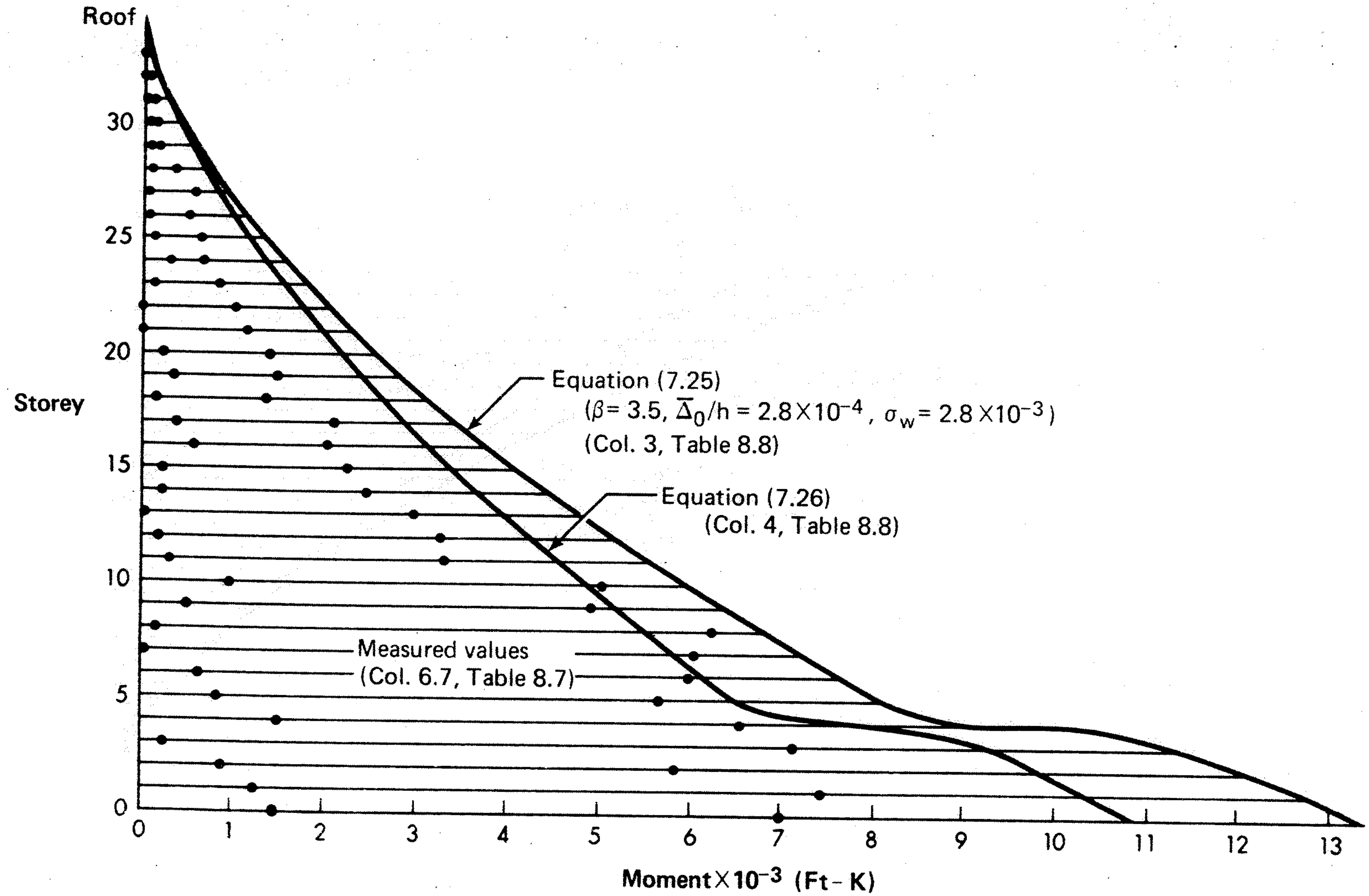


Figure 8.17 Moment due to wall out-of-plumbs in the core of building B

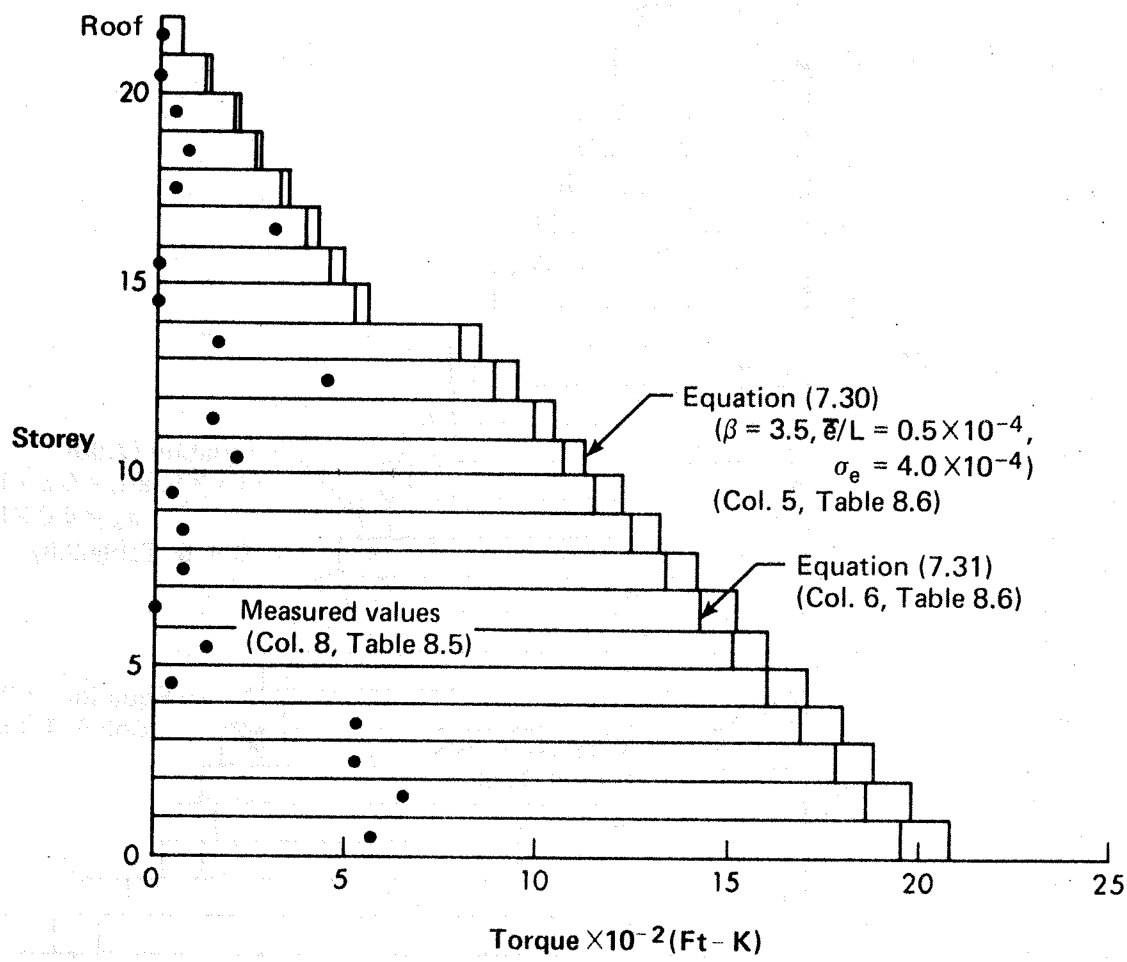


Figure 8.18 Torque due to wall out-of-plumbs in the core of building A

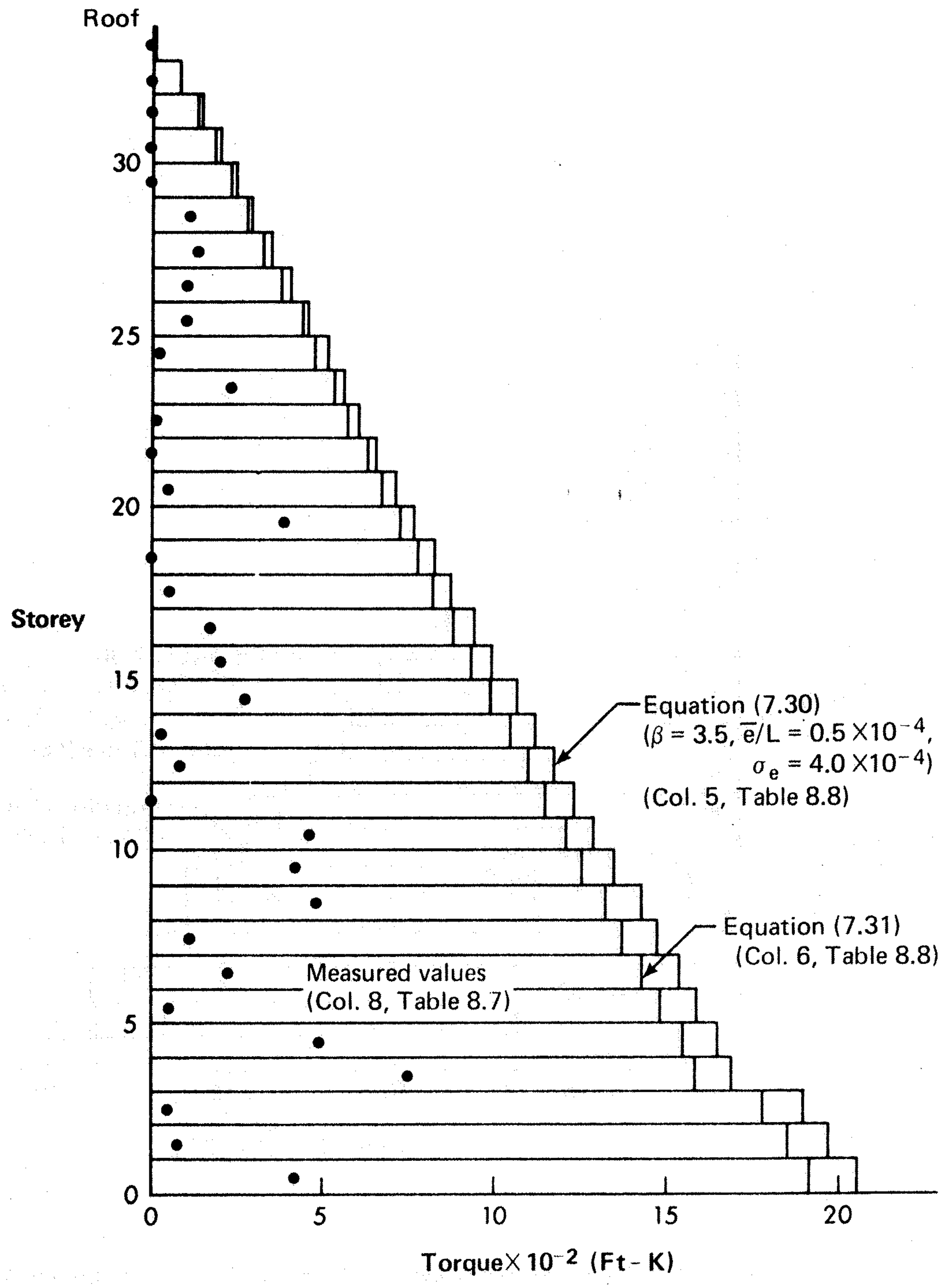


Figure 8.19 Torque due to wall out-of-plumbs in the core of building B

8.2.3 Lateral Deflections

Equation (7.37), which gives an equivalent wall out-of-plumb for use in an overall stability analysis of a structure, has the same form and consequently the same characteristics as Eq. (7.18) derived for column out-of-plumbs. Therefore, the results obtained in section 8.1.5 can be extended to the present case.

As applied to the 22-storey building A, the equivalent slope of the walls forming the core is $\Delta_d/h = 0.00028 + 3.5 \times 0.0028/\sqrt[2.2]{88} =$ 0.00156 Rad. and the slope calculated for building B (where n = 136) is 0.00133 Rad.

CHAPTER IX

COMPARATIVE STUDY

Design equations have been derived and their applicability has been confirmed in the previous two chapters, but the significance of the effects in terms of the overall structural design has not been determined. The importance of the out-of-plumb effects in design can be evaluated in terms of corresponding wind effects.

The study in this chapter is limited in scope but gives interesting results when applied to core-braced structures. Where appropriate, the design recommendations given in section 5.4 are compared with the results of the techniques developed in this report. A certain consistency is achieved by using the specified loads in the calculations. No attempt is made to include the wind effects on the column axial loads, which would have resulted in increased axial loads on the leeward side and reduced axial loads on the windward side, with no net difference in the total gravity load.

9.1 Horizontal Forces at Connection Point

Wind pressures of 35.0 psf in the upper sections of buildings A and B and 22.0 psf at ground level were obtained from the simple procedure prescribed by the National Building Code of Canada[60]. Of these pressures, a proportion 8/13 is applied directly to the windward side of the building and a proportion 5/13 acts as a suction on the leeward side. The basic wind pressure with a return period of 30 years is 8.5 psf for these buildings.

The shears caused by wind and transmitted to the core were calculated at specific beam-to-column connections in both buildings. The values presented in column 4 of Table 9.1 were obtained from the windward pressures given above applied to the particular connection tributary areas (see Figs. 6.2 and 6.3). The horizontal forces created in these connections by the out-of-plumbs of the columns are presented in the same table. A sample calculation using Eq. (7.5) is given below the table. The out-of-plumb to wind shear ratios given in column 6 indicate that, in a tall braced building, the out-of-plumb shears generally govern in the design of the connections, while in the top storeys they still account for an important fraction of the wind shears.

The out-of-plumb shears become even more critical in a frame of the type shown in Fig. 8.1 where the forces are transferred to a bracing system in the plane of the frame. The shears due to column out-of-plumbs are increased from bent to bent during the transfer but the wind forces remain constant. It is understood that other significant forces, such as the P-Δ forces described in Chapter IV, are also present.

The model shown in Fig. 9.1(a) was used as an example in Ref. 23 to estimate the out-of-plumb forces in girder-to-column connections. All columns are assumed to be erected with initial out-of-plumbs of 0.002 Rad.[36] (see section 5.4.2). The constant forces of value 1.02 kips created at each connection are added algebraically when transferred to the bracing system.

1	2	3	4	5	6
Building	Floor Level	Column Location	Wind Shear (kips)	Out-of-Plumb Shear (Eq. 7.5) (kips)	Out-of-Plumb Shear / Wind Shear
A	1*	Exterior Corner	6.2 3.3	13.6† 8.2	2.19 2.48
	23**	Exterior Corner	7.5 3.9	2.8 1.7	0.37 0.44
B	1*	Exterior Corner	3.9 2.0	16.6 11.8	4.26 5.90
	31**	Exterior Corner	5.0 2.6	2.1 1.0	0.42 0.38

* Design wind pressure = 8/13 x 22 = 13.54 psf.

** Design wind pressure = 8/13 x 35 = 21.54 psf.

† **Sample** calculation:

$$F_d = 3.5 \times .0017\sqrt{1643^2 + 1585^2} = 13.6 \text{ kips.}$$

TABLE 9.1 RATIO OF OUT-OF-PLUMB SHEAR TO WIND SHEAR AT BEAM-TO-COLUMN CONNECTIONS

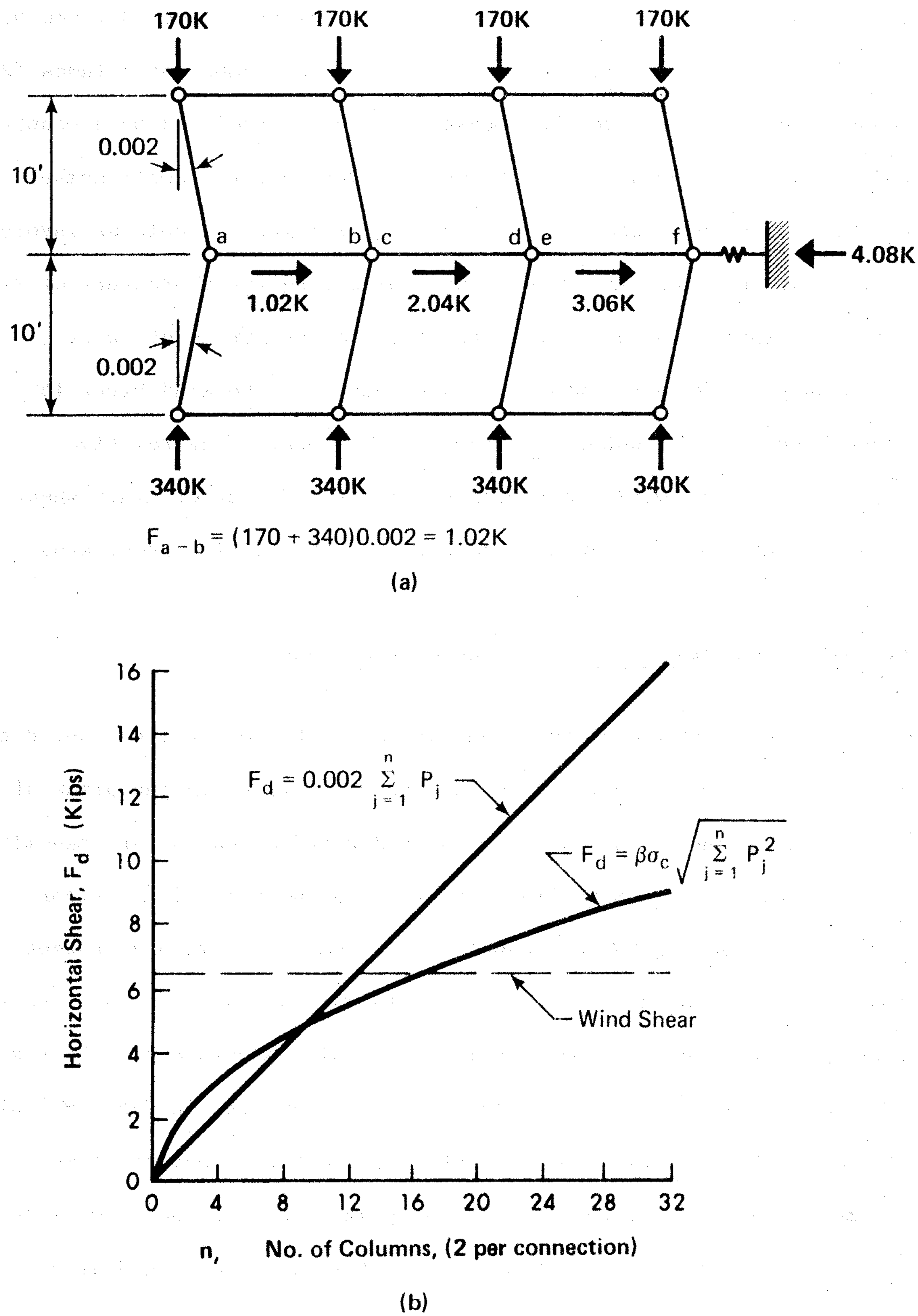


Figure 9.1 Distribution of horizontal shear in a braced bent

The same frame has been used in the example of Fig. 8.1 for an eventual comparison. The horizontal shears from Figs. 8.1 and 9.1(a) are plotted in Fig. 9.1(b) as a function of the number of columns (2 per connection). The figure shows that the shears predicted by a constant column out-of-plumb of 0.002 are unconservative for a small number of columns but become excessively large as the number of columns involved increases. An arbitrary wind shear, calculated for a pressure of 21.54 psf and a span perpendicular to the plane of the frame of 30 ft., has been plotted on the graph to point out that the wind shear is independent of the number of columns in the bent. The relative importance of the wind shear with respect to the out-of-plumb shear is given by the gravity-to-wind load ratio at the section under study.

9.2 Shear and Moment in the Plane of the Floor

The portion of the floor delimited by column lines No. 8 to 15 in Fig. 6.2 must be designed for shears and moments in the plane of the floor. Assuming a wind force in the x direction, the shear transmitted to the core at a specific floor level by the portion of the floor described above, is roughly equal to the sum of the lateral forces concentrated at connections No. 8, 9, 14 and 15. A shear equal to 17.6 kips is calculated at floor level 1 for a wind pressure of 22 psf and a tributary area of 800 ft^2. This value is compared in Table 9.2 with the shear calculated at the same storey from the 16 out-of-plumb columns on lines 8 to 15. The out-of-plumb shear is about twice as large as the wind shear at that level. The values obtained at floor level 23 show that the wind controls in the upper storeys.

1	2	3	4	5
	Floor Level	Wind Effect	Out-of-Plumb Effect	Out-of-Plumb Effect / Wind Effect
Shear In Floor	1* 23**	17.6 k. 21.0 k.	36.0 k.† 7.4 k.	2.0 0.35
Moment In Floor	1* 23**	222 ft.-k. 265 ft.-k.	2206 ft.-k.†† 452 ft.-k.	9.9 1.71

* Design wind pressure = 22.0 psf

** Design wind pressure = 35.0 psf

† $F_d = 3.5 \times .0017 \sqrt{\sum_{j=1}^{16} P_j^2}$ (Eq. 7.7)

†† $M_d = 3.5 \times .0017 \sqrt{\sum_{j=1}^{16} [P^2 (L_x^2 + L_y^2)]_j}$ (Eq. 7.9)

TABLE 9.2 COMPARISON OF OUT-OF-PLUMB AND WIND EFFECTS IN FLOOR SYSTEMS

The moments calculated at the same floor sections, under the same conditions, are listed in the second half of the table. The out-of-plumb moments obtained from Eq. (7.9) far exceed the wind moments at every storey of building A.

9.3 Shear, Moment, and Torque in Core

The simplified method of Ref. 60 was used to calculate the moments and shears caused by a basic wind pressure of 8.5 psf at each storey of buildings A and B. The results are presented in Tables 9.3 to 9.5. The tabulated values have been calculated for a wind applied perpendicular to the short face of the buildings. The wind loads (windward and leeward components combined) and resulting shears and moments in the orthogonal direction can be obtained by factoring the tabulated values by the appropriate building length-to-width ratio given below the tables.

A comparison of the shears given in column 4 of Tables 9.3 and 9.5 with those given in column 5 of Tables 8.2 and 8.4 reveals that the column out-of-plumbs create shears in the cores of buildings A and B which do not exceed 4.5 and 3.3 percent of the wind shears (depending on the direction of the wind).

The out-of-plumb moments given in column 6 of Table 8.2 are compared in Fig. 9.2 with the corresponding wind moments for building A. Although still small, the fraction of moment due to column out-of-plumbs is relatively larger in the upper section of the structure than at the base. This is in part due to the gravity-to-wind load ratio but is largely a reflection of the fact that wind

1	2	3	4	5
Storey No.	Storey Height (Ft.)	Wind Load (Short Span) (kips)	Shear (kips)	Moment (Ft.-K.)
27	20	35.0	35.0	0.0
26	12	56.0	91.0	700
25	12	42.0	133.0	1792
24	12	42.0	175.0	3388
23	12	42.0	217.0	5488
22	12	39.4	256.4	8092
21	12	39.4	295.8	11169
20	12	39.4	335.2	14718
19	12	39.4	374.6	18741
18	12	39.4	414.0	23236
17	12	39.4	453.4	28204
16	12	39.4	492.8	33645
15	12	36.8	529.6	39558
14	12	36.8	566.4	45914
13	12	36.8	603.2	52710
12	12	36.8	640.0	59949
11	12	36.8	676.8	67629
10	12	34.1	710.9	75750
9	12	34.1	745.0	84281
8	12	34.1	779.1	93221
7	12	31.5	810.6	102570
6	12	31.5	842.1	112298
5	12	28.9	871.0	122403
4	12	28.9	899.9	132855
3	12	26.3	926.2	143654
2	12	26.3	952.5	154768
1	20	35.0	987.5	166198
				185948

Wind pressure, $q(\frac{1}{30}) = 8.5$ psf.

Building dimensions: Long span, 147' Ratio = 1.485
Short span, 99'

TABLE 9.3 SHEAR AND MOMENT DUE TO WIND IN CORE OF (27-STOREY) BUILDING A

1	2	3	4	5
Storey No.	Storey Height (Ft.)	Wind Load (Short Span) (kips)	Shear (kips)	Moment (Ft.-K.)
22	12	19.7	19.7	0.0
21	12	39.4	59.1	236
20	12	39.4	98.5	946
19	12	39.4	137.9	2128
18	12	39.4	177.3	3782
17	12	39.4	216.7	5910
16	12	39.4	256.1	8510
15	12	36.8	292.9	11584
14	12	36.8	329.7	15098
13	12	36.8	366.5	19055
12	12	36.8	403.3	23453
11	12	36.8	440.1	28292
10	12	34.1	474.2	33574
9	12	34.1	508.3	39264
8	12	34.1	542.4	45364
7	12	31.5	573.9	51872
6	12	31.5	605.4	58759
5	12	28.9	634.3	66024
4	12	28.9	663.2	73636
3	12	26.3	689.5	81594
2	12	26.3	715.8	90184
1	20	35.0	750.8	98773
				113789

Wind pressure, q $(\frac{1}{30})$ = 8.5 psf

Building Dimensions:

- Long span, 147' Ratio = 1.485
- Short span, 99'

TABLE 9.4 SHEAR AND MOMENT DUE TO WIND IN CORE OF (22-STOREY) BUILDING A

1	2	3	4	5
Storey No.	Storey Height (Ft.)	Wind Load (Short Span) (kips)	Shear (kips)	Moment (Ft.-K.)
34	24	54.4	54.4	0.0
33	12	81.6	136.0	1306
32	12	54.4	190.4	2938
31	12	48.4	238.8	5222
30	12	48.4	287.2	8088
29	12	48.4	335.6	11534
28	12	48.4	384.0	15562
27	12	48.4	432.4	20170
26	12	48.4	480.8	25358
25	12	48.4	529.2	31128
24	12	48.4	577.6	37478
23	12	48.4	626.0	44410
22	12	48.4	674.4	51922
21	12	48.4	722.8	60014
20	12	48.4	771.2	68688
19	12	45.3	816.5	77942
18	12	45.3	861.8	87740
17	12	45.3	907.1	98082
16	12	45.3	952.4	108967
15	12	45.3	997.7	120396
14	12	45.3	1043.0	132368
13	12	45.3	1088.3	144884
12	12	42.3	1130.6	157944
11	12	42.3	1172.9	171511
10	12	42.3	1215.2	185586
9	12	42.3	1257.5	200168
8	12	42.3	1299.8	215258
7	12	39.3	1339.1	230856
6	12	39.3	1378.4	246925
5	24	59.0	1437.4	263466
4	27	77.1	1514.5	297964
3	16	59.6	1574.1	338855
2	15	39.1	1613.2	364041
1	15	37.8	1651.0	388239
				413004

Wind pressure, $q(\frac{1}{30}) = 8.5$ psf.

Building Dimensions : Long Span, 152' Ratio = 1.333
Short Span, 114'

TABLE 9.5 SHEAR AND MOMENT DUE TO WIND IN CORE OF BUILDING B

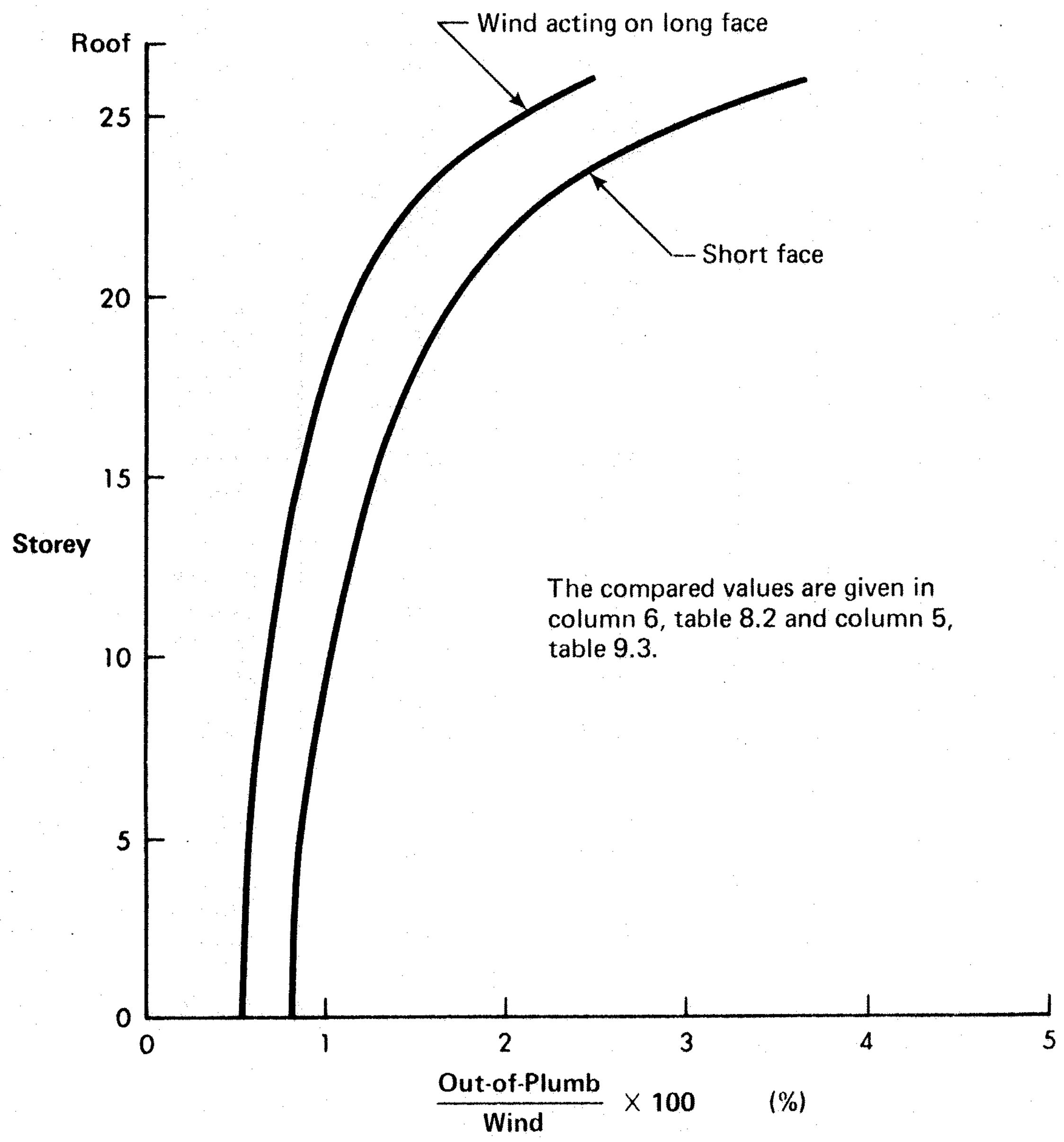


Figure 9.2 Comparision of moments caused by column out-of-plumbs and moments due to wind in Luilding A

moments are added algebraically from storey to storey while out-of-plumb moments are combined statistically according to Eq. (7.13).

A similar comparison is presented in Fig. 9.3 for the moments caused by the wall out-of-plumbs of building A. The figure indicates that the out-of-plumb moments can be proportionally quite large in the upper storeys of a tall building. Fortunately, these results are not significant. A core is designed for the forces at its base and in general for the forces at one or two other locations along the core. Therefore, the forces present in the upper section of the core are resisted by a stiffer and stronger core than required.

The actual buildings, A and B, had cores formed of eight and nine orthogonal walls respectively. The moments, torques, and lateral deflections calculated for only four walls are consequently larger than in the real case. Assuming that the actual eight walls of building A carry an equal share of the total vertical load, it can be shown that the moments predicted by Eq. (7.26) are reduced by 55 percent. Thus, in the case of the wind acting on the short face of the building as shown in Fig. 9.3, the moment is reduced from 39 to about 19 percent at the top of the building and from 6.5 to about 3.5 percent at the base.

The results presented in Fig. 9.4 show that the moments in the core due to column and wall out-of-plumbs are negligible in the case of building B. A reduction of 55 percent also exists when the calculations are based on the actual nine walls.

There are no recommendations related to wind in the Canadian National Building Code[60] which allow a calibration of the torques

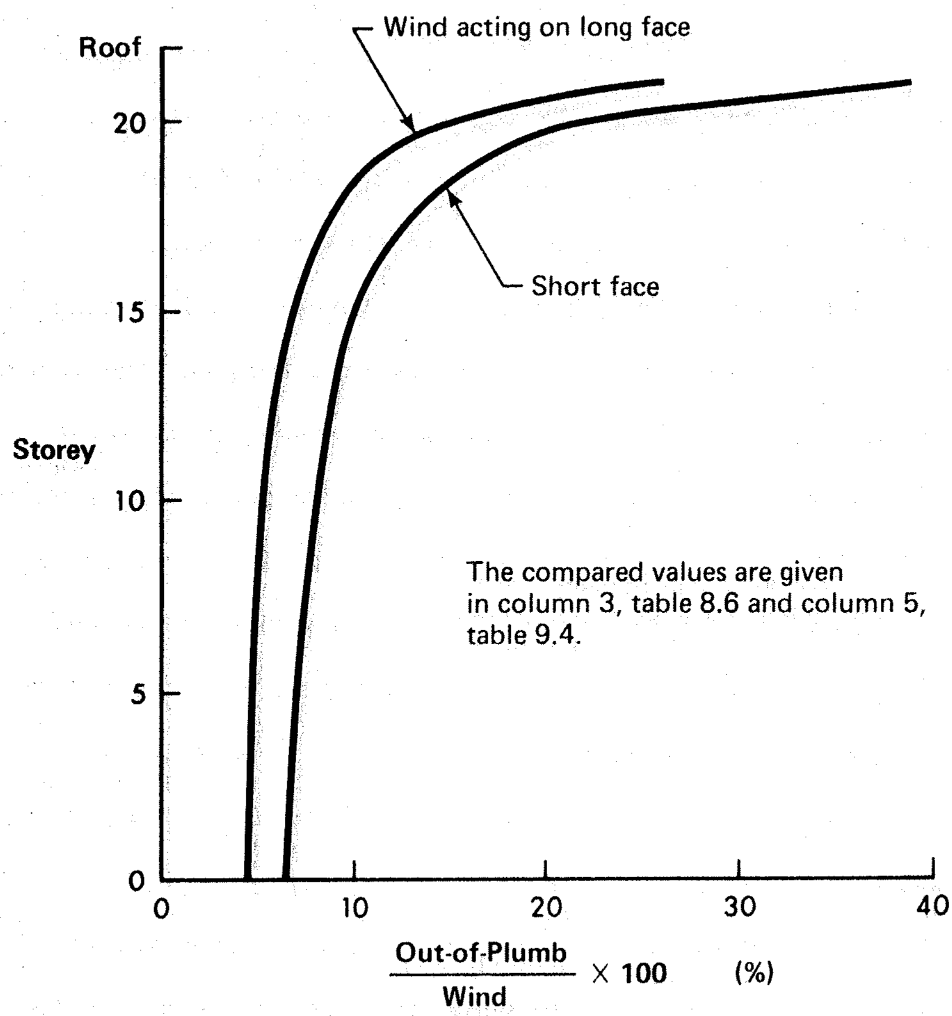


Figure 9.3 **Comparison of moments caused by wall out-of-plumbs and moments due to wind in building A**

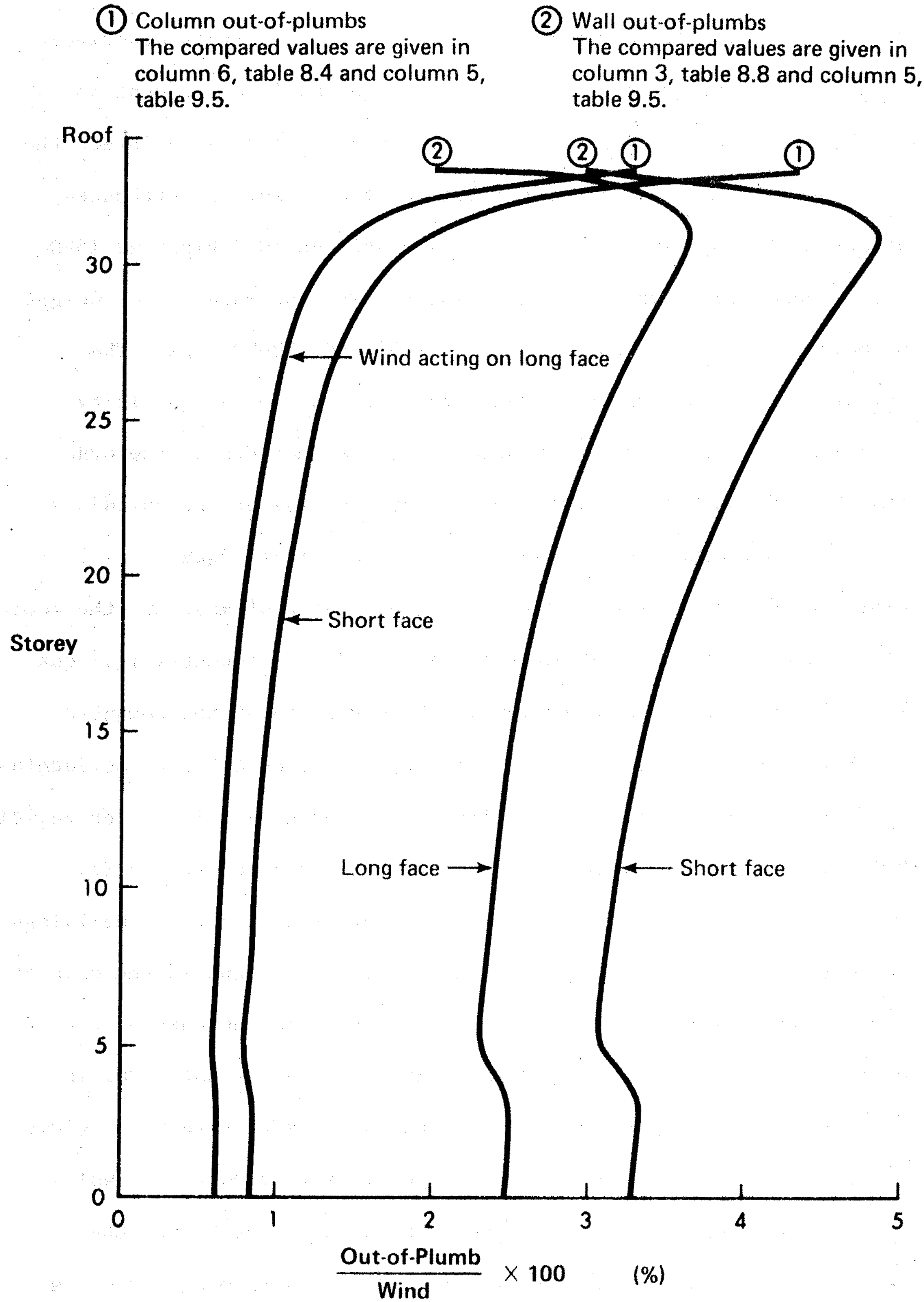


Figure 9.4 **Comparision of moments caused by column and wall out-of-plumbs and moments due to wind in building B**

given in column 7 of Tables 8.2 and 8.4 and in columns 5 and 6 of Tables 8.6 and 8.8. However, the simple static analysis described in the commentary on the effects of earthquakes in Supplement No. 4 to the National Building Code of Canada can be used to calibrate the out-of-plumb torques. The total lateral forces for an earthquake occurring in a seismic zone 2 are calculated as 1032 kips and 1560 kips, respectively, for buildings A and B. The estimated total weights of both buildings are approximately 61450 and 100000 kips. The calculated lateral forces acting through the design eccentricity recommended by the code[60] produce torsional moments in the order of 7585 and 28236 ft.-k., respectively, at the base of the buildings.

The code recommends a design eccentricity equal to 1.5 times the distance between the calculated center of mass and the center of resistance of the structure, plus an accidental eccentricity equal to 0.05 times the plan dimension in the direction of the computed eccentricity. By reason of its symmetry, building A has an accidental eccentricity equal to 7.35 ft. The calculated eccentricity for building B is 7.0 ft. and the accidental eccentricity is equal to 7.6 ft.

The torques created by the out-of-plumbs in these buildings account for 33 and 38 percent of the accidental torque in the case of the columns and 27 and 17 percent, respectively, in the case of the walls. When the calculated eccentricity is also accounted for in building B, the proportions are reduced to 16 percent for the columns and 7 percent for the walls. The proportions related to the walls in both buildings are further reduced by a factor of 2.5 of the calculations based on the actual number of walls forming the cores. The out-of-plumbs could therefore induce torsional effects that are not necessarily negligible.

9.4 Lateral Deflections

It has been shown in sections 5.1 and 5.4 that the most common technique in an overall stability analysis for initial out-of-plumbs consists in assuming that columns and walls all lean in the same direction, as in Fig. 5.1(a). The standards described in section 5.4, which refer to that model, were applied to building A and compared with the results of Eq. (7.16) and (7.18) in Fig. 9.5. It was shown in section 8.1.5 that curves 1 and 2 in Fig. 9.5 were upper bounds on the actual lateral deflections for this structure. In fact, the actual deflection curves obtained from the measured column out-of-plumbs in the x and y directions were shown in Fig. 8.12 to be well within these limits. By superimposing Fig. 8.12 on Fig. 9.5, the conservative nature of the various code recommendations is evident.

The West German expression (No. 5)[47], which is a function of the building height, gives the closest estimate. The lateral deflections given by curves 4 and 6 are about 13 times larger than the limit given by Eq. (7.18) at the top of the building. Curve No. 6, suggested by the Swedish Concrete Regulations (B7-1968)[46], has been obtained for 6 columns with a slope of 0.007 Rad. and 10 columns with a slope of 0.0035 Rad., resulting in a slope of 0.0048 Rad. at each storey.

By extending the study, it can be ascertained whether some of these recommendations still have the same relationship when applied to structures of different heights. Figs. 9.6 and 9.7 present curves obtained for a 10-storey building (E.3) and a 5-storey building (E.1) respectively. It is observed that the different code requirements are

① $\Delta_d/h = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2} \Big/ \sum_{j=1}^{n} P_j$ .... (Eq. 7.16)

② $\Delta_d/h = \beta\sigma_c \Big/ \sqrt[2.2]{n}$ .... (Eq. 7.18)

③ $\Delta_d/h = 0.002$ ..... [CSA-S16.1] ... (Section 5.4.2)

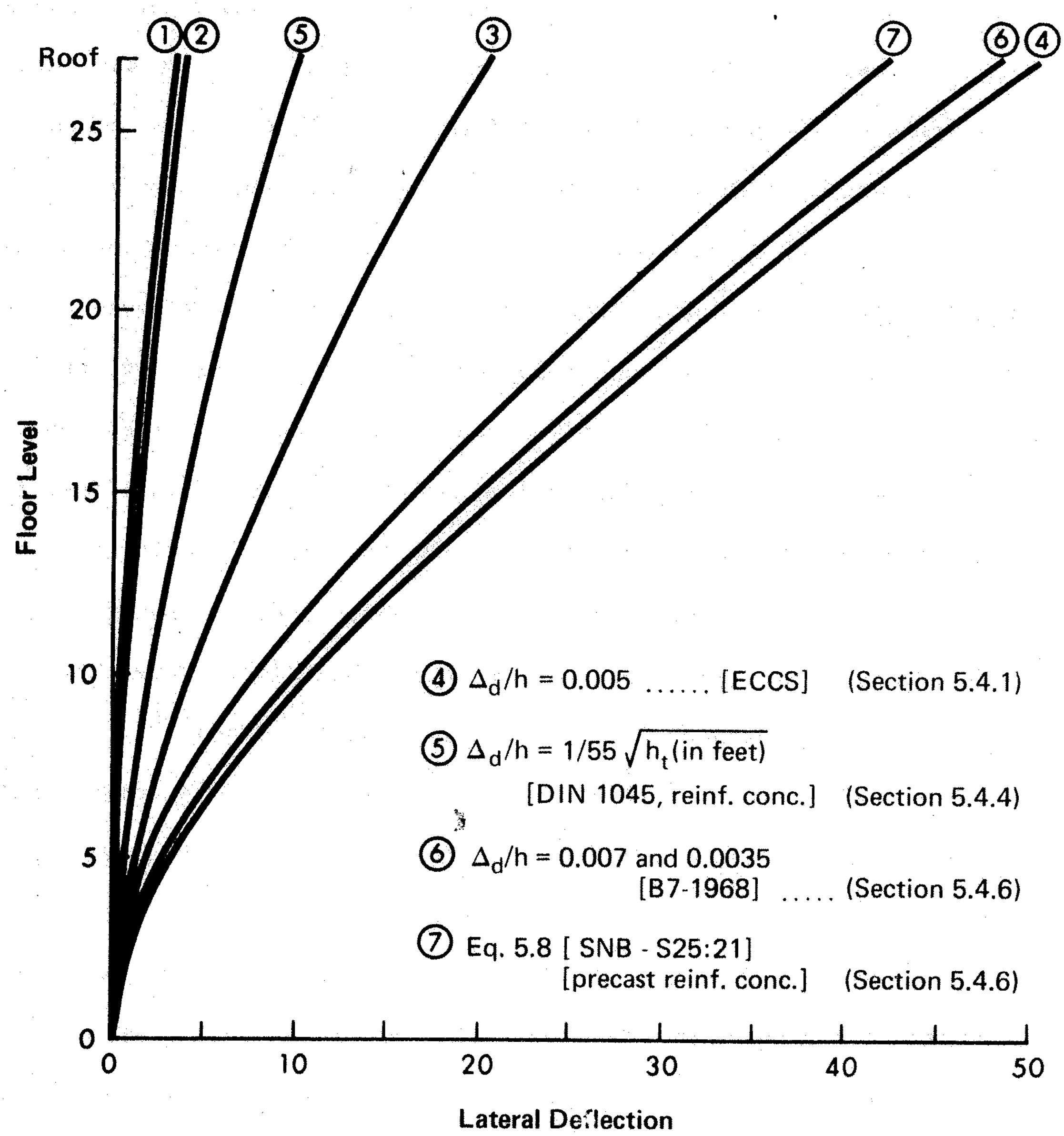


Figure 9.5 Comparison of lateral deflections derived from different code specifications for building A

① $\Delta_d/h = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2} \Big/ \sum_{j=1}^{n} P_j$ .... (Eq. 7.16)

② $\Delta_d/h = \beta\sigma_c \Big/ \sqrt[2.2]{n}$ .... (Eq. 7.18)

③ $\Delta_d/h = 0.002$ .... [CSA-S16.1] .... (Section 5.4.2)

④ $\Delta_d/h = 0.005$ .... [ECCS] .... (Section 5.4.1)

⑤ $\Delta_d/h = 1/55 \sqrt{h_t \text{ (in feet)}}$

[DIN 1045, reinf. concrete] (Section 5.4.4)

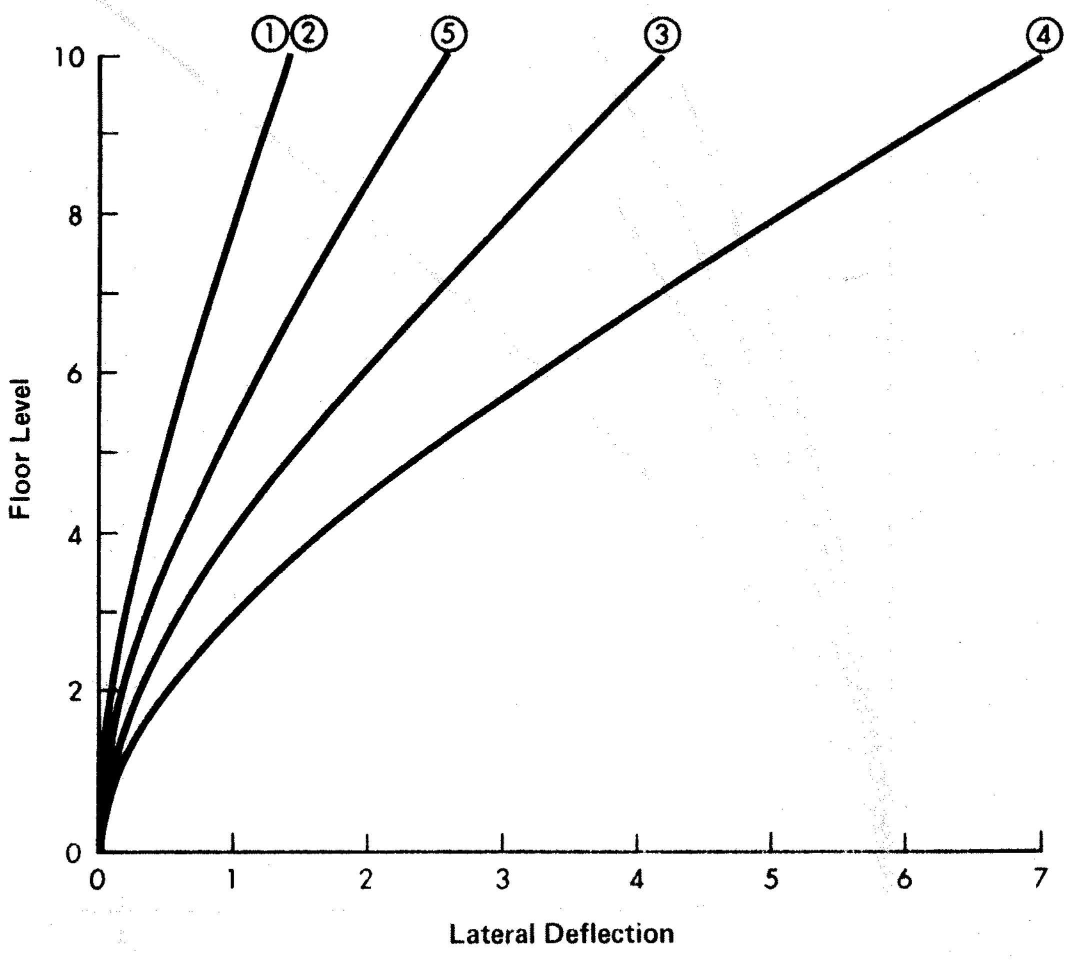


Figure 9.6 Comparison of lateral deflections derived from different code specifications for building E.3

① $\Delta_d/h = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2} \Big/ \sum_{j=1}^{n} P_j$ .... (Eq. 7.16)

② $\Delta_d/h = \beta\sigma_c \Big/ \sqrt[2.2]{n}$ .... (Eq. 7.18)

③ $\Delta_d/h = 0.002$ .... [CSA - S16.1] .... (Section 5.4.2)

④ $\Delta_d/h = 0.005$ .... [ECCS] .... (Section 5.4.1)

⑤ $\Delta_d/h = 1/55 \sqrt{h_t \text{ (in feet)}}$ [DIN 1045, reinf. concrete] (Section 5.4.4)

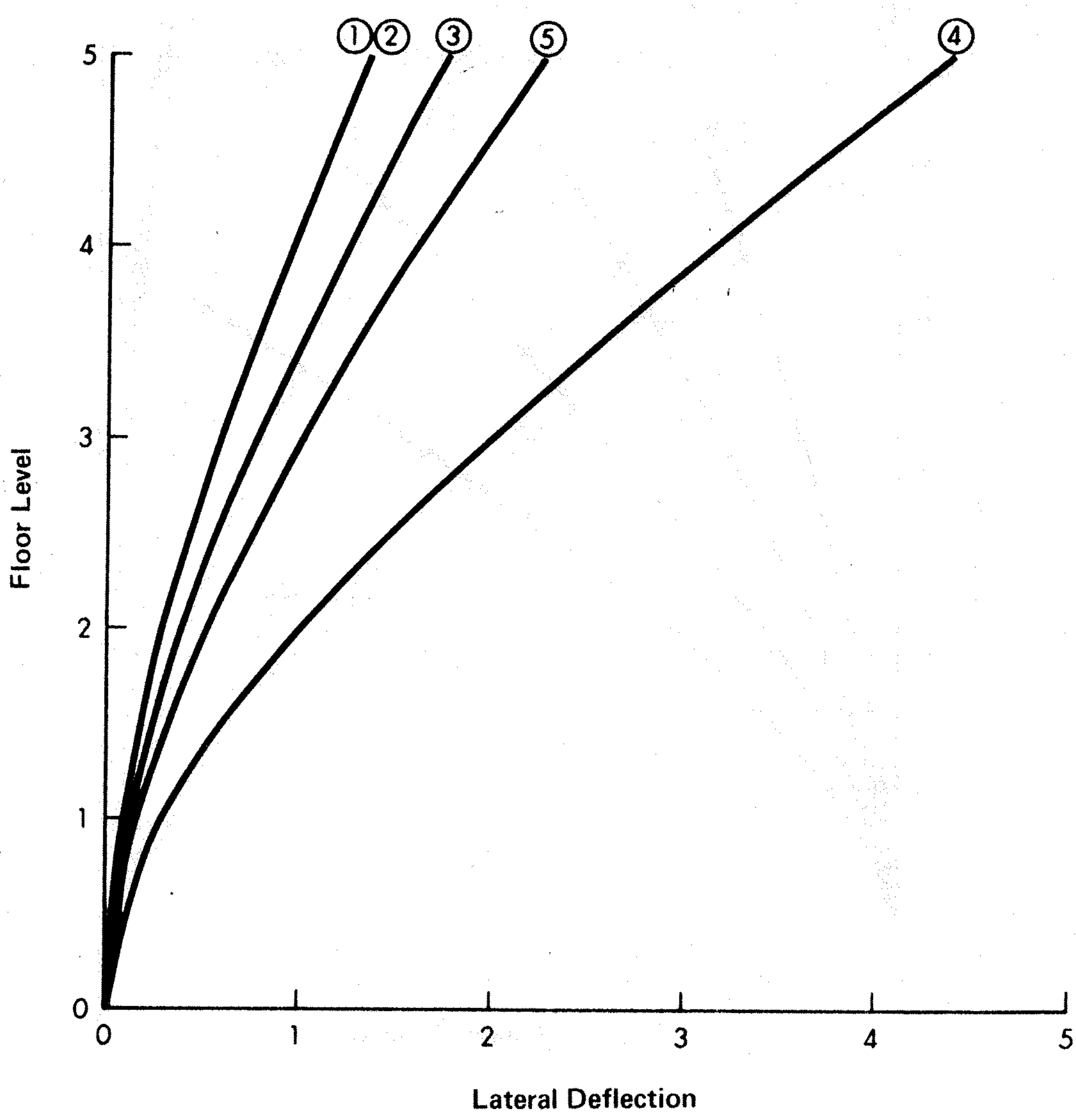


Figure 9.7 **Comparison of lateral deflections derived from different code specifications for building E.1**

more appropriate to low structures in the sense that the disparities between their predictions and the results of Eq. (7.18) are significantly reduced. The West German recommendation[47] in general estimates the deflections reasonably well while the European recommendation[41] still remains excessively conservative. Any comparison between the different code recommendations, however, must be carefully interpreted for the reasons given in section 5.1.

For the buildings under study, the deflections predicted by Eqs. (7.18) and (7.37) account for a small percentage of the deflections due to wind, as shown in Table 9.6. The column out-of-plumbs have apparently no significant effect on the overall stability of the structures. The sway induced by the wall out-of-plumbs accounts for less than 5 percent of the sway due to wind when the actual number of walls is used in the calculations. Although the lateral deflections are small for these buildings of 20 storeys and over, it cannot immediately be concluded that the overall stability of other buildings is not affected by out-of-plumbs. The gravity-to-wind load ratio of a building, the type of building, and the number of columns and/or walls present in the building are all significant factors to be taken into account.

An example is given in Fig. 9.8 where a one-storey braced structure is analyzed for two different combinations of columns and walls. Eqs. (7.16) and (7.36) are used to calculate the horizontal forces due to out-of-plumbs for the given axial loads. The forces H_c and H_w obtained for column and wall out-of-plumbs, respectively, are combined according to Eq. (7.38). Since the deflections are directly related to the applied lateral loads in this case, the ratios given in Fig. 9.8 are a measure of the sway induced by out-of-plumbs to the sway

Column Out-of-Plumbs:

Building	No. of Columns n	Δ_d/h* ($\times 10^4$ Rad.)	$\frac{\text{Out-of-Plumb}}{\text{Wind}} \times 100$ (%)
A	458	3.67	1.0
B	880	2.73	1.0

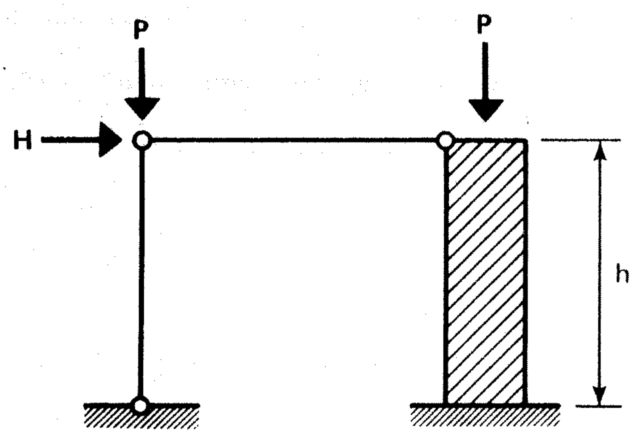
* $\Delta_d/h = 3.5 \times .0017/\sqrt[2.2]{n}$ (Eq. 7.18)

Wall Out-of-Plumbs:

Building	No. of Walls n	Δ_d/h** ($\times 10^3$ Rad.)	$\frac{\text{Out-of-Plumb}}{\text{Wind}} \times 100$ (%)
A	(4 walls/storey) 88	1.56	6.0
	(8 walls/storey) 176	1.20	4.6
B	(4 walls/storey) 136	1.33	3.5
	(8 walls/storey) 306	1.00	2.6

** $\Delta_d/h = 0.00028 + 3.5 \times .0028/\sqrt[2.2]{n}$ (Eq. 7.37)

TABLE 9.6 LATERAL DEFLECTIONS CAUSED BY OUT-OF-PLUMBS AS A PERCENTAGE OF DEFLECTIONS DUE TO WIND



1 Column:

$$H_c = \frac{P\,\Delta_d}{h} = 80\left[3.5\times 0.0017\,\frac{\sqrt{80^2}}{80}\right] = 0.48\text{ k}$$

1 Wall:

$$H_w = \frac{P\,\Delta_0}{h} = 80\left[0.00028 + 3.5\times 0.0028\,\frac{\sqrt{80^2}}{80}\right] = 0.8\text{ k}$$

$$\frac{\sqrt{H_c^2 + H_w^2}}{H} = 0.052$$

4 Columns:

$$H_c = \frac{\Sigma\,P\,\Delta_0}{h} = 80\left[3.5\times 0.0017\,\frac{\sqrt{4\times 20^2}}{80}\right] = 0.24\text{ k}$$

2 Walls:

$$H_w = \frac{\Sigma\,P\,\Delta_0}{h} = 80\left[0.00028 + 3.5\times 0.0028\,\frac{\sqrt{2\times 40^2}}{80}\right] = 0.58\text{ k}$$

$$\frac{\sqrt{H_c^2 + H_w^2}}{H} = 0.035$$

Figure 9.8 Example—proportion of sway

caused by the lateral loads. The 5.2 percent out-of-plumb sway obtained for the one-column, one-wall structure is reduced to 3.5 percent for the four-column, two-wall structure carrying the same total load.

CHAPTER X

DISCUSSION

The previous chapters have been devoted to the presentation and discussion of aspects of structural stability related to P-Δ and out-of-plumb effects. The general discussion presented in this chapter should establish a logical link between these different sections and emphasize the major characteristics of the study.

10.1 P-Δ Effects

The P-Δ effects were briefly discussed in the first sections of the thesis with emphasis placed on the creation and transfer of additional horizontal forces in structures. Practical techniques for including second order effects in analysis were presented. It was demonstrated that when an approximate second order analysis is performed, the horizontal forces in the structure are not distributed in a proper manner. Generally, the errors in these forces do not affect the design of structural members but do affect the design of connections and floor diaphragms when the second order effects are important. Such a situation may occur in a pin-connected frame which relies on a separate lateral support system for stability. The transfer of horizontal forces in this type of structure is more critical than in other types of structures.

As a general rule, the correct horizontal force distribution should be evaluated at specific storeys in any type of structure when the storey shears given in Fig. 3.3 are significant compared to the applied wind forces. A redistribution of the forces based on the free-body diagram method described in section 4.3 is then recommended, since the concept of the free-body diagram is familiar to most designers. When the sway forces are not significant, the horizontal forces given by a first order analysis can be used in the design of the connections and floor systems.

10.2 Out-of-Plumb Effects

The study of the effects of out-of-plumbs on the stability of structures was primarily based on core-braced buildings in which a pin-connected steel frame is supported by a central reinforced concrete core. Although the transfer of the horizontal forces is more critical in this type of structure, the results obtained on core-braced buildings can be extrapolated to other types of structures including moment resisting frames. The sway of a continuous structure creates extra moments in the members and the additional forces produced are reduced to a minimum. The out-of-plumb effects in moment resisting frames are not investigated as such in this thesis.

Statistical methods are essential to describe the nature of the out-of-plumb forces. A certain probability of occurrence can then be associated with the selected factor of safety. A safety index of 3.5, corresponding to a probability of failure of 4.6×10^{-4}, has been adopted, based on discussions presented in section 7.1 and in Appendices B and C. The adequacy of the selected safety index and the applicability of the equations derived in Chapter VII have been confirmed in Chapter VIII.

The out-of-plumb measurements taken as a part of this research project and presented in Chapter VI compare quite well with the published measurements listed in section 6.1. The statistical populations obtained are normally distributed, resulting in greatly simplified calculations. The calculated means are generally small enough to be neglected when the sample dimensions are sufficiently large to present a realistic estimate of the distribution characteristics. Unfortunately, the standard deviations obtained in this thesis are specific to the building measured. They are apparently typical for steel columns and cast-in situ reinforced concrete walls,as observed by comparing the results of the three different buildings A, B, and C. The standard deviation for the reinforced concrete walls is almost double that measured for the steel columns, as shown in Table 7.4.

Data from other structures are required to estimate the effects on the standard deviation of variables like:

- the structural material,
- the type of structure,
- the erection and plumbing techniques,
- the skill and experience of the constructor.

The population mean may also be large in some cases and the possibility of systematic variations due to erection techniques or errors caused by the use of a faulty instrument, for instance, should be evaluated. While the mean of the column population is almost zero, the corresponding value for the walls is not negligible.

10.2.1 Connection and Floor Design in Braced Buildings

It was demonstrated in sections 9.1 and 9.2 that the action of column out-of-plumbs in a core-braced building generally controls the design of beam-to-column connections and of floor diaphragms for horizontal shears and moments. The forces and moments given by Eqs. (7.7) and (7.9) should be computed for the loading cases of Ref. 36, using the appropriate load combination factors. In tall buildings with uniformly varying gravity loads, the shears and moments can be calculated at specific levels and the remaining values obtained by interpolation.

The model shown in Fig. 5.1(b) was incorrect for the assessment of forces in the plane of the floors. With a suitable uniform slope equal to $\beta\sigma_c$, as given by Eq. (7.2), the model would produce an upper bound on the forces, which would be equivalent to totally neglecting the random nature of column out-of-plumbs (See Appendix C). The application shown in Fig. 9.1(b) provides a graphical representation of the problem. The straight line is the result of the algebraic summation of the individual column forces suggested by the model of Fig. 5.1(b). The curve, obtained from a statistical summation, represents more exactly the actual forces in the structure. In this particular example, since $\Delta_d/h = 0.002$ is lower than $\beta\sigma_c = 0.006$, the actual forces are underestimated by the simple model for frames having fewer than 10 columns. Beyond this limit, a significant reduction takes place.

Each beam-to-column connection in a moment resisting frame can be conservatively designed for an extra horizontal force equal to 0.85 percent of the largest axial load in the two columns above and below the floor (see Eq. 7.5). This assumes that no significant transfer of force exists in the structure and that a minimum bracing force is required to stabilize the columns. A slight reduction of this value might be expected in future from an appropriate study on the effects of column out-of-plumbs in continuous frames.

The basic equation used in Russia for the calculation of out-of-plumb horizontal forces follows the pattern observed in the present study[50]. Eq. (5.4) should be compared to Eq. (7.18), considering that ε_1 is equal to three times the standard deviation obtained from measurements taken on concrete structures. The design value $\varepsilon_1 = 0.012$, defined as the total change in slope between two columns (at their intersection), is twice as large as $\beta\sigma_c = 0.006$ for one column in this thesis and has been described as too large in Ref. 48.

The need for a variable safety index, which has been observed in the study presented in Fig. 5.4, is most likely to compensate for the neglect of the variable axial loads P_J combined as in Eq. (7.16) or (7.7). It has been shown in Appendix D that an expression of the form of Eq. (7.18) underestimates the "exact" force given by Eq. (7.16) for a small number of columns. The Russians observed this trend in their study and compensated by imposing a larger factor of safety.

The Swedish Building Regulations[51] described in section 5.4.6 also present the results of a comprehensive statistical approach. The regulations are, in some respects, in good agreement with the findings of this thesis. However, the force in a connection, given by Eq. (5.6), is equal to about 3.5 percent of the average axial load in the load bearing elements; which is large when compared to the 0.84 percent found in this study.

10.2.2 Core Design

Shears and moments in a core from both column and wall out-of-plumbs can be neglected, according to the calculations presented in Chapter IX. However, as shown in section 9.3, the torques account for a significant percentage of the minimum torsional moments prescribed by the Canadian National Building Code[60]. The empirical minimum eccentricity prescribed by the code has been intended to account for possible additional torsion arising from various sources listed in Supplement No. 4 to the Canadian National Building Code. It is not mentioned, however, that the minimum eccentricity is also intended to account for the possibility of a torque caused by out-of-plumbs but there is no apparent reason why it should not.

In view of the significance of the out-of-plumb torques it might be more reasonable to include the torsional moments given by Eqs. (7.15) and (7.31) specifically. In a three-dimensional analysis of a structure under earthquake loading, the out-of-plumb torques would be added to the prescribed calculated and accidental torsional moments. In an analysis for wind loads, the out-of-plumb torques would be considered alone since, in this case, there is no provision for a minimum eccentricity of load application. In mixed construction (core-braced building), the torques would be evaluated at specific storeys for the factored axial loads and combined according to the expression $\sqrt{T_c^2 + T_w^2}$ to account for their random nature.

It is common practice, however, to neglect the three-dimensional effects in most buildings, based on a recognition of the fact that the torques are comparatively small and that a structure is generally much stiffer and stronger in torsion than necessary.

10.2.3 Overall Stability

When the overall stability of a structure under combined lateral and gravity loads is to be assessed, the extra lateral deflections caused by column and wall out-of-plumbs can be disregarded. The deflections caused by the out-of-plumbs account for a very small percentage of the deflections due to wind, as demonstrated in section 9.4 (Table 9.6).

The uniform slopes given by Eqs. (7.18) and (7.37) are suitable for the assessment of the overall stability of a structure when the loading case considered is associated with the vertical loads acting alone. The presence of initial imperfections in a structure gives rise to an initial sway of the structure. Lateral forces are computed and applied to the structure to produce additional deflections and corresponding P-Δ shears and forces. The concept has been developed in Ref. 23 and has been adapted for the Canadian Standards S16.1[36] for a constant slope of 0.002. A discussion of the standard is given in section 5.4.2.

Check how?

In a core-braced building or an equivalent composite structure, the forces obtained from Eqs. (7.18) and (7.37) should be combined together according to the expression $\sqrt{H_c^2 + H_w^2}$ to account for their random nature. As before, the forces can be evaluated at specific floors and the intermediate values obtained by interpolation. The factored axial loads from the gravity load case are used in the equations[36].

The fictitious horizontal load principle discussed in section 3.2 and applied to a cantilevered member in section 7.2.3 can be used to justify the application of Eq. (7.18) to moment resisting frames.

The extra moments created in the structure in this manner resist the sway induced by column out-of-plumbs.

Taking for granted that Eqs. (7.16) and (7.18) provide an upper bound on the lateral deflections, the different code recommendations described in section 5.4 were shown to be conservative in Figs. 9.5 to 9.7. The study demonstrated that a fixed slope, such as 0.002 or 0.005, is not an appropriate model for general application. The West German expression given by equation No. 5 in Fig. 9.5 provided a close estimate in every case. The variable h_t suggested by the Germans is in some ways equivalent to the variable n in Eq. (7.18).

The prescription of the Swedish Building Code for reinforced concrete[51], Eq. (5.8), conforms in principle to the views of the present study. However, the expression assumes that 20 percent of the maximum inclination obtained from field measurements is a systematic variation and 80 percent is random. The maximum inclination of 0.015 used in the equation seems slightly high, as demonstrated in Fig. 9.5. The reduction factor γ, which accounts for the tolerance requirements and the degree of control, has no real significance, in view of the discussion presented in section 5.3.

The entire provision for out-of-plumbs summarized in Fig. 5.5 is, in general, acceptable. The most curious statement is that the three types of forces given by Eqs. (5.6) to (5.8) cannot be combined.

10.3 Concluding Remarks

The stability of a structure cannot be properly ensured unless all the major destabilizing forces in the structure are properly resisted. For this reason, P-Δ as well as out-of-plumb forces should be given consideration in design.

The statistical method presented herein for steel frames and cast-in situ reinforced concrete walls can also be applied to cast-in situ or precast concrete frames,as long as the characteristics of the respective member out-of-plumb populations are known or estimated. A similar type of approach can be adopted for the evaluation of forces in the bracing members which provide lateral support to the compression flange of initially crooked beams and girders. An assembly of bracing members and beams in a horizontal plane can be treated as an assembly of beams and out-of-plumb columns in the vertical. The required statistical characteristics would be obtained from a survey of beam deviations in buildings under construction.

CHAPTER XI

SUMMARY AND CONCLUSIONS

11.1 Summary

Different forces which are likely to affect the strength and stability of buildings and their components were investigated in this thesis. The forces were classified in three categories: the first order forces, the P-Δ forces, and the forces due to initial out-of-plumbs.

The P-Δ forces were discussed briefly and approximate methods for their determination were presented.

The thesis essentially concentrated on the investigation of out-of-plumb effects and the development of suitable design procedures. Statistical methods provided an appropriate means of defining the problems of stability and strength related to structural out-of-plumbs.

Measurements were made on steel columns and concrete walls in two tall core-braced buildings and one large industrial building under construction to determine actual characteristics of out-of-plumbs for use in the statistical calculations.

Equations were derived for the design of connections, floor diaphragms, and vertical bracing systems affected by the out-of-plumb forces and methods were suggested for the evaluation of the building sway movements.

Comparisons with corresponding first order effects have demonstrated that while some out-of-plumb effects are negligible, others may be very significant. Moreover, when compared to the results based on present standards, the effects were generally found to be either under- or overestimated.

The investigation resulted ultimately in the creation of more rational clauses for design standards which are related to the stability of structures and individual members. The proposed clauses are listed in the following section.

11.2 Conclusions

The investigation is concluded by presenting the results of the research in the form of proposed clauses for design standards. The actual sections of the Canadian Standard S16.1 "Steel Structures for Buildings - Limit States Design"[36] which relate to the overall stability of structures and of individual members are rewritten in view of the present findings. Some recommendations which relate to concrete structures are presented separately for consideration for the appropriate concrete standards.

The section numbering adopted below corresponds to that of the Canadian Standard but the nomenclature used is that of this thesis.

12.2.1 Proposed Clauses for CSA-S16.1 "Steel Structures for Buildings"

8.6 Stability Effects

8.6.1 The analyses referred to in Clauses 8.4 and 8.5 shall include the sway effects produced by the vertical loads acting on the structure in its displaced configuration, unless the structure is designed in accordance with the provisions of Clause 8.6.3.

For certain types of structures where the vertical loads are small, where the structure is relatively stiff and where the lateral load resisting elements are well distributed, the sway effects may not have a significant influence on the design of the structure (See clause 9.3.2(b)).

8.6.2 For structures in which the sway effects have been included in the analysis to determine the design moments and forces (see Appendix J), the effective length factors for members shall be based on the sidesway prevented condition (See clause 9.3.2(a)).

(a) Where a loading combination produces significant relative lateral displacements of the column ends, the sway effects shall include the effect of the vertical loads acting on the displaced structure but need not include the sway effects produced by initial column out-of-plumbs.

(b) However, in a steel frame the sway effects shall not be less than those produced by the vertical loads acting on the structure assumed displaced an amount equal to $0.006/\sqrt[2.2]{n}$, where n is the total number of columns in the structure (see section J-3 of Appendix J).

(c) In mixed construction, composed of steel columns and cast-in situ reinforced concrete walls arranged orthogonally, the sway effects shall not be less than those produced by the vertical loads acting on

(i) the columns assumed out-of-plumb by the amount given in clause 8.6.2(b) and

(ii) the walls assumed out-of-plumb by an amount equal to $0.0003 + 0.01/\sqrt[2.2]{n}$, where n is the total number of one-storey walls in the structure.

The storey forces obtained in this manner shall be combined according to $\sqrt{H_c^2 + H_w^2}$ and applied to the structure as in Appendix J. In this expression H_c and H_w are the forces obtained from the column and wall out-of-plumbs respectively.

8.6.3 For structures in which the sway effects have not been included in the analysis, the use of effective length factors greater than 1.0 (sidesway permitted case) for the design of columns, provides an approximate method of accounting for the sway effects in moment resisting frames (see clause 9.3.3). This provision shall not be used for structures analyzed in accordance with Clause 8.5.

19. Stability of Structures and Individual Members

19.1 General

19.1.1 In the design of a steel structure, care shall be taken to ensure that the structural system is adequate to resist the forces caused by the factored loads and to ensure that a complete structural system is provided to transfer the factored loads to the foundations, particularly when there is a dependence on walls, floors, and roofs acting as shear resisting elements or diaphragms. (See also Clause 8.6).

19.1.2 Design drawings shall indicate all load resisting elements essential to the integrity of the completed structure and shall show details necessary to ensure the effectiveness of the load resisting system. Design drawings shall also indicate the requirements for roofs and floors used as diaphragms.

19.1.3 Erection drawings shall indicate all load resisting elements essential to the integrity of the completed structure. Permanent and temporary load resisting elements essential to the integrity of the partially completed structure shall be clearly specified on the erection drawings.

19.1.4 Where the portion of the structure under consideration does not provide adequate resistance to applied lateral forces and other destabilizing forces, provision shall be made for transferring the forces to adjacent lateral load-resisting elements.

(a) Beam-to-column connections and floor diaphragms shall be designed to resist horizontal forces due to column out-of-plumbs given by

$$F_d = 0.006 \sqrt{\sum_{j=1}^{n} P_j^2}$$

where n = number of participating columns above and below floor level.

P_j = factored axial loads in the individual columns.

(b) Individual sections of floor diaphragms shall also be designed for in-plane moments given by

$$M_d = 0.006 \sqrt{\sum_{j=1}^{n} [P^2(L_x^2 + L_y^2)]_j}$$

where n and P are defined as in clause 19.1.4(a) and L_x and L_y are lever arms, taken in two orthogonal directions, from the column to the point at which the moment is calculated.

19.1.5 The structure shall be analysed to ensure that adequate resistance to torsional deformations has been provided. As a minimum, the bracing system in a structure shall be capable of resisting a torsional moment, T_c, at each storey, given by the expression of Clause 19.1.4(b), with the summation applied to the total number of columns in the storey. The torsional moment is calculated with respect to the center of resistance of the structure and shall account for the effects of the gravity loads acting on the out-of-plumbs of the columns.

(a) A steel structure shall be designed for the above torsional moment.

(b) Mixed construction, composed of steel columns and cast-in situ reinforced concrete walls arranged orthogonally shall be able to resist a torsional moment at a specific storey given by:

(i) the expression defined in Clause 19.1.5(a) as applied to the columns and,

(ii) the expression below as applied to the walls:

$$T_w = 0.0015 \sqrt{\sum_{j=1}^{n} (PL)^2_j}$$

where n = total number of walls in the storey

P = factored axial load in each individual wall

L = length of the wall.

The torques obtained in (b) shall be combined according to $\sqrt{T_c^2 + T_w^2}$ at each storey.

19.2 Stability of Columns

19.2.1 Beam-to-column connections shall have adequate strength to transfer the applied forces, the sway forces (see Appendix J), plus the forces calculated as follows:

(a) In a simple braced frame, the forces described in Clause 19.1.4(a).

(b) In a continuous frame, 0.85 percent of the largest factored axial load in the two columns above and below the floor at a specific connection.

These forces shall be computed for the loading cases of Clause 7.2.4 using the appropriate load combination factors.

APPENDIX J - Guide To Calculation Of Stability Effects

J.1 This guide provides one approach to the calculation of the additional bending moments and forces generated by the vertical loads acting through the deflected shape of the structure. By this approach, the above moments and forces are incorporated into the results of the analysis of the structure. However, due to the approximate nature of the method, the horizontal forces to be used in the design of floor diaphragms and beam-to-column connections are inexact but can be easily corrected when required.

Alternatively, a second order analysis, which formulates equilibrium on the deformed structure, may be used to include the stability effects.

J.2 Combined Loading Case

Step 1 - Apply the factored load combination to the structure (Clause 7.2.2).

Step 2 - Compute the lateral deflections at each floor level (Δ_i) by first order elastic analysis.

Step 3 - Compute the artificial storey shears V_i' due to the sway forces.

$$V_i' = \frac{\Sigma P_i}{h_i} (\Delta_{i+1} - \Delta_i)$$

Step 4 - Compute the artificial lateral loads H_i'.

$$H_i' = V_{i-1}' - V_i'$$

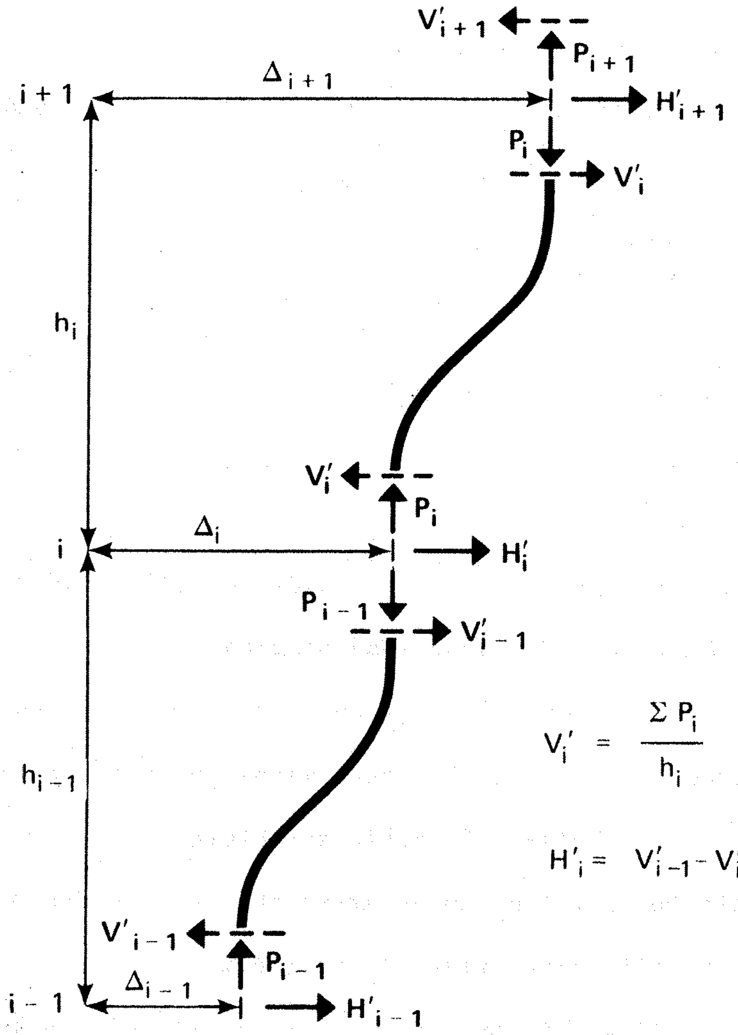
Step 5 - Repeat Step 1 applying the artificial lateral loads H_i' in addition to the factored load combination.

Step 6 - Repeat Steps 2 through 5 until satisfactory convergence is achieved. Lack of convergence within 5 cycles may indicate an excessively flexible structure. In no case shall the building sway exceed the recommended maximum values for deflections given in Appendix I.

Convergence can be achieved in one cycle by using the expression given below instead of the equation given in step 3 in the calculation of the artificial storey shears.

$$V_i' = \frac{1}{\dfrac{1}{\dfrac{\Sigma P_i(\Delta_{i+1} - \Delta_i)}{h_i}} - \dfrac{1}{\Sigma V_i}}$$

where ΣV_i = total first order shear at storey i; the other terms are defined in Figure 11.1.



$$V_i' = \frac{\Sigma P_i}{h_i} (\Delta_{i+1} - \Delta_i)$$

$$H'_i = V'_{i-1} - V'_i$$

where:

V'_i = Artificial shear at storey i due to sway forces,

ΣP_i = Sum of the column axial loads at storey i,

h_i = Height of storey i, and

Δ_{i+1}, Δ_i = Displacements of levels i + 1 and i respectively

Figure 11.1 Sway forces due to vertical loads

J.3 Vertical Loads Only

Since vertical loads do not normally produce significant sway deflections of the structure, the initial sway forces are computed on the basis of the sway displacements in each storey produced by initial column out-of-plumbs.

$$\Delta_i = \frac{0.006\ h_i}{2.2\sqrt{n}}$$

where, Δ_i = net sway displacement at storey i (equivalent to $\Delta_{i+1} - \Delta_i$).

h_i = height of storey i

n = total number of columns in the building.

Using these deflections, the calculations are started at step 3 of the procedure described in J.2.

J.4 Horizontal Force Distribution

The procedure described in section J.2 produces horizontal forces slightly in error. The correct forces should be evaluated at specific storeys when the artificial storey shears, V_i', are significant compared to the applied lateral loads at these storeys. The individual column shears are calculated from equilibrium of each column with the moments, axial loads, and lateral deflections obtained from the second order analysis. The correctly distributed horizontal forces are then determined by equilibrium of these shears and applied forces at floor levels. In all other cases, the horizontal forces given by a first order analysis should be used.

12.2.2 Proposed Clauses for Concrete Buildings

(a) A structure composed exclusively of load bearing cast-in situ concrete walls should be designed for an extra sway produced by the vertical loads acting on the walls assumed out-of-plumb an amount equal to $0.0003 + 0.01/\sqrt[2.2]{n}$, where n is the total number of one-storey walls in the structure.

(b) A structure composed of load bearing cast-in situ concrete walls arranged orthogonally should be able to resist a torsional moment at a specific storey given by:

$$T_w = 0.0015 \sqrt{\sum_{j=1}^{n} (PL)^2_j}$$

where, n = total number of walls in the storey

P = factored axial load in each individual wall

L = length of the wall

LIST OF REFERENCES

1. Galambos, T.V., "Structural Members and Frames", Prentice Hall Inc., Englewood Cliffs, N.J., 1968.

2. Column Research Council, "Guide to Stability Design Criteria for Metal Structures", 3rd Edition, B.G. Johnston Editor, John Wiley and Sons, Inc., New York, 1976.

3. McGuire, W., "Steel Structures", Prentice-Hall Inc., Englewood Cliffs, N.J., 1968.

4. Timoshenko, S.P. and Gere, J.M., "Theory of Elastic Stability", 2nd Edition, McGraw-Hill Book Co., Inc., 1961.

5. Driscoll, G.C. et al., "Plastic Design of Multi-Storey Frames", Fritz Engineering Laboratory Report No. 273.20, Lehigh University, 1965.

6. Davison, J.H., and Adams, P.F., "Stability of Braced and Unbraced Frames", Journal of the Structural Division, American Society of Civil Engineers, Vol. 100, No. ST2, Feb. 1974.

7. Canadian Standards Association, Standard S16-1969, "Steel Structures for Buildings", Canadian Standards Association, Rexdale, Ontario, 1969.

8. Majumdar, S.N.G., Nikhed, R.P., MacGregor, J.G., and Adams, P.F., "Approximate Inelastic Analysis of Shear-Wall Frame Structures", Proceedings of the 8th Congress of the International Association for Bridge and Structural Engineering, Sept. 1968.

9. Khan, F.R. and Sbarounis, J.A., "Interaction of Shear Walls and Frames", Journal of the Structural Division, American Society of Civil Engineers, Volume 90, No. ST3, June 1964.

10. Fintel, M. et al, "Response of Buildings to Lateral Forces", Reported by the American Concrete Institute Committee 442, American Concrete Institute Journal, Feb. 1971.

11. Springfield, J. and Adams, P.F., "Aspect of Column Design in Tall Steel Buildings", Journal of the Structural Division, American Society of Civil Engineers, Vol. 98, No. ST5, May 1972, pp. 1069-1083.

12. Rosman, R., "The Continuum Analysis of Shear-Wall Structures", Discussion 4, Technical Committee 21, American Society of Civil Engineers - International Association for Bridge and Structural Engineering, International Conference on Tall Buildings, August, 1972, Proceedings: Vol. III-21.

13. Clough, R.W., King, I.P. and Wilson, E.L., "Structural Analysis of Multi-Storey Buildings", Journal of the Structural Division, American Society of Civil Engineers, Vol. 90, No. ST3, June 1964.

14. Logcher, R.D. et al, "ICES STRUDL II, Engineering User's Manual", Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, June 1969.

15. Schwaighofer, J. and Microys, H.F., "Analysis of Shear-Walls Using Standard Computer Programs", American Concrete Institute Journal, Vol. 66, Dec. 1969, p. 1005.

16. Stafford Smith, B., "Modified Beam Method for Analyzing Symmetrical Interconnected Shear Walls", American Concrete Institute Journal, Vol. 67, Dec. 1970, p. 977.

17. Rosman, R., "Torsion of Perforated Concrete Shafts", Journal of the Structural Division, American Society of Civil Engineers, Vol. 95, No. ST5, May 1969, pp. 991-1010.

18. Tso, W.K. and Biswas, J.K., "General Analysis of Non-Planar Coupled Shear-Walls", Journal of the Structural Division, American Society of Civil Engineers, Vol. 99, No. ST3, March 1973, pp. 365-380.

19. Taranath, B.S. and Stafford Smith, B., "Torsion Analysis of Shear Core Structures", Analysis of Structural Systems for Torsion, American Concrete Institute Journal, SP-35, 1973, pp. 239-263.

20. American Society of Civil Engineers - International Association for Bridge and Structural Engineering, Monograph on Planning and Design of Tall Buildings (First Draft), Chapter A41, Mixed Construction, H. Iyengar (Ed.), 1976.

21. Stamato, M.C., "Three-Dimensional Analysis of Tall Buildings", State-of-Art 4, Technical Committee 24, American Society of Civil Engineers - International Association for Bridge and Structural Engineering, International Conference on Tall Buildings, August, 1972, Proceedings: Vol. III-24.

22. Manning, T.A. and Weaver, W., "The Analysis of Tier Buildings with Shear-Walls", Technical Report No. 128, Department of Civil Engineering, Stanford University, April 1970.

23. Adams, P.F., "The Design of Steel Beam-Columns", A Publication of the Canadian Steel Industries Construction Council, Willowdale, Ontario, 1974.

24. Rosenblueth, E., "Slenderness Effects in Buildings", Journal of the Structural Division, American Society of Civil Engineers, Vol. 91, No. ST1, Jan. 1965, pp. 229-252.

25. Goel, S.C., "P-Δ and Axial Column Deformation in Aseismic Frames", Journal of the Structural Division, American Society of Civil Engineers, Vol. 95, No. ST8, Aug. 1969, pp. 1163-1711.

26. Wood, B., Beaulieu, D., and Adams, P.F.,"Column Design by the P-Δ Method", Journal of the Structural Division, American Society of Civil Engineers, Vol. 102, No. ST2, Feb. 1976.

27. Wood, B., Beaulieu, D., and Adams, P.F., "Further Aspects of Column Design by the P-Δ Method", Journal of the Structural Division, American Society of Civil Engineers, Vol. 102, No. ST3, March 1976.

28. MacGregor, J.G., "Stability of Reinforced Concrete Building Frames", State-of-Art 1, Technical Committee 23, American Society of Civil Engineers - International Association for Bridge and Structural Engineering, International Conference on Tall Buildings, August, 1972, Proceedings: Vol. III-23.

29. Nixon, D., Beaulieu, D., and Adams, P.F., "Simplified Second Order Frame Analysis", Canadian Journal of Civil Engineering, Vol. 2, No. 4, Dec. 1975.

30. Adams, P.F., "Stability of Asymmetric Structures - Research Needs", Journal of the Structural Division, American Society of Civil Engineers, Vol. 100, No. ST11, Nov. 1974.

31. Adams, P.F., "Stability of Three-Dimensional Building Frames", State-of-Art 4, Technical Committee 16, American Society of Civil Engineers - International Association for Bridge and Structural Engineering, International Conference on Tall Buildings, August, 1972, Proceedings: Vol. II-16.

32. Beck, H., and Konig, G., "Characteristics Relating to the Torsional Stiffness of Multi-Storey Buildings", Beton- und Stahlbetonbau, Dec. 1968.

33. American Institute of Steel Construction, "Specifications for the Design, Fabrication and Erection of Structural Steel for Buildings", The American Institute of Steel Construction, New York, 1969.

34. Iffland, J.S.B., "Design for Stability in High Rise Buildings", State-of-Art 2, Technical Committee 16, American Society of Civil Engineers - International Association for Bridge and Structural Engineering, International Conference on Tall Buildings, August, 1972, Proceedings: Vol. II-16.

35. Beck, H., "Stability in Design of Tall Concrete Buildings", Theme Report, Technical Committee 23, American Society of Civil Engineers - International Association for Bridge and Structural Engineering, International Conference on Tall Buildings, August, 1972, Proceedings: Vol. III-23.

36. Canadian Standards Association, Standard S16.1-1974, "Steel Structures for Buildings - Limit States Design", Canadian Standards Association, Rexdale, Ontario, 1974.

37. Fiorato, A.E., "Geometric Imperfections in Concrete Structures - A Literature Survey", National Swedish Building Research Summaries, Document D5:1973, Ill. 32 Sw.Kr., Stockholm, 219 p.

38. Latta, J.K., "Inaccuracies in Construction", Canadian Building Digest, Division of Building Research, National Research Council of Canada, April 1975, 4 p.

39. Berg, J. Van den, "The Dimensional Accuracy of the Building Councils New Building Systems", (In Swedish), Report R18:1971, Statens Institut för byggnadsforskning, Stockholm, 1971.

40. European Convention for Structional Steelwork (ECCS), "European Recommendations for Steel Constructions", Vol. II, Recommendations, 1976.

41. Committee 8 of the European Convention for Constructional Steelwork (ECCS), "Manual on the Stability of Steel Structures", July 1976.

42. Massonnet, C., "Perspective of Steel Design in Europe", Proceedings of the Canadian Structural Engineering Conference, Montreal, 1976, 41 p.

43. Column Research Council, "Guide to Design Criteria for Metal Compression Members", 2nd Ed., B.G. Johnston, Editor, John Wiley and Sons, New York, 1966, Section 2.10.

44. Winter, G., "Lateral Bracing of Columns and Beams", Transactions, American Society of Civil Engineers, Vol. 125, 1960.

45. Fey, T., "Approximate Second-Order Analysis of Reinforced Concrete Frames", (In German), Bauingenieur, June 1966.

46. Lorentsen, M., "Consideration of Imperfections in the Design of Prefabricated Buildings", State-of-Art 6, Technical Committee 23, American Society of Civil Engineers - International Association for Bridge and Structural Engineering, International Conference on Tall Buildings, August, 1972, Proceedings: Vol. III-23, pp. 87-105.

47. DIN 1045; DIN 4224 German Codes, "Design of Reinforced Concrete Structures".

48. Sno, V.E., "Determination of the Loads on Floor Slabs Caused by Imperfections in the Assembly of Concrete Elements", (In Russian), Stroitelnaja mechanika i rascet sooruzenij, Vol. 11, No. 6, pp. 62-65.

49. Mazin, M.R., "Accuracy in Erection of Reinforced Concrete Columns in Industrial Buildings", Translated from Russian by P.A. Ross, Ministry of Technology Building Research Station, Library Communication No. 1386, Garston, Watford, April 1967.

50. Chandzi, V.V., "Determination of the Horizontal Loads on the Floor Slabs of Skeletal Buildings", (in Russian) Stroitelnaja mechanika i rascet sooruzenij, Vol. 9, No. 4, pp. 31-33.

51. Statens Planverk, "Regulations for Prefabricated Reinforced Concrete Columns" (In Swedish), SBN-S25:21 Publ. No. 25, Stockholm, 1970.

52. Lewicki, B., "Building with Large Prefabricates", Elsevier Publishing Co., Amsterdam, 1966, p. 128.

53. Klingberg, L., "How to Ensure Accuracy in Assembly", In Tolerances and Accuracy in Building, SBI-Report 81, Danish Building Research Institute, Copenhagen, pp. 94-99.

54. Hahn, G.J. and Shapiro, S.S., "Statistical Models in Engineering", John Wiley and Sons, Inc., New York, 1967.

55. Benjamin, J.R. and Cornell, C.A., "Probability, Statistics and Decision for Civil Engineers", McGraw Hill Book Company Inc., New York, 1970.

56. Ahlgren, B.E. and Ringvall, S., "Measurements of the Accuracy of Assembly of Prefabricated Concrete Elements", (In Swedish), Thesis 151, Institutionen for Brobyggnast, Kungliga Tekniska Högskolan, Stockholm, 1969.

57. Haris, A.A.K., "Minimum Eccentricity Requirements in the Design of Reinforced Concrete Columns", Ph.D. Thesis, Civil Engineering Department, University of Texas, Austin, University Microfilms, Ann Arbor, Michigan, 1972.

58. MacGregor, J.G., "Safety and Limit States Design for Reinforced Concrete", Canadian Journal of Civil Engineering, Vol. 3, No. 4, Dec. 1974, pp. 484-513.

59. Allen, D., "Limit States Design - A Probabilistic Study", Canadian Journal of Civil Engineering, Vol. 2, No. 1, March 1975, pp. 36-49.

60. National Building Code of Canada, 6th Ed., NRCC No. 13982, National Research Council of Canada, Ottawa, Canada, 1975.

61. Galambos, T.V., and Ravindra, M.K., "Load and Resistance Factor Design Criteria for Steel Buildings", Research Report No. 18, Structural Division, Civil and Environmental Engineering Department, Washington University, St. Louis, Mo., June 1973.

62. Mitchell, G.R., and Woodgate, R.W., "A Survey of Floor Loadings in Office Buildings", Construction Industry Research and Association, Report No. 25, London, Aug. 1970.

63. Peir, J.C., and Cornell, C.A., "Spatial and Temporal Variability of Live Loads", Journal of the Structural Division, American Society of Civil Engineers, Vol. 99, No. ST5, May 1973, pp. 903-922.

64. McGuire, R.K., and Cornell, C.A., "Live Load Effects in Office Buildings", Journal of the Structural Division, American Society of Civil Engineers, Vol. 100, No. ST7, July 1974, pp. 1351-1366.

APPENDIX A

PROBABILISTIC AND STATISTICAL CONCEPTS

The following appendix presents a summary of the probabilistic and statistical concepts introduced in the report. Although there are many excellent texts available to fulfill this purpose, it is thought that a condensed summary of the essential concepts will greatly facilitate comprehension of the material. For more detailed discussion with illustrations from civil engineering practice, Refs. 54 and 55 are recommended.

A-1 Probability

If an experiment is conducted N times, and a particular attribute A occurs n times, then the limit of n/N as N becomes large is defined as the probability of the event A, denoted Pr(A).

However, a more general definition is needed to cover the case in which an estimate of the outcome of an event is principally intuitive. In this case: "The probability Pr(A) is a measure of the degree of belief held in a specified proposition A". This interpretation of probability is a broader concept and includes the first definition.

A-2 Probability Rules

1. If Pr(A) and Pr($\overline{A}$) represent respectively the probabilities of the event A occurring and not occurring, then

$$Pr(\overline{A}) = 1 - Pr(A) \qquad (A\text{-}1)$$

2. If A and B are two independent events, then the probability that both A and B will happen, known as the "joint probability" denoted by "and", is the product of the respective individual probabilities - that is,

$$Pr(A \text{ and } B) = Pr(AB) = Pr(A)\,Pr(B) \qquad (A\text{-}2)$$

3. If A and B are two mutually exclusive events - that is Pr(AB) = 0 - then the probability denoted by "or" that one of these two events will take place is given by the sum of their individual probabilities:

$$Pr(A \text{ or } B) = Pr(A+B) = Pr(A) + Pr(B) \qquad (A\text{-}3)$$

4. The probability of an event A is a number greater than or equal to zero but less than or equal to unity. The probability of a certain (absolute) event B is unity.

$$0 \leq Pr(A) \leq 1 \qquad (A\text{-}4)$$

$$Pr(B) = 1 \qquad (A\text{-}5)$$

A-3 Random Variables

A random variable is a function defined on a sample space. For example, in the toss of two dice, the sample space consists of the

36 possible pairs of outcomes. The sum or the average or even the square of the summed value for each pair of tosses is a random variable, because it is a function defined for every point in the sample space.

A sample space involving either a finite number or a countable infinity of elements is said to be "discrete". A discrete random variable is one that can take on only a countable number of values. A second type of random variable is a "continuous" variable. A continuous random variable may take on any value in one or more intervals and results from measured, rather than counted data.

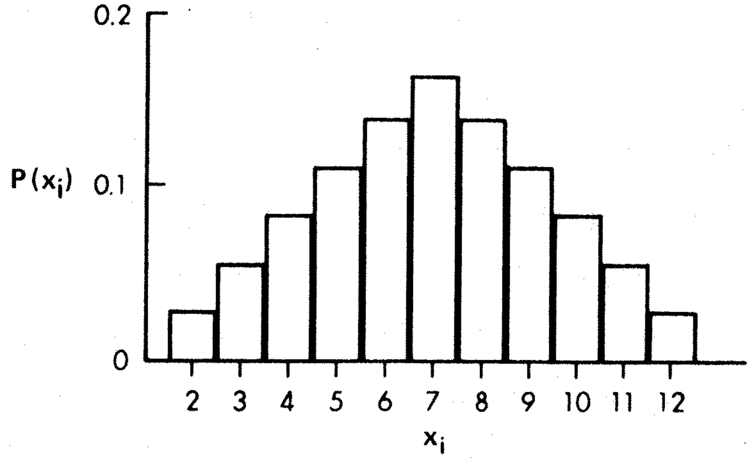
A-4 Probability Function and Cumulative Distribution

A-4.1 Discrete Random Variable

In the two dice example, the probability of each value of the random variable x representing the sum of the results of the two tosses is obtained by adding the probabilities of appropriate points in the sample space. Because each of the 36 points is equally likely and their total probability must add to 1, each point has associated with it a probability of 1/36. The "probability function", $P(x_i)$, obtained is sketched in Fig. A-1(a).

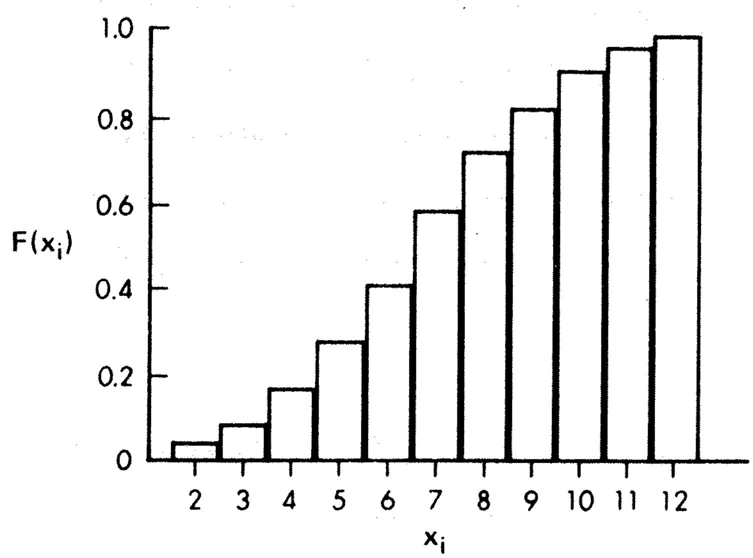
The function $F(x_i)$ plotted in Fig. A-1(b) gives the probability of obtaining a value smaller than or equal to some value x_i of the discrete random variable x and is known as the "cumulative distribution function" of that random variable. $F(x_i)$ can be obtained by summing the values of the probability function over those points in the sample space for which the random variable takes on a value less than or equal to x_i - that is,

x_i	2	3	4	5	6	7	8	9	10	11	12
$P(x_i)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36



a) Probability function

x_i	< 2	2	3	4	5	6	7	8	9	10	11	≥ 12
$F(x_i)$	0	1/36	3/36	6/36	10/36	15/36	21/36	26/36	30/36	33/36	35/36	1



b) Cumulative distribution function

Figure A.1 Statistical distribution for sum of values in tossing two dice

$$Pr(x \leq x_i) = F(x_i) = \sum_{x \leq x_i} P(x_i) \tag{A-6}$$

Clearly,

$$0 \leq F(x_i) \leq 1 \qquad \text{for all } x_i \tag{A-7}$$

$$F(x_i) \geq F(x_j) \qquad \text{for } x_i \geq x_j \tag{A-8}$$

The complement of the distribution function gives the probability that the random variable exceeds a specified value - that is,

$$P_r(x > x_i) = 1 - F(x_i) \tag{A-9}$$

Also,

$$\Sigma\, P(x_i) = 1 \tag{A-10}$$

A-4.2 Continuous Random Variable

The case of a continuous random variable is treated in a manner similar to the discrete variable. Here, the distributions are represented by smooth continuous curves and the discrete summations are replaced by integrations.

If $F(x_i)$ is the cumulative distribution of a continuous random variable x, then

$$\begin{aligned} \lim_{x_i \to -\infty} F(x_i) &= F(-\infty) = 0 \\ \lim_{x_i \to \infty} F(x_i) &= F(\infty) = 1 \end{aligned} \tag{A-11}$$

For a discrete random variable the probability function $P(x_i)$ was defined as the probability associated with the value x_i. Such a direct definition is clearly no longer meaningful for a continuous random variable. Instead, the definition of the cumulative distribution function is used to define the "probability density function" f(x) of a continuous variate x as follows:

$$f(x) = \lim_{\Delta_x \to 0} \frac{Pr(x_i \leq x \leq x_i + \Delta_x)}{\Delta_x} = \frac{d}{dx}[F(x)] \qquad \text{(A-12)}$$

Probability for a continuous random variable may thus be interpreted in terms of relative area under the curve defined by the probability density function. As an example, different probabilities are represented by the shaded areas on Fig. A-2 for a continuous random variable x with probability density function f(x).

A-5 Expected Value or Mean

The best known measure of central tendency is the "expected value", more frequently called the "arithmetic mean", or sometimes "the mean".

When the mathematical form of the distribution is known, the expected value is defined as

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx \qquad x = \text{continuous random variable}$$

$$E(x) = \sum_i x_i p(x_i) \qquad x = \text{discrete random variable} \qquad \text{(A-13)}$$

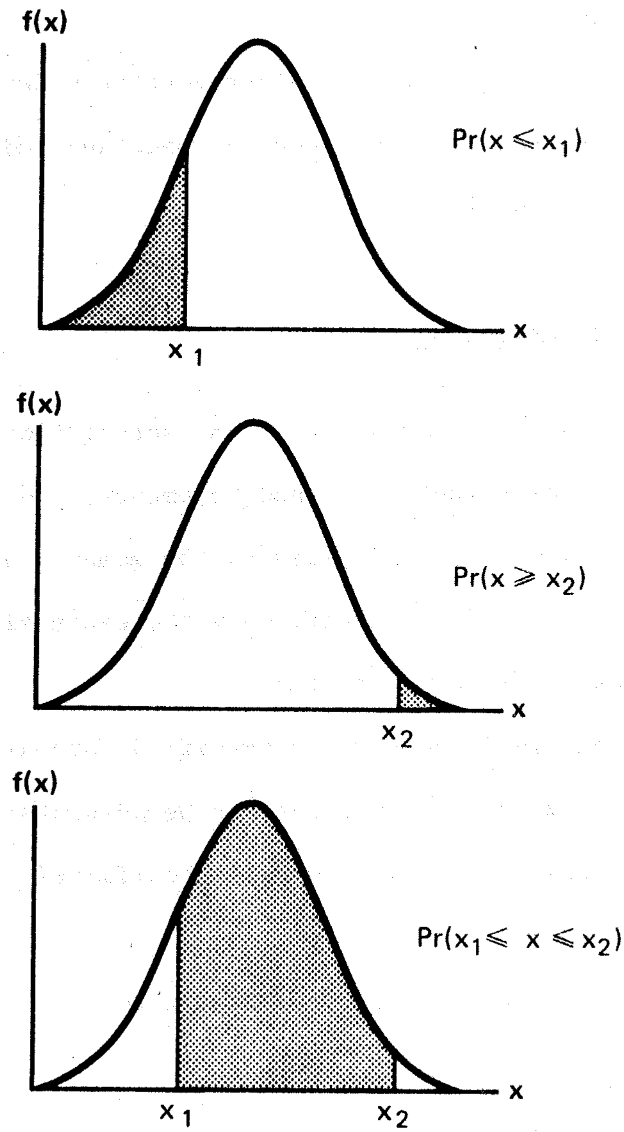


Figure A.2 Probability density function, f(x)

The mean is more frequently estimated from the values of n observations. The "data mean", denoted by $\bar{x}$ or μ, is calculated as

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{A-14}$$

where x_i, i = 1, 2, ..., n, are the values for the n data points. Other measures of central tendency, such as mode and median are described in Refs. 54 and 55.

A-6 Moments of a Distribution

In addition to the mean, other characteristics are frequently used to describe the distribution spread, symmetry, and peakedness. These characteristics may be summarized by the moments of the distribution. For the purpose of simplicity only the expressions defining the moments from data will be presented.

A distribution is completely specified once all its moments are known. However, many distributions can be adequately described by the first four moments, and discussion will be limited to these moments.

The first central moment is always zero

$$m_1 = 0 \tag{A-15}$$

and is the difference between the mean and itself.

A-6.1 Variance and Standard Deviation

The second moment about the mean is a measure of dispersion. It is known as the "variance" m_2, var(x) or σ_x^2.

$$\sigma_x^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2 \qquad \text{(A-16)}$$

Eq. (A-16) leads to what statisticians call a "biased estimate". The corresponding unbiased formula is

$$s^2 = \frac{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)} \qquad \text{(A-17)}$$

Eq. (A-17) is generally used instead of (A-16) as an estimate of the variance where only a small number of observations is available.

The square root of the variance is known as the "standard deviation" and is denoted by the symbol σ_x. A non-dimensional characteristic called "coefficient of variation" is of special importance and is defined as,

$$v = \frac{\sigma_x}{\bar{x}} \qquad \text{(A-18)}$$

A-6.2 Skewness

The third moment about the mean is related to the asymmetry or "skewness" of a distribution.

$$m_3 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^3}{n}$$

$$= \frac{\sum_{i=1}^{n} x_i^3}{n} - 3\,\frac{\sum_{i=1}^{n} x_i^2}{n}\,\frac{\sum_{i=1}^{n} x_i}{n} + 2\left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^3 \qquad \text{(A-19)}$$

The third moment is generally standardized in order to compare the symmetry of two distributions where the scales of measurement differ.

$$\alpha_3 = \frac{m_3}{\sigma^3} \tag{A-20}$$

A single peaked distribution with $\alpha_3 < 0$ is said to be skewed to the left, that is, it has a left "tail" as shown in Fig. A-3(a). If $\alpha_3 > 0$, the distribution is skewed to the right. For symmetric distribution, $\alpha_3 = 0$.

A-6.3 Kurtosis

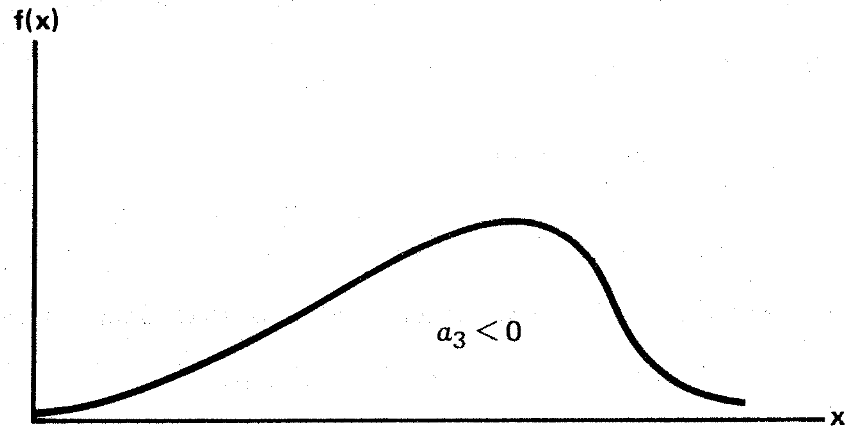
The fourth moment about the mean is related to the peakedness, also called "kurtosis" of the distribution, and is defined as

$$\begin{aligned} m_4 &= \frac{\sum_{i=1}^{n} (x_i - \bar{x})^4}{n} \\ &= \frac{\sum_{i=1}^{n} x_i^4}{n} - 4\,\frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i^3}{n^2} + 6\left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^2 \frac{\sum_{i=1}^{n} x_i^2}{n} - \\ &\quad - 3\left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^4 \end{aligned} \tag{A-21}$$

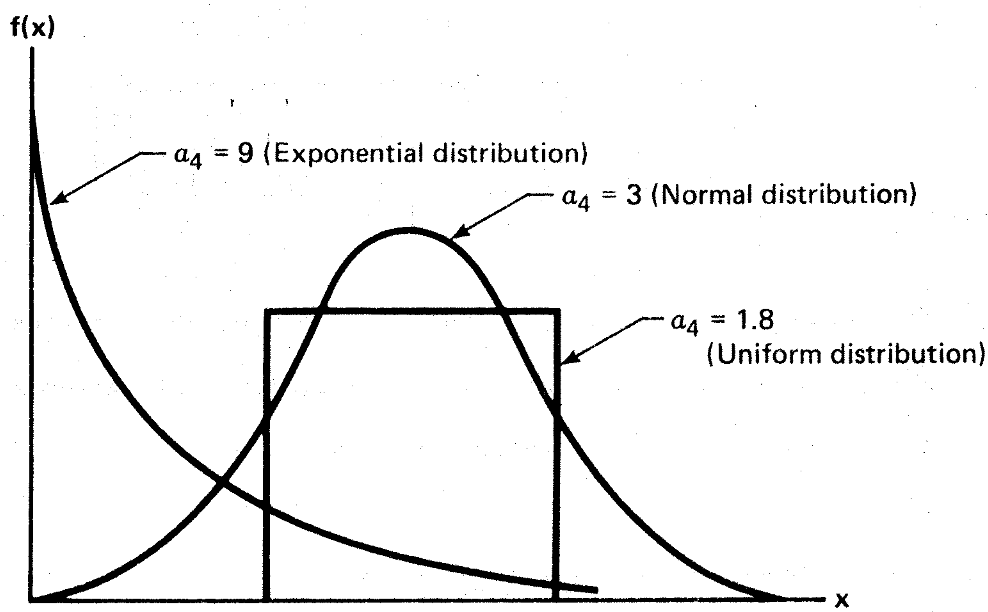
The quantity

$$\alpha_4 = \frac{m_4}{\sigma^4} \tag{A-22}$$

is a relative measure of kurtosis. As shown in Fig. A-3(b), α_4 is 1.8 for a uniform distribution, 3.0 for a perfectly normal distribution, and 9.0 for an exponential distribution.



(a) Distribution skewed to the left



(b) Relative measure of kurtosis

Figure A.3 Moments of a distribution

A-7 Covariance and Coefficient of Correlation

The joint behavior of two random variables x and y is usually summarized by the "covariance", $\sigma_{x,y}$.

$$\sigma_{x,y} = E(xy) - E(x)E(y) \qquad \text{(A-23)}$$

If x and y are independent, $\sigma_{x,y} = 0$.

The standardized measure of the linear relationship between two variates is the "coefficient of correlation", ρ,

$$\rho = \frac{\sigma_{x,y}}{\sigma_x \sigma_y}$$

$$= \frac{\sum_{i=1}^{n} x_i y_i - \frac{\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n}}{\sqrt{\left[\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right]\left[\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}\right]}} \qquad \text{(A-24)}$$

ρ lies between and includes -1 and 1. If $\rho = 0$, the variates are said to be uncorrelated. The correlation coefficient only gives a measure of the linear relationship between two variables.

A-8 First Order Probabilistic Approach

In a first order probabilistic approach, the first two moments are used to characterize a random variable. The mean, standard deviation, and correlation coefficient concisely describe the best predictions, the uncertainty, and the joint behavior of the variables.

A-9 Moment Algebra

Properties of Expectation

$$E(c) = c \tag{A-25}$$

$$E(cx) = \overline{cx} \tag{A-26}$$

where c is a deterministic constant.

$$\text{var}\,(c) = 0 \tag{A-27}$$

$$\text{var}\,(cx) = c^2\,\sigma_x^2 \tag{A-28}$$

Sum of Random Variables

$$\text{Let } z = x + y \tag{A-29}$$

$$\text{then } \bar{z} = \bar{x} + \bar{y} \tag{A-30}$$

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 + 2\sigma_{x,y} \tag{A-31}$$

If x and y are uncorrelated

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 \tag{A-32}$$

Difference of Random Variables

$$\text{Let } z = x - y \tag{A-33}$$

$$\text{Then } \bar{z} = \bar{x} - \bar{y} \tag{A-34}$$

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 - 2\sigma_{x,y} \tag{A-35}$$

If x and y are uncorrelated

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 \tag{A-36}$$

If x and y are uncorrelated, whether z is the sum or the difference of x and y, the variances σ_x^2 and σ_y^2 always add to give σ_z^2.

Product of Random Variables

$$\text{Let } z = xy \tag{A-37}$$

$$\text{Then } \bar{z} = \bar{x}\,\bar{y} + \sigma_{x,y} \tag{A-38}$$

If x and y are uncorrelated

$$\bar{z} = \bar{x}\,\bar{y} \tag{A-39}$$

$$\sigma_z^2 = \bar{x}^2\sigma_y^2 + \bar{y}^2\sigma_x^2 + \sigma_x^2\sigma_y^2 \tag{A-40}$$

which is simplified to

$$v_z^2 = v_x^2 + v_y^2 + v_x^2 v_y^2 \tag{A-41}$$

A-10 Normal Distribution

The "normal (or Gaussian) distribution" is the most widely used model in applied probability theory. Its probability density function as shown in Fig. A-4(a) is,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] \tag{A-42}$$

$$-\infty < x < \infty, \; -\infty < \mu < \infty, \; \sigma > 0$$

The mean, μ, and the variance, σ^2, of the normal distribution are estimated by Eqs. (A-14) and (A-16) respectively. The cumulative normal distribution is

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(z-\mu)^2}{2\sigma^2}\right] dz \tag{A-43}$$

This expression gives the probability of a randomly selected value from a normal distribution. Most text-books provide a table of the cumulative distribution function of a "standardized" normal random variable, y, which is defined as,

$$y = \frac{x-\mu}{\sigma} \tag{A-44}$$

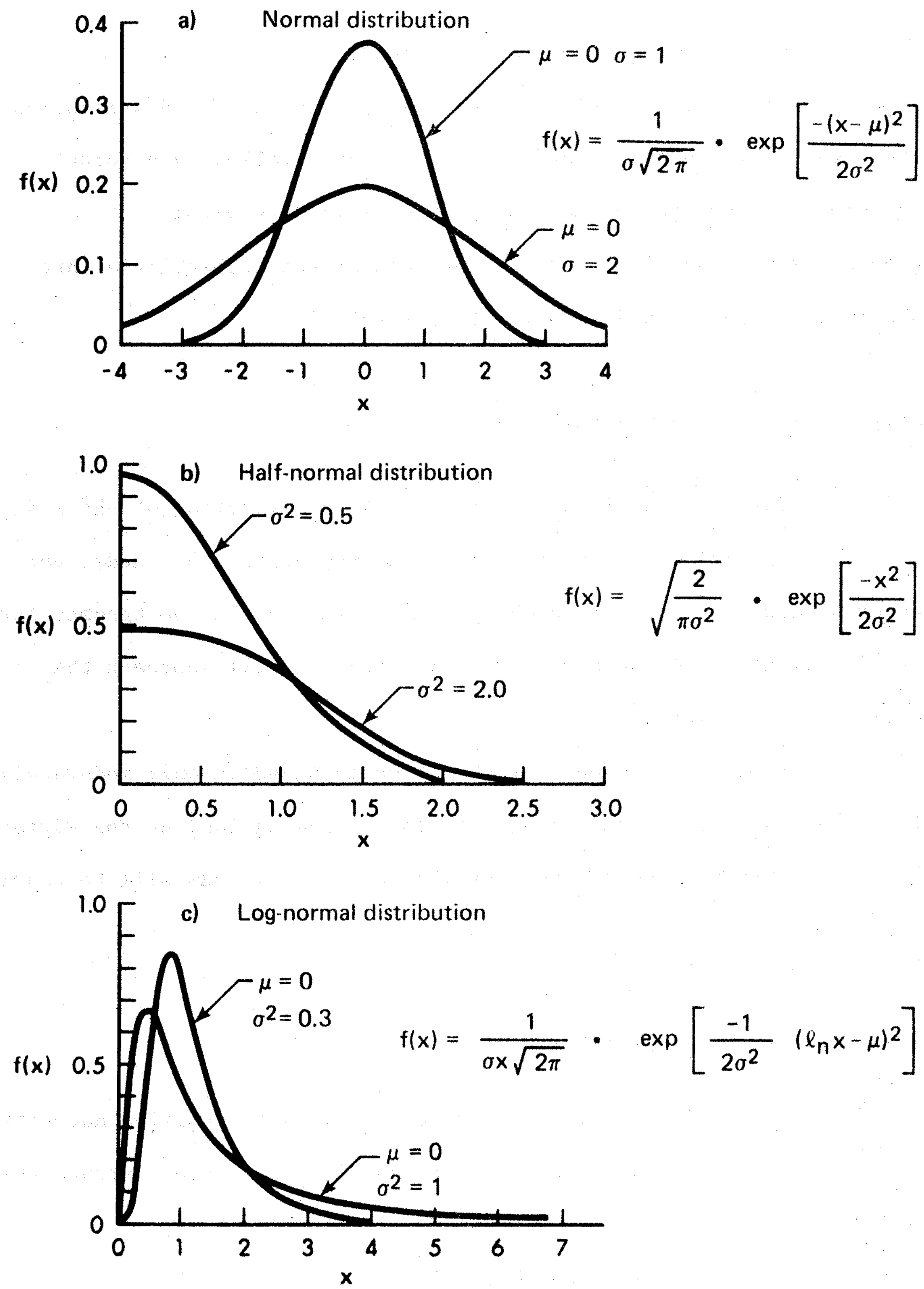


Figure A.4 Continuous statistical distributions

and has a mean 0 and a standard deviation 1.

The cumulative probabilities of the standardized normal distribution are given in Table A-1. For a random variable following a normal distribution, 68.3 percent of the probabilities are within $\pm 1\sigma$ around the mean and 95.5 and 99.7 percent of the probabilities are within the $\mu \pm 2\sigma$ and $\mu \pm 3\sigma$ ranges respectively.

A-11 Central Limit Theorem

The "central limit theorem" is a justification of the wide use of the normal distribution. This theorem states that under very general conditions, as the number of variables in the sum becomes large, the distribution of the sum of random variables will approach the normal distribution.

Even if the number of variables involved is only moderately large, as long as no one variable dominates and as long as the variables are not highly dependent, the distribution of their sum will be nearly normal[55].

A-12 Half-Normal Distribution

The "half-normal distribution" is used to describe normally distributed variates in which only the absolute deviations around the mean are known.

The probability density function is

$$f(x) = \sqrt{\frac{2}{\pi\sigma^2}} \exp\left[-\frac{x^2}{2\sigma^2}\right] \qquad \text{(A-45)}$$

$$F(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{z^2}{2}} \, dz$$

y	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	·9^{2}0097	·9^{2}0358	·9^{2}0613	·9^{2}0863	·9^{2}1106	·9^{2}1344	·9^{2}1576
2·4	·9^{2}1802	·9^{2}2024	·9^{2}2240	·9^{2}2451	·9^{2}2656	·9^{2}2857	·9^{2}3053	·9^{2}3244	·9^{2}3431	·9^{2}3613
2·5	·9^{2}3790	·9^{2}3963	·9^{2}4132	·9^{2}4297	·9^{2}4457	·9^{2}4614	·9^{2}4766	·9^{2}4915	·9^{2}5060	·9^{2}5201
2·6	·9^{2}5339	·9^{2}5473	·9^{2}5604	·9^{2}5731	·9^{2}5855	·9^{2}5975	·9^{2}6093	·9^{2}6207	·9^{2}6319	·9^{2}6427
2·7	·9^{2}6533	·9^{2}6636	·9^{2}6736	·9^{2}6833	·9^{2}6928	·9^{2}7020	·9^{2}7110	·9^{2}7197	·9^{2}7282	·9^{2}7365
2·8	·9^{2}7445	·9^{2}7523	·9^{2}7599	·9^{2}7673	·9^{2}7744	·9^{2}7814	·9^{2}7882	·9^{2}7948	·9^{2}8012	·9^{2}8074
2·9	·9^{2}8134	·9^{2}8193	·9^{2}8250	·9^{2}8305	·9^{2}8359	·9^{2}8411	·9^{2}8462	·9^{2}8511	·9^{2}8559	·9^{2}8605
3·0	·9^{2}8650	·9^{2}8694	·9^{2}8736	·9^{2}8777	·9^{2}8817	·9^{2}8856	·9^{2}8893	·9^{2}8930	·9^{2}8965	·9^{2}8999
3·1	·9^{3}0324	·9^{3}0646	·9^{3}0957	·9^{3}1260	·9^{3}1553	·9^{3}1836	·9^{3}2112	·9^{3}2378	·9^{3}2636	·9^{3}2886
3·2	·9^{3}3129	·9^{3}3363	·9^{3}3590	·9^{3}3810	·9^{3}4024	·9^{3}4230	·9^{3}4429	·9^{3}4623	·9^{3}4810	·9^{3}4991
3·3	·9^{3}5166	·9^{3}5335	·9^{3}5499	·9^{3}5658	·9^{3}5811	·9^{3}5959	·9^{3}6103	·9^{3}6242	·9^{3}6376	·9^{3}6505
3·4	·9^{3}6631	·9^{3}6752	·9^{3}6869	·9^{3}6982	·9^{3}7091	·9^{3}7197	·9^{3}7299	·9^{3}7398	·9^{3}7493	·9^{3}7585
3·5	·9^{3}7674	·9^{3}7759	·9^{3}7842	·9^{3}7922	·9^{3}7999	·9^{3}8074	·9^{3}8146	·9^{3}8215	·9^{3}8282	·9^{3}8347
3·6	·9^{3}8409	·9^{3}8469	·9^{3}8527	·9^{3}8583	·9^{3}8637	·9^{3}8689	·9^{3}8739	·9^{3}8787	·9^{3}8834	·9^{3}8879
3·7	·9^{3}8922	·9^{3}8964	·9^{4}0039	·9^{4}0426	·9^{4}0799	·9^{4}1158	·9^{4}1504	·9^{4}1838	·9^{4}2159	·9^{4}2468
3·8	·9^{4}2765	·9^{4}3052	·9^{4}3327	·9^{4}3593	·9^{4}3848	·9^{4}4094	·9^{4}4331	·9^{4}4558	·9^{4}4777	·9^{4}4988
3·9	·9^{4}5190	·9^{4}5385	·9^{4}5573	·9^{4}5753	·9^{4}5926	·9^{4}6092	·9^{4}6253	·9^{4}6406	·9^{4}6554	·9^{4}6696
4·0	·9^{4}6833	·9^{4}6964	·9^{4}7090	·9^{4}7211	·9^{4}7327	·9^{4}7439	·9^{4}7546	·9^{4}7649	·9^{4}7748	·9^{4}7843
4·1	·9^{4}7934	·9^{4}8022	·9^{4}8106	·9^{4}8186	·9^{4}8263	·9^{4}8338	·9^{4}8409	·9^{4}8477	·9^{4}8542	·9^{4}8605
4·2	·9^{4}8665	·9^{4}8723	·9^{4}8778	·9^{4}8832	·9^{4}8882	·9^{4}8931	·9^{4}8978	·9^{5}0226	·9^{5}0655	·9^{5}1066
4·3	·9^{5}1460	·9^{5}1837	·9^{5}2199	·9^{5}2545	·9^{5}2876	·9^{5}3193	·9^{5}3497	·9^{5}3788	·9^{5}4066	·9^{5}4332
4·4	·9^{5}4587	·9^{5}4831	·9^{5}5065	·9^{5}5288	·9^{5}5502	·9^{5}5706	·9^{5}5902	·9^{5}6089	·9^{5}6268	·9^{5}6439
4·5	·9^{5}6602	·9^{5}6759	·9^{5}6908	·9^{5}7051	·9^{5}7187	·9^{5}7318	·9^{5}7442	·9^{5}7561	·9^{5}7675	·9^{5}7784
4·6	·9^{5}7888	·9^{5}7987	·9^{5}8081	·9^{5}8172	·9^{5}8258	·9^{5}8340	·9^{5}8419	·9^{5}8494	·9^{5}8566	·9^{5}8634
4·7	·9^{5}8699	·9^{5}8761	·9^{5}8821	·9^{5}8877	·9^{5}8931	·9^{5}8983	·9^{6}0320	·9^{6}0789	·9^{6}1235	·9^{6}1661
4·8	·9^{6}2067	·9^{6}2453	·9^{6}2822	·9^{6}3173	·9^{6}3508	·9^{6}3827	·9^{6}4131	·9^{6}4420	·9^{6}4696	·9^{6}4958
4·9	·9^{6}5208	·9^{6}5446	·9^{6}5673	·9^{6}5889	·9^{6}6094	·9^{6}6289	·9^{6}6475	·9^{6}6652	·9^{6}6821	·9^{6}6981

TABLE A-1 STANDARD CUMULATIVE NORMAL DISTRIBUTION where $x \geq 0$ and $\sigma > 0$ is a scale parameter which does not equal the standard deviation of the distribution. A plot of a half-normal distribution is given in Fig. A-4(b).

A-13 Log-Normal Distribution

The "log-normal distribution" is the model for a random variable having a logarithm which follows the normal distribution with parameters μ and σ. Thus the probability density function for x is

$$f(x) = \frac{1}{\sigma x \sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}(\ell n\, x - \mu)^2\right] \qquad \text{(A-46)}$$

where $x > 0$, $-\infty < \mu < \infty$, $\sigma > 0$

The log-normal distribution as shown in Fig. A-4(c) is skewed to the right, the degree of skewness increasing with increasing values of σ. Note that μ and σ are scale and shape parameters respectively and not location and scale parameters as in the normal distribution.

By the central limit theorem, it can be shown that the distribution of the product of n independent positive variates approaches a log-normal distribution.

The cumulative values for $y = \ell n\, x$ can be obtained from the tabulation of the standardized normal distribution and the corresponding values of x are found by taking antilogs.

APPENDIX B

EFFECTS OF NON-DETERMINISTIC GRAVITY LOADS

B-1 Deterministic Gravity Loads

The horizontal force created by the gravity loads acting on the out-of-plumb of one column is,

$$F = P \frac{\Delta_0}{h} \qquad \text{(B-1)}$$

where P is a deterministic axial load and Δ_0/h is a normally distributed out-of-plumb variable with a mean, μ_c, equal to zero and a standard deviation, σ_c, equal to 0.0017 Rad. (see Fig. 6.7).

$$\frac{\Delta_0}{h} \sim> N(0,0.0017) \qquad \text{(B-2)}$$

For this case, the resulting design force is given by Eq. (7.3) in section 7.1.1:

$$F_d = \beta \, \sigma_c \, P \qquad \text{(B-3)}$$

The safety index β has been selected as 3.5. The absolute value of the force F_d then has a probability of not being exceeded (given by Table A-1) of 0.99954, assuming that P is not a variable.

B-2 Non-Deterministic Gravity Loads

A random gravity load with a known distribution would produce the horizontal force given by Eq. (B-4) when acting on an out-of-plumb column.

$$F = P_0 \frac{\Delta_0}{h} \tag{B-4}$$

The force in Eq. (B-4) has a distribution with a mean μ_f and a variance σ_f^2 given by expressions (A-38) and (A-40) in Appendix A.

$$\mu_f = \mu_c \mu_p + \sigma_{c,p} \tag{B-5}$$

$$\sigma_f = \sqrt{\mu_c^2 \sigma_p^2 + \mu_p^2 \sigma_c^2 + \sigma_c^2 \sigma_p^2} \tag{B-6}$$

where, μ_c = 0.0

μ_p = mean of gravity load population

$\sigma_{c,p}$ = covariance = 0.0

σ_c = 0.0017

σ_p = standard deviation of gravity load population

The covariance $\sigma_{c,p}$ is zero since obviously there is no correlation between the axial load and the out-of-plumb of a column. Then,

$$\mu_f = 0.0$$

$$\sigma_f = \sigma_c \sqrt{\mu_p^2 + \sigma_p^2} \tag{B-8}$$

The gravity load is the sum of dead and live loads. Since the dead and live loads are not correlated[59], Eqs. (A-30) and (A-32) are used to define μ_p and σ_p:

$$\mu_p = \mu_d + \mu_\ell \tag{B-9}$$

$$\sigma_p = \sqrt{\sigma_d^2 + \sigma_\ell^2} \tag{B-10}$$

In these expressions, the subscripts d and ℓ define the dead and live load distribution characteristics respectively. It can be assumed that the standard deviation of the dead load is not very significant compared to the standard deviation of the live load in Eq. (B-10)[58,59]. The dispersion of the gravity load distribution will be very close to the dispersion of the live load distribution.

The sustained live load distributions for the instantaneous load, the lifetime maximum load, and the loads for two intermediate time periods are given in Fig. B-1[61]. Of these, only the instantaneous distribution is known from live load surveys[62]. Various theoretical models have been proposed to derive the lifetime maximum live load distribution from load survey data[63,64]. Using the live load model of Ref. 63 and the live load data of Ref. 62, the statistics of lifetime maximum live load have been derived through simulation in Ref. 64.

What is needed in the present case is the distribution of the lifetime maximum live load intensities given by the dashed curve in Fig. B-1 and not the arbitrary point-in-time loads obtained from the live load surveys. For all practical purposes, the lifetime maximum live load distribution can be reasonably approximated by a normal distribution. A computer simulation has shown that the product of two normal variables is also very close to a normal. The design horizontal force for the case of variable gravity loads can then be defined by Eq. (A-44):

$$F_d = \mu_f + \lambda \sigma_f \qquad \text{(B-11)}$$

The safety index λ in this expression is not necessarily 3.5 as for β. Since $\mu_f = 0$,

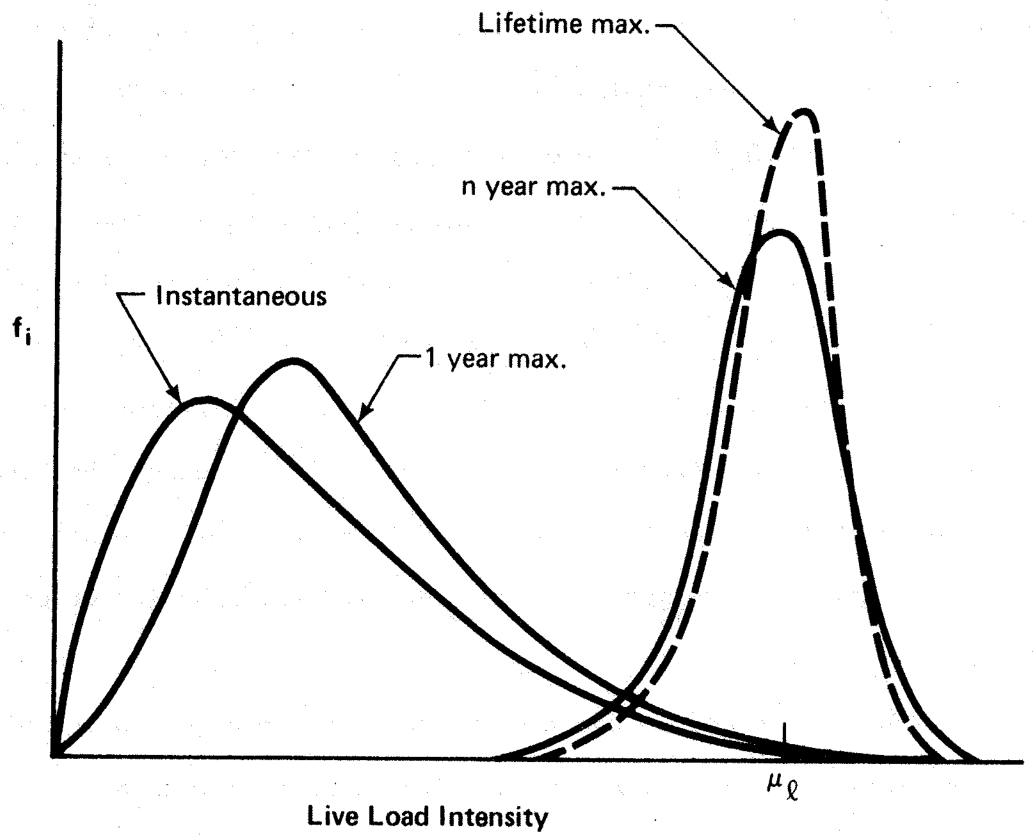


Figure B.1 Probability distributions of live load intensities

$$F_d = \lambda \sigma_c \sqrt{\mu_p^2 + \sigma_p^2} \qquad \text{(B-12)}$$

Eq. (B-12), derived for a non-deterministic load, should be compared to Eq. (B-3) to determine the value of the factor λ that would make the horizontal forces given by the two equations equal.

B-3 Estimation of Dead and Live Load Parameters

The four parameters μ_d, μ_ℓ, σ_d, and σ_ℓ must be estimated in order to determine the safety index λ.

Approximations given in Ref. 59 are listed in Table B-1. The lifetime maximum dead load is assumed to have an average value equal to the design value with a coefficient of variation of 0.07[58,59]. This implies that the maximum dead load will be within ±14 percent of the design value in 95 percent of all structures.

For an office building floor designed for 50 psf, the expected maximum live load (not including partitions) over a 30-year life would be reduced to about 35 psf[59]. It has been indicated in Ref. 64 that the Canadian National Building Code formula[60] for reduction of floor load with tributary area, $0.3 + 10/\sqrt{A}$, is fairly consistent with calculated maximum lifetime loads based on measurements. Therefore, the ratio of expected 30-year load to National Building Code design load will be assumed to be 0.7, independent of tributary area. The results of load surveys[62] indicate that the coefficient of variation for maximum floor loads is about 0.3 and is unchanged with increasing area.

For office and residential buildings, the expected load at a given time is approximately equal to the 30-year load for an infinite area. For office buildings, this corresponds to $0.7(0.3 + 10/\sqrt{\infty})$ 50 psf =

	Mean μ / Specified Load	Coeff. of Variation σ/μ
Dead Load	1.0	0.07
Live Load		
- Maximum 30 years	0.7	0.3
- At any time	$0.21/(0.3 + 10/\sqrt{A})$	$0.3 + 0.4/\sqrt{A}$

A is the Tributary Area in sq. ft.

TABLE B-1 PROBABILISTIC ASSUMPTIONS FOR GRAVITY LOADS

10.5 psf, a value confirmed by survey results[59]. On the other hand the coefficient of variation of a load at any time increases with a decrease in area. The equation given is based on Table 7 of Ref. 62.

Approximate formulas are given in Ref. 64 for the mean and standard deviation of the maximum sustained live load during a structure's life plus the largest extraordinary event which occurs during the random duration of this maximum sustained load. The approximations fit the actual distributions in the upper fractiles of these loads.

$$\mu_\ell = 14.9 + 763/\sqrt{A_1} \text{ psf} \quad \text{(B-13)}$$

$$\sigma_\ell = \sqrt{11.3 + 15000/A_1} \text{ psf} \quad \text{(B-14)}$$

where A_1 is the "influence area" which corresponds to four times the more common "tributary area", A, in the case of single-storey column loads.

These results have shown that for columns, the prescribed design loads as a function of area for the Canadian National Building Code[60] correspond approximately to the 0.9 fractile of the maximum total load. As shown in Fig. B-2, the NBC prescribed load is 50 psf for office buildings and for $A > 200$ ft.^{2}, this value is reduced by a factor $0.3 + 10/\sqrt{A}$. The prescribed live load P in a column is then estimated by Eq. (B-15) for a column tributary area larger than 200 ft.^{2}.

$$P = \mu_\ell + \psi\sigma_\ell \quad \text{(B-15)}$$

The value for the safety index ψ which corresponds to the 0.9 fractile is 1.3 and μ_ℓ and σ_ℓ are given by Eqs. (B-13) and (B-14) or are taken from Table B-1.

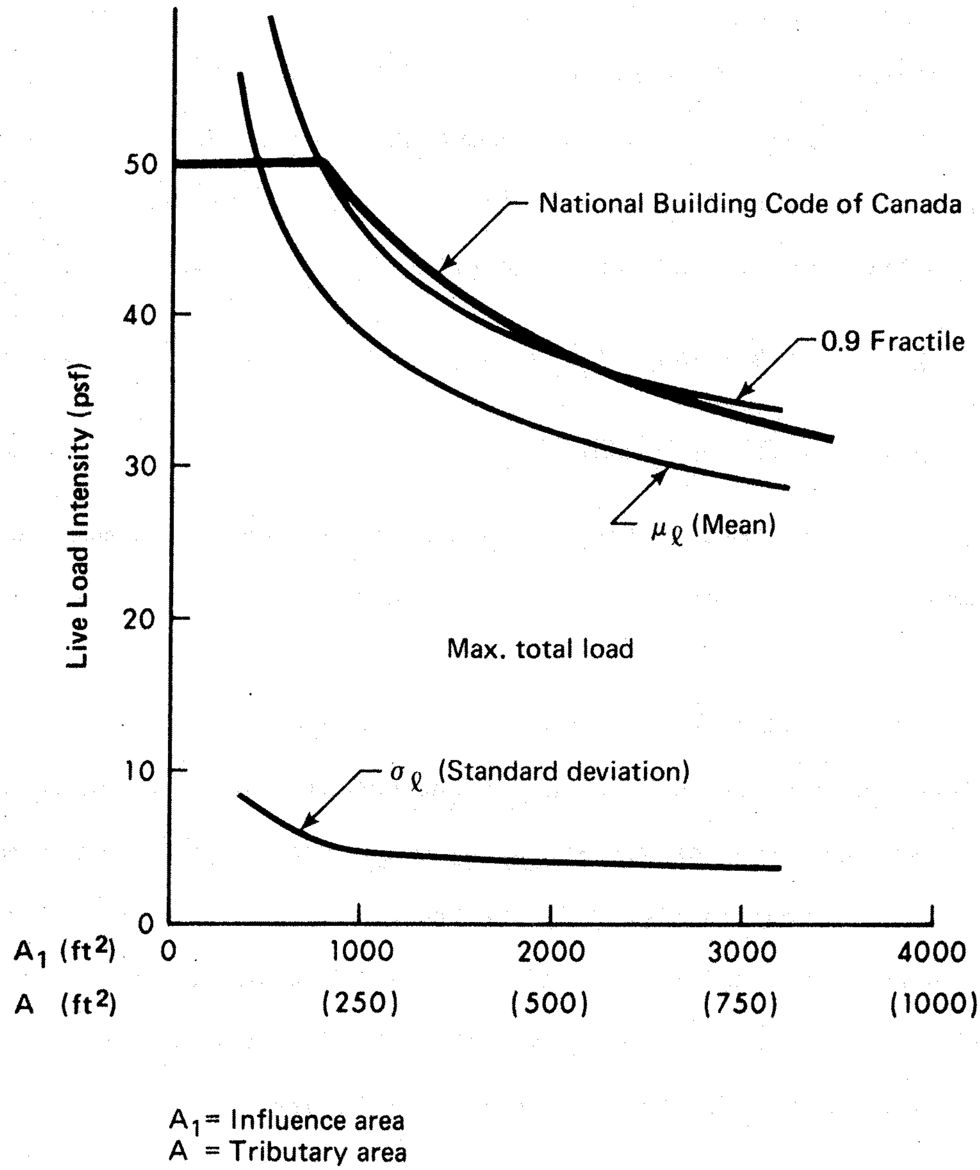


Figure B.2 Prescribed live loads for column design

Example:

$$A = 200 \text{ ft.}^2, \quad A_1 = 800 \text{ ft.}^2$$

Prescribed load $= 50 \times 200 = 10000$ lbs.

$$\mu_\ell = (14.9 + 763/\sqrt{800})\ 200 = 8375 \text{ lbs.}$$

$$\sigma_\ell = \sqrt{11.3 + 15000/800} \times 200 = 1096 \text{ lbs.}$$

$$P = 8375 + 1.3 \times 1096 = 9800 \text{ lbs.}$$

$\therefore$ 2 percent difference.

Using the values of Table B-1,

$$\mu_\ell = 0.7 \times 50 \times 200 = 7000 \text{ lbs.}$$

$$\sigma_\ell = 0.3 \times 7000 = 2100 \text{ lbs.}$$

$$P = 7000 + 1.3 \times 2100 = 9730 \text{ lbs.}$$

$\therefore$ 2.8 percent difference.

B-4 Probability Calculations

The distributions of the two random variables in Eq. (B-4) are given in Fig. B-3(a,b) with their corresponding probabilities. The dead load is not included in the load distribution shown in (b) in order to simplify the calculations. The results should not be changed significantly. The distribution represented by the continuous curve in Fig. B-3(c) is the distribution of the variable horizontal force, $P\Delta_0/h$, for a deterministic axial load P. The shape of the distribution is the same as in (a) but the scale is different. The variance in this case is $P^2\sigma_c^2$ according to Eq. (A-28). The horizontal

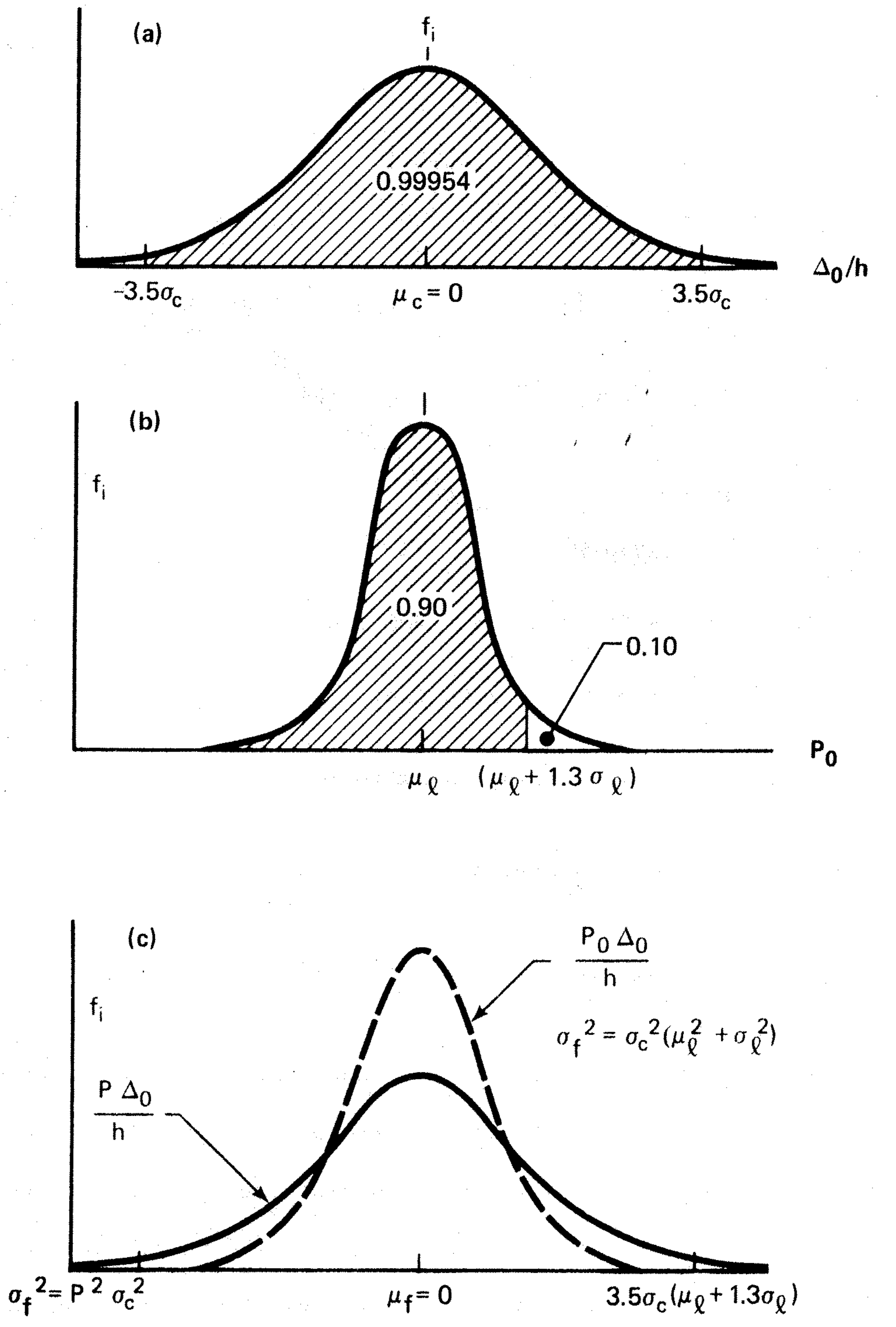


Figure B.3 Distributions

shear distribution for a variable axial load P_0 is represented by the dotted curve on the same figure. The variance is now equal to $\sigma_c^2(\mu_\ell^2 + \sigma_\ell^2)$ and is always smaller than the variance in the case of the deterministic load. In terms of standard deviations,

$$\sigma_c(\mu_\ell + 1.3\ \sigma_\ell) > \sigma_c\sqrt{\mu_\ell^2 + \sigma_\ell^2} \qquad \text{(B-16)}$$

The variance of the distribution assuming P non-deterministic is reduced from the variance assuming P deterministic. The probability is then greater under the dotted curve that the shear force will be less than $3.5\sigma_c(\mu_\ell + 1.3\sigma_\ell)$. By assuming P deterministic, the absolute value of the shear has a 99.954 percent chance of being less than this limit.

$$\Pr\left[\left|\frac{P\Delta_0}{h}\right| < 3.5\sigma_c\ (\mu_\ell + 1.3\sigma_\ell)\right] = 0.99954 \qquad \text{(B-17)}$$

When the axial load is random,

$$\Pr\left[P_0\frac{\Delta_0}{h} < 3.5\ \sigma_c\ (\mu_\ell + 1.3\sigma_\ell)\right] = ?$$

Dividing both sides of the inequality by the standard deviation of the population, gives

$$\Pr\left[\frac{P_0\Delta_0/h}{\sigma_c\sqrt{\mu_\ell^2+\sigma_\ell^2}} < \frac{3.5\ (\mu_\ell + 1.3\sigma_\ell)}{\sqrt{\mu_\ell^2+\sigma_\ell^2}}\right] = ? \qquad \text{(B-18)}$$

As shown in section A-10 of Appendix A, when a normally distributed variable with a mean equal to zero is divided by the standard deviation of the population, the variable is said to be standardized. The expression on the left hand side of the inequality is then the standardized horizontal shear for a non-deterministic load and can be called F_{st}. The new safety index λ for the force obtained in Eq. (B-12) is given by the expression on the right hand side. The factor λ can be evaluated from the values of Table B-1 or from Eqs. (B-13) and (B-14). From Table B-1,

$$\mu_\ell = 0.7\ P$$

$$\sigma_\ell = 0.3\ \mu_\ell = 0.3 \times 0.7\ P = 0.21\ P$$

$$\lambda = \frac{3.5\ (0.7 + 1.3 \times 0.21)\ P}{\sqrt{0.7^2 + 0.21^2}\ P} = 4.66$$

The probability of F_{st} being lower than this value is obtained from the table of the Standard Cumulative Normal Distribution (Table A-1).

$$Pr\ [F_{st} < 4.66] = 0.9999984$$

$$Pr\ [|F_{st}| < 4.66] = 0.9999968$$

Using Eqs. (B-13) and (B-14) with A = 200 ft.^{2} and A_1 = 800 ft.^{2}, μ_ℓ = 8375 lbs. and σ_ℓ = 1096 lbs.

$$\lambda = \frac{3.5\ (8375 + 1.3 \times 1096)}{\sqrt{8375^2 + 1096^2}} = 4.06$$

$$Pr\ [F_{st} < 4.06] = 0.999975$$

$$Pr\ [|F_{st}| < 4.06] = 0.999951$$

Due to the random nature of the gravity loads, the horizontal shear given by Eq. (B-3) has a real probability which corresponds to $\beta \simeq 4.2$ when the loads prescribed by the Canadian National Building Code are used in combination with $\beta = 3.5$ in Eq. (B-3).

It is possible to calculate the value of ψ in Eq. (B-15) which would have held the probability at 0.99954 (β = 3.5) in Eq. (B-3). This occurs when $(\mu_\ell + \psi\sigma_\ell) = \sqrt{\mu_\ell^2 + \sigma_\ell^2}$ in Eq. (B-18). Then,

$$\Pr\,[F_{st} < 3.5] = 0.99954$$

The quadratic equation obtained has a root equal to

$$\psi = \frac{\sqrt{\mu_\ell^2 + \sigma_\ell^2} - \mu_\ell}{\sigma_\ell} \qquad \text{(B-19)}$$

The index ψ will be close but never equal to zero according to this equation. An indeterminate result is obtained when the variation σ_ℓ^2 is zero. The values in Table B-1 for the maximum live load give ψ = 0.15, while Eqs. (B-13) and (B-14), for the example presented previously, yield ψ = 0.065. Thus, if a gravity load given by the mean shown in Fig. B-2 was used, the resulting horizontal shear calculated by Eq. (B-3) would have a 99.95 percent chance of not being exceeded.

APPENDIX C

DEGREE OF DEPENDENCE OF COLUMN OUT-OF-PLUMBS

It seems likely that a certain correlation exists between the out-of-plumbs of the columns in a structure. Whether it significantly affects the results of the theory developed in Chapter VII has yet to be verified.

Assuming that z is the sum of two random variables, x and y, the variance of the new variable z is given by Eq. (A-31) in Appendix A:

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 + 2\sigma_{x,y} \qquad \text{(C-1)}$$

The variance σ^2 is defined in section A-6.1 and the covariance $\sigma_{x,y}$ in section A-7. The horizontal force at a connection point, as given by Eq. (7.4) in the case of two out-of-plumb columns, is now a normally distributed variable of the form:

$$F \sim> N(P_1\mu_x + P_2\mu_y, \sqrt{P_1^2\sigma_x^2 + P_2^2\sigma_y^2 + 2P_1P_2\sigma_{x,y}})$$

For $\mu_x = \mu_y = 0$ and $\sigma_x = \sigma_y = \sigma$,

$$F \sim> N(0, \sigma\sqrt{P_1^2 + P_2^2 + 2\rho\, P_1P_2})$$

where $\rho = \sigma_{x,y}/\sigma^2$ is defined in section A-7 of Appendix A as the coefficient of correlation, which is the standardized measure of the joint behavior of two random variables. When $\rho = 1.0$, the variates are positively perfectly correlated and when $\rho = -1.0$, they are perfectly negatively correlated. On a graph, these conditions are represented by straight lines of slope +1 and -1 respectively. If $\rho = 0.0$, the variates are said to be uncorrelated or perfectly independent.

The horizontal force caused by n out-of-plumb columns, assuming a certain degree of correlation between the columns, is then given by:

$$F_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2 + 2\rho \sum_{j=1}^{n-1} P_j\, P_{j+1}} \qquad \text{(C-2)}$$

The second term under the root sign is the summation of all the possible independent combinations of pairs of adjacent columns. An upper bound is found when $\rho = 1$, i.e. when there exists a positive linear dependence between the variables. Then,

$$F \sim> N(0, \sigma\sqrt{(P_1 + P_2)^2})$$

$$F \sim> N(0, \sigma\,(P_1 + P_2))$$

The resulting design horizontal force for the general case of n columns is:

$$F_d = \beta\sigma_c \sum_{j=1}^{n} P_j \qquad \text{(C-3)}$$

This is equivalent to the model shown in Fig. 5.1(b) with all the columns out-of-plumb by $\beta\sigma_c$. The lower bound is obtained by assuming perfect independence between the variables ($\rho = 0$).

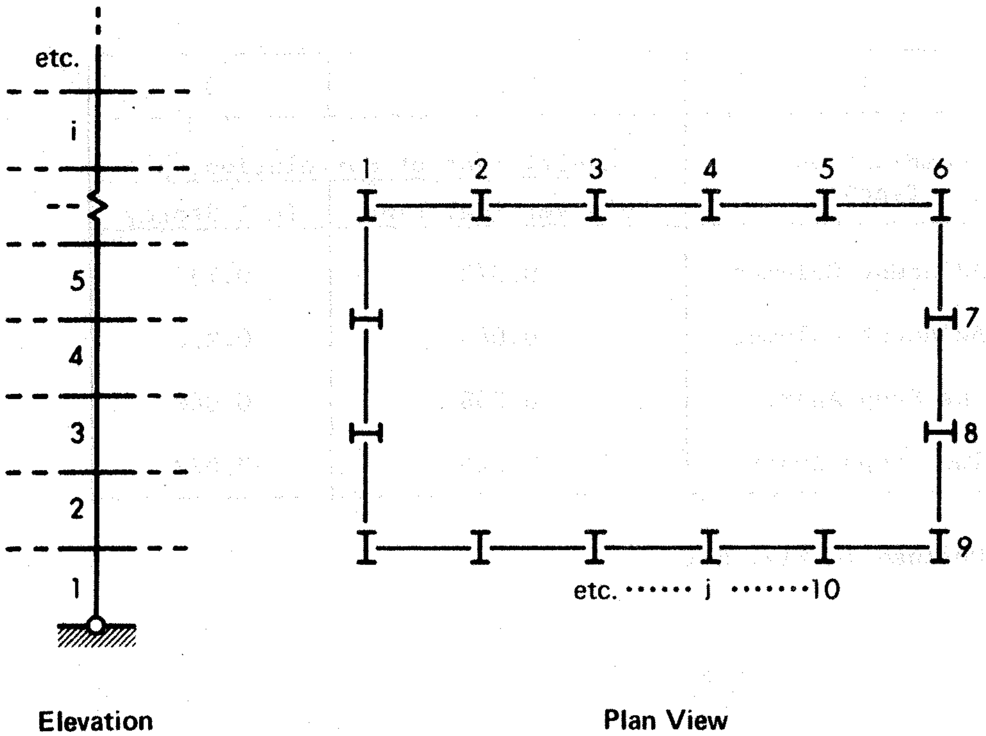
$$F \sim> N(0, \sigma\sqrt{P_1^2 + P_2^2})$$

$$F_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2} \qquad \text{(C-4)}$$

This formulation is used in Chapter VII, sections 7.1.1 and 7.1.2.

Different combinations of out-of-plumb columns taken in a vertical line and in a storey are shown in Fig. C-1. The correlations corresponding to the different combinations, denoted as a, b, c, and d, can be calculated and can be visualized graphically. An estimation of the actual coefficient of correlation for each case can be obtained from Eq. (A-24). The results obtained for building B are listed in Table C-1 and the graphical representations of the two different correlations pertaining to group 'a' are given in Figs. C-2 and C-3. Fig. C-2 shows the correlation between columns adjacent in vertical lines and Fig. C-3 shows the correlation between adjacent columns at each storey. The values plotted on each figure represent a sample of the total number of observations. The out-of-plumbs in the x and y directions are considered together.

The results shown in column 2 of Table C-1 for groups a and b indicate a slight but non-significant dependence between columns in vertical lines. The scatter of the points shown in Fig. C-2 confirms these results. The coefficients of correlation obtained in cases c and d where the pairs are one and two steps apart are even closer to zero. This result was expected. It indicates a decreased dependence of the variables as the compared columns are taken farther apart. The usual practice of erecting tier columns does not seem to induce a significant degree of correlation between the column out-of-plumbs from one floor to another. The correlation that could have existed initially from floor to floor is apparently wiped out during



Combinations

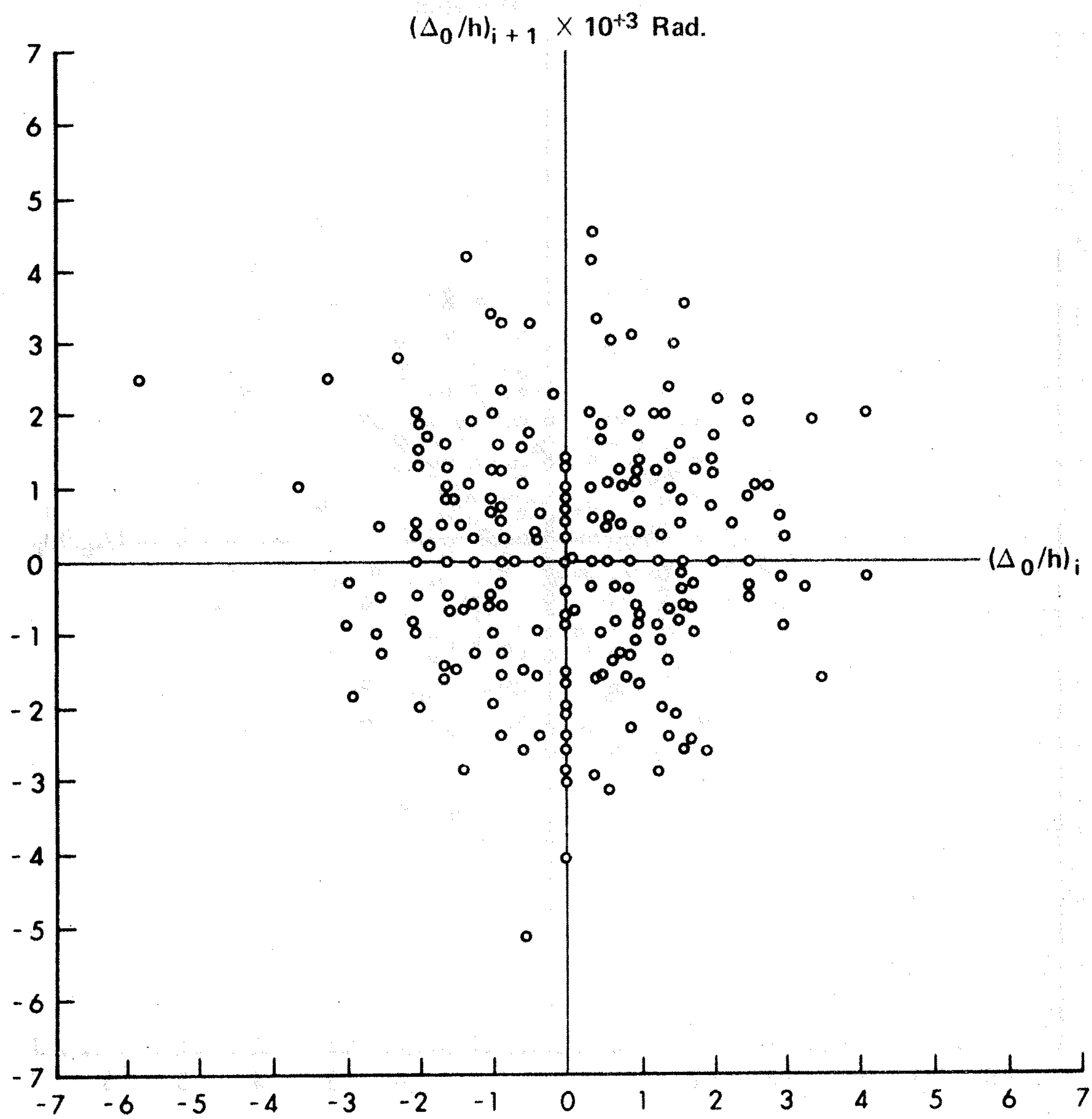
a – 1-2, 3-4, 5-6, etc . . . (adjacent columns)
b – 2-3, 4-5, 6-7, etc . . . (adjacent columns)
c – 1-3, 4-6, 7-9, etc . . . (1 step apart)
d – 1-4, 5-8, 9-12, etc . . . (2 steps apart)

Figure C.1 Column combinations for the evaluation of the degree of correlation between out-of-plumbs

1	2	3
Combination Type*	Coefficient of Correlation, ρ	
	In A Vertical Line	In A Storey
a - Adjacent Columns	0.072	0.133
b - Adjacent Columns	0.063	0.238
c - One Step Apart	-0.008	0.068
d - Two Steps Apart	0.024	-0.022

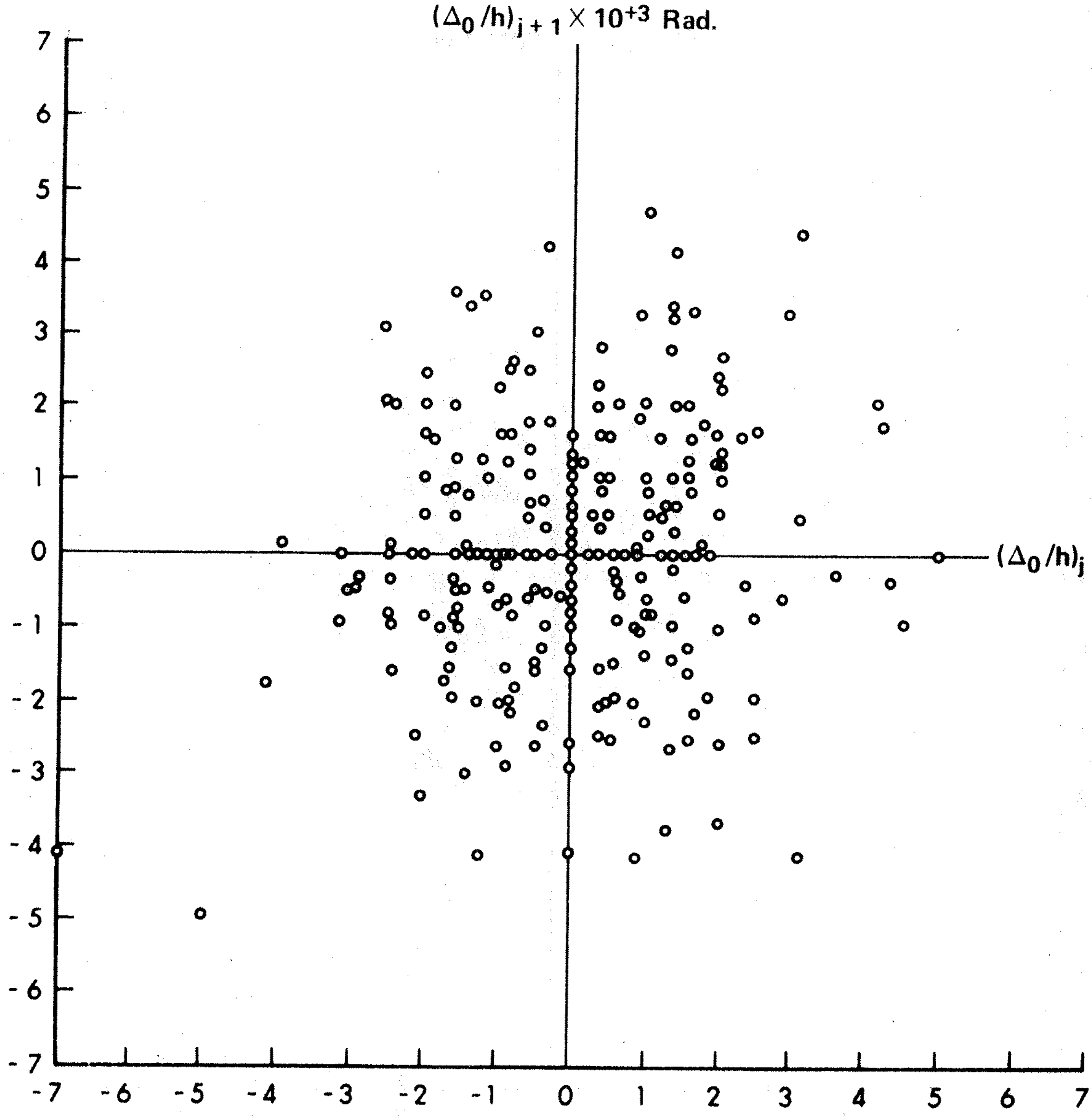
* Defined in Fig. B-1

TABLE C-1 COEFFICIENTS OF CORRELATION FOR BUILDING B



Coefficient of correlation = 0.072 (from data)

Figure C.2 Correlation between two out-of-plumb columns adjacent in a vertical line in building B



Coefficient of correlation = 0.133 (from data)

Figure C.3 Correlation between two out-of-plumb columns adjacent in a storey in building B

the construction of the building by the effects described earlier in section 5.3.

Coefficients of correlation of 0.133 and 0.238 for adjacent columns in a storey were obtained for building B. The plot corresponding to $\rho = 0.133$ is given in Fig. C-3 and shows considerable scatter. A positive coefficient of correlation in this case indicates that two adjacent columns in a storey lean in the same positive or negative direction more often than predicted by the theory developed in Chapter VII based on total independence. Since an unsafe situation could result, an evaluation of this effect is mandatory.

The results obtained for combination types c and d in column 3 of Table C-1 confirm earlier observations that the correlation decreases rapidly as the columns forming the pairs are taken farther apart. It is believed that the correlation within a storey is in great part tied in to fabrication errors. When girders for a specific storey are cut slightly shorter or longer, the adjustment of these girders between the columns in a bent might force the columns to lean in the same direction in the plane of the bent.

In summary, a correlation does exist between column-of-plumbs. Although it is negligible from storey to storey, it is significant within a storey. For all practical purposes, $\rho = 0.0$ between columns in vertical lines, $\rho = 0.2$ between adjacent columns in a storey, $\rho = 0.1$ for pairs one step apart in a storey, and $\rho = 0.0$ for pairs more than one step apart.

Considering the case of two adjacent columns at a same storey Eq. (C-2) becomes:

$$F_d = \beta\sigma_c \sqrt{P_1^2 + P_2^2 + 2(0.2)\, P_1 P_2}$$

Assuming equal axial load in the columns,

$$F_d = \sqrt{2.4}\ \beta\sigma_c\ P$$

The lower bound given by Eq. (C-4) is

$$F_d = \sqrt{2}\ \beta\sigma_c\ P$$

Then, the correlation existing between two adjacent columns increases the horizontal force predicted by Eq. (C-4) by 10 percent.

For the general case of n columns at the same storey, Eq. (C-2) yields:

$$F_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2 + 2[0.2 \sum_{j=1}^{n-1} P_j P_{j+1} + 0.1 \sum_{j=1}^{n-2} P_j P_{j+2} + 0.0]}$$

Assuming that

$$\sum_{j=1}^{n-1} P_j P_{j+1} \simeq \sum_{j=1}^{n} P_j^2$$

and

$$\sum_{j=1}^{n-2} P_j P_{j+2} \simeq \sum_{j=1}^{n} P_j^2 ,$$

$$F_d = \beta\sigma_c \sqrt{1.6 \sum_{j=1}^{n} P_j^2} = 1.26\ \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2}$$

which constitutes an increase of 26 percent from the lower bound given by Eq. (C-4). The calculations show that the 26 percent limit is attained at 15 columns.

It remains to check whether this effect is reduced or increased when columns from different storeys are combined. By summing separately within each storey, Eq. (C-2) becomes:

$$F_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n_i} P_j^2 + \sum_{j=1}^{n_{i+1}} P_j^2 + 2[0.2(\sum_{j=1}^{n_i-1} P_jP_{j+1} + \sum_{j=1}^{n_{i+1}-1} P_jP_{j+1}) + + 0.1(\sum_{j=1}^{n_i-2} P_jP_{j+2} + \sum_{j=1}^{n_{i+1}-2} P_jP_{j+2})]}$$

where n_i is the number of columns at storey i.

This equation can be simplified within each storey as before:

$$F_d = \beta\sigma_c \sqrt{1.6 \sum_{j=1}^{n_i} P_j^2 + 1.6 \sum_{j=1}^{n_{i+1}} P_j^2}$$

$$F_d = 1.26\ \beta\sigma_c \sqrt{\sum_{j=1}^{n_i} P_j^2 + \sum_{j=1}^{n_{i+1}} P_j^2}$$

Eq. (C-4), applied to the same case, becomes:

$$F_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n_i} P_j^2 + \sum_{j=1}^{n_{i+1}} P_j^2}$$

This shows that the increase of 26 percent from the lower bound remains when columns from different storeys are combined.

Eq. (C-2), however, is not practical in a design for the horizontal out-of-plumb forces. It seems more logical to use the expression giving the lower bound with an increased safety index which would account for the correlation effects and other factors. When a factor β of 3.5 is used in Eq. (C-4), the real probability of being exceeded is not 4.6×10^{-4} but 5×10^{-3} corresponding to a β

value of 2.8. In other words, in five out of a thousand times, the horizontal forces calculated for β = 3.5 would exceed the predicted values.

The applications given later in Chapter VIII, where several measured and predicted quantities are directly compared, will justify the choice of a safety index equal to 3.5 under the above conditions. More important is the fact that the safety index β is actually increased from 3.5 to 4.2, as shown in Appendix B, because of the random nature of the gravity loads. The combination of the effects observed in Appendices B and C results in an average safety index of 3.5 with a corresponding probability of 0.99954.

APPENDIX D

LATERAL DEFLECTIONS DUE TO COLUMN OUT-OF-PLUMBS

Expression (7.16) reproduced below as (D-1) requires excessive computational efforts for structures with a large number of columns. An investigation is needed to determine whether this equation can be simplified.

$$\frac{\Delta_d}{h} = \beta\sigma_c \frac{\sqrt{\sum_{j=1}^{n} P_j^2}}{\sum_{j=1}^{n} P_j} \tag{D-1}$$

When the column axial loads are assumed to be constant, the variable P_j disappears and Eq. (D-1) is reduced to:

$$\frac{\Delta_d}{h} = \frac{\beta\sigma_c}{\sqrt{n}} \tag{D-2}$$

In practical structures the axial loads differ and cause Eq. (D-2) to be unconservative. A more general formulation would be,

$$\frac{\Delta_d}{h} = \frac{\beta\sigma_c}{\sqrt[x]{n}} \tag{D-3}$$

where the variable x is a function of the number of columns in a structure and the variations in column axial loads. The influence of these two factors on the variable x can be evaluated.

Equating (D-1) and (D-3) yields:

$$\sqrt[x]{n} = \frac{\sum_{j=1}^{n} P_j}{\sqrt{\sum_{j=1}^{n} P_j^2}}$$

which in turn becomes:

$$x = \frac{\ln n}{\ln R} \qquad \text{(D-4)}$$

where

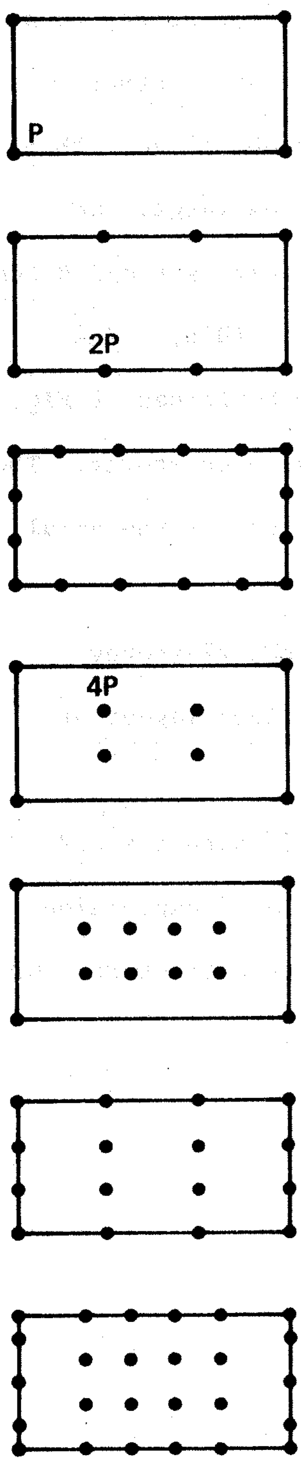
$$R = \frac{\sum_{j=1}^{n} P_j}{\sqrt{\sum_{j=1}^{n} P_j^2}} \qquad \text{(D-5)}$$

The minimum x value is 2.0 and is obtained when the axial loads in Eq. (D-4) are constant.

$$x = \frac{\ln n}{\ln\left(\frac{nP}{\sqrt{nP^2}}\right)} = \frac{\ln n}{\ln \sqrt{n}} = \frac{\ln n}{1/2 \ \ln n} = 2 \qquad \text{(D-6)}$$

The upper limit is not defined but is in the order of 2.5 in practical structures. Larger values are obtained only in very unusual cases.

Fig. D-1 shows the column layouts of seven different building cross-sections. The variable x is calculated for typical 1, 2, 6, and 10-storey buildings by assigning the relative axial loads P, 2P, and 4P to the corner, exterior, and interior columns respectively. The column axial loads are increased uniformly from floor to floor. For instance, if the top column of a 3-storey column stack carries 2P, the middle one and the lower one carry 4P and 6P respectively.



	1 Storey n/R	1 Storey X	2 Storeys n/R	2 Storeys X	6 Storeys n/R	6 Storeys X	10 Storeys n/R	10 Storeys X
1	$\frac{4}{2.00}$	2.00	$\frac{8}{2.68}$	2.11	$\frac{24}{4.4}$	2.14	$\frac{40}{5.61}$	2.14
2	$\frac{8}{2.68}$	2.11	$\frac{16}{3.60}$	2.16	$\frac{48}{5.91}$	2.18	$\frac{80}{7.52}$	2.17
3	$\frac{16}{4.13}$	2.04	$\frac{32}{5.21}$	2.10	$\frac{96}{8.55}$	2.13	$\frac{160}{10.88}$	2.13
4	$\frac{8}{2.43}$	2.34	$\frac{16}{3.25}$	2.35	$\frac{48}{5.34}$	2.31	$\frac{80}{6.80}$	2.29
5	$\frac{12}{3.13}$	2.18	$\frac{24}{4.20}$	2.21	$\frac{72}{6.90}$	2.21	$\frac{120}{8.78}$	2.20
6	$\frac{16}{3.6}$	2.16	$\frac{32}{4.83}$	2.20	$\frac{96}{7.93}$	2.20	$\frac{160}{10.09}$	2.20
7	$\frac{26}{4.67}$	2.11	$\frac{52}{6.26}$	2.15	$\frac{156}{10.28}$	2.17	$\frac{260}{13.08}$	2.16

Figure D.1 X values for different column layouts

In all cases, except in the very unusual case No. 4, x does not exceed 2.2. A similar pattern is observed in all arrangements: The factor x increases slightly when passing from a one-storey to a two-storey building; the value remains fairly constant as other storeys are added and finally decreases when n becomes large. This behavior is explained by the fact that when n becomes large, the difference between the axial loads becomes less significant and R tends towards $\sqrt{n}$. At the limit, x is 2.0 as shown in Eq. (D-6). The notable increase of the variable x in the 2-storey buildings of Fig. D-1 reflects the factor of 2 between the axial loads at each storey. The number of columns is then too small to hide the effect of the axial load variations.

As applied to the actual axial loads in the 27-storey building A (Fig. 6.2), $x = \ell n\ 18.62 = 2.10$. The column layout of building A is given in example No. 3 in Fig. D-1.

In view of these considerations, Eq. (D-3) with x = 2.2 is recommended for use in design. However, the "exact" expression (D-1) should be used in the case of one or two-storey structures for a more accurate evaluation of Δ_d/h.

APPENDIX E

EFFECT OF WALL THICKNESS VARIATIONS

ON MEASUREMENTS OF OUT-OF-PLUMBS

The exact deviation from plumb at a specific section of a wall is obtained by using the average of two measurements, one taken on either side of the wall. Measurements taken on one side only do not account for the unavoidable thickness variations of the wall. However, it is physically impossible to take double measurements at each wall section.

It is possible to determine to what extent the measurements are affected by estimating the distribution of the wall thickness variations and combining the resulting variance with the variance of the wall out-of-plumb population. The variance is defined as the squared value of the standard deviation.

Thickness measurements were taken wherever possible with a measuring tape on the core of building B. At least two measurements were taken per vertical section of the wall.

The variables that must be distributed and used in the calculations are the deviations from the mean at each individual section. The wall section shown in Fig. E-1(a) is thicker at the bottom and the measurement taken as shown results in an out-of-plumb value smaller than the actual. In (b) the recorded deviation is larger than it should be while in (c) the actual out-of-plumb is recorded. In other cases, as in (d), the thickness variation does

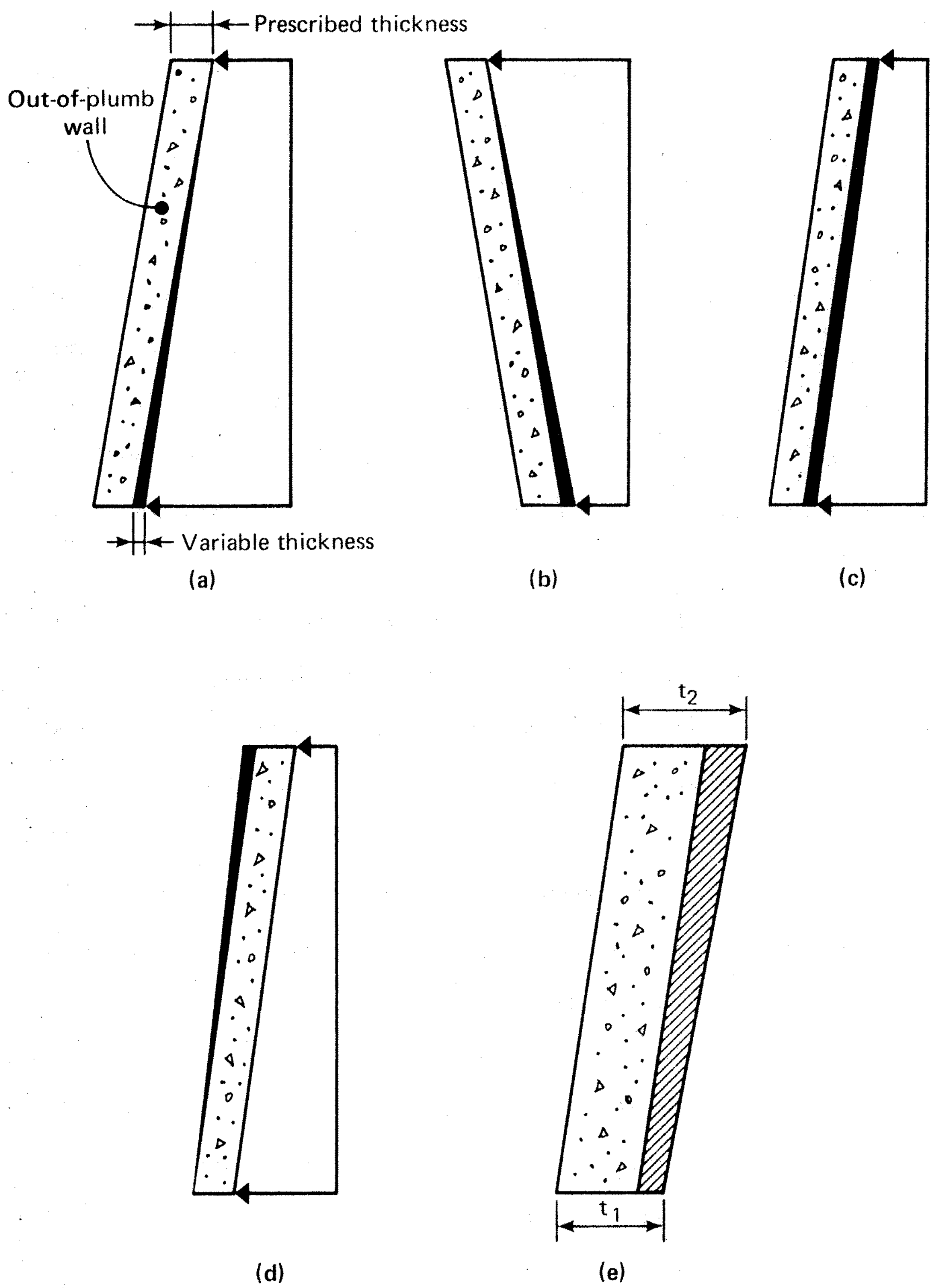


Figure E.1 Wall thickness variations

not affect the measurements. For a set of two measurements at the section shown in Fig. E-1(e), the mean thickness is $(t_i + t_2)/2$ and the values to be distributed are $t_1' = t_1$ - mean and $t_2' = t_2$ - mean.

The distribution obtained is given in Fig. E-2 together with the mean, the standard deviation, and other characteristics. The distribution is close to normal. The mean, of course, is zero and the standard deviation is 0.16". The variance of the measured out-of-plumbs is the sum of the variance of the actual out-of-plumbs and the variance of the deviations of the wall thickness from the mean at specific sections.

$$\text{var}\ (\Delta_0)_m \;=\; \text{var}\ (\Delta_0)_{act} + \text{var}\ (\delta_t)$$

or

$$\text{var}\ (\Delta_0)_{act} \;=\; \text{var}\ (\Delta_0)_m - \text{var}\ (\delta_t) \qquad \text{(E-1)}$$

The standard deviation of the measured out-of-plumbs for building B is given in Table 6.11 and is approximately 0.0023. A representative value in units of inches is obtained by multiplying the standard deviation by the standard storey height in practical structures, say 144". The variance is then $(144'' \times 0.0023)^2 = (0.33'')^2$ and

$$\text{var}\ (\Delta_0)_{act} \;=\; (0.33)^2 - (0.16)^2 \;=\; 0.08\ \text{in.}^2$$

The actual standard deviation should therefore be

$$\sigma \;=\; \sqrt{0.08} \;=\; 0.29''$$

which constitute an insignificant reduction from 0.33". Since the effect is slightly on the conservative side when neglected, a reduction will not be applied to the measured standard deviation in this report.

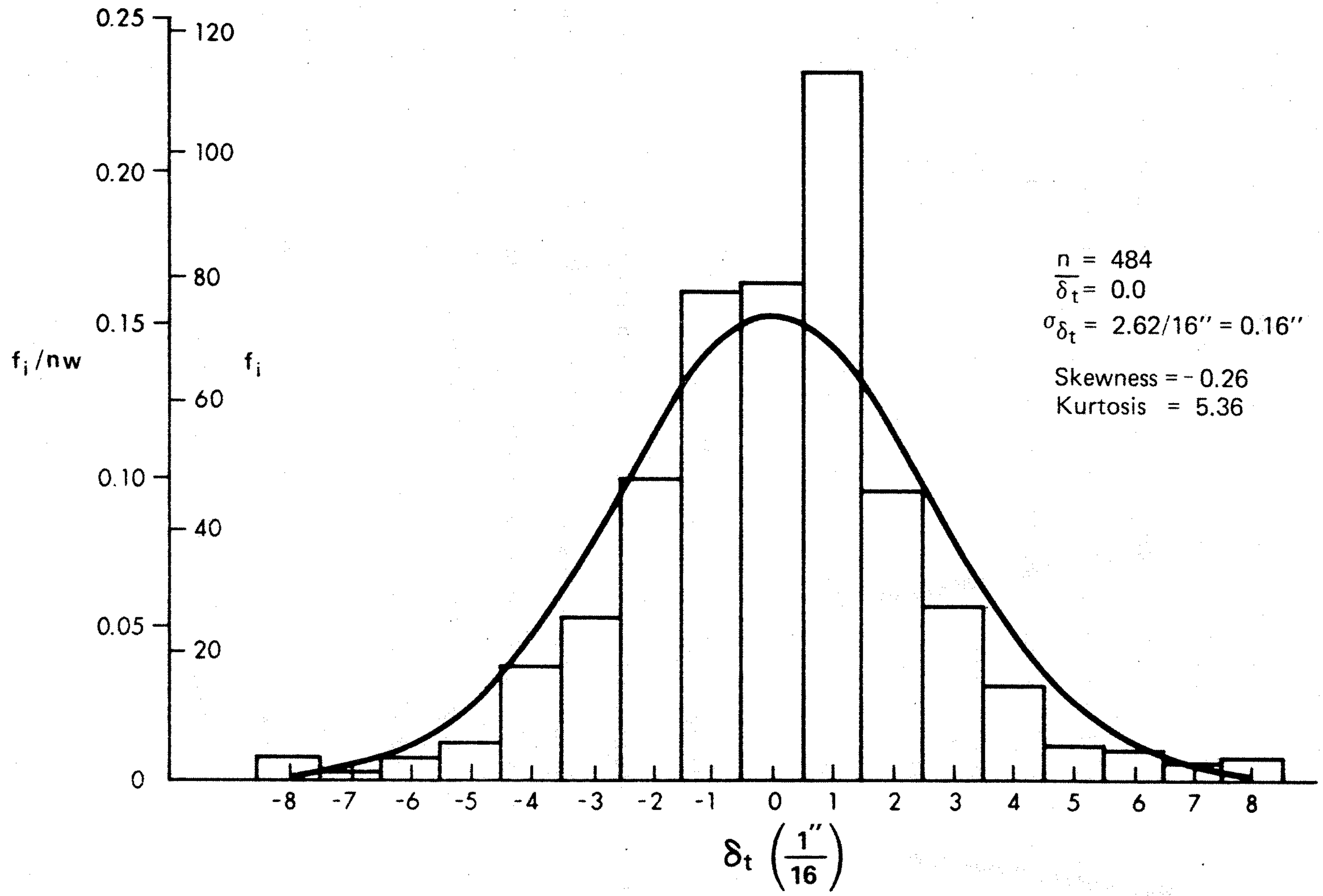


Figure E.2 Distribution of wall thickness deviations from the mean at a cross section