

A Review of Mechanisms and Models for Dynamic Strength, Dynamic Failure, and Fragmentation

K.T. Ramesh^a, James D. Hogan^a, Jamie Kimberley^b, Angela Stickle^c, Andrew L. Tonge^a

^a*Hopkins Extreme Materials Institute, The Johns Hopkins University, Baltimore, MD 21218*

^b*New Mexico Institute of Mining and Technology, Socorro, NM 87801*

^c*Johns Hopkins Applied Physics Laboratory, Laurel, MD 20723*

Abstract

Modeling of catastrophic disruption requires understanding the processes of dynamic failure and fragmentation. This paper summarizes current mechanisms and models for dynamic failure, strength, and fragmentation, reviewing these from a mechanics perspective and with an emphasis on making links to the developing advances in these areas in the engineering and computational mechanics communities. We describe dynamic failure processes, examine size and rate effects, articulate the scaling concepts that arise naturally from these processes, and examine the influences of these processes on effective strength and fragmentation.

Keywords: disruption, strength, failure, dynamic fragmentation, planetary materials, brittle behavior

1. Introduction

1 There is evidence throughout the solar system of catastrophic disruption, particularly as the
2 result of large impacts into asteroids (Williams, 1989; Zappal et al., 1995; Binzel and Xu, 1993;
3 Burbine et al., 2001) and comets (Podolak and Prialnik, 1996; Chapman, 1975). Missions in recent
4 decades (e.g., NEAR, Deep Impact, Hayabusa, Dawn, Rosetta, soon OSIRIS-REx) have revealed
5 new and interesting details about the nature of small bodies in the solar system. Many asteroids
6 have been observed to have impact craters with diameters comparable to the body size (Bottke Jr
7 et al., 2002), suggesting that very large impacts can be below the disruption threshold. In addition,
8 internal structure and damage has been observed on the surface of most small bodies, as is evident

Email addresses: ramesh@jhu.edu (K.T. Ramesh), jd.hogan@jhu.edu (James D. Hogan),
jamiiek@nmt.edu (Jamie Kimberley), angela.stickle@jhuapl.edu (Angela Stickle),
andy.tonge@jhu.edu (Andrew L. Tonge)

9 from large-scale fractures observed on Ida: (Sullivan et al., 1996); Eros: (Veveřka et al., 2000); and
10 Vesta: (Le Corre et al., 2012; Schenk et al., 2012; Scully et al., 2012). It appears that much of this
11 structure is the consequence of impact-dominated processes.

12 Our understanding of impact processes is typically limited by two things: first, our understand-
13 ing of the dynamic material properties and failure processes of the planetary materials that are
14 involved in the impact, and second, our ability to capture these complex processes within com-
15 putational simulations of these extreme events. Laboratory experiments have been very useful in
16 improving our understanding of impact processes and catastrophic disruption (e.g., Gault (1973);
17 Gault and Wedekind (1969); Fujiwara and Tsukamoto (1980); Nakamura and Fujiwara (1991);
18 Martelli et al. (1994)). Laboratory-scale experiments, however, are orders of magnitude smaller
19 than most of the collisions in the solar system. Thus computational simulations (e.g., Melosh et al.
20 (1992); Benz and Asphaug (1994); Jutzi et al. (2010)) and scaling relationships (Holsapple, 1993;
21 Davis et al., 1994; Housen et al., 1983) have become increasingly critical in understanding large-
22 scale impacts. Hydrocode-based simulations have been shown to be capable of handling many of
23 the complexities of the major impact event, particularly with respect to shockwave propagation
24 and interactions with boundary conditions, and are typically benchmarked against laboratory ex-
25 periments. Improvements to such simulations and the development of new scaling relationships
26 should result from advances in the understanding and modeling of the failure processes that occur
27 during impact events.

28 Most of the impact events of interest to planetary science represent extreme dynamic events,
29 which are characterized by the deposition of large amounts of energy in very short times. Because
30 the speeds at which energy can propagate away from the location of deposition are finite (for
31 example, shock speeds), the local energy density rises very rapidly, and so the material seeks
32 new internal pathways to dissipate the energy (for example, fracture, melting, and vaporization).
33 These internal energy pathways are typically referred to as “mechanisms.” Which mechanisms are
34 available depends on the materials involved, and which of these mechanisms are exercised depends
35 also on the severity of the impact, and the generalized loading conditions (e.g. impact parameters
36 such as obliquity).

37 A schematic of the main variables, failure mechanisms and processes in planetary impact events
38 is presented in Figure 1 in terms of the conventional domains associated with a major impact.

39 Consider an impactor of diameter a impacting a much larger target body at a high velocity V . One
40 characteristic timescale for the event is then given by $\tau_i = a/V$, and the likely phenomena can
41 then be categorized in terms of time after impact, described in terms of multiples of τ_i . This type
42 of domain decomposition can also be performed in terms of length scales using multiples of the
43 impactor diameter. Four domains can be identified, denoted generally as the source, strong shock,
44 strength/flow and structural domains. Within each of these domains, we can identify the expected
45 pressures, strain rates, and temperatures. These variables typically characterize the conditions that
46 are developed in those domains. For example, the domain just under the impactor (called the
47 contact domain) perceives the highest pressures, strain rates and temperatures. For each domain,
48 we also identify critical macroscale processes, the deformation and failure mechanisms, and the
49 kinds of models needed to describe these mechanisms. The material models that are used should
50 be able to account for this range of pressure, strain rate and temperature histories, be able to
51 incorporate these deformation and failure mechanisms in an effective way, and provide the key
52 parameters necessary for the modeling of the critical processes in each domain.

53 The domains of primary interest to us in this paper are in the shaded region in Figure 1. The
54 materials in this region are under pressures of the order of 10 MPa to 1 GPa, deforming at in-
55 termediate strain rates of $10^{-6} - 10^4 \text{ s}^{-1}$, and undergoing massive failure through the collective
56 behavior of cracks, voids and shear bands. Modeling the response of the material involves the
57 coupling of these damage mechanisms with the rapidly varying stress states associated with the
58 propagating shock, the development of rarefaction fans and subsequent spall from shock release
59 down isentropes, massive fragmentation at intermediate strain rates, fragmentation-induced dilata-
60 tion (“bulking”) and the overall rates of deformation. In practice, few simulation approaches are
61 able to handle all of these phenomena within the same simulation with high fidelity, with particular
62 difficulties often arising from the fragmentation and bulking components.

63 This paper summarizes current mechanisms and models for dynamic failure, strength, and
64 fragmentation, reviewing these from a mechanics perspective and with an emphasis on making
65 links to the developing advances in these areas in the engineering and computational mechanics
66 communities. Note that the background of the authors is that of experiments and modeling in
67 dynamic mechanics of solids, rather than planetary science, and so it is likely that we cannot
68 adequately represent the extensive literature in this area in planetary science. We therefore do

69 not attempt to capture all of the excellent work already published within the planetary science
70 literature in this area, providing instead representative examples of related work. Finally, we note
71 that reviews such as this are inevitably biased towards the works that are most familiar to the
72 authors, and we apologize in advance to any who feel that their work has been slighted: this is
73 certainly not our intention.

74 **2. Dynamic Failure Mechanics**

75 The specific failure processes that are developed within any given region of the target depend
76 primarily on the material class (e.g. brittle versus ductile) and the current stress state. Figure 2
77 provides some of the typical failure processes that are developed within each material class as a
78 function of the multiaxial stress state. Note for example, that the influence of confining pressures
79 on failure processes can also be dramatic. Confining stresses can reduce available driving forces
80 for some failure processes like fracture. However, confining stresses can also change the *mode*
81 of failure. For example, Hu et al. (2011) demonstrated that the application of a bi-axial confining
82 stress can re-orient the principal direction of crack growth during compression, producing diffuse
83 shear-dominated failure zones (Figure 3). Some of these mechanisms are not intuitively expected:
84 for example, a homogeneous ductile material subjected to hydrostatic compression generally does
85 not fail, whereas the same ductile material containing inclusions/heterogeneities can develop adi-
86 abatic shear bands under hydrostatic compression because the inclusions break symmetry and act
87 as nucleation sites for the failure process. As a consequence, the possibility of large-scale shear
88 localization should be considered in most planetary materials even in regions of nearly hydrostatic
89 compression. Additional details on selected mechanisms in Figure 2 may be found in the follow-
90 ing references: necking and void growth (Wu et al., 2003); adiabatic shear bands (Wright, 2002);
91 bifurcation to nucleate voids (Wu et al., 2003); wing cracks (Horii and Nemat-Nasser, 1985); crack
92 shutdown under pressure (Hu et al., 2011); and spallation (Wright and Ramesh, 2009).

93 The major dynamic failure processes that are developed during large-scale impacts include (but
94 are not limited to):

- 95 • Dynamic fracture (nucleation, growth and coalescence of cracks) (Freund, 1998).
- 96 • Dynamic void nucleation and growth (leading to spallation) (Meyers, 1994).

- 97 • Void collapse (e.g., (Carroll and Holt, 1972; Molinari and Mercier, 2001)). Note that pore
98 compaction can also be a localizing process, e.g. (Issen and Rudnicki, 2000).
- 99 • Adiabatic shear banding (dynamic localization of shearing deformations) (Wright, 2002;
100 DORAZIO et al., 2011).
- 101 • Amorphization or phase change of some crystal structures (Chen et al., 2003).

102 Some examples of these failure processes are presented in Figure 4 for a variety of materials
103 (references are noted in the sub-figures). The morphology of the failures is distinct in each case.
104 Since each failure process is typically developed under a particular stress state, the presence of
105 such failures is often used as a signature of the prior existence of that stress state. The associated
106 length scales include the failure size and the failure spacing.

107 We provide foundational models for each failure mechanism in the next sections. In general,
108 such models typically prescribe five major components: (i) a nucleation criterion; (ii) an onset of
109 growth criterion (sometimes called an initiation criterion), if nucleation has already occurred; (iii)
110 an equation for the growth dynamics; (iv) a description of the interactions with other failures or
111 with boundaries; and (v) a coalescence or runaway instability criterion. The nucleation question is
112 the least well-understood of the five components, and is an active area in multiscale modeling (e.g.,
113 Rudd and Belak (2002)). Growth, on the other hand, is relatively well described, although growth
114 dynamics continues to be an area of intense research (e.g., Wilkerson and Ramesh (2014)). In most
115 cases the interaction and coalescence questions are also poorly-understood and are the source of
116 many recent models (e.g., Jacques et al. (2012)).

117 A consequence of the activation of any of the failure processes presented in Figure 2 are in-
118 herent length scales and timescales, which then are manifested in the macroscopic impact event.
119 These scales should ideally be resolved in simulations if the failure process is to be captured, an
120 issue of great importance in engineering because of the need for design of protection systems.
121 However, resolving these scales is often difficult to do from a computational resources viewpoint,
122 particularly for the scales associated with small bodies in the solar system. As an example, the
123 spacing of macroscopic shear bands will be reflected in the fragment sizes (Zhou et al., 2006c),
124 and so the computational scheme must be able to resolve this spacing if fragment sizes are to be
125 predicted. These fragment length scales change with loading path, e.g. when a region of material

126 undergoes hydrostatic compression followed by hydrostatic tension, the initial compressive state
127 may result in shear banding, and the subsequent tensile state is then felt by a damaged material
128 containing shear bands with a spacing that will affect the tensile fragmentation. In this section, we
129 will explore some key failure mechanisms that are activated during catastrophic disruption, and
130 we present some examples of models for these mechanisms. A relatively recent compendium of
131 many of these mechanisms and associated models can be found in the Proceedings of the IUTAM
132 Symposium on Dynamic Fracture and Fragmentation of 2009, with the papers appearing in the
133 International Journal of Fracture in 2010.

134 An area of particular current growth in the mechanics literature is that of multiscale compu-
135 tational models of failure processes such as shear localization and dynamic fracture. Since these
136 processes *localize* in both space and time, they are intrinsically multiscale. Significant advances in
137 multiscale modeling capabilities are under development, through both sequential (coarse-graining
138 or fine-scaling) and concurrent (hierarchical and partitioned-domain) approaches (a discussion of
139 these is provided by Tadmor and Miller (2012)). Snapshots of the state of the art can now be ob-
140 served in all of the major mechanics conferences. Much that was considered simply not possible
141 a decade ago is now feasible, e.g. the coupled handling of plasticity and fracture across multiple
142 scales within computational frameworks (Chakravarthy and WA, 2010), and the efficient compu-
143 tational solution of multiscale damage problems (Liu, 2014). An excellent example of these ideas
144 as applied to modeling materials across multiple scales is provided by Phillips (2001), and a more
145 recent discussion of the computational aspects is provided by Fish (2013). Many of these advances
146 are able to bridge across six or more orders of magnitude in length scales, and arose because
147 of the engineering need for microstructure-aware computational mechanics schemes (e.g., Ghosh
148 (2011)) for failure-resistant product design (e.g. jet engines). These theoretical frameworks and
149 computational methods lend themselves also to the much larger scales examined by the planetary
150 science community, although significant challenges remain. Improved models of the disruption
151 and fragmentation of planetary bodies should benefit from such multiscale approaches.

152 We discuss the major failure mechanisms briefly et seq. The literature on the mechanics of
153 individual failure processes is vast, and so we focus on the key concepts rather than provide a
154 comprehensive review.

155 2.1. Dynamic fracture

156 An excellent discussion of fracture mechanics that includes dynamic fracture is presented by
157 Broberg (1999). The elastodynamic solutions corresponding to fast cracks were first presented by
158 Freund, and are summarized in his book on Dynamic Fracture Mechanics (Cambridge University
159 Press). These works generally focus on the propagation of pre-existing cracks. Nucleation of
160 cracks from heterogeneities is a complex problem with solutions that vary widely depending on
161 the nature of the heterogeneity and on the local stress state. A common, if imperfect, approach
162 to nucleation is to define the local maximum tensile stress (say maximum principal stress) σ_{max} ,
163 define the heterogeneity size l_h , and then to assume that nucleation occurs when $\sigma_{max} = \alpha \frac{K_{IC}}{\sqrt{2\pi l_h}}$,
164 where K_{IC} is a property (the “fracture toughness”) of the matrix material surrounding the hetero-
165 geneity and α is a prefactor that is used to identify the “strength” of the heterogeneity (i.e. the pre
166 factor is different for inclusions, pores, and so forth).

167 For a pre-existing crack of a given length, the onset of tensile crack growth (crack initiation)
168 occurs when the stress intensity factor K_I (which represents the driving force on the crack tip,
169 depends on the stress state and is tabulated for a variety of problems, e.g. Anderson (2004) and the
170 DTD Handbook online) reaches a critical value (the aforementioned fracture toughness): $K_I = K_{IC}$.
171 Fracture toughness data for a number of geological materials is presented by Zhang and Zhao
172 (2014). The stress intensity factor is the variable through which the multiaxial stress state affects
173 the crack tip, and such analyses motivate the development of pressure-dependent behavior of brittle
174 rocks.

175 Once the crack is growing, the dynamics of crack growth for fast cracks is defined by the rate
176 of change of crack length \dot{l} , and is given by

$$\dot{l} = v_c \left(\frac{K_I - K_{IC}}{K_I - K_{IC}/2} \right)^\beta, \quad (1)$$

177 where v_c is the crack speed (an important parameter that must be measured) and β is a parameter
178 that defines the increase in effective crack inertia with crack speed. Crack speeds can vary substan-
179 tially, but are limited by the Rayleigh wave speed except under pathological conditions (Broberg,
180 1999). In geomaterials, they are usually of the order of 200-2,000 m/s (Zhang and Zhao, 2014).

181 Once the cracks are sufficiently large, they will begin to interact (the works of Kachanov (2003)

182 in this area are particularly useful). The general interactions of multiple dynamic cracks are very
183 difficult mechanics problems. Many approaches to describe this interaction have been used, such
184 as assuming a periodic array of cracks (Deng and Nemat-Nasser, 1992; Deshpande et al., 2011),
185 considering self-consistent solutions (Paliwal and Ramesh, 2008), or making structural approx-
186 imations such as buckling columns (Ashby and Cooksley), 1986). In general, crack interactions
187 result in increased driving force on crack tips, and the crack speed increases (equation 1). Crack
188 interactions are particularly important when estimating the strength of brittle materials.

189 Eventually the growing cracks will coalesce. Coalescence of cracks is typically only modeled
190 in an approximate manner through either an instability analysis of the ligament between inter-
191 acting cracks (e.g., Benzerga and Leblond (2010)), through empirical functions that parameterize
192 the instability, or through mode-specific rules (Tang et al. (2001)). Fragmentation follows after
193 coalescence, and so fragment sizes and shapes depend on the full set of nucleation, growth and
194 coalescence behaviors.

195 Computational modeling of fracture processes is well developed for the growth phase, and
196 there are several commercial and public domain software packages that handle fracture mechanics,
197 including dynamic fracture. These include general purpose commercial finite element packages
198 like Abaqus that include techniques such as cohesive zone modeling, and downloadable software
199 such as FRANC3D developed by academic groups (in this case the Cornell Fracture Group). Such
200 codes are generally capable of tracking crack fronts and crack paths through solids during dynamic
201 failure processes, but the computational cost increases rapidly with increasing numbers of cracks,
202 and convergence with respect to fragmentation remains a major research problem.

203 *2.2. Adiabatic shear localization*

204 Shear localization or shear banding is an instability brought about through the large shearing
205 deformations of materials. Shear bands are of two types: (a) resulting from deformation instabil-
206 ities associated with evolving parameters in the constitutive equations (Rudnicki and Rice, 1975),
207 and (b) thermal instabilities associated with evolving temperature, adiabatic heating and subse-
208 quent thermal softening (Molinari and Clifton, 1987). Deformation instabilities are mathemati-
209 cally easily described in terms of bifurcation analyses, are commonly observed in slow loading
210 problems, and are not discussed further here.

211 Thermal instabilities lead to a kind of shear band known as an adiabatic shear band (Molinari
212 and Clifton, 1987). These only arise under dynamic loading and are a result of the competition
213 between the timescales associated with the loading dynamics and the timescales associated with
214 the thermal conduction. An excellent summary of the conditions for nucleation of adiabatic shear
215 localization (in terms of the onset of instability) is provided by Wright (2002). Criteria for the onset
216 of adiabatic shear bands typically examine the derivatives of the stress in the stress-strain curve of
217 the material e.g., in terms of strain-hardening, strain-rate-sensitivity and thermal softening of the
218 shear stress, or in terms of the evolution of effective frictional stresses. The canonical works in the
219 area of shear localization and adiabatic shear band development are the books by Wright (2002)
220 and Dodd and Bai (2012).

221 Once an adiabatic shear band has nucleated, the strain localization evolves in two directions:
222 along the direction of shear (in the form of a shear band tip), and normal to the direction of shear
223 (in the form of the band thickness). Unlike the crack tip, the tip of the shear band is typically poorly
224 defined; however, the propagation of the shear band tip along the direction of shear appears to be
225 similar to that of a shear crack, and so a tip velocity is sometimes identified as equal to a shear crack
226 speed for modeling purposes. A now-classic set of experiments that describes this behavior, with
227 associated analyses, is presented by Zhou et al. (1996). The rate of localization depends strongly
228 on material behavior and the macroscopic stress state (Wright, 2002). For many materials, it is
229 possible to define a finite band thickness that depends on material properties (Dodd and Bai, 2012).
230 The final microstructure within this shear band thickness is typically very different from the initial
231 microstructure of the material, because very large shear strains must be accommodated by the
232 material within the band. An additional mechanism that may be very important in fluid-saturated
233 geophysical materials is that of an effective thermal pressurization produced by the constrained
234 relative thermal expansion of fluids within the band (Platt et al., 2014). Adiabatic shear bands in
235 brittle materials have been recently examined by citeGradySB2011.

236 The interactions of shear bands define the shear band spacing, and essentially constitute com-
237 petitions between momentum transport and thermal transport . A detailed analysis of shear band
238 interactions is provided by Zhou et al. (2006c). The shear band spacing may control the apparent
239 fragment size under some conditions, because the shear localization is often the first failure mech-
240 anism that is developed during the compressive states that initiate after impact loading, with cracks

241 often following along the shear bands.

242 From a computational viewpoint, adiabatic shear localization represents a major challenge be-
243 cause of the dynamics involved, the associated length scales, and the evolving local microstructure
244 in the band. The majority of simulations of adiabatic shear localization do not account for the non-
245 linear evolution of the material behavior within the shear band. Mesh-insensitive computations of
246 shear localization can be obtained by incorporating the thermal conduction length scales in thermo-
247 mechanical simulations, and by incorporating enriched numerical techniques such as XFEM and
248 its relatives. This failure mode remains one of the most challenging dynamic failure mechanisms
249 for simulations.

250 *2.3. Spall failures under dynamic loading*

251 There is not, unfortunately, an accessible standard reference on the mechanics of spall failure,
252 but an excellent discussion can be found in the book by Meyers (1994). In the engineering sense
253 used here, spallation is the result of the dynamic nucleation, growth and coalescence of voids or
254 cracks (Meyers, 1994)) under a macroscopically hydrostatic tensile state, and are typically observed
255 on the opposite side of the target body from the impact face. The tensile states are generated by the
256 interaction of rarefaction or tension waves coming off free surfaces, and so body geometry plays a
257 big role in the location of spall. Full brittle spall is rare, and so we focus on the ductile (void-driven)
258 spall problem. Void nucleation typically occurs over a very wide range of length scales because of
259 the scales of heterogeneities. For the case of an elastic-plastic solid, it can be rigorously shown that
260 even homogeneous nucleation of voids will occur through a bifurcation process at sufficiently high
261 hydrostatic tensile stresses (Wright and Ramesh, 2009). The presence of a heterogeneity reduces
262 the critical stress that is needed, and thus in general geophysical materials have a distribution of
263 nucleation sites with a corresponding distribution of nucleation stresses. Nucleation can occur
264 at, for example, vacancy clusters (Mori and Meshii, 1964), precipitates (Embury and Nicholson,
265 1965), inclusions (Thompson and Weihrauch, 1976), large xenoliths and so forth. Most materials
266 of interest to the planetary science community contain pre-existing voids and pores, although the
267 initial porosity will typically be modified by the compressive shocks from the impact.

268 The classical picture of spall mechanics is the following: voids are nucleated as described
269 above, grow through the development and growth of a plastic zone around the void, and then

270 the interactions of the growing voids leads to the macroscopic spall failure. The “spall strength”
271 is the maximum hydrostatic tensile stress (Antoun et al., 2003) that the material can withstand,
272 is associated with these dynamically growing and interacting voids, and is viewed as a material
273 property (akin to the fracture toughness described earlier). For any given void, the void growth
274 rate is limited by two factors (Wright and Ramesh, 2009): (1) the inertia associated with moving
275 the mass of material away from the current void surface, and (2) the viscoplastic inertia associated
276 with developing plastic flow at sufficiently high rates. The consequence is that the spall strength
277 increases rapidly with the volumetric strain rate (Wright and Ramesh, 2009). The spall strengths
278 of engineering materials are typically measured using plate impact spallation experiments (Meyers,
279 1994), but there is limited data on the spall strengths of geological materials (e.g., Field (2007)).
280 A first-order and somewhat conservative estimate of the spall strength for effective elastic-plastic
281 solids can be obtained using the analytical procedures described by (Wu et al., 2003). A recent
282 discussion of the theoretical mechanics issues associated with spall (albeit in metals) can be found
283 in Wilkerson and Ramesh (2014).

284 Spall failures that develop in numerical simulations are typically captured either by explicitly
285 incorporating a spall failure criterion (such as a spall strength) or develop naturally through the
286 evolution of an internal (porosity-type) damage model (such as localization of growing porosity).
287 Validation of computational models of spall is difficult because of the dearth of time-resolved
288 experimental spall data on the materials of interest, and issues of length-scale dominate such com-
289 putational analyses. An example of the state-of-the-art in time-resolved experimental spall data is
290 provided by the Line VISAR work of Furnish et al. (2009), and demonstrates the statistical nature
291 of the mechanism.

292 **3. Dynamic Failure Mechanics and The Effective Strength of Planetary Materials**

293 The icy, basaltic and chondritic materials that dominate small bodies are generally very hetero-
294 geneous, containing multiple constituent phases that may be individually either brittle or ductile.
295 The local behavior may include cracking, shear banding and void growth, while the macroscopic
296 representation of the averaged behavior is usually the “strength,” defined as the limiting stress that
297 the material can undergo before measurable permanent deformation, and represented by a limit
298 surface in a six-dimensional stress space (Nemat-Nasser, 2009). The effective strength is a conse-

299 quence of the local behaviors averaged over some representative volume element or RVE (which
300 scales with mesh/cell size in simulations). The review article by Holsapple (2009) provides an
301 excellent discussion of the strength of planetary materials. In this paper, we focus only on mak-
302 ing connections between the underlying failure mechanisms and the effective strength (and later
303 fragmentation).

304 Generally, a strength model captures (a) the onset of permanent deformation, by defining the
305 yield surface, (b) the amount of incremental plastic deformation that will occur when the stress
306 moves outwards from the yield surface (this is called a flow law) and (c) whether the yield surface
307 itself evolves as a result of deformation (this is called hardening). The evolution of other internal
308 variables (for example, porosity or crack damage) may also need to be defined (Jutzi et al., 2008).
309 The strength model must be defined for all possible loading paths in stress space (e.g. loading
310 in compression, followed by shear, and followed by unloading). Given the variety of possible
311 paths, it is extremely difficult to find validated phenomenological models that describe all of the
312 possible behaviors. This motivates the development of strength models that contain evolving inter-
313 nal variables, where the evolution equation for the internal variable can describe specific physical
314 mechanisms and thus reduce the need for massive suites of experiments for parameter identifica-
315 tion. The review article by Holsapple (2009) lists some of the strength models commonly used
316 by the planetary impact community. A model that is used in engineering and that should be con-
317 sidered by the broader community is the Sandia Geomodel (Fossum and Brannon, 2004), which
318 is also efficient and is available in some high-performance codes, and its subsequent development
319 in the form of Kayenta, which provides a generalized framework for complex plasticity models
320 (datasets are available for limestone, tuff and granite). Another model of interest is that developed
321 originally for concrete by de Borst and Gutierrez (1999). The transition from smeared cracks to
322 discrete cracks in a finite element framework has been examined by de Borst et al. (2004). Major
323 challenges remain in obtaining parameters for these models for any given material. A recent sub-
324 stantial data set and review of experimental methods to probe the dynamic strength and failure of
325 geological materials is provided by Zhang and Zhao (2013).

326 As an alternative to phenomenological models, micromechanics-based models may be used
327 (Paliwal and Ramesh, 2008). These models are based on the underlying deformation mechanisms
328 (such as cracking or twinning) that can be activated in materials subjected to any given loading.

329 One benefit of these physics-based approaches is that they naturally suggest ways to incorpo-
330 rate strain rate, size scale and variability effects into the strength model. The extension to scales
331 and loading regimes outside of the range of experimental data is a continuing hurdle for prob-
332 lems in planetary and space science. The incorporation of fundamental sub-scale physics through
333 micromechanics-based models provides promise for obtaining more representative outcomes than
334 phenomenological models when simulations are performed in regimes where test data is not avail-
335 able.

336 Failure mechanisms such as cracking are subscale mechanisms that are averaged over the vol-
337 ume in estimating the effective strength of most geomaterials, and the length scales associated
338 with these mechanisms are such that the effective strength is now a function of sample size. The
339 consequences of the dynamic subscale processes on the strength of planetary materials are three-
340 fold. First, the strength of geomaterials is a function of the discretization size, with larger RVEs
341 corresponding to weaker materials. Second, the *variability* in the strength is also a function of the
342 discretization size, with smaller RVEs having greater variability (Graham-Brady, 2010). Third,
343 both the strength and the variability, at any given discretization size, depend on the effective rate
344 of deformation (higher rates lead to higher mean strengths and lower variabilities (Daphalapurkar
345 et al., 2011)). We demonstrate these implications of dynamic failure mechanics for the influence
346 of strain rate, sample size, and sample variability on the effective strength in the next section.

347 *3.1. Defect distributions, failure dynamics and effective strength*

348 A key characteristic of all planetary materials is the kind and degree of heterogeneity (e.g.
349 inclusions and pores), since these typically control the nucleation of failure processes. We refer
350 to these heterogeneities as “defects” in the subsequent discussion. When and how must a defect
351 be considered in the discussion of material behavior? At any given scale, any effective property
352 of a material is determined by volume averaging over RVEs at smaller scales. Thus, for example,
353 pores are averaged to obtain porosity and, therefore, density. However, two materials with the
354 same effective porosity may have very different pore size distributions. Consider one material with
355 a multitude of small pores and one with a few large voids. The failure mechanisms of these two
356 materials may be different because the size of the pores can have a strong impact on the failure
357 process (Katcoff and Graham-Brady, 2014). Thus it may be important to capture not just the

358 average defect density but also the defect distribution in determining effective behavior.

359 The incorporation of defect distributions and the associated micromechanics can simplify the
360 treatment of disruption problems. The influence of defect-driven nucleation on the failure dynam-
361 ics introduces a length scale (based on the defect spacing), a nucleation stress scale (based on
362 the defect size), and two time scales: the timescale for the failure to propagate and the timescale
363 for the failures from individual defects to communicate. By comparing these timescales with the
364 timescale associated with the loading dynamics (e.g. the reciprocal of the applied strain rate), we
365 can define different regimes of behavior that guide our development of a material model.

366 The connections between the defect population and the rate-dependent strength are demon-
367 strated in Figure 5, which considers a brittle solid containing a population of flaws/defects of
368 varying severity (that is, a defect distribution). We move clockwise from the top right quadrant of
369 Figure 5, which shows a probability distribution function $g(s)$ of defect sizes s within the material.
370 These defects are not necessarily internal cracks, but rather heterogeneities in the material that are
371 potential sites for crack nucleation. Fracture mechanics tells us that the stress needed to activate a
372 defect (such as a slit microcrack) decreases as $\frac{1}{\sqrt{s}}$, as shown in the bottom right quadrant of Figure
373 5. Two different rates of loading, slow and fast, are represented on the bottom left quadrant of
374 Figure 5, which shows the stress as a function of time. At any given time, under slow loading (the
375 red solid line), the most severe (largest) flaw is triggered first, and the growth of the corresponding
376 crack might lead to failure of the structure. Under dynamic loading (the green solid line), while the
377 most severe flaw is still triggered first, the finite-velocity growth of the corresponding crack can
378 be outpaced by the rate of increase of the loading, so that the next-most-severe flaw is triggered
379 before macroscopic failure occurs, and so on. At any given time, the dashed lines show that at low
380 loading rates only the largest defects are activated, while at high loading rates the majority of the
381 defect distribution will be activated. Thus the entire distribution of flaws may be activated in the
382 material under dynamic loading (the concept of higher loading rates initiating the smaller flaws of
383 a distribution is utilized in many studies (Holsapple, 1994a; Melosh et al., 1992).

384 Full micromechanics calculations (Paliwal and Ramesh, 2008) show that when the crack growth
385 dynamics and self-consistent crack interactions are accounted for, the distribution of activated de-
386 fects (which is different from the distribution $g(s)$ of available defects) is not necessarily described
387 by the two-parameter Weibull-type function $n = k\varepsilon^m$ (where n is the number of activated defects, ε

388 is a strain measure and k, m are effective material parameters that must be estimated independent of
389 fracture mechanics). The latter approach using the two-parameter Weibull is used extensively, e.g.,
390 (Melosh et al., 1992) and more recently (Huang and Subhash, 2003). The effective strength of the
391 overall material, containing an independently prescribed distribution of defects from which cracks
392 may be activated, can be computed directly from the crack interactions (Paliwal and Ramesh,
393 2008). This distinction is important in engineering brittle materials, where the defect population
394 is controllable and can be designed, but is generally less useful for rock masses where the initial
395 distribution of defects may not be known a priori.

396 The modeling frameworks described above allow for crack growth rates that are determined
397 via dynamic fracture mechanics (as opposed to assuming constant crack speeds), determines the
398 population of activated defects using fracture mechanics rather than phenomenology, accounts
399 for self-consistent interactions of cracks and computes effective strength therefrom, and naturally
400 incorporates confinement effects on the dynamics of crack growth and the strength. These concepts
401 lead to an important insight: when it comes to impact loading, bigger is not necessarily weaker.
402 Eventually the effect that bigger bodies are more likely to have bigger defects is outpaced by the
403 effect that bigger bodies take much longer times to break.

404 The distribution of flaws is commonly characterized using a power law distribution of flaw
405 sizes over many orders of magnitude (Holsapple et al., 2002). The geophysical processes that
406 produce the rock mass typically place an upper bound (s_{max}) on the flaw size in the rock mass,
407 and we assume that the lower bound on the flaw size is s_{min} (in engineered ceramics, entirely
408 different defect distributions are generated by the different processing conditions used to make the
409 material, and the maximum defect size and total defect density are dominated by these processing
410 conditions). Any impact into this rock mass will be modeled using a numerical approach which has
411 some finite spatial resolution. The length scale h introduced by the computational discretization is
412 likely to fall within the limits $s_{min} - s_{max}$ of the flaw size distribution and divides the distribution
413 into “subscale” flaws and “super scale flaws,” the former being smaller than h and the latter being
414 larger than h . Flaws which are larger than h can be resolved explicitly by the computational mesh.
415 The sub scale flaws, smaller than h , cannot be resolved by the computational mesh and therefore
416 must be effectively homogenized and represented using a “strength” model, such as a continuum
417 damage model. The super scale flaws, larger than h , result in failure processes that can be explicitly

418 captured by the simulation, but these failure processes need additional equations (e.g., equations of
 419 fracture mechanics) to describe their energetics and their dynamics. Essentially, strength models
 420 are used at the subscale while failure models supplement the strength models at larger scales. A
 421 variety of techniques have been developed (Pandolfi and Ortiz, 2012; Guy et al., 2012; Moës et al.,
 422 2003; Camacho and Ortiz, 1996; Xu and Needleman, 1994) for addressing those flaws which *can*
 423 be resolved by the computational method.

424 Because the strength model used at scales below numerical resolution must average in some
 425 sense mechanisms occurring at smaller scales, the strength should generally be anisotropic with
 426 evolving anisotropy. As a simple illustration, if a brittle material is first loaded in compression,
 427 we should expect that the material will now contain a collection of cracks that are aligned along
 428 the principal stress direction (Horii and Nemat-Nasser, 1985). If this sample is now subsequently
 429 loaded in tension along the perpendicular direction, the effective strength of the material in that
 430 direction will be much lower than the effective strength in the direction of the original compression,
 431 so that the strength is now anisotropic. The engineering mechanics community is moving rapidly
 432 towards the incorporation of anisotropic strength models in simulations of impact events, but many
 433 challenges remain in both formulation and implementation.

434 3.2. *Scaling of rate-dependent strength of geophysical materials*

435 An example of the use of micromechanics to develop a simplified strength model is presented
 436 here. A recently developed rate-dependent strength model that incorporates the interaction of a dis-
 437 tribution of preexisting flaws and crack growth dynamics has been shown to reasonably describe
 438 the dynamic strength of a wide range of brittle solids (Kimberley et al., 2013). By identifying criti-
 439 cal time and length scales involved in the problem, a universal relationship between the unconfined
 440 compressive strength of a brittle solid, σ_c , and the applied equivalent strain rate, $\dot{\epsilon}$, is found:

$$\frac{\sigma_c}{\sigma_0} = 1 + \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right)^{2/3}. \quad (2)$$

441 Here $\sigma_0 = \alpha \frac{K_{IC}}{\bar{s}\eta^{1/4}}$ and $\dot{\epsilon}_0 = \alpha \frac{c_d K_{IC}}{\bar{s} E} \eta^{1/4}$ are the characteristic stress and strain rate based on the
 442 mechanical (K_{IC} is the fracture toughness, c_d is the dilatational wave speed, E is Young's Modulus),
 443 and microstructural properties of the material (\bar{s} is the average flaw size, η is the flaw density).
 444 Physical interpretations of these characteristic quantities are given in Kimberley et al. (2013). The

Table 1: Characteristic stress and strain rate parameters for the compressive strength data on the brittle solids presented in Figure 6. Details are presented in (Kimberley et al., 2013)

Material	σ_0 (MPa)	$\dot{\epsilon}_0$ (s^{-1})
Limestone (Frew et al., 2001)	70	2.0×10^2
Limestone (Green and Perkins, 1969)	300	5.0×10^3
Basalt (Kumar, 1968)	200	1.0×10^3
MAC 88118 (Kimberley and Ramesh, 2011)	50	2.0×10^2
Concrete (Ross et al., 1995)	53	2.5×10^2

445 associated parameters are presented in Table 1 (for compression) for some geological materials.
 446 The equations describing the characteristic stress and strain rate provide useful tools for examining
 447 the effect of material properties and microstructure on the rate dependent strength. This strength
 448 scaling captures (Figure 6) the rate-insensitive response of geological materials at low rates as
 449 well as the sharp increase in compressive strength observed when these materials are compressed
 450 at high rates. Preliminary results show that it can be used to describe tensile failure as well with
 451 adjustments to the characteristic stress and strain rate, for reasons discussed in that paper. A similar
 452 fracture-based approach to predicting the rate dependent strength was used by Grady and Lipkin
 453 (1980) which resulted in a power law dependence of strength with strain rate with an exponent
 454 of 1/3. The difference in scaling exponent observed here is a result of the defect distribution and
 455 crack interactions. It is also possible to develop scaling exponents for rate-dependence that relate
 456 to assumed or measured Weibull distributions of size-dependent strength. In this regard, however,
 457 we note that the apparent Weibull modulus of engineering brittle solids is known to be a function
 458 of the rate of loading, as discussed by Daphalapurkar et al. (2011).

459 The associated micromechanics model also demonstrates that the superimposition of confin-
 460 ing pressure will result in a linear dependence (Hu et al., 2011) of the deviatoric strength on the
 461 pressure (as observed in a number of materials), and suggests that there will be a reduction of
 462 the rate dependence of the strength with increasing pressure. The latter prediction has not yet
 463 been tested experimentally but is consistent with the success of a number of simulations that use
 464 rate-independent strength at high pressures.

465 3.2.1. Implementation of micromechanics-derived strength models in disruption

466 Many catastrophic disruption studies define a measure Q^* of the disruption, called the catas-
 467 trophic disruption threshold and defined as the specific kinetic energy per unit target mass at which

468 the ratio of the mass of the largest fragment to the original target mass becomes 0.5 (Ryan, 2000).
 469 Here we present a revised disruption scaling where the results of Kimberley et al. (2013) are in-
 470 corporated in traditional analytical treatments of catastrophic disruption. In the strength regime,
 471 Holsapple (1994b) has shown that

$$Q^* \propto \left(\frac{S}{\rho}\right)^{3\mu/2} U^{(2-3\mu)} \quad (3)$$

472 where S is a material property with units of stress describing the strength of the target, ρ is the
 473 target density, U is the impact velocity and μ is an exponent in the coupling parameter (typically
 474 taken to be 0.55 for rocky bodies (Holsapple, 1993)). Making the strength measure in (3) the
 475 failure strength described by Kimberley et al. (2013), and approximating the strain rate in the body
 476 by $\dot{\epsilon} = U/R$ the following scaling is obtained:

$$Q^* \propto \left(\frac{\sigma_0}{\rho} + \frac{\sigma_0}{\rho} \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0}\right)^{2/3}\right)^{3\mu/2} U^{(2-3\mu)} \quad (4)$$

477 The size scaling in the strength regime articulated above has significant shortcomings related
 478 to the strength model (e.g. it is developed for uniaxial stress states, and ignores explicit size de-
 479 pendence) and the assumptions related to the disruption process itself (lack of strain rate history,
 480 oversimplification of the strain rate distribution in the target body). However, it does provides a
 481 view of how micromechanics models may be utilized to evaluate scaling in the strength regime.
 482 Ideally the micro mechanical model would be run in a concurrent numerical framework leading
 483 to a fully coupled simulation (such simulations are underway, with preliminary results presented
 484 by Tonge et al. (2014)). Our current numerical implementations are not efficient enough to allow
 485 for full scale simulations of the disruption process, and so we have resorted to incorporating the
 486 strength model of Kimberley et al. (2013), which incorporates the key features of the full microme-
 487 chanics.

488 Equation (4) is plotted in Figure 7 for an impact velocity of 1 km/s (using a bold black line)
 489 together a with summary of earlier scaling laws described by Holsapple et al. (2002). Equation (4)
 490 predicts that the strength regime consists of two regions as illustrated in Figure 7. For very small
 491 bodies there is a decrease in threshold specific energy with a slope of $-\mu$ in the log-log plot. This
 492 corresponds to the high strain rate regime of the strength-rate relationship, and agrees with the

493 experimental measurements on small targets conducted by Housen and Holsapple (1999). As the
494 target size increases the average strain rate in the body decreases and the specific energy needed
495 to disrupt a body reaches a plateau corresponding to the strength observed at low strain rates (as
496 previously noted, crack growth dynamics dominates defect probability at large sizes). This feature
497 is notably absent in all other scaling predictions (other than Durda et al. (1998)), and is a direct
498 result of the strength predicted based on micromechanical approaches. For bodies in the 100m
499 - 10km range this scaling predicts higher catastrophic disruption thresholds as compared to most
500 other predictions in the plot. As the size of the target body increases further, the scaling should of
501 course transition from strength dominated to gravity dominated in which a body may be shattered
502 and reaccumulate, or dispersed.

503 One possible implication of the disruption threshold presented here is that it would be harder
504 to generate rubble pile bodies from targets in the 1km -10km range. Bodies with higher disruption
505 thresholds are less likely to accumulate after impact because fragments have high enough velocity
506 to escape gravitation of the rest of the fragments. The existence of small rubble pile bodies such as
507 Asteroid 25143 Itokawa could imply that this disruption scaling is simply wrong, or alternatively
508 suggest that Itokawa is the result of reaccumulation of a portion of a larger body that was disrupted.
509 This latter suggestion is supported by observational evidence that suggests that Itokawa is likely
510 the result of fragmentation of a larger parent body (Tsuchiyama et al., 2011).

511 As can be seen in Figure 7, there is great variation in the strength regime scaling of various
512 authors, all of which adequately agree with the very small scale laboratory observations used to tie
513 the strength scaling to the axes. This highlights the effect that the assumptions regarding strength
514 and failure can have on a predicted scaling outcome. It is our hope that simulations incorporating
515 micromechanical approaches that account for the multiaxial stress states and damage anisotropy
516 will provide new insight into the impact processes shaping our solar system.

517 **4. Dynamic Fragmentation**

518 The rapid deposition of impactor kinetic energy to the interacting bodies during impact results
519 in the initiation of cracks, voids or shear bands from internal defects, and these failure processes
520 then interact and coalesce to form fragments. Under quasi-static loading, fragmentation is domi-
521 nated by the growth of a few dominant cracks (Rong et al., 1979). Tens of thousands of fragments

522 may still be generated under nominally low-rate conditions (Hogan et al., 2012). Under dynamic
523 loading, many more nucleation sites are activated (Zhou et al., 2006b), resulting in decreasing
524 fragment sizes for increasing strain rates (Grady, 2009b). This fragmentation occurs across many
525 length scales, ranging from the order of the body size down to the micro-scale (e.g., order of
526 minimum defect spacing).

527 The inherent limits on experimental, numerical and observational resolution results in atten-
528 tion being primarily given to larger scales, but the smaller scales dominate the nucleation problem,
529 and may carry a significant part of the energy at ultra-high strain rates. In experiments, stud-
530 ies are often concerned with quantifying the largest fragment size as, for example, a measure of
531 catastrophic disruption (Fujiwara et al., 1977; Ryan et al., 1991). Measurements of complete dis-
532 tributions are less frequent and characterization of fragments < 1 mm are not widely performed. In
533 simulations, the numerical resolution sets the minimum resolvable fragment size in the absence of
534 a post-processing fragmentation step. For planetary and space science observational data, instru-
535 ment resolution limitations typically prevent documenting sub-meter fragments on, for example,
536 Itokawa (Fujiwara et al., 2006). Examining these larger fragmentation scales may be sufficient for
537 interpreting some planetary impacts (e.g., Sudbury (Zieg and Marsh, 2005)). Better links between
538 laboratory experiments, numerical simulations, and observational data are needed to bridge these
539 fragmentation length scales. In this section we explore some of these links by examining two im-
540 portant parts of dynamic brittle fragmentation: the average fragment size, and size distributions.
541 A further discussion of many of the topics may be found in the book by Grady (2006), and an
542 additional resource on both ductile and brittle fragmentation is provided by Grady (2009a).

543 *4.1. Fragment size distributions*

544 Statistical and geometric approaches have been pursued to predict fragmentation distributions
545 when experimental results were not available. Lienau (1936) randomly partitioned lines to in-
546 vestigate fragment distributions. Mott and Linfoot (1943) predicted distributions by randomly
547 partitioning geometric shapes with lines. In later work, Grady and Kipp (1985) noted Poisson,
548 binomial, log normal, and Weibull fragmentation distributions can be obtained using similar geo-
549 metric approaches. Grady (2008) noted that these distribution shapes have a strong dependence on
550 material type, where ductile materials appear to be better characterized with an exponential form

551 and brittle materials follow a power-law shape. Combinations of exponential and power-law func-
552 tions have also been explored in the literature. The origins of these various forms for the fragment
553 size distribution are presented in the reviews by Åström (2006) and the book chapter by Grady
554 (2009a).

555 Much interest in brittle fragmentation has been focused on power-law fragment size distribu-
556 tions because of the link to scale-invariance (Turcotte, 1993). Power-law distributions can be mod-
557 eled as a cascade of breakups (Astrom, 2006) and have the form $N(L) \propto L^{-n}$ where the exponent
558 n is the fractal dimension. In the planetary and space science community, the slopes of power-law
559 distributions is used to constrain collision evolution (Davis et al., 1979; Mazrouei et al., 2014).
560 This fractal dimension ranges between 1.5 to 2.5 for experiments with brittle materials (Hogan
561 et al., 2013a, 2012; Grady, 2009a), 3.1 to 3.5 on Itokawa (Mazrouei et al., 2014), and between 2.2
562 and 2.7 in catastrophic disruption simulations (Jutzi et al., 2010). Values of approximately 2 may
563 indicate the fragmentation process is mainly surface driven, whereas fractal dimensions closer to
564 3 suggest the damage is more spatially distributed (Taşdemir, 2009). We emphasize that power-
565 law exponents are strongly related to the measurement or numerical resolution. Grady (2009b)
566 suggested that there exist two governing length scales, λ_e and λ_c , which bound the region of the
567 cumulative distribution described by a power-law function. This idea was explored for dynamic
568 fragmentation of granite by Hogan et al. (2012).

569 *4.2. Characteristic fragment sizes*

570 In addition to computing the fragment size distribution, the prediction of a characteristic length
571 scale, λ , is central to understanding brittle fragmentation events. The prediction of λ enables, for
572 example, fragmentation distributions to be normalized and linked across laboratory experiments
573 and numerical simulations of much larger scales. This characteristic length scale is often taken to
574 be the average or median fragment size. Early works on brittle fragmentation sought to develop
575 relationships between energy input and resulting fragment size (Bond, 1961; Hukki, 1961; von
576 Rittinger, 1876; Kick, 1885; Bergstrom et al., 1961; Bergstrom, 1962; Gilvarry, 1961). Linking
577 energy dissipation with fragmentation was determined to be more robust than geometric argu-
578 ments, although underlying microstructural effects were not yet considered. In this review, we
579 consider theories developed for predicting rate-dependent average fragment sizes developed by

580 Grady (2009b), Glenn and Chudnovsky (1986), and Zhou et al. (2006a,b). These are developed for
 581 fracture-dominated failure processes. Fragment sizes associated with adiabatic shear band failure
 582 follow a different scaling law than that for brittle fracture, as discussed by Zhou et al. (2006c),
 583 following on from the work of Wright and Ockendon (1996) and Grady and Kipp (1987).

584 The average fragment size according to Grady is (Grady, 2009b):

$$L_{Grady} = \left(\frac{48G_c}{\rho\dot{\epsilon}^2} \right)^{1/3} \quad (5)$$

585 where the fracture energy is related to the square of the fracture toughness K_{IC} . This strain-rate
 586 scaling of the average fragment size is used extensively in the community, but, as we will show
 587 later, experimental data on the fragmentation of some planetary materials has demonstrated that
 588 this approximation strongly overestimates mean fragment sizes at low rates of deformation ($<$
 589 10^3 s^{-1}) and indeed scales incorrectly with strain rate at these low rates. In contrast, the approach
 590 seems to have the correct scaling with strain rate at very high strain rates ($> 10^5 \text{ s}^{-1}$). Glenn and
 591 Chudnovsky (1986) extended the work by Grady to include the elastic strain energy contribution,
 592 which is important at low rates, and predicted a quasi-static average fragment size that is essentially
 593 independent of strain rate for low strain rates. They suggested that the average fragment size, L_{GC} ,
 594 be calculated as:

$$L_{GC} = 4 \sqrt{\frac{3}{\alpha}} \sinh\left(\frac{\phi}{3}\right) \quad (6)$$

595 where

$$\phi = \sinh^{-1} \left[\beta \left(\frac{3}{\alpha} \right)^{3/2} \right] \quad (7)$$

596 and $\alpha = \frac{3\sigma_t^2}{\rho E \dot{\epsilon}^2}$, $\beta = \frac{3G_c}{2\rho \dot{\epsilon}^2}$, with E the Young's modulus and σ_t is the tensile strength of the material.

597 A different approach to predicting average rate-dependent fragmentation sizes uses computa-
 598 tional tools that explicitly capture both energetics and dynamics, and that account for the residual
 599 damage within fragments as well as the residual kinetic energy associated with wave propagation
 600 within fragments. These are process-driven models rather than end-state models. Miller et al.
 601 (1999) used cohesive elements in a one-dimensional finite element scheme to investigate brittle
 602 fragmentation. These simulations predicted average fragment sizes an order of magnitude smaller
 603 than those obtained using the energy-based models. Drugan (2001) accounted for wave interac-

604 tions for one-dimensional fragmentation of a bar, and predicted average fragment sizes smaller
605 than the Glenn and Chudnovsky (1986) predictions at intermediate rates, while converging with
606 the Grady (2009b) and Glenn and Chudnovsky (1986) models at very high strain rates. This work,
607 as well as the study by Shenoy and Kim (2003) using cohesive elements as simulated defects, es-
608 tablished the potential importance of wave interactions during fragmentation of brittle materials.
609 The incorporation of elastic wave propagation and interactions, crack nucleation and growth, as
610 well as defect distributions in a fragmentation simulation was pursued by Zhou et al. (2006a,b)
611 (ZMR) for a much larger range of strain rates than Drugan (2001) and Shenoy and Kim (2003).
612 A range of brittle material properties was examined, and a normalizing strain rate measure was
613 identified. All of the material behaviors were shown to collapse to a single rate-dependent curve
614 for the rate average fragment size. In addition to fragment sizes, Zhou et al. (2006a,b) also derive
615 characteristic time ($t_0 = \frac{EG_c}{\sigma_c^2}$), length ($L_0 = ct_0$) and strain rate ($\dot{\epsilon}_0 = \frac{\sigma_c}{Et_0}$) terms used to nor-
616 malize rate-dependent size predictions, and this resulted in collapsing of size predictions across a
617 wide range of brittle material properties. At low rates, the ZMR model predicts average fragment
618 sizes that are larger than those predicted by Glenn and Chudnovsky (1986) and at higher rates, the
619 ZMR model predicts fragment sizes that are approximately 1/3 to 1/5 the average size predicted by
620 Grady. More sophisticated simulations by Levy and Molinari (2010) extended these computational
621 approaches and included the effect of the initial defect distribution to normalize the average frag-
622 ment size predictions. Very recent work by Bazant and Caner (2014) provides further advances in
623 understanding of the scaling of fragmentation with respect to strain rate.

624 *4.3. Comparing experiments and fragmentation models*

625 It is nontrivial to compare fragmentation experiments with fragmentation models, because the
626 experimental approaches used to generate fragments almost always result in stress states that are
627 much more complicated than the stress states assumed in the models. For example, most ex-
628 perimental methods associated with impact generate initially compressive stress states that are
629 expected to produce internal failures and this seeds the subsequent fragmentation process. Frag-
630 ments can be developed from macroscopically compressive states within which very large amounts
631 of strain energy can be stored, and the release of that strain energy can generate very fine fragments.
632 Most experimental methods produce a wide range of strain rates as well, so it can be difficult to

633 decide what strain rate to use for comparison with a model.

634 Published fragment-size distributions exist, e.g. (Hogan et al., 2013a, 2012) for impact test-
635 ing. However, the corresponding average fragment sizes were plotted at a strain rate estimated
636 by the ratio of impact velocity to target thickness, without accounting for the difference between
637 compressive and tensile states or the variation in strain rate within the sample. This demonstrates
638 the difficulty of comparison of experiments and models arising from the complexity of the exper-
639 imental stress and strain rate distributions. Similarly, in their experimental study on the uniaxial
640 compression of SiC-N, Wang and Ramesh (2004) plot their average fragment size against the com-
641 pressive loading rates. However, the equivalent tensile rate is not equal to the compressive loading
642 rate. In another study, Hogan et al. (2013b) plotted median fragment sizes using a mass-size rep-
643 resentation but compared the sizes to a model that used a number-size representation. The lesson
644 here is that there are many sizes and strain rates for which to compare models with, and one should
645 keep this in mind when conclusions are made.

646 Here we define a specific approach to compare experimental fragmentation results obtained
647 from compression Kolsky bar experiments on basalt (Hogan et al., 2015) with theoretical predic-
648 tions. Grady and Lipkin (1980) also used Kolsky bar experiments to study the fragmentation of
649 planetary materials. This particular testing technique allows fragmentation distributions to be di-
650 rectly linked with material strength measurements under well-defined stress-state and strain-rate
651 loading conditions, but the comparison of compressive and tensile states must still be made. We
652 choose to define an equivalent tensile problem by converting the strain energy in the compression
653 problem to the kinetic energy in an equivalent expanding ring. A comparison of experimentally
654 measured sizes with models also requires deciding what strain-rates to use for comparison. Here
655 we define an equivalent tensile strain rate ($\dot{\epsilon}_{equi}$), since the models all assume tension. We define
656 the equivalent tensile strain rate by defining an equivalent expanding ring problem with:

$$\dot{\epsilon}_{equi} = \frac{V}{R} \quad (8)$$

657 where R (m) is the equivalent expanding ring radius and V (m/s) is the velocity of the expan-
658 sion of the equivalent expanding ring. We can estimate V by assuming that the strain energy in
659 compression is converted to the kinetic energy of an expanding ring. The strain energy (W) in

660 compression is given as:

$$W = \left[\frac{1}{2} \int \bar{\sigma} d\epsilon \right] \nabla = \frac{1}{2} \frac{\sigma t^3}{E} \quad (9)$$

661 where ϵ is the strain, $\bar{\sigma}$ is the effective stress (Pa), ∇ is the volume (m^3) and t is the specimen size
 662 (m) (here we are assuming a cube). In uniaxial compression $\bar{\sigma}$ is equal to the compressive strength.
 663 The kinetic energy of an equivalent expanding ring is given as:

$$KE_{ring} = \frac{1}{2} m V_{ring}^2 = \frac{1}{2} \rho (2\pi r) R t V_{ring}^2 = \pi \rho R^2 t V_{ring}^2 \quad (10)$$

664 where m is the mass (kg). Equating these energies ($W=KE$) and solving for V_{ring} , we find:

$$V_{ring} = \sqrt{\frac{\bar{\sigma}^2 t^2}{\pi \rho R^2 E}} \quad (11)$$

665 and correspondingly

$$\dot{\epsilon}_{equi} = \frac{V_{ring}}{R} = \sqrt{\frac{\bar{\sigma}^2 t^2}{\pi \rho R^4 E}} \quad (12)$$

666 We assume that R is 10x the specimen length. Other radii may be assumed (e.g., 30x speci-
 667 men length), but our results are relatively insensitive to this change because the applied rate is so
 668 low. In the calculation of the model predictions, we use material properties of basalt of $\rho=2,870$
 669 kg/m^3 (Stickle et al., 2013), $E=70$ GPa (Stickle et al., 2013) and $K_{1c}=1.6$ MPa \sqrt{m} (Balme et al.,
 670 2004). Following convention, we take σ_t to be 1/10 of the material's quasi-static compressive
 671 strength, which is approximately 400 MPa (Stickle et al., 2013).

672 Shown in Figure 8 are the experimental results from Hogan et al. (2015) and comparisons with
 673 the models using the equivalence arguments made above. Note we normalize by the characteristic
 674 length (L_0) and strain rates ($\dot{\epsilon}_0$) proposed by Zhou et al. (2006a,b). We see from Figure 8 that
 675 current models all over-estimate average fragment sizes. We also point out that the models of
 676 Grady (2009b) predict much larger sizes than those of Glenn and Chudnovsky (1986), and Zhou
 677 et al. (2006a,b) at these strain rates. In a major planetary impact event, the strain rates produced
 678 can be very high over relatively small volumes (close to the source domain in Figure 1), but the
 679 rates are much lower over much larger volumes that can participate in the fragmentation process.
 680 Thus models such as the Grady-Kipp model may significantly overestimate the fragment sizes over

681 much of the volume. For example, Fig 9A shows the results of computed strain rates from a CTH
682 simulation (Ernst et al., 2009) of a 1km body impacting a half-space at 5km/s, and Fig 9B shows
683 the corresponding strain rate domains in comparison with two fragmentation models. In this case
684 the ZMR model predicts the development of much smaller fragments that are more in line with the
685 large amounts of fines observed near many impact craters. Fig 9B also shows the importance of
686 using updated fragmentation models in the consideration of such events.

687 Increasingly sophisticated models incorporating, for example, continuum elasticity, inelastic-
688 ity, and damage mechanics have been developed to capture the behavior of brittle materials (Clay-
689 ton, 2008; Tonge et al., 2013). However, none of these approaches directly couple fragmentation to
690 the deformation, so that in practice fragment sizes are estimated by stopping the simulation at some
691 ad hoc time, estimating the strain rate distribution, and then using post-processing calculations to
692 extract fragment distributions. This decoupled approach has the major disadvantage of providing
693 solutions that depend on the time chosen to estimate the onset of fragmentation. The inclusions
694 of fracture-inducing heterogeneities (Kraft et al., 2008; Kraft and Molinari, 2008) to naturally ac-
695 count for nucleation, the coupling of fragmentation models to the internal variable theory, and the
696 ability to obtain further fragmentation through granular flow would greatly improve the reliability
697 of such simulations.

698 **5. Summary**

699 This paper summarizes current mechanisms and models for dynamic failure, strength, and frag-
700 mentation, reviewing these from a mechanics perspective and with an emphasis on making links to
701 the developing advances in these areas in the engineering and computational mechanics commu-
702 nities. We believe that the effective incorporation of failure processes into large-scale impact sim-
703 ulations through micromechanics-based approaches presents a great opportunity for advancement
704 in the fidelity of simulations of impact and disruption. In this paper, we advocate for a consistent
705 multiscale approach to modeling strength, failure and fragmentation in the context of large-scale
706 numerical simulations, with particular attention paid to the handoff between strength models and
707 failure models in relation to the numerical resolution. When the failure processes are subscale
708 to the computational resolution, the consequence is effective behavior (such as strength) that is
709 anisotropic, size-dependent and rate-dependent. Advances in theoretical descriptions of these be-

710 haviors and advances in computational mechanics approaches to multiscale modeling have great
711 promise for producing higher-fidelity simulations of large-scale impact and disruption events.

712 **6. Acknowledgements**

713 This work was supported in part by the NASA Planetary Geology and Geophysics program,
714 and in part by the Hopkins Extreme Materials Institute. KTR acknowledges also the support of the
715 Materials in Extreme Dynamic Environments Collaborative Research Alliance through Coopera-
716 tive Agreement Number W911NF-12-2-0022.

717 **7. Reference**

- 718 Anderson, T., 2004. *Fracture Mechanics: Fundamentals and Applications*. CRC Press.
- 719 Antoun, T., Seaman, L., Curran, D., Kanel, G., Razorenov, S., A.V., U., 2003. *Spall Fracture*
720 *(Shock Wave and High Pressure Phenomena)*. Springer.
- 721 Ashby, M., Cooksley, S. H. N., 1986. The failure of brittle solids containing small cracks under
722 compressive stress states. *Acta Metallurgica* 34 (3), 497 – 510.
- 723 Astrom, J. A., 2006. Statistical models of brittle fragmentation. *Advances in Physics* 55 (3-4),
724 247–278.
- 725 Balme, M., Rocchi, V., Jones, C., Sammonds, P., Meredith, P., Boon, S., 2004. Fracture toughness
726 measurements on igneous rocks using a high-pressure, high-temperature rock fracture mechan-
727 ics cell. *Journal of Volcanology and Geothermal Research* 132 (23), 159 – 172, parameterisation
728 and Modelling of Lava Flows.
- 729 Bazant, Z., Caner, F., 2014. Impact comminution of solids due to local kinetic energy of high shear
730 strain rate: I. continuum theory and turbulence analogy. *Journal of the Mechanics and Physics*
731 *of Solids* 64 (17), 223–235.
- 732 Benz, W., Asphaug, E., 1994. Impact simulations with fracture. i. method and tests. *Icarus* 107 (1),
733 98–116.
- 734 Benzerga, A., Leblond, J.-B., 2010. Ductile fracture by void growth to coalescence. *Advances in*
735 *Applied Mechanics* 44, 169–305.
- 736 Bergstrom, B., 1962. Energy and size distribution aspects of single particle crushing. In: *Proc. 5th*
737 *Symp. Rock Mechanics*. pp. 155–72.
- 738 Bergstrom, B., Sollenberger, C., Mitchell Jr, W., 1961. Energy aspects of single particle crushing.
739 *Trans. AIME* 220, 367–372.
- 740 Binzel, R. P., Xu, S., 1993. Chips off of asteroid 4 vesta: Evidence for the parent body of basaltic
741 achondrite meteorites. *Science* 260 (5105), 186–191.

742 Bond, F. C., 1961. Crushing and grinding calculations. Allis-Chalmers Manufacturing Company.

743 Bottke Jr, W. F., Vokrouhlický, D., Rubincam, D. P., Broz, M., 2002. The effect of yarkovsky
744 thermal forces on the dynamical evolution of asteroids and meteoroids. *Asteroids III* 395.

745 Broberg, K., 1999. *Cracks and Fracture*. Academic Press.

746 Burbine, T., Buchanan, P., Binzel, R., Bus, S., Hiroi, T., Hinrichs, J., Meibom, A., McCoy, T.,
747 2001. Vesta, vestoids, and the howardite, eucrite, diogenite group: Relationships and the origin
748 of spectral differences. *Meteoritics and Planetary Science* 36 (6), 761–781.

749 Camacho, G., Ortiz, M., 1996. Computational modelling of impact damage in brittle materials.
750 *International Journal of Solids and Structures* 33 (20), 2899–2938.

751 Carroll, M., Holt, A., 1972. Static and dynamic pore-collapse relations for ductile porous materials.
752 *Journal of Applied Physics* 43 (4), 1626–1636.

753 Chakravarthy, S., Wang, C., 2010. Origin of plasticity length-scale effects in fracture. *Physical Re-*
754 *view Letters* 105, 115502.

755 Chapman, C. R., 1975. The nature of asteroids. *Scientific American* 232, 24–33.

756 Chen, M., McCauley, J. W., Hemker, K. J., 2003. Shock-induced localized amorphization in boron
757 carbide. *Science* 299 (5612), 1563–1566.

758 Clayton, J. D., 2008. A model for deformation and fragmentation in crushable brittle solids. *Inter-*
759 *national Journal of Impact Engineering* 35 (5), 269 – 289.

760 Daphalapurkar, N. P., Ramesh, K. T., Graham-Brady, L., Molinari, J.-F., Feb. 2011. Predicting
761 variability in the dynamic failure strength of brittle materials considering pre-existing flaws.
762 *Journal of the Mechanics and Physics of Solids* 59 (2), 297–319.

763 Davis, D., Chapman, C., Greenberg, R., Weidenschilling, S., Harris, A., 1979. Collisional evolu-
764 tion of asteroids-populations, rotations, and velocities. *Asteroids 1*, 528–557.

765 Davis, D. R., Ryan, E. V., Farinella, P., 1994. Asteroid collisional evolution: Results from current
766 scaling algorithms. *Planetary and Space Science* 42 (8), 599–610.

767 de Borst, R., Gutierrez, M., 1999. Discrete vs smeared crack models for concrete fracture: bridging
768 the gap. *International Journal of Fracture* 95, 261–277.

769 de Borst, R., Remmers, J., Needleman, A., Abellan, M., 2004. Discrete vs smeared crack mod-
770 els for concrete fracture: bridging the gap. *International Journal for Numerical and Analytical*
771 *Methods in Geomechanics* 28, 583–607.

772 Deng, H., Nemat-Nasser, S., 1992. Dynamic damage evolution in brittle solids. *Mechanics of*
773 *Materials* 14, 83–104.

774 Deshpande, V. S., Gamble, E., Compton, B. G., McMeeking, R. M., Evans, A. G., Zok, F. W.,
775 2011. A constitutive description of the inelastic response of ceramics. *Journal of the American*
776 *Ceramic Society* 94 (s1), s204–s214.

777 Dodd, B., Bai, Y., 2012. *Adiabatic shear localization: frontiers and advances*. Elsevier.

778 DORAZIO, M., FOLCO, L., ZEOLI, A., CORDIER, C., 2011. Gebel kamil: The iron meteorite
779 that formed the kamil crater (egypt). *Meteoritics and Planetary Science* 46 (8), 1179–1196.
780 URL <http://dx.doi.org/10.1111/j.1945-5100.2011.01222.x>

781 Drugan, W., 2001. Dynamic fragmentation of brittle materials: analytical mechanics-based mod-
782 els. *Journal of the Mechanics and Physics of Solids* 49 (6), 1181 – 1208.

783 Durda, D. D., Greenberg, R., Jedicke, R., 10 1998. Collisional models and scaling laws: A new
784 interpretation of the shape of the main-belt asteroid size distribution. *Icarus* 135 (2), 431–440.

785 Embury, J., Nicholson, R., 1965. The nucleation of precipitates: the system al-zn-mg. *Acta Metal-*
786 *lurgica* 13 (4), 403–417.

787 Ernst, C. M., Barnouin-Jha, O. S., Ramesh, K. T., Swaminathan, P. K., Kimberley, J., 2009. Strain
788 rate and dynamic fracturing in planetary-scale impacts. In: *Lunar and Planetary Institute Science*
789 *Conference Abstracts*. Vol. 40.

790 Field, J., 2007. Spall strength of geomaterials. In: *Shock Compression of Condensed Matter*.

791 Fish, J., 2013. *Practical Multiscaling*. Wiley.

- 792 Fossum, A. F., Brannon, R. M., 2004. The sandia geomodel: theory and users guide.
- 793 Freund, L., 1998. Dynamic Fracture Mechanics. Cambridge University Press.
- 794 Frew, D. J., Forrestal, M. J., Chen, W., Mar 2001. A split hopkinson pressure bar technique to
795 determine compressive stress-strain data for rock materials. *Experimental Mechanics* 41 (1),
796 40–46.
- 797 Fujiwara, A., Kamimoto, G., Tsukamoto, A., 1977. Destruction of basaltic bodies by high-velocity
798 impact. *Icarus* 31 (2), 277 – 288.
- 799 Fujiwara, A., Kawaguchi, J., Yeomans, D., Abe, M., Mukai, T., Okada, T., Saito, J., Yano, H.,
800 Yoshikawa, M., Scheeres, D., et al., 2006. The rubble-pile asteroid itokawa as observed by
801 hayabusa. *Science* 312 (5778), 1330–1334.
- 802 Fujiwara, A., Tsukamoto, A., 1980. Experimental study on the velocity of fragments in collisional
803 breakup. *Icarus* 44 (1), 142 – 153.
- 804 Furnish, M., Chhabildas, L., Reinhart, W., Trott, W., Vogler, T., 2009. Determination and inter-
805 pretation of statistics of spatially resolved waveforms in spalled tantalum from 7 to 13 gpa.
806 *International Journal of Plasticity* 25, 587–602.
- 807 Gault, D., 1973. Displaced mass, depth, diameter, and effects of oblique trajectories for impact
808 craters formed in dense crystalline rocks. *The moon* 6 (1-2), 32–44.
- 809 Gault, D., Wedekind, J., 1969. The destruction of tektites by micrometeoroid impact. *J. Geophys.*
810 *Res.* 74 (27), 6780–6794, cited By (since 1996)30.
- 811 Ghosh, S., 2011. Multiscale Characterization and Domain Partitioning for Multiscale Analysis of
812 Heterogeneous Materials. Springer.
- 813 Gilvarry, J., 1961. Fracture of brittle solids. i. distribution function for fragment size in single
814 fracture (theoretical). *Journal of Applied Physics* 32 (3), 391–399.
- 815 Glenn, L. A., Chudnovsky, A., Feb. 1986. Strain and energy effects on dynamic fragmentation.
816 *Journal of Applied Physics* 59 (4), 1379 –1380.

- 817 Grady, D., 2009a. Dynamic fragmentation of solids. In: Shock Wave Science and Technology
818 Reference Library, Vol. 3. Springer Berlin Heidelberg, pp. 1–108.
- 819 Grady, D., Kipp, M., 1985. Geometric statistics and dynamic fragmentation. *Journal of Applied*
820 *Physics* 58 (3), 1210–1222.
- 821 Grady, D., Kipp, M., 1987. The growth of unstable thermoplastic shear with application to steady-
822 wave shock compression in solids. *Journal of the Mechanics and Physics of Solids* 35, 95119.
- 823 Grady, D. E., 2006. *Fragmentation of rings and shells: the legacy of N.F. Mott*. Springer.
- 824 Grady, D. E., 2008. Fragment size distributions from the dynamic fragmentation of brittle solids.
825 *International Journal of Impact Engineering* 35 (12), 1557 – 1562, hypervelocity Impact Pro-
826 ceedings of the 2007 Symposium - HVIS 2007.
- 827 Grady, D. E., 2009b. Length scales and size distributions in dynamic fragmentation. *International*
828 *Journal of Fracture* 163 (1–2), 85–99.
- 829 Grady, D. E., Lipkin, J., 1980. Criteria for impulsive rock fracture. *Geophysical Research Letters*
830 7 (4), 255–258.
- 831 Graham-Brady, L., 2010. Statistical characterization of meso-scale uniaxial compressive strength
832 in brittle materials with randomly occurring flaws. *International Journal of Solids and Structures*
833 47 (18-19), 2398–2413.
- 834 Green, S. J., Perkins, R. D., 1969. Uniaxial compression tests at strain rates from 0.0001/sec. to
835 1000/sec. on three geologic materials. Technical Report MSL-68-6, General Motors Technical
836 Center Warren Mich Materials and Structures Lab, United States.
- 837 Guy, N., Seyed, D. M., Hild, F., 2012. A probabilistic nonlocal model for crack initiation and
838 propagation in heterogeneous brittle materials. *International Journal for Numerical Methods in*
839 *Engineering* 90 (8), 1053–1072.
- 840 Hogan, J., Plescia, J., El Mir, C., Ramesh, K., Apr. 2015. Dynamic brittle fragmentation: Probing
841 the byproducts of hypervelocity impact in space. In: 2015 Hypervelocity Impact Symposium
842 (HVIS 2015). Boulder, USA.

843 Hogan, J. D., Castillo, J. A., Rawle, A., Spray, J. G., Rogers, R. J., 2013a. Automated microscopy
844 and particle size analysis of dynamic fragmentation in natural ceramics. *Engineering Fracture*
845 *Mechanics* 98 (0), 80 – 91.

846 Hogan, J. D., Rogers, R. J., Spray, J. G., Boonsue, S., 2012. Dynamic fragmentation of granite for
847 impact energies of 6 to 28 j. *Engineering Fracture Mechanics* 79 (0), 103 – 125.

848 Hogan, J. D., Spray, J. G., Rogers, R. J., Vincent, G., Schneider, M., 2013b. Dynamic fragmenta-
849 tion of planetary materials: Ejecta length quantification and semi-analytical modelling. *Interna-*
850 *tional Journal of Impact Engineering* 62 (0), 219 – 228.

851 Holsapple, K., 1993. The scaling of impact processes in planetary sciences. *Annual Review of*
852 *Earth and Planetary Sciences* 21, 333–373.

853 Holsapple, K., Dec. 1994a. Catastrophic disruptions and cratering of solar-system bodies - a review
854 and new results. *Planetary and Space Science* 42 (12), 1067–1078.

855 Holsapple, K., 1994b. Catastrophic disruptions and cratering of solar system bodies: A review and
856 new results. *Planetary and Space Science* 42 (12), 1067–1078.

857 Holsapple, K., Giblin, I., Housen, K., Nakamura, A., Ryan, E., 2002. Asteroid impacts: Laboratory
858 experiments and scaling laws. *Asteroids III* 1, 443–462.

859 Holsapple, K. A., 2 2009. On the "strength" of the small bodies of the solar system: A review
860 of strength theories and their implementation for analyses of impact disruptions. *Planetary and*
861 *Space Science* 57 (2), 127–141.

862 Horii, H., Nemat-Nasser, S., 1985. Compression-induced microcrack growth in brittle solids: Axial
863 splitting and shear failure. *Journal of Geophysical Research: Solid Earth (1978–2012)* 90 (B4),
864 3105–3125.

865 Housen, K., Schmidt, R., Holsapple, K., 1983. Crater ejecta scaling laws: Fundamental forms
866 based on dimensional analysis. *Journal of Geophysical Research: Solid Earth (1978–2012)*
867 88 (B3), 2485–2499.

- 868 Housen, K. R., Holsapple, K. A., 1999. Scale effects in strength-dominated collisions of rocky
869 asteroids. *Icarus* 142 (1), 21 – 33.
- 870 Hu, G., Ramesh, K., Cao, B., McCauley, J., 2011. The compressive failure of aluminum nitride
871 considered as a model advanced ceramic. *Journal of the Mechanics and Physics of Solids* 59 (5),
872 1076–1093.
- 873 Huang, C., Subhash, G., 2003. Influence of lateral confinement on dynamic damage evolution
874 during uniaxial compressive response of brittle solids. *Journal of the Mechanics and Physics of*
875 *Solids* 51, 1089–1105.
- 876 Hukki, R., 1961. Proposal for a solomonic settlement between the theories of von rittinger, kick,
877 and bond. *Trans. AIME* 220, 403–408.
- 878 Issen, K., Rudnicki, J., 2000. Conditions for compaction bands in porous rock. *Journal of Geo-*
879 *physical Research* 105, 21529–21536.
- 880 Jacques, N., Mercier, S., Molinari, A., 2012. Void coalescence in a porous solid under dynamic
881 loading conditions. *International Journal of Fracture* 173, 203–213.
- 882 Jutzi, M., Benz, W., Michel, P., 2008. Numerical simulations of impacts involving porous bodies:
883 I. implementing sub-resolution porosity in a 3d sph hydrocode. *Icarus* 198 (1), 242–255.
- 884 Jutzi, M., Michel, P., Benz, W., Richardson, D. C., 2010. Fragment properties at the catastrophic
885 disruption threshold: The effect of the parent bodys internal structure. *Icarus* 207 (1), 54 – 65.
- 886 Kachanov, M., 2003. On the problems of crack interactions and crack coalescence. *International*
887 *Journal of Fracture* 120, 537–543.
- 888 Katcoff, C., Graham-Brady, L., 2014. Modeling dynamic brittle behavior of materials with circular
889 flaws or pores. *International Journal of Solids and Structures* 51 (3), 754–766.
- 890 Kick, F., 1885. *Das Gesetz der proportionalen Widerstände und seine Anwendungen*. Felix.
- 891 Kimberley, J., Ramesh, K., Daphalapurkar, N., 2013. A scaling law for the dynamic strength of
892 brittle solids. *Acta Materialia* 61 (9), 3509 – 3521.

893 Kimberley, J., Ramesh, K. T., 2011. The dynamic strength of an ordinary chondrite. *Meteoritics*
894 *and Planetary Science* 46 (11), 1653–1669.

895 Kraft, R., Molinari, J., 2008. A statistical investigation of the effects of grain boundary properties
896 on transgranular fracture. *Acta Materialia* 56 (17), 4739 – 4749.

897 Kraft, R., Molinari, J., Ramesh, K., Warner, D., 2008. Computational micromechanics of dynamic
898 compressive loading of a brittle polycrystalline material using a distribution of grain boundary
899 properties. *Journal of the Mechanics and Physics of Solids* 56 (8), 2618 – 2641.

900 Kumar, A., 06 1968. The effect of stress rate and temperature on the strength of basalt and granite.
901 *Geophysics* 33 (3), 501–510.

902 Le Corre, L., Reddy, V., Becker, K., O’Brien, D., Palmer, E., Li, J., Gaskell, R., Denevi, B.,
903 Nathues, A., Sierks, H., et al., 2012. Nature of orange ejecta around oppia and octavia craters on
904 vesta from dawn framing camera. In: *AAS/Division for Planetary Sciences Meeting Abstracts*.
905 Vol. 44.

906 Levy, S., Molinari, J., 2010. Dynamic fragmentation of ceramics, signature of defects and scaling
907 of fragment sizes. *Journal of the Mechanics and Physics of Solids* 58 (1), 12 – 26.

908 Lienau, C., 1936. Random fracture of a brittle solid. *Journal of the Franklin Institute* 221 (4), 485
909 – 494.

910 Liu, Y. e. a., 2014. A regularized phenomenological multiscale damage model. *INTERNATIONAL*
911 *JOURNAL FOR NUMERICAL METHODS IN ENGINEERING* 99, 867–887.

912 Martelli, G., Ryan, E., Nakamura, A., Giblin, I., 1994. Catastrophic disruption experiments: Re-
913 cent results. *Planetary and Space Science* 42 (12), 1013–1026.

914 Mazrouei, S., Daly, M., Barnouin, O., Ernst, C., DeSouza, I., 2014. Block distributions on itokawa.
915 *Icarus* 229, 181–189.

916 Melosh, H., Ryan, E., Asphaug, E., 1992. Dynamic fragmentation in impacts: Hydrocode simu-
917 lation of laboratory impacts. *Journal of Geophysical Research: Planets* (1991–2012) 97 (E9),
918 14735–14759.

- 919 Meyers, M., 1994. *Dynamic Behavior of Materials*. Wiley Interscience.
- 920 Miller, O., Freund, L., Needleman, A., 1999. Modeling and simulation of dynamic fragmentation
921 in brittle materials. *International Journal of Fracture* 96 (2), 101–125.
- 922 Moës, N., Cloirec, M., Cartraud, P., Remacle, J.-F., 2003. A computational approach to handle
923 complex microstructure geometries. *Computer methods in applied mechanics and engineering*
924 192 (28), 3163–3177.
- 925 Molinari, A., Clifton, R., 1987. Analytical characterization of shear localization in thermovis-
926 coplastic materials. *Journal of Applied Mechanics* 54, 806–812.
- 927 Molinari, A., Mercier, S., 2001. Micromechanical modelling of porous materials under dynamic
928 loading. *Journal of the Mechanics and Physics of Solids* 49, 1497–1516.
- 929 Mori, T., Meshii, M., 1964. On the nucleation of vacancy clusters in gold. *Acta metallurgica* 12 (1),
930 104–106.
- 931 Mott, N. F., Linfoot, E. H., February 1943. A theory of fragmentation. Tech. Rep. vol. AC3348,
932 United Kingdom Ministry of Supply, UK.
- 933 Nakamura, A., Fujiwara, A., 1991. Velocity distribution of fragments formed in a simulated colli-
934 sional disruption. *Icarus* 92 (1), 132–146.
- 935 Nemat-Nasser, S., 2009. *Plasticity: A Treatise on Finite Deformation of Heterogeneous Inelastic*
936 *Materials*. Cambridge University Press.
- 937 Paliwal, B., Ramesh, K., 2008. An interacting micro-crack damage model for failure of brittle
938 materials under compression. *Journal of the Mechanics and Physics of Solids* 56 (3), 896–923.
- 939 Pandolfi, A., Ortiz, M., 2012. An eigenerosion approach to brittle fracture. *International Journal*
940 *for Numerical Methods in Engineering* 92 (8), 694–714.
- 941 Phillips, R., 2001. *Crystals, Defects and Microstructures: Modeling Across Scales*. Cambridge
942 University Press.

- 943 Platt, J., Rudnicki, J., Rice, J., 2014. Stability and localization of rapid shear in fluid-saturated fault
944 gouge: 2. localized zone width and strength evolution. *Journal of Geophysical Research Solid*
945 *Earth* 119, 43344359.
- 946 Podolak, M., Prialnik, D., 1996. Models of the structure and evolution of comet p/wirtanen. *Plan-*
947 *etary and Space Science* 44 (7), 655 – 664.
- 948 Rong, C., Xiao-Xin, Y., Hung-Sen, X., 1979. Studies of the fracture of gabbro. *International Jour-*
949 *nal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts* 16 (3), 187 – 193.
- 950 Ross, C., Tedesco, J., Kuennen, S., 1995. Effects of strain rate on concrete strength. *ACI Materials*
951 *Journal* 92 (1).
- 952 Rudd, R., Belak, J., 2002. Void nucleation and associated plasticity in dynamic fracture of poly-
953 crystalline copper: an atomistic simulation. *Computational Materials Science* 24, 148–153.
- 954 Rudnicki, J., Rice, J., 1975. Conditions for the localization of deformation in pressure-sensitive
955 dilatant materials. *Journal of Mechanics and Physics of Solids* 23, 371–394.
- 956 Ryan, E. V., 2000. Asteroid fragmentation and evolution of asteroids. *Annual Review of Earth and*
957 *Planetary Sciences* 28 (1), 367–389.
- 958 Ryan, E. V., Hartmann, W. K., Davis, D. R., 1991. Impact experiments 3: Catastrophic fragmenta-
959 tion of aggregate targets and relation to asteroids. *Icarus* 94 (2), 283 – 298.
- 960 Schenk, P., O'Brien, D. P., Marchi, S., Gaskell, R., Preusker, F., Roatsch, T., Jaumann, R.,
961 Buczowski, D., McCord, T., McSween, H. Y., et al., 2012. The geologically recent giant impact
962 basins at vestas south pole. *Science* 336 (6082), 694–697.
- 963 Scully, J., Russell, C., Yin, A., Williams, D., Blewett, D., Buczowski, D., Ammannito, E.,
964 Roatsch, T., Preusker, F., Le Corre, L., et al., 2012. Geologic mapping of the av-4 domitia
965 quadrangle of asteroid 4 vesta. In: *EGU General Assembly Conference Abstracts*. Vol. 14. p.
966 3251.
- 967 Shenoy, V., Kim, K.-S., 2003. Disorder effects in dynamic fragmentation of brittle materials. *Jour-*
968 *nal of the Mechanics and Physics of Solids* 51 (1112), 2023 – 2035.

- 969 Stickle, A., Kimberley, J., Ramesh, K., 2013. Dynamic strength experiments on basalt with appli-
970 cations to cratering on mercury. In: Lunar and Planetary Institute Science Conference Abstracts.
971 Vol. 44. p. 3021.
- 972 Sullivan, R., Greeley, R., Pappalardo, R., Asphaug, E., Moore, J., Morrison, D., Belton, M., Carr,
973 M., Chapman, C., Geissler, P., Greenberg, R., Granahan, J., III, J. H., Kirk, R., McEwen, A.,
974 Lee, P., Thomas, P., Veverka, J., 1996. Geology of 243 ida. *Icarus* 120 (1), 119 – 139.
- 975 Tadmor, R., Miller, R., 2012. *Modeling Materials: Continuum, Atomistic and Multiscale Tech-*
976 *niques*. Cambridge University Press.
- 977 Tang, C., Lina, P., Wong, R., Chau, K., 2001. Analysis of crack coalescence in rock-like materials
978 containing three flaws, part ii: numerical approach. *International Journal of Rock Mechanics &*
979 *Mining Sciences* 38, 925–939.
- 980 Taşdemir, A., 2009. Fractal evaluation of particle size distributions of chromites in different com-
981 minution environments. *Minerals Engineering* 22 (2), 156–167.
- 982 Thompson, A. W., Weihrauch, P. F., 1976. Ductile fracture: nucleation at inclusions. *Scripta Met-*
983 *allurgica* 10 (2), 205–210.
- 984 Tonge, A., Ramesh, K., Barnouin, O., 2014. Large impacts on airless bodies: The himeros event
985 on eros. In: Lunar and Planetary Institute Science Conference Abstracts. Vol. 45. p. 1998.
- 986 Tonge, A. L., Kimberley, J., Ramesh, K., 2013. A consistent scaling framework for simulating high
987 rate brittle failure problems. *Procedia Engineering* 58 (0), 692 – 701.
- 988 Tsuchiyama, A., Uesugi, M., Matsushima, T., Michikami, T., Kadono, T., Nakamura, T., Ue-
989 sugi, K., Nakano, T., Sandford, S. A., Noguchi, R., et al., 2011. Three-dimensional structure of
990 hayabusa samples: origin and evolution of itokawa regolith. *Science* 333 (6046), 1125–1128.
- 991 Turcotte, D. L., 1993. *Fractals and chaos in geology and geophysics*. Cambridge University Press,
992 United Kingdom.

993 Veverka, J., Robinson, M., Thomas, P., Murchie, S., Bell, J., Izenberg, N., Chapman, C., Harch, A.,
994 Bell, M., Carcich, B., et al., 2000. Near at eros: Imaging and spectral results. *Science* 289 (5487),
995 2088–2097.

996 von Rittinger, R., 1876. *Textbook of mineral dressing*. Ernst and Korn.

997 Wang, H., Ramesh, K., 2004. Dynamic strength and fragmentation of hot-pressed silicon carbide
998 under uniaxial compression. *Acta Materialia* 52 (2), 355 – 367.

999 Wilkerson, J., Ramesh, K., 2014. A dynamic void growth model governed by dislocation kinetics.
1000 *Journal of the Mechanics and Physics of Solids* 70, 262–280.

1001 Williams, J., 1989. In: R. P. Binzel, T. G., Matthews, M. S. (Eds.), *Asteroid family identification*
1002 *and proper elements*. In *Asteroids II*. Univ. of Arizona Press, Tucson, p. 10341072.

1003 Wright, T., 2002. *The physics and mathematics of adiabatic shear bands*. Cambridge University
1004 Press.

1005 Wright, T., Ockendon, H., 1996. A scaling law for the effect of inertia on the formation of adiabatic
1006 shear bands. *International Journal of Plasticity* 12, 927–934.

1007 Wright, T., Ramesh, K., 2009. Statistically informed dynamics of void growth in rate dependent
1008 materials. *International Journal of Impact Engineering* 36 (10), 1242 – 1249.

1009 Wu, X., Wright, T., Ramesh, K., 2003. The effects of thermal softening and heat conduction on the
1010 dynamic growth of voids. *International Journal of Solids and Structures* 40 (17), 4461 – 4478.

1011 Xu, X.-P., Needleman, A., 1994. Numerical simulations of fast crack growth in brittle solids.
1012 *Journal of the Mechanics and Physics of Solids* 42 (9), 1397–1434.

1013 Zappal, V., Bendjoya, P., Cellino, A., Farinella, P., Froeschl, C., 1995. Asteroid families: Search of
1014 a 12,487-asteroid sample using two different clustering techniques. *Icarus* 116 (2), 291 – 314.

1015 Zhang, Q., Zhao, J., 2013. A review of dynamic experimental techniques and mechanical behaviour
1016 of rock materials. *Rock Mechanics and Rock Engineering*, 1–68.

1017 Zhang, Q., Zhao, J., 2014. Quasi-static and dynamic fracture behaviour of rock materials: phe-
1018 nomena and mechanisms. *International Journal of Fracture* 189, 1–32.

1019 Zhou, F., Molinari, J. F., Ramesh, K., 2006a. Analysis of the brittle fragmentation of an expanding
1020 ring. *Computational Materials Science* 37 (1-2), 74 – 85.

1021 Zhou, F., Molinari, J. F., Ramesh, K. T., 2006b. Effects of material properties on the fragmentation
1022 of brittle materials. *International Journal of Fracture* 139, 169–196.

1023 Zhou, F., Wright, T., Ramesh, K., 2006c. The formation of multiple adiabatic shear bands. *Journal*
1024 *of the Mechanics and Physics of Solids* 54 (7), 1376 – 1400.

1025 Zhou, M., Rosakis, A., Ravichandran, G., 1996. Dynamically propagating shear bands in impact-
1026 loaded prenotched plates-i. experimental investigations of temperature signatures and propaga-
1027 tion speed. *Journal of Mechanics and Physics of Solids* 44, 981–1006.

1028 Zieg, M. J., Marsh, B. D., 2005. The sudbury igneous complex: Viscous emulsion differentiation
1029 of a superheated impact melt sheet. *Geological Society of America Bulletin* 117 (11-12), 1427–
1030 1450.

1031 **List of Tables**

1032 1 Characteristic stress and strain rate parameters for the compressive strength data
1033 on the brittle solids presented in Figure 6. Details are presented in (Kimberley
1034 et al., 2013) 17

1035 **List of Figures**

1036 1 Schematic of mechanistic domains and associated mechanisms in catastrophic dis-
1037 ruption and asteroid impact. Note that $\tau_i \sim \frac{a}{v}$ is an impactor-dependent timescale. 43

1038	2	Failure processes that may be developed during various stress states within dynamic loading. Each region of the target body goes through a complex history of multiaxial stress states and deformation states during a catastrophic disruption event. The typical failure processes that are developed under example stress states are shown. Note that the strength of a material element will also be affected by the failure processes developed over the stress path.	44
1039			
1040			
1041			
1042			
1043			
1044	3	Influence of confinement on failure process in brittle solids (aluminum nitride in this case) (Hu et al., 2011). Photographs taken every 2 microseconds with exposure times of 500 nanoseconds. (a) Unconfined uniaxial dynamic compressive loading in the horizontal direction. (b) Failure during planar confinement (in the vertical direction) and dynamic compressive loading (in the horizontal direction). Note the development of axial cracks, propagating at speeds of several hundred m/s in (a), but none of these are observed in (b).	45
1045			
1046			
1047			
1048			
1049			
1050			
1051	4	Examples of failure processes: (a) brittle fracture, (b) shear bands, (c) void growth and spallation, and (d) amorphization. Note that a variety of length scales are represented in this figure, and all of these processes can lead to fragmentation. . . .	46
1052			
1053			
1054	5	Multipart schematic showing the influence of rate of loading on the activation of defects in a material containing a distribution of defects. The probability of finding a defect of size s is shown in the top right quadrant. The bottom right quadrant shows how the activation stress depends on the defect size. The bottom left quadrant shows how the applied stress might vary with time, with the red solid line showing a low loading rate and the green solid line showing a high loading rate. At any given time, the dashed lines show that at low loading rates only the largest defects are activated, while at high loading rates the majority of the defect distribution will be activated.	47
1055			
1056			
1057			
1058			
1059			
1060			
1061			
1062			
1063	6	All brittle solids, including geophysical and engineering materials, appear to follow a universal dependence of the (a) compressive and (b) tensile strengths on the strain rate (Kimberley et al., 2013). This results from the KRD scaling analysis of the influence of a defect distribution on fracture dynamics.	48
1064			
1065			
1066			

1067	7	Disruption model incorporating micro-mechanics-based scaling of strength (bold black line).	48
1068			
1069	8	Experimental fragment size averages of structure-dominated fragmentation compared with the models of Grady (2009b), Glenn and Chudnovsky (1986), Zhou et al. (2006b).	49
1070			
1071			
1072	9	A. Computed strain rate domains from the impact of a 1km quartz body at 5km/s into a quartz half-space (Ernst et al., 2009). B. Comparison of two different fragmentation models over the computed strain rate domain and the typical laboratory strain rate domain. The ZMR model predicts a much larger amount of fines as a consequence of the impact.	50
1073			
1074			
1075			
1076			

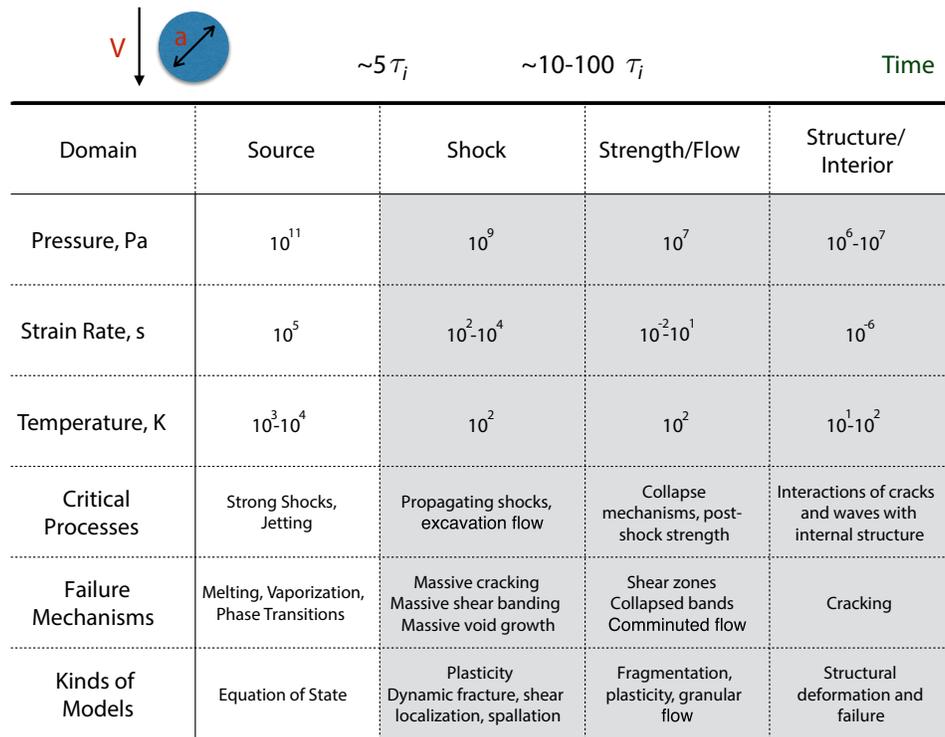


Fig. 1: Schematic of mechanistic domains and associated mechanisms in catastrophic disruption and asteroid impact. Note that $\tau_i \sim \frac{a}{V}$ is an impactor-dependent timescale.

	Uniaxial Tension	Uniaxial Compression	Hydrostatic Compression	Hydrostatic Tension	Shear	Geologic Material Example
Homogeneous Ductile Material	Necking, void growth	Adiabatic shear bands	None	Bifurcation to nucleate voids	Adiabatic shear bands	Deep heated rocks, planetary cores, iron meteorites
Homogeneous Brittle	Bifurcation to nucleate cracks	None	None	Bifurcation to nucleate cracks	Amorphization?	Bedrock (e.g., lava flows); some minerals
Brittle and Cracked	Tensile fracture	Wing cracks	Shutdown cracks, shear bands	Brittle Spallation	Mixed Mode fracture	Bedrock; sedimentary rocks; megaregolith
Brittle and Porous	Tensile fracture	Wing cracks	Shear bands, microbuckling, kink bands	Spallation	Shear cracks	Sedimentary rocks; regolith; fractured bedrock; ice; asteroids
Brittle with Inclusions	Tensile fracture	Wing cracks	Shear bands	Spallation	Shear cracks	Chondrites, Asteroids
Ductile with Inclusions	Void nucleation	Void nucleation	None	Spallation	Shear bands	Lava flows
Ductile and Porous	Necking, void growth	Compaction, microbuckling	Compaction, microbuckling	Spallation	Void shear mechanisms	Metallic meteorites?

Fig. 2: Failure processes that may be developed during various stress states within dynamic loading. Each region of the target body goes through a complex history of multiaxial stress states and deformation states during a catastrophic disruption event. The typical failure processes that are developed under example stress states are shown. Note that the strength of a material element will also be affected by the failure processes developed over the stress path.

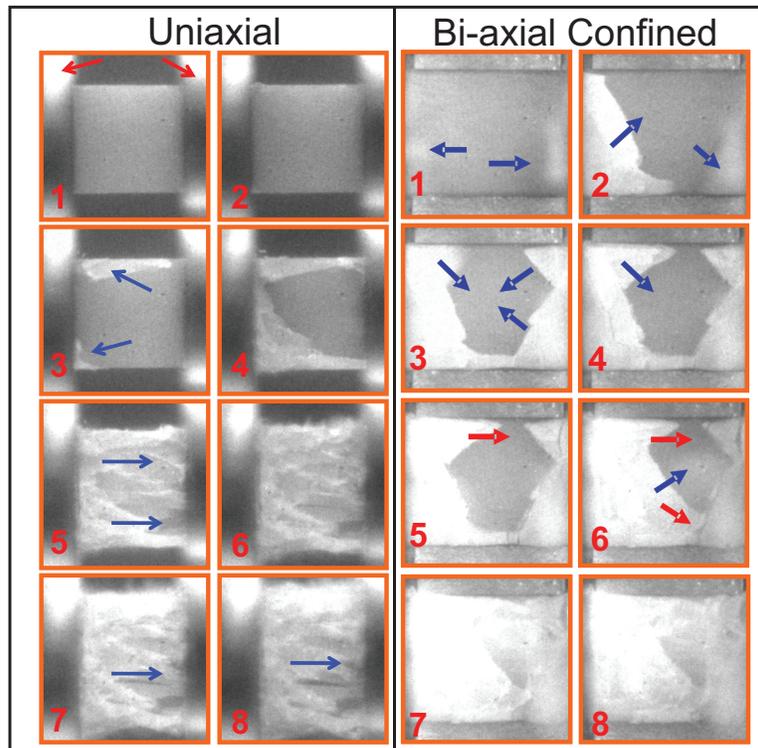


Fig. 3: Influence of confinement on failure process in brittle solids (aluminum nitride in this case) (Hu et al., 2011). Photographs taken every 2 microseconds with exposure times of 500 nanoseconds. (a) Unconfined uniaxial dynamic compressive loading in the horizontal direction. (b) Failure during planar confinement (in the vertical direction) and dynamic compressive loading (in the horizontal direction). Note the development of axial cracks, propagating at speeds of several hundred m/s in (a), but none of these are observed in (b).

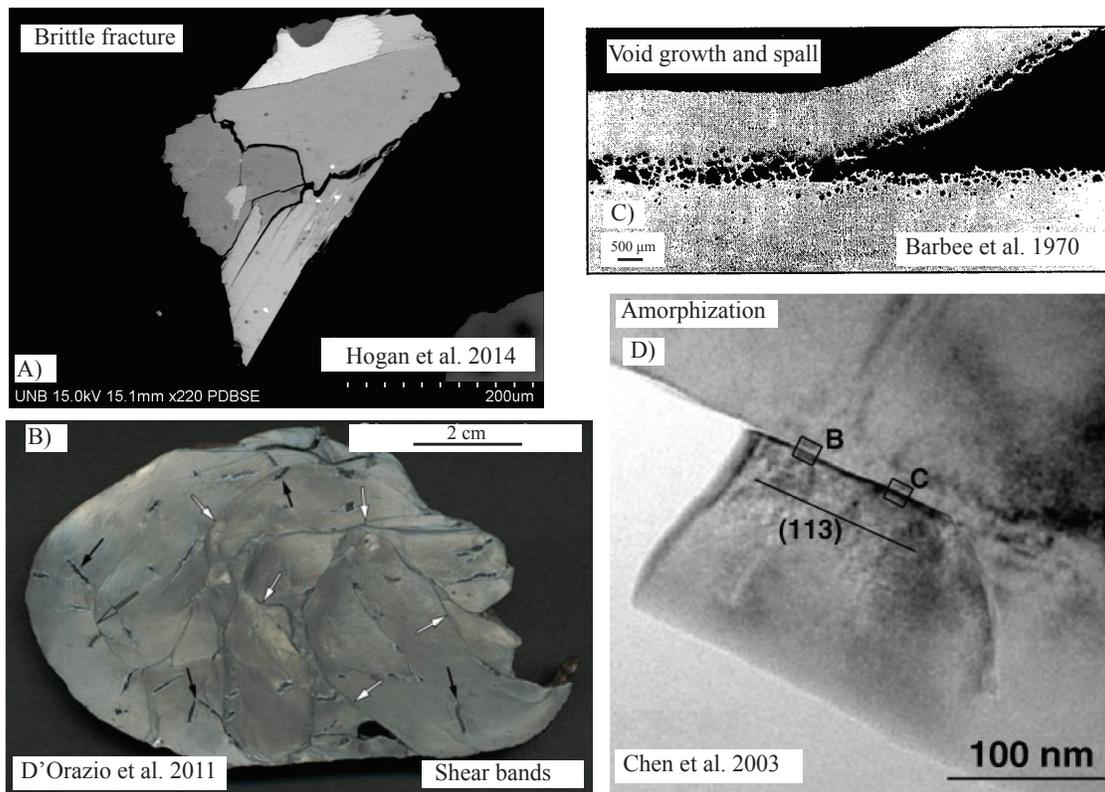


Fig. 4: Examples of failure processes: (a) brittle fracture, (b) shear bands, (c) void growth and spallation, and (d) amorphization. Note that a variety of length scales are represented in this figure, and all of these processes can lead to fragmentation.

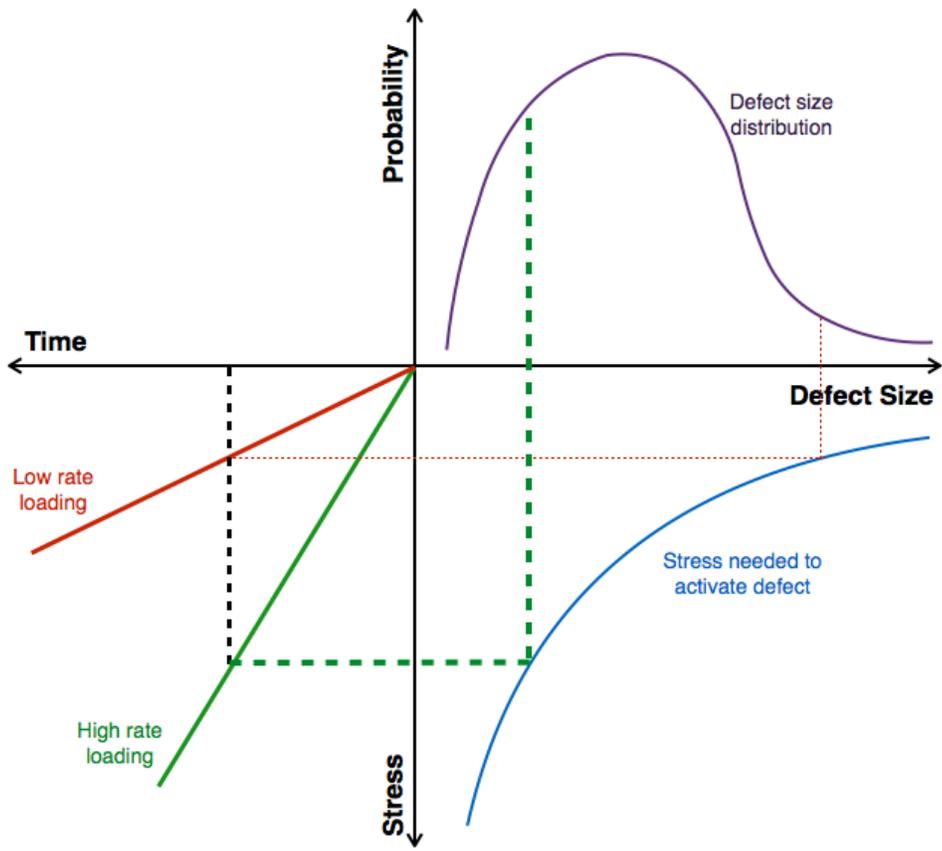


Fig. 5: Multipart schematic showing the influence of rate of loading on the activation of defects in a material containing a distribution of defects. The probability of finding a defect of size s is shown in the top right quadrant. The bottom right quadrant shows how the activation stress depends on the defect size. The bottom left quadrant shows how the applied stress might vary with time, with the red solid line showing a low loading rate and the green solid line showing a high loading rate. At any given time, the dashed lines show that at low loading rates only the largest defects are activated, while at high loading rates the majority of the defect distribution will be activated.

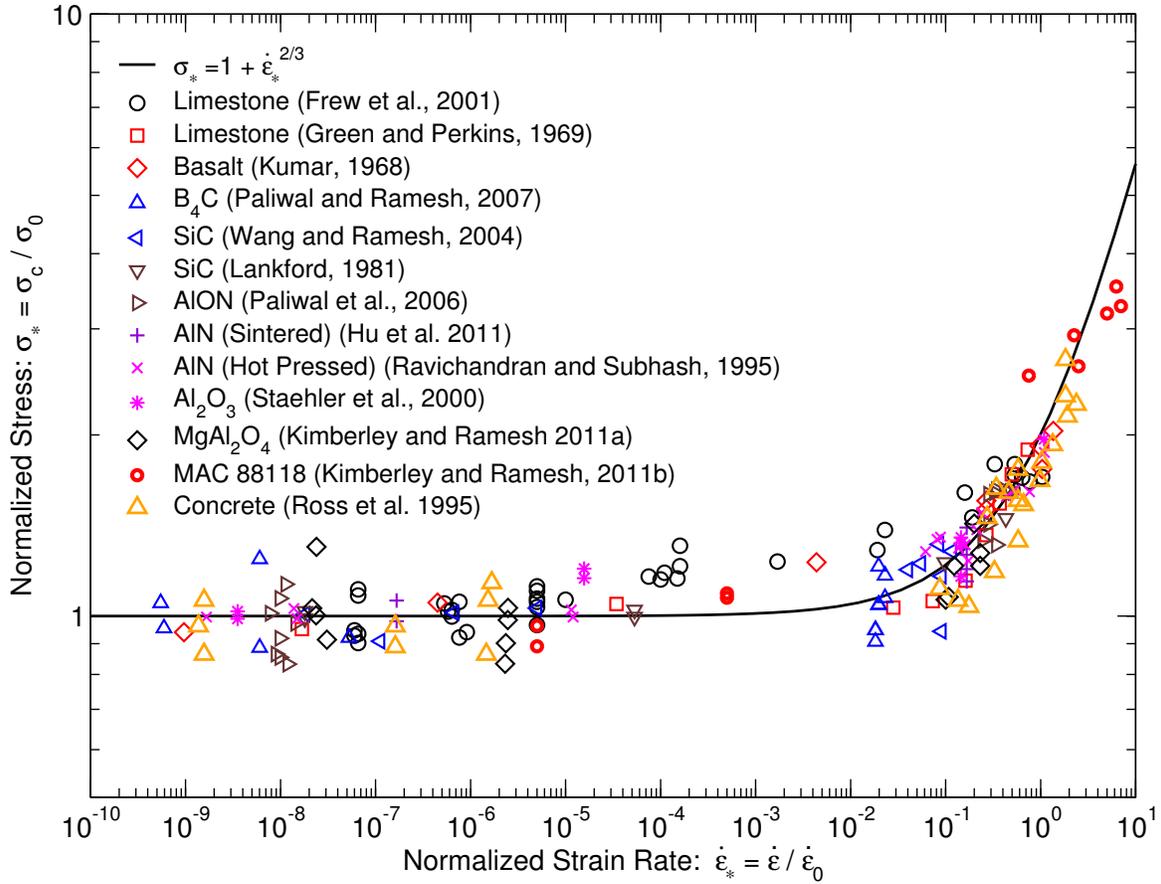


Fig. 6: All brittle solids, including geophysical and engineering materials, appear to follow a universal dependence of the (a) compressive and (b) tensile strengths on the strain rate (Kimberley et al., 2013). This results from the KRD scaling analysis of the influence of a defect distribution on fracture dynamics.

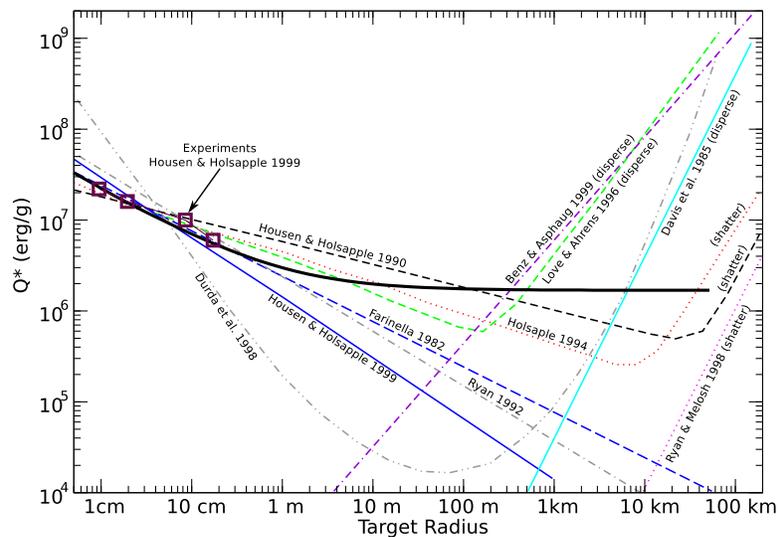


Fig. 7: Disruption model incorporating micro-mechanics-based scaling of strength (bold black line).

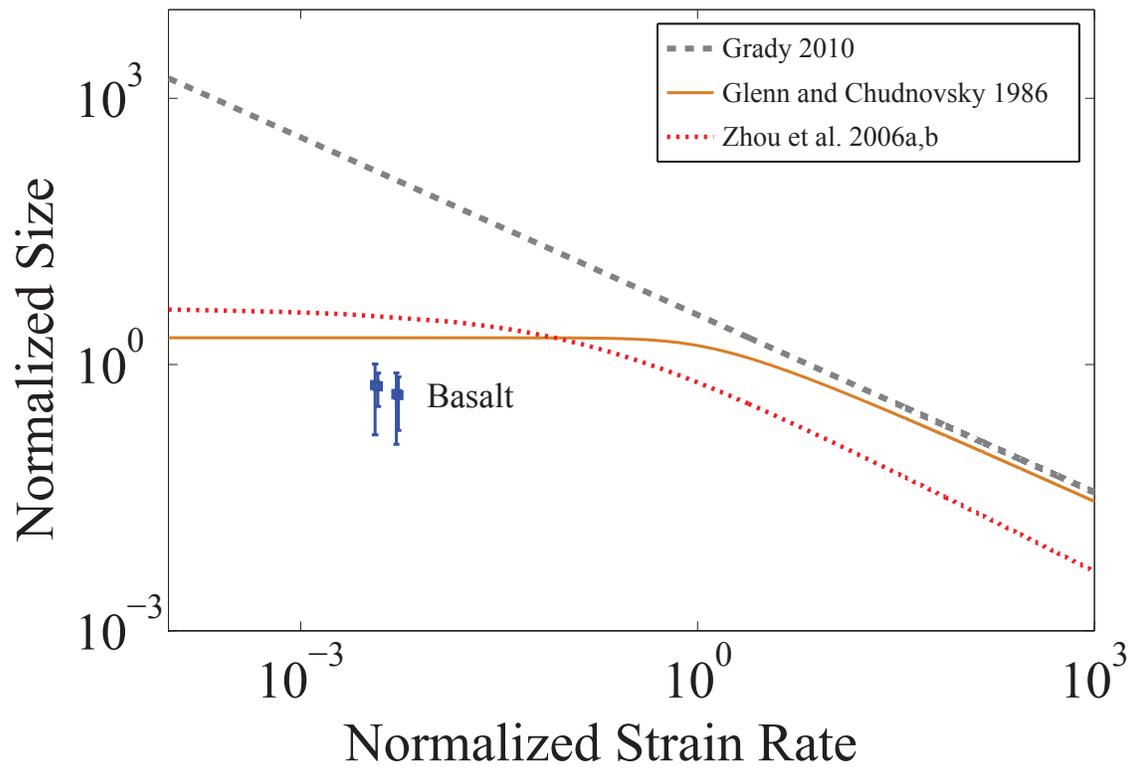


Fig. 8: Experimental fragment size averages of structure-dominated fragmentation compared with the models of Grady (2009b), Glenn and Chudnovsky (1986), Zhou et al. (2006b).

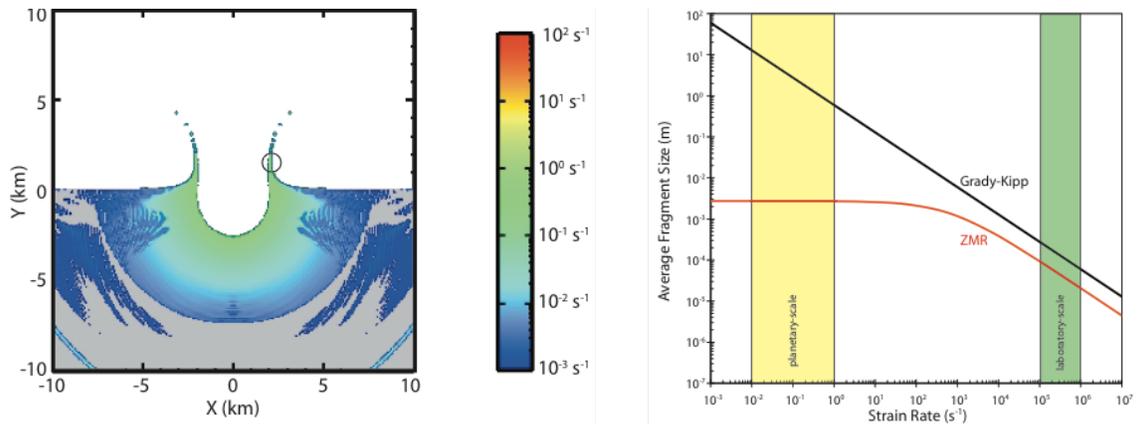


Fig. 9: A. Computed strain rate domains from the impact of a 1km quartz body at 5km/s into a quartz half-space (Ernst et al., 2009). B. Comparison of two different fragmentation models over the computed strain rate domain and the typical laboratory strain rate domain. The ZMR model predicts a much larger amount of fines as a consequence of the impact.