

RURAL ECONOMY

ECONOMIC STRUCTURAL ANALYSIS OF THE CANADIAN
AGRICULTURAL PRODUCTION SECTOR

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Staff Paper 92-08

STAFF PAPER



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ABSTRACT

This study develops a model to analyze the changes in the Canadian agricultural production sectors over the period 1926 to 1988. Economic analysis is focused on the estimation of certain parameters including elasticities of substitution between the factors of production, own-price elasticities, scale effects of production and biases in technical change. Aggregate agricultural output is a function of labor, capital, intermediate goods, and land and building structures. The application of duality theory allows for the use of aggregate cost functions.

Unlike earlier studies, a dynamic model of a system of share equations is fitted to the data. Selection of the model is based on tests of adequacy (statistical diagnostic tests) and of restrictions from underlying economic theory. Potential models include a static model (long-run equilibrium model), a partial adjustment model and an autoregressive process of order one for disturbances model, all of which are rejected. However, a system of share equations of a multivariate autoregressive process of order one of the translog functional form fits the data satisfactorily.

One feature of the dynamic model utilized in this study is the ready availability of estimates of the long-run parameters. One significant difference between this study and earlier studies is the calculating of standard errors of estimates of elasticities, which are obtained using a statistical simulation procedure. Confidence intervals for the estimates of elasticities are constructed using the simulated results. Most earlier studies either did not estimate standard errors or approximated them with standard error estimates of the coefficients, which are also used to calculate the elasticities; an approach which is faulty.

The results of this study show that the long-run production structure for the Canadian agricultural sectors is non-homothetic. Scale of production is intermediate goods using and labor saving. Technical change is intermediate goods using and land saving.

Labor and land and building structures, and intermediate goods and land and building structures are substitutes for Canada.

All factors except land and building structures are inelastic to their own prices. Land and building structures appears to be elastic to its own price for Canada.

1. INTRODUCTION

This study develops a model to analyze changes in the Canadian agricultural sector over time. The agricultural sector is an integral part of the Canadian economy and from a policy and continuing research point of view, it is important to recognize the long term behavior in the sector.

The Canadian agricultural industry has visibly changed in this century. The development of the sector can be credited to the invention and adoption of new technology for the sector, in particular: farm mechanization, the adoption and development of improved varieties of crops and livestock, the greater use of fertilizers for increased yield and the use of chemicals for weed and pest control etc. [Manning (1985), Furniss (1964,1970)]. Farm management skills have improved as well. This study employs a cost function to assess structural characteristics of the Canadian agricultural sector. The popularity of a cost function in the study of technological change and the substitution behavior of inputs of the production process can be attributed to the development of duality theory. In this study, the following parameters defining the technology in agricultural production for Canada will be estimated:

- Elasticity of substitution between factors of production;
- Own-price elasticities;
- Scale effects of production; and
- Bias of technological change.

Several studies of this nature have been undertaken to analyze the U.S and the Canadian agricultural sectors. Islam and Veeman (1980), Lopez (1980) and Adamowicz (1986) analyzed the Canadian agricultural sector. Binswanger (1974a, 1974b), Brown and Christensen (1981), Ray (1982), Antle (1984) analyzed United States agricultural time series data. Except for Lopez (1980), each of these studies imposed the integrability restrictions i.e., homogeneity and symmetry on the estimating equations without testing their prior validity. Further, all of the above studies estimated long-run static models. In estimating a long-run static model, it is assumed that the production sector is in equilibrium and it adjusts instantaneously to the changes in the exogenous factors.

Furthermore, most of earlier studies did not perform tests of statistical goodness of fit. The goodness of fit tests are to check whether the error terms are white noise. These tests indicate how well a model fits data statistically and whether the model is misspecified. A desirable model is the one which does not reject the tests of economic restrictions and of statistical diagnostic checks.

One of the other serious problems with the earlier studies is that inferences about the substitution behavior between the factor inputs in response to the changes in relative prices are based on the sign and magnitude of the estimates of elasticities alone. These inferences could be completely misleading and wrong as will be discussed later. Therefore, in this study, confidence interval for the estimates of elasticities are constructed.

Our study differs from the above in three important respects. First, it tests rather than imposing the restrictions of consistency of economic theory. Second, it provides a frame work to test whether the data on the Canadian agricultural production sector are generated from a dynamic process. Third, inference from the estimates of elasticities is based on their confidence intervals rather than the point estimates alone which was the case with the earlier studies discussed above.

2. METHODOLOGY

A system of cost share equations is utilized to estimate the parameters of interest. Further a translog functional form is arbitrarily chosen. A system of the cost share equations of translog form of a long-run static model can be written as

$$S_i(t) = \alpha_i + \sum_{j=1}^n \beta_{ij} \ln p_j + \gamma_{iq} \ln q + \theta_{i\tau} \tau + v_i(t)$$

where $i = 1, 2, \dots, n$ and $v_i(t)$ is the random error term. The above system of share equations can be rewritten as

$$S(t) = \alpha + \Pi X(t) + \theta\tau + v(t) \quad (1)$$

where $S(t)$, $X(t)$, $v(t)$, α , Π and θ are matrices of order $n \times 1$, $(n+1) \times 1$, $n \times 1$, $n \times (n+1)$ and $n \times 1$ respectively.

From duality theory, the following restrictions have to be satisfied by the estimated parameters to insure that the estimated cost function describes the same technology as the production function which, in turn, generates the data for the production sectors in question:

1. $C(t)$ is linear homogeneous in prices;
2. $C(t)$ is non-decreasing in prices; and
3. $C(t)$ is concave in prices which implies symmetry of the β_{ij}

parameters i. e., $\beta_{ij} = \beta_{ji}$ for $i, j = 1, 2, \dots, N$ and the Hessian matrix is negative semi-definite.

Most earlier studies imposed these restrictions on the system of share equations without testing the validity of such actions. To determine whether the cost function is correctly specified, these restrictions should be tested. If the restrictions are rejected by a cost function then it is likely misspecified. In this case, the inferences made on the data generating process of the producing sector may be incorrect based on an estimated model with restrictions imposed that would otherwise be rejected, if tested.

Misspecification of a system of share equations may result due to (a) an incorrect specification of the functional form of the share equations (e.g., using a generalized Leontief form, when technology of the generating process is translog form or vice versa), and/or to (b) a correctly specified functional form but with some of the relevant explanatory variables omitted (e.g., lagged dependent and independent variables). The type (b) misspecification is discussed below in detail.

It is possible that the integrability conditions can be rejected even though the long-run system of share equations is correctly specified. This situation may occur when the observed share values do not correspond to long-run optimum values. The assumption that agricultural production sectors are operating at a long-run equilibrium state cannot be easily supported. The data are more likely being generated from a dynamic production process. The presence of dynamics in the production process may result from the inability of producers to adjust instantaneously to changes in the economy. Adjustment lags occur because of the existence of costs of adjustment and costs of obtaining information (Anderson and Blundell (1982)). Furthermore, outputs are not

realized instantly at the time of input decisions. Therefore, producers tend to forecast output and prices on the basis of past experiences.

Based on the above discussion, dynamics enter the production process in two ways (Antle (1986)): output may be a function of current and past inputs because of costs of adjustment (Lucas (1967)) or time taken to build (Kydland and Prescott (1982)); and output may depend on past output produced because of the models of learning-by-doing (Arrow (1962)) and multi-stage production (Long and Plosser (1983), Antle (1983)).

Integrability conditions can be rejected as discovered in this study (Section 3) in circumstances when a static system of share equations are estimated when in fact the data seem to have been generated by a dynamic production process. Therefore, a model based on a dynamic system of share equations like those used in the studies by Anderson and Blundell (1982) and Nakamura (1985) are considered for this study. One of the interesting properties of this model is in fact that a static model is nested within it and thus restricted model suitability can be tested. Further long-run (static) parameters are also readily available from the estimated parameters of the dynamic model.

When a static model is fitted, it is assumed that the data is generated from a production process operating at a long-run optimum (equilibrium) state. Therefore, in the estimation of the system of equations (5), it assumed that

$$E_s [S(t) | X(t)] = S^0 \quad (2)$$

where the subscript s indicates that the expectation is taken for the static model (1). However, based on the discussion above, the data may have been generated by a dynamic process. Thus, if a static model is estimated, some of the explanatory variables (lagged input price and output quantity indexes) may have been omitted and assumption (5) may not hold.

Based on Nakamura (1985), assumption (2) is replaced by³

$$E_d [S(t) | X(s) = X^*, \tau = \tau^*, s \leq t] = S^0 \quad (3)$$

where d denotes that expectation is taken under a dynamic model. Assumption (3) means that if prices, state of technology and output for periods preceding the current one are constant, then the expected share values of a dynamic model are long-run optimum values. This assumption also implies that

$$E_d [S(t) | X(s) = X^*, \tau = \tau^*, s \leq t] = E_s [S(t) | X(t) = X^*] \quad (4)$$

This follows from the fact that if the exogenous variables are constant for all preceding periods and the present period, then the production process is in fact static.

There is a group of models, known as stationary multivariate autoregressive models, ARX(r, r) of order r which satisfy assumption (3). These models were first considered by Anderson and Blundell (1982) for the estimation of a singular system of share equations. One of the

³ Nakamura (1985) did not have time trend variable τ as an explanatory variable in his model. However, he stated that inclusion of the time trend variable in the assumption (equation (11)) is consistent with the assumption without the time trend variable.

interesting properties of these models is that estimates of the long-run parameters are readily available from the estimated parameters. A general form of system of equations based on ARX(r,r) is given by

$$S(t) = \gamma + \Phi_1 S(t-1) + \dots + \Phi_r S(t-r) + \Gamma_1 X(t) + \Gamma_2 X(t-1) + \dots + \Gamma_r X(t-r) + \delta\tau + \varepsilon(t) \quad (5)$$

To fit an ARX model of order $r \geq 2$, a very large data set is required. For a time series data set it is usually not possible. For this study, with four aggregated factor system categories, only $r = 1$ is considered because of data limitations. A system of share equations based on ARX(1,1) is given by

$$S(t) = \gamma + \Phi_1 S(t-1) + \Gamma_0 X(t) + \Gamma_1 X(t-1) + \delta\tau + \varepsilon(t) \quad (6)$$

where γ , Φ_1 , Γ_0 , Γ_1 , δ are matrices of unknown constants of order $n \times 1$, $n \times n$, $n \times (n+1)$, $n \times (n+1)$ and $n \times 1$ respectively. $\varepsilon(t)$ is a $n \times 1$ [n is equal to four for this study] vector of random errors of $N(0, \Omega)$, where Ω is a variance-covariance matrix.

Additivity of share equations

$$i^T S(t) = 1$$

implies the following restrictions on the parameters (unknown constants) of equation (6) where i is a sum vector of order $n \times 1$.

$$i^T \gamma = 1 - k$$

$$i^T \Phi_1 = k i^T$$

These specifications imply that every column of Φ_1 sums up to the constant k which is unknown.

$$i^T \Gamma_0 = i^T \Gamma_1 = 0 i^T = (0, 0, \dots, 0)$$

$$i^T \varepsilon(t) = 0$$

This specification and the assumption that $\varepsilon(t)$ follows $N(0, \Omega)$ implies that

$$i^T \Omega i = 0$$

In other words, the variance-covariance matrix, Ω , is singular. Therefore, the system of equations represented by model (6) is singular. To estimate the parameters of model (6), a transformation of the parameters must be made so that the transformed system is non-singular. This issue is considered below. A relationship between the parameters of the system of equations specified by (1) and (6) exists because of assumption (3). Assumption (3) is required to estimate the parameters of (1) from the estimated model (6). If $X(s) = X^*$ and $\tau = \tau^*$ for all $s \leq t$, then from (6) one has

$$E[S(t) | X(s) = X^*, \tau = \tau^*, s \leq t] = E[S(t-1) | X(s) = X^*, \tau = \tau^*, s \leq t]$$

and

$$E[S(t) | X(s) = X^*, \tau = \tau^*, s \leq t] = \lambda + \Gamma_0 X^* + \Gamma_1 X^*$$

$$+ \Phi_1 E[S(t-1) | X(s) = X^*, \tau = \tau^*, s \leq t] + \delta \tau^*$$

Assuming $(I - \Phi_1)$ is non-singular, where I is an identity matrix of order n , the above can be written as

$$E[S(t) | X(s) = X^*, \tau = \tau^*, s \leq t] = (I - \Phi_1)^{-1} \lambda + (I - \Phi_1)^{-1} (\Gamma_0 + \Gamma_1) X^* + (I - \Phi_1)^{-1} \delta \tau^* \quad (7)$$

From (1) one has

$$E[S^*(t) | X(t) = X^*] = \alpha + \Pi X^* + \theta \tau^* \quad (8)$$

Under assumption (3), equations (7) and (8) imply that

$$\alpha = (I - \Phi_1)^{-1} \lambda, \Pi = (I - \Phi_1)^{-1} (\Gamma_0 + \Gamma_1), \text{ and } \theta = (I - \Phi_1)^{-1} \delta \quad (9)$$

Substituting the conditions of (9) into model (6), one obtains

$$S(t) = (I - \Phi_1) \alpha + \Gamma_0 X(t) + \Phi_1 S(t-1) + ((I - \Phi) \Pi - \Gamma_0) X(t-1) + (I - \Phi_1) \theta \tau + \varepsilon(t) \quad (10)$$

or

$$S(t) = (I - \Phi_1) \alpha + \Phi_1 S(t-1) + \Gamma_0 \Delta X(t) + (I - \Phi) \Pi X(t-1) + (I - \Phi_1) \theta \tau + \varepsilon(t) \quad (11)$$

Model (11) has several interesting properties. The estimation of long-run parameters, α , Π and τ , is readily available. Further, several well known models used in some previous studies of this kind are nested within model (11) [Anderson and Blundell (1982), Nakamura (1985)].

2.1 MODELS NESTED WITHIN THE DYNAMIC MODEL

Three models nested within the dynamic models are discussed below.

2.1.1 PARTIAL ADJUSTMENT MODEL

The dynamic system of equations (11) reduces to a system of share equations corresponding to the partial adjustment model, if $\Gamma_0 = (I - \Phi_1) \Pi$ and is given by

$$S(t) = (I - \Phi_1) \alpha + \Phi_1 S(t-1) + (I - \Phi_1) \Pi X(t) + (I - \Phi_1) \theta \tau + \varepsilon(t) \quad (12)$$

The more popular form of this partial adjustment model used in the literature is

$$(I - \Phi_1) = \Phi.$$

2.1.2 AUTOREGRESSIVE PROCESS FOR THE DISTURBANCES

If $\Gamma_0 = \Pi$, the dynamic model (11) reduces to a system of share equations with errors following an autoregressive process of order one. AR(1) is given by

$$S(t) = (I - \Phi_1) \alpha + \Phi_1 S(t-1) + \Pi X(t) - \Phi_1 \Pi X(t-1) + (I - \Phi_1) \theta \tau + \varepsilon(t) \quad (13)$$

This is the model developed by Berndt and Savin (1975) with notation difference that Φ_1 in (13) is equal to R .

2.1.4 STATIC MODEL

If $\Gamma_0 = \Pi$ and $\Phi_1 = 0$, then dynamic model (11) reduces to static model (1).

2.2 ESTIMATION

An objective of this study is to estimate a model following the parsimony principle. In other words, the estimated model should be the one which fits the data adequately, but involves the least number of parameters. The competing models are (11) through (13) and (1). All of the models are singular because of additivity. Further, the maximum likelihood estimation procedure is used to estimate these models because of non-linearity of the parameters. Models (1), (11) through (13) can be made non-singular by dropping one of the equations. In this study n^{th} equation is dropped. The estimates of the parameters are invariant to which equation is dropped (for detailed discussion see Sandhu (1991)). By dropping the n^{th} equation of the system (16) and using the additivity restriction the system of equations reduces to,

$$\begin{aligned} S^n(t) = & (I-D) \alpha^n + D S^n(t-1) + \Gamma_0^n \Delta X(t) \\ & + (I-D) \Pi^n X(t-1) + (I-D) \theta^n \tau + \varepsilon^n(t) \end{aligned} \quad (14)$$

D is an $(n-1) \times (n-1)$ matrix obtained from $\Phi_1 G$ by dropping its last row and is given by

$$D = \begin{bmatrix} \Phi_{11} - \Phi_{1n} & \Phi_{12} - \Phi_{1n} & \cdots & \Phi_{1n-1} - \Phi_{1n} \\ \Phi_{21} - \Phi_{2n} & \Phi_{22} - \Phi_{2n} & \cdots & \Phi_{2n-1} - \Phi_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \Phi_{(n-1)1} - \Phi_{(n-1)n} & \Phi_{(n-1)2} - \Phi_{(n-1)n} & \cdots & \Phi_{(n-1)(n-1)} - \Phi_{(n-1)n} \end{bmatrix}$$

The system of equations (14) is the non-singular transformation of model (11). Model (14) and its subsets are estimated by using the maximum likelihood estimation procedure. However, there is the question of whether the estimates of the model corresponding to (11) can be identified from the estimates of (14). The answer is no because the estimates of Φ cannot be identified from the estimates of D (Berndt and Savin (1975), Anderson and Blundell (1982) and Nakamura (1985)). The estimates of remaining parameters can uniquely be obtained from the estimated parameters of model (14). The objective of this study is to obtain estimates of the long-run parameters, the non-identification of Φ has no serious effect on such estimation.

2.3 MODEL SPECIFICATION, ESTIMATION AND DIAGNOSTIC TESTING

A correct theoretical model can be specified only if the physical

and economic nature of the agricultural sector to be analyzed is completely understood. This, in reality, is not practical to achieve. A purely empirical model is one which is selected entirely on the basis of statistical procedures applied to data available for the Canadian agricultural sector. However, a purely empirical model will not be of much use for economic analysis of the sectors. A model building technique that lies between an exact theoretical model and an exclusively empirical model; an iterative model building technique based on work by Box and Jenkins (1976) will be used here.

The iterative model building technique to be used involves the following stages:

- (a) Postulation of a class of models based on information of the economic and physical nature of the agricultural sector;
- (b) Tentative selection of a model from the postulated class of models with the objective of selecting the simplest and most satisfactory model possible following the parsimony principle;
- (c) Estimation of the tentative model; and
- (d) Model diagnostic checking.

2.4 STAGES OF THE ITERATIVE MODEL BUILDING TECHNIQUE

Each of the stages of the iterative model building technique are described in more detail below.

2.4.1 POSTULATION OF A CLASS OF MODELS

Based on the methodology of this study, a class of postulated models consists of the following models which are defined in this chapter:

- a system of static translog share equations;
- a system of partial adjustment translog share equations;
- a system of share equations with autoregressive errors; and
- a system of multivariate autoregressive translog share equations, $ARX(1,1)$.

2.4.2 IDENTIFICATION OF A TENTATIVE MODEL

The simplest model from the class of postulated models is selected as a tentative model. The relative simplicity of a model is defined here as one involving the least number of parameters to be estimated and the ease of estimation. The models listed above are ordered by increasing complexity with the exception that partial adjustment and autoregressive errors models are of equal complexity as both involve the same number of parameters. Further, neither of these two models is nested in the other. At stage 1 of iteration 1, a system of static translog share equations is selected as a tentative first model for estimation because of its relative simplicity.

2.4.3 ESTIMATION OF THE TENTATIVE MODEL

The parameters of the tentative model are estimated using the maximum likelihood estimation procedure. At each iteration three forms of the tentative model are estimated; an unrestricted form, a homogeneous form, and a homogeneous and symmetric form. One of the share equations is dropped for each form of the model to take care of the singularity of the system of equations caused by their additivity.

2.4.4 DIAGNOSTIC CHECKING

The objective is to select and estimate the model which satisfies the statistical assumptions and the regularity conditions defined earlier in this chapter. A further objective of this study is to undertake economic analysis based on the estimated model. Therefore, a model which satisfies all the desirable statistical properties but fails to satisfy economic restrictions will still be rejected. Economic restrictions, therefore, will be tested first to narrow down the choices. In addition the desirable model is the one which satisfies the economic restrictions. Therefore, statistical tests of goodness fit will be made only on the homogeneous and symmetric form of each model. However, it should be noticed that if a model is not a statistical good fit then the tests of economic restriction may not make any sense.

Diagnostic tests for a statistically good fit are however discussed first. To test whether errors for each equation are white noise, residual autocorrelation and partial autocorrelation plots are used. Other tests that could have been used were the multivariate portmanteau tests by Hosking (1980) and Li and McLeod (1981) which are developed to check for adequacy of fitted multivariate autoregressive moving average models. However, these tests are not utilized in this study. Normal probability plots are used to test for normality of the error terms. These tests are performed only on the homogeneous, symmetric model because it is this model that is of most interest.

A homogeneous model is nested in the most unrestricted model and a symmetric model is nested in the homogeneous model. Homogeneity is automatically imposed due to additivity when symmetry is imposed on the model. The objective is to check whether homogeneity and symmetry restrictions are satisfied by the model in question. Therefore, first a test of homogeneity and symmetry (imposed simultaneously on the unrestricted model) is performed. If the test results in rejection then homogeneity and symmetry are tested separately to see whether one or both of these restriction are rejected. The likelihood ratio tests are used since the restricted models are nested in the model without the restrictions in question. The following hypotheses are defined to test for these regularity conditions:

H_0 : The model satisfies the homogeneity and symmetry restrictions;

against

H_A : The model does not satisfy the homogeneity and symmetry restrictions.

The hypotheses to test for the regularity restrictions separately are:

H_{01} : The model is homogeneous; against

H_{A1} : The model is not homogeneous; and

H_{02} : The homogeneous model is symmetric; against

H_{A2} : The homogeneous model is not symmetric.

Let Θ denote the parametric set of the model. Then $\text{Max loglik } \{\Theta_{01}\}$

denotes the maximum likelihood of the model under H_{01} . Let

$$\log \lambda = \text{Max loglik } \{\Theta_0\} - \text{Max loglik } \{\Theta_A\}$$

$$\log \lambda_1 = \text{Max loglik } \{\Theta_{01}\} - \text{Max loglik } \{\Theta_{A1}\}$$

$$\log \lambda_2 = \text{Max loglik } \{\Theta_{02}\} - \text{Max loglik } \{\Theta_{01}\}$$

Further $-2 \log \lambda$, $-2 \log \lambda_1$ and $-2 \log \lambda_2$ are likelihood ratio test statistics for tests of the hypotheses stated above respectively. They follow χ^2 distributions with q , q_1 and q_2 degrees of freedom respectively. Here, q is the number of restrictions applied to the unrestricted model to obtain a homogeneous and symmetric model, q_1 is the number of restrictions applied to the unrestricted model to obtain the homogeneous model and q_2 is the number of independent restrictions imposed on the homogeneous model to obtain the homogeneous and symmetric model.

If the model passes all of the above tests, it is selected for further economic analysis. Otherwise, some other model from the class of postulated models is selected as a new tentative model (stage 2.6.2). Stages 2.6.3 and 2.6.4 are repeated until a satisfactory model is determined. If there is no satisfactory model, a new class of postulated models is specified and the iterative model building technique is performed again.

2.4.5 GOODNESS OF FIT TESTS FOR THE NESTED MODELS

Once a model is selected which satisfies the economic restrictions and diagnostic checks, it can be taken as the maintained hypothesis model to further check whether this model fits the data significantly better than the nested model (with economic restrictions). The objective of this test is to evaluate the model and decide whether the maintained hypothesis model is just a marginally better or a significantly better fit than the nested models. The likelihood test ratio test will be used to test the goodness of fit of the maintained hypothesis model as compared to the nested models. Akaike Information Criterion (AIC) will also be used to perform the goodness of fit test. The AIC can be used for testing nested as well as non-nested models. AIC will be useful if partial adjustment or autoregressive error for disturbances models are selected since these models are not nested in each other. AIC is defined as (Harvey 1981)

$$\text{AIC} = -2 \log \text{lik } \{\hat{\Theta}\} + 2K$$

where K is the total number of parameters estimated and $\log \text{lik } \{\hat{\Theta}\}$ is the maximum log likelihood estimate. The decision rule is to accept the model with the minimum AIC. This test makes appropriate allowance for parsimony of the model since it involves both max likelihood estimate as well as the number of parameters estimated (Harvey 1981).

However, it should be noted that some of the nested models considered are obtained by putting restrictions on D (e.g., a static model is obtained from the ARX(1,1) model if $D = 0$ and $\Gamma_0 = \Pi$). As noted earlier Φ is not identifiable (Nakamura 1985, Berndt and Savin 1975). Following Nakamura (1985) and Berndt and Savin (1975), restrictions on D are only necessary and are not sufficient. For example $\Phi = 0$ implies that $D = 0$ and the converse is not true since $D = 0$ implies that elements of Φ

in each row are equal for the n-1 rows (i.e., without the dropped nth row, i.e., for n-1 rows only).

2.5 ECONOMIC STRUCTURAL ANALYSIS

In this section parameters of the structure of technology are discussed. The parameters under investigation are technological change, homotheticity, and substitution and own-price elasticities.

2.5.1 TECHNOLOGICAL CHANGE AND HOMOTHETICITY

Hypothesis that long-run technical change is factor neutral will be tested. If this hypothesis is rejected then inference is made regarding technical change biases to the factors.

Hypothesis of homotheticity of the production function will be tested. Most of the earlier studies tested for homotheticity. Rejection of homotheticity corresponds to the rejection of increasing, constant, and decreasing return to scale hypotheses. For the case of a translog cost function, homotheticity of the long-run production structure implies that $\gamma_{iq} = 0$, for $i = 1, 2, \dots, n$. Therefore, likelihood ratio test will be used to the hypothesis.

2.5.3 ELASTICITIES

Estimates of own-price elasticities and elasticities of substitution between factor inputs of an industry are important to structural and policy analyses.

In economic analysis, it is useful to have a measure of how "substitutable" one factor input is for another. The most frequently used measure of substitutability is elasticity of substitution. The two best known measures are *the direct elasticity of substitution* and *the Allen partial elasticity of substitution*. The Allen partial elasticity of substitution estimators have been used in various studies; Binswanger (1974b), Islam and Veeman (1980), Lopez (1980), Moroney and Tripani (1981), Adamowicz (1986). For factor i and j the elasticity of substitution is defined as

$$\sigma_{ij} = \frac{C_i C_{ij}}{C_i C_j}$$

where $C_i = \frac{\partial C}{\partial p_i}$ and $C_{ij} = \frac{\partial C_i}{\partial p_j}$ and for the long-run translog cost function, it can be easily written as

$$\sigma_{ij} = 1 + \frac{\Pi_{ij}}{S_i S_j}$$

where S_i is the i^{th} share and Π_{ij} is the i, j^{th} element of the matrix Π of the long-run parameters of the share equations. The σ_{ij} is the estimator of the long-run elasticities of substitution between factors i and j ($i < j$ and $= 1, 2, 3, 4$).

From Moroney and Tripani (1981) and the various other studies mentioned above, own-price elasticities can be written as

$$\eta_{ii} = S_i + \frac{\Pi_{ii}}{S_i} - 1$$

The above definitions do not incur any difficulty provided that a static system of share equations fits the data in question. However, if a static model does not fit the data well enough and one of the other models (ARX(1,1), partial adjustment model or the autoregressive error) is selected, a question arises whether it is valid to use the observed values of $S(t)$ or estimated values of shares from the model fitted. As discussed in a previous section, the observed share values may not correspond to the optimum long-run share values of the production process. Recall that objective of this study is to analyze the long term economic structure of the Canadian agricultural sector. To estimate long-run elasticities, the long-run values of the shares have to be used. However, the long-run values of the shares cannot be observed and have to be estimated. The long-run values of shares are obtained as

$$\hat{S}(t) = \hat{\alpha} + \hat{\Pi} X(t) + \hat{\theta} \tau \quad (15)$$

Here, $\hat{\alpha}$, $\hat{\Pi}$ and $\hat{\theta}$ are maximum likelihood estimates of their respective parameters. In other words, estimates of long-run cost minimizing values of shares are obtained by using estimates of long-run parameters from the fitted model. Elasticities are calculated at mean expected values of shares $\hat{S}(t)$ for four sub-sample groups: 1959-68; 1969-78; 1979-88. Mean expected values of long-run shares are given by

$$\bar{\hat{S}}(r) = \sum_{t=t_r}^{t_R} S(t) / (t_R - t_r) \quad (16)$$

where r stands for the sub-sample ($r=1,2,3$ and 4), t_r and t_R denotes respective lower and higher index year for the range. Estimates of elasticities of substitution and own-price are obtained by

$$\hat{\sigma}_{ij}(r) = 1 + \frac{\Pi_{ij}}{\bar{\hat{S}}_i(r) \bar{\hat{S}}_j(r)} \quad (17)$$

and

$$\hat{\eta}_{ii}(r) = \bar{\hat{S}}_i(r) + \frac{\hat{\Pi}_{ii}}{\bar{\hat{S}}_i(r)} - 1 \quad (18)$$

To make statistical tests and inferences regarding the elasticities, the distributions and/or standard errors of the estimators of the elasticities are required. Making statistical decisions on the basis of point estimates (MLE estimates) alone could be misleading. For example, suppose the estimated elasticity of substitution between any two arbitrary inputs is a large positive value indicating that the two inputs are good substitutes. If the hypothesis of zero substitution is not statistically rejected because of a large estimated standard error, the above conclusion will be wrong.

In general, the distribution of the above estimators of elasticities is not a well behaved one. The estimators are the ratios of the normal

variable ($\hat{\Pi}_{ij}$) and the product of two normal variables (\hat{S}_i and \hat{S}_j). Under certain conditions, the distribution of the product of two normal variables is normal (see Anderson and Thurnsby (1986)). Further, the ratio of two independent normal variables follows the Cauchy distribution (Hogg and Craig (1978), p. 142), however, the mean and higher moments of the Cauchy density do not exist (Mood, Graybill and Boes (1974) p.117). Therefore, the estimators of elasticities do not have nice small sample properties. Therefore, some procedure other than a parametric method have to be used to obtain standard errors of estimates of elasticities since their analytic expressions are not available. In the past an asymptotic (Taylor's series) approximation has been used, however, Green and Hahn (1987) argued against this method. They used a non-parametric Efron's bootstrap method (Efron and Gong (1983)) to derive the standard errors of estimates of elasticities for a linear expenditure system. However, the bootstrap procedure is not suitable to obtain estimates of standard errors of elasticities for the system of share equations of ARX(1,1) and other dynamic models of translog functional form used in this study due to the complexity of the models. Therefore, a statistical simulation procedure will be used to obtain the standard errors of elasticities. A similar procedure had been utilized by Adamowicz et. al. (1989a, 1989b) to obtain variance of consumer's surplus for several functional forms of demand relations.

Let θ denote a vector of the parameters α and Π with $V(\theta)$ as the associated variance-covariance matrix. Further, let $\hat{\theta}$ and $V(\hat{\theta})$ denote the maximum likelihood estimators of θ and $V(\theta)$. Under certain regularity conditions $\sqrt{T}(\hat{\theta}-\theta)$ is asymptotic normal with mean zero and has an asymptotic variance-covariance matrix $I^{-1}(\theta)$ (see Judge et. al (1985) p.178 and Cox and Hinkley (1974) Ch.9.2). This result is used to generate samples corresponding to the parameter θ and the variance-covariance matrix $V(\theta)$. Specifically, 1000 samples from a multivariate normal distribution with mean θ and variance-covariance matrix $V(\theta)$ are generated. Expected share values corresponding to each sample are obtained using equation (28). One thousand sets of elasticities are calculated using these expected shares and equations (29) and (30). Standard errors of estimates of elasticities are approximated to the standard deviations of the 1000 simulated elasticities.

The well known Central limit theorem (Hogg and Craig (1978)) states that "if X_1, X_2, \dots, X_n denote a random sample of size n from any distribution having positive variance σ^2 (and hence finite mean μ), then the random variable $\sqrt{n}(\bar{x} - \mu)/\sigma$ has a limiting normal distribution with mean 0 and unit variance." Therefore, large sample confidence intervals for estimates of elasticities can be constructed using the $(1-\alpha)$ percentile from a normal distribution. Hence, the $(1-\alpha)\%$ confidence intervals for the estimated elasticities are given by

$$\bar{\sigma}_{ij} \pm Z_{\alpha} \cdot \text{S.E.}(\sigma_{ij})$$

$$\bar{\eta}_{ii} \pm Z_{\alpha} \cdot \text{S.E.}(\eta_{ii})$$

Where $\bar{\sigma}_{ij}$ and $\bar{\eta}_{ii}$ are the average elasticities (for all i, j) and $\text{S.E.}(\bar{\sigma}_{ij})$ and $\text{S.E.}(\bar{\eta}_{ii})$ are their respective standard errors generated from the simulated set of one thousand values of elasticities. If zero lies in the

confidence interval, then the hypothesis of zero elasticity is not rejected at $(1-\alpha)\%$ level of significance.

The methods detailed here dictate the data requirements necessary to achieve the estimation of parameters for both the Canadian and Alberta agricultural production sectors. The following section describes the data acquired and their transformations for use in methods application.

2.6 DATA DESCRIPTION

This study utilizes secondary data which were originally assembled by T. W. Manning for his work [Manning (1985, 1986a, 1986b)] on the Alberta and Canadian agricultural sectors. Most of the data were obtained from various Statistics Canada publications but augmented by unpublished data from Alberta Agriculture and Statistics Canada. The original data sets covered the period 1926 to 1984. For the purposes of this study, the data are updated to 1988. A complete list of input and output commodities and their sources is given in Appendix 1 of Sandhu (1991).

Data on prices and quantities of outputs produced and inputs used in the agricultural production sector are required. Actual production data for most crops were available until 1984. Prices for Canada are estimated using weighted average processes on the information available in Statistics Canada publication 22-200. Data on the values of most inputs are available from Statistics Canada publications. If average prices are not available then price indexes from various Statistics Canada publications are used. Data on quantities of inputs are obtained by dividing values by prices. Annual values of unpaid labor and owned land are estimated from the residual values reflecting the differences between total output values and total values of remaining inputs [for details see Appendix 1 of Sandhu (1991)].

Estimation of a system of share equations of a translog cost function is undertaken in this study. Therefore, data on output indexes, as well as share and input price indexes of selected aggregated input categories are required. The four input categories are capital, intermediate goods, labor, and land and building structures.

Capital input is the aggregate of depreciation, repairs on machinery and 15% of the beginning inventories of livestock. Labor is the aggregate of hired and unpaid (including family) labor. Land and building structures is the aggregate of land owned and rented, and depreciation and repairs of building structures on farms. All other inputs such as feed, seed, fuel, fertilizers, chemicals, electricity, financial expenses etc., are aggregated into the intermediate goods categories. Complexity, especially with regard to a model of a system of share equations of a multivariate autoregressive process of order one creates a degrees of freedom problem if more than four input aggregates are used.

The Tornqvist indexing procedure, which is an approximation of the Divisia continuous procedure [Christensen (1975), Diewert (1976), Islam and Veeman (1980), Manning (1985, 1986b)], is used to obtain the four aggregated input price and share values, along with output quantity indexes.

3. A MODEL FOR THE CANADIAN AGRICULTURAL SECTOR

In this section, a model of the Canadian agricultural production sector is developed based on methodology presented in section 2.

Maximum log-likelihood estimates for the models under consideration and likelihood ratio test statistics for various tests are presented in Table 3.1a below. Hypothesis H_0 postulated in section 2 is rejected at 1%

level of significance for the static, partial adjustment, AR(1) error models since likelihood statistics are respectively 77.75, 23.40 and 20.31. For the static model the plots of estimated residuals, residual autocorrelation and partial autocorrelation, and normal probability (Figures 1, 2 and 3 respectively) are contained in Appendix 1. Plots of the estimated residuals obtained from the three share equations indicate systematic behavior. Residuals from the capital share equation are negative for years 1937 to 1956, are positive for 1957 to 1982, and are again negative from 1983 onward. The pattern followed by the residuals indicates non-random behavior of the residuals. A similar pattern, but not to the same extent, is indicated by the estimated residuals for the other two share equations. From Fig. 2, Appendix 1, residual autocorrelation for the capital share equation is significant for the first few lags but dampens out instead of truncating. This suggests that the residuals from the capital share equation follow an autoregressive process instead of a white noise pattern. The residual partial autocorrelation function truncates after the first lag, suggesting the case of an autoregressive process of order one. A similar conclusion can be drawn from the residual autocorrelation and partial autocorrelation plots for labor. From the analysis of the intermediate share equation, indications are that residuals may follow an autoregressive process pattern of order one or two or even a mixed autoregressive moving average process pattern (Box and Jenkins 1976, McLeod et. al 1977). Therefore, residuals from the fitted share equations for the homogeneous and symmetric static model are not randomly distributed. Normal probability plots (Figure 3, Appendix 1) appear to be satisfactory, since the plotted points are very close to the straight line.

Further, diagnostic tests for a partial adjustment model based on residuals (Figures 4, 5, 6, Appendix 1) suggest that the model is inadequate. Residual and normal probability plots do not indicate any significant deficiency. However, the residual autocorrelation and partial autocorrelation plots for the share equations of intermediate goods and labor indicate that the residuals do not follow white noise process. Instead, the residuals from the share equations may follow either an autoregressive or a mixed autoregressive moving average process.

Diagnostic tests for AR(1) error model based on Figures 7, 8 and 9, Appendix 2, do not indicate a significant lack of fit. Residual autocorrelation and partial autocorrelation functions appear to be non-significant, indicating that residuals are white noise. The normal probability plots do not indicate deviance from normality. Therefore, it can be concluded from a diagnostic check of residuals from a model with autoregressive process of order one for the disturbances, that the model fits the data statistically well.

Therefore, a static model which has been used by most of the earlier studies for analyzing the Canadian and U.S. agricultural production

sectors is found to be very inadequate for the present case. A partial adjustment model again seems to be inadequate since it rejects the economic restrictions as well as statistical diagnostic tests. AR(1) model though fits statistically better than the above two models but the hypothesis of homogeneity and symmetry still is rejected.

Estimated results and diagnostic tests indicate that the ARX(1,1) provides a reasonable estimate of the data generating process of the Canadian agricultural sector. The likelihood ratio test statistic for the simultaneous test of homogeneity and symmetry is 15.95, which is less than 16.812 ($\chi^2_6(.01)$). Therefore, the hypothesis of homogeneity and symmetry is not rejected at 1% level of significance. The hypotheses of homogeneity and symmetry, when tested separately, show a slight evidence against the former at 1% level of significance. However, when tested simultaneously, null hypothesis is not rejected. Therefore, it can be concluded that homogeneity and symmetry restrictions are satisfied by the multivariate autoregressive model of a system of share equations of translog form.

Residual plots (Figure 10, Appendix 1) of the multivariate autoregressive process of order one, ARX(1,1) for the homogeneous symmetric model does not show any significant non-randomness. Residual autocorrelation and partial autocorrelation plots (Figure 11, Appendix 2) suggest that, the residuals follow a white noise process, since the autocorrelation and partial autocorrelation functions are not significant. Normal probability plots (Figure 12, Appendix 2) also suggest that the residuals are normally distributed.

Table 3.1a: Maximum Likelihood Estimates and Likelihood Ratio Statistics for Canadian Agricultural Sector.

Model		Estimated Max. Log-Likelihood	Likelihood Ratio Statistics	
			$-2\log\lambda_3$	$-2\log\lambda_6$
STATIC	UNRESTRICTED	717.19	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 0 5px; margin: 0 5px;"> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px;"></div> </div> <div style="margin: 0 5px;">→</div> <div style="border-bottom: 1px solid black; padding: 0 5px;"> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px;"></div> </div> </div>	77.75
	HOMOGENEOUS	704.90		
	SYMMETRIC	678.32		
PARTIAL ADJUSTMENT	UNRESTRICTED	738.09	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 0 5px; margin: 0 5px;"> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px;"></div> </div> <div style="margin: 0 5px;">→</div> <div style="border-bottom: 1px solid black; padding: 0 5px;"> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px;"></div> </div> </div>	23.40
	HOMOGENEOUS	733.70		
	SYMMETRIC	726.39		
AR(1) ERROR	UNRESTRICTED	806.65	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 0 5px; margin: 0 5px;"> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px;"></div> </div> <div style="margin: 0 5px;">→</div> <div style="border-bottom: 1px solid black; padding: 0 5px;"> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px;"></div> </div> </div>	20.31
	HOMOGENEOUS	800.44		
	SYMMETRIC	796.50		
ARX(1,1)	UNRESTRICTED	835.50	<div style="display: flex; align-items: center;"> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 0 5px; margin: 0 5px;"> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px;"></div> </div> <div style="margin: 0 5px;">→</div> <div style="border-bottom: 1px solid black; padding: 0 5px;"> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px; margin-bottom: 2px;"></div> <div style="border-bottom: 1px solid black; height: 10px;"></div> </div> </div>	15.95
	HOMOGENEOUS	829.09		
	SYMMETRIC	827.50		

$\chi^2_3(.05) = 7.815$, $\chi^2_3(.01) = 11.345$, $\chi^2_6(.05) = 12.592$ and $\chi^2_6(.01) = 16.812$

3.1.5 GOODNESS OF FIT TESTS FOR NESTED MODELS OF ARX(1,1)

The model of a system of share equations desired is one which satisfies economic restrictions and has a good fit on the basis of statistical tests of the residuals. The above tests indicate that a multivariate autoregressive model fits the data generated from the Canadian agricultural production sector. There are other tests that can be performed to further check whether the model fits the data generating process of the Canadian agricultural sector. Goodness of fit tests will give an indication as to whether the ARX(1,1) model is a good fit compared to the nested homogeneous and symmetric models. The likelihood ratio test and Akaike Information Criterion (AIC) will be used to test whether this model is an improvement over the static model, the partial adjustment model, and the model with autoregressive process of order one for the disturbances.

The AIC and maximum log-likelihood estimates for the various models (with symmetric and homogeneity restrictions) are presented in Table 3.1b below.

Table 3.1b: Results on Goodness of Fit Tests of the Canadian Models.

Model	Maximum Log-likelihood	AIC
ARX(1,1)	827.50	-1577.00
AR(1) ERROR	796.50	-1544.00
PARTIAL ADJUSTMENT	726.39	-1404.78
STATIC	678.32	-1296.64

Based on the AIC, ARX(1,1) is the best model. Now taking the ARX(1,1) model as the maintained hypothesis, the likelihood ratio statistics to test the hypotheses that the AR(1) error model, the partial adjustment model and the static model fit the data better are 62.0 (D.F.⁴ 15), 202.22 (D.F. 15), and 298.37 (D.F. 24) respectively. The hypotheses of all the nested models are rejected at 1% level of significance, since the critical values are $\chi^2_{15}(.01) = 30.58$ and $\chi^2_{24}(.01) = 42.98$. Therefore, the multivariate autoregressive process of order one fits the data for the Canadian agricultural sector adequately and the above analysis suggests that this model can be used for further economic analysis.

The estimated values of the parameters and their asymptotic standard errors for the ARX(1,1) model for Canada are presented in Table 3.1c.

The numbers in the subscripts in the notations of the parameters in Table 3.1c represent 1 for capital, 2 for intermediate goods, 3 for labor, and 4 for land and building structures.

⁴ D.F. denotes number of independent restrictions to obtain the nested model from the maintained hypothesis model, ARX(1,1).

Table 3.1c: Estimates of the Parameters of the Homogeneous-Symmetric ARX(1,1) Model for Canada.

Parameter	Estimated Value	Standard Error	Parameter	Estimated Value	Standard Error
D_{11}	0.9383*	0.1773	Γ_{14}	-0.0136	0.0157
D_{12}	0.0547	0.1329	Γ_{1q}	-0.0069	0.0180
D_{13}	0.0493	0.1380	Γ_{21}	0.0080	0.0142
D_{21}	0.6764	0.4562	Γ_{22}	0.2484*	0.0245
D_{22}	0.8666*	0.3407	Γ_{23}	-0.1754*	0.0269
D_{23}	0.2416	0.3566	Γ_{24}	-0.0520	0.0426
D_{31}	-0.8489*	0.3387	Γ_{2q}	0.1240*	0.0480
D_{32}	-0.1786	0.2600	Γ_{31}	-0.0478*	0.0107
D_{33}	0.5091	0.2771	Γ_{32}	-0.1849*	0.0185
Γ_{11}	0.0494*	0.0053	Γ_{33}	0.1889*	0.0200
Γ_{12}	-0.0281*	0.0090	Γ_{34}	0.0277	0.0319
Γ_{13}	-0.0155	0.0099	Γ_{3q}	-0.1181*	0.0358
Long-Run Parameters					
α_1	0.0568	0.0576	Π_{14}^\dagger	-0.0683	0.0127
α_2	0.5393*	0.0496	Π_{22}	0.1899*	0.0192
α_3	0.2725*	0.1115	Π_{23}	-0.1437*	0.0303
α_4^\dagger	0.1315	0.0237	Π_{24}^\dagger	-0.3078	0.0459
Π_{11}	0.0372	0.0264	Π_{33}	0.0952	0.0762
Π_{12}	0.0228	0.0163	Π_{34}^\dagger	-0.1986	0.0174
Π_{13}	0.0083	0.0406	Π_{44}^\dagger	0.5747	
Scale Parameters			Technical Change Parameter		
Π_{1q}	-0.0287	0.0691	θ_1	0.0054	0.0955
Π_{2q}	0.3183*	0.0574	θ_2	0.0105	0.0082
Π_{3q}	-0.2787*	0.1344	θ_3	-0.0071	0.0185
Π_{4q}^\dagger	-0.0109	0.0286	θ_4^\dagger	-0.0088*	0.0039

¹ The estimated values marked with * indicate the estimated values are significant at 5% level of significance ($t(.05)=1.96$).

² The parameters marked with \dagger indicate that estimated values are obtained using additivity restrictions.

³ The long-run parameters are α , π , and θ (Section 2, equation 15).

3.2 TECHNOLOGICAL CHANGE AND HOMOTHETICITY

In this section the share equations, estimated from the multivariate autoregressive process with the restrictions of homogeneity of order one in prices and symmetry, are used to test the hypotheses of factor neutral technical change and of homotheticity of the production process. The following two hypotheses are tested:

H_{OT} : Technological change in the Canadian agricultural sector for the period 1926 to 1988 is factor neutral;

H_{OY} : Data for the Canadian agricultural sector for the period 1926 to 1988 are generated by a homothetic production process.

Table 3.2: Tests of Neutral Technical Change and Homotheticity For Canada

Model	Estimated Max. Log-Likelihood	$-2\log \lambda_3$	Decision $\chi^2_3(.01) = 11.3$
ARX(1,1)	827.50		
ARX(1,1) ¹	820.98	13.05	H_{OT} is rejected
ARX(1,1) ²	813.87	27.27	H_{OY} is rejected
ARX(1,1) ³	802.99		

¹ ARX(1,1) model under H_{OT} i.e., without time variable.

² ARX(1,1) model under H_{OY} i.e., without output variable.

³ ARX(1,1) model without time and output variables.

3.2.1 TECHNOLOGICAL CHANGE

The hypothesis H_{OT} is tested using the likelihood ratio test. The system of share equations of the homogeneous-symmetric ARX(1,1) model of translog form are estimated with and without a time trend variable. The maximum likelihood estimates are given in the Table 3.2. The hypothesis H_{OT} of factor neutral technical change is rejected, since $-2\log \lambda_3 = 13.05$ is greater than the 1% critical level of $\chi^2_3(.01) = 11.345$.

Therefore, technical progress in the Canadian agriculture production sector is factor biased.

The technical change is factor using/saving if the sign of the estimated coefficient of the time trend variable, θ of the respective factor share equation is positive/negative, and provided the estimated value is statistically significant. From Table 3.1c, the estimated values of the bias of the technical change parameters for capital and labor are non-significant. The bias of technical change (estimated using the additivity restriction) for land and building structures is significant. The significance level for bias of technical change for intermediate goods is 20%. The sign of the estimates of the biases indicate technical change is intermediate goods using and land saving. Therefore, it can be

concluded that the technical change in the Canadian agricultural production process is factor biased and is land and building saving and may be intermediate goods using.

3.2.2 HOMOTHETICITY

The hypothesis H_{oy} is tested by dropping the output index variable from the system of share equations. The maximum likelihood estimate for this model is 813.87 with a corresponding likelihood ratio statistic of 27.27 ($-2\log\lambda$). The hypothesis H_{oy} is rejected since the value of 27.29 is statistically significant. Therefore, the production process for the Canadian agricultural sector is non-homothetic. This implies technology of the Canadian agricultural sector does not exhibit constant returns to scale. As discussed in the methodology chapter, a hypothesis of constant returns to scale is automatically rejected when the hypothesis of homotheticity is rejected. The relationship between scale and the factors of production is discussed below.

From Table 3.1c the long-run coefficients of scale of production for intermediate goods and labor are statistically significant. Scale of production is intermediate goods using since, the long-run coefficient is positive, while the scale of production is labor saving since the coefficient is negative.

3.3. OWN-PRICE AND SUBSTITUTION ELASTICITIES

Estimates of substitution and own-price elasticities are presented in Tables 3.3.1 and 3.3.2, respectively. Elasticities are estimated for the time periods 1959-68, 1969-78, 1979-88, and 1926-88. The expected values of shares to be used in equations (17) and (18) are calculated using equations (15) and (16) for their respective time periods in order to estimate their elasticities. Using the maximum likelihood estimates of the long-run parameters and the variance-covariance matrix, a sample of the size of one thousand is simulated. Long-run expected shares are calculated for each value of the sample. The elasticities are then calculated for each time period as described above. The average and standard deviations of the simulated set of one thousand elasticities are calculated. Further, by applying the Central Limit Theorem, 95% and 90% asymptotic confidence intervals of the elasticities are constructed using the simulated results. These results are presented in the Tables 3.3.1 and 3.3.2 below. The economic analysis based on the estimates and their confidence intervals is presented in the subsections below.

3.3.1 ELASTICITIES OF SUBSTITUTION

The relationship between the factors of production is determined on the basis of the signs and magnitudes of the statistically significant estimates of the elasticity of substitution. If an estimate of elasticity of substitution is not significant, then it can be concluded that the pair of factors does not exhibit substitution or complementary properties.

From Table 3.3.1, the estimates of elasticities of substitution between labor and land and building structures and between intermediate goods and land and building structures are statistically significant at the 5% level of significance for all periods under consideration. The elasticity of substitution between capital and intermediate goods is

significant at the 10% level of significance for the period 1979-88 only. The rest of the estimates of elasticities of substitution are not different from zero even at the 10% level of significance.

The elasticity of substitution between labor and land and building structures is positive and largest among all of the estimates. These estimates also increased from approximately 2.46 in the earlier periods to 3.41 for the last period of 1979-88. Therefore, labor and land and building structures are substitutes and the degree of substitution has increased for the last decade. This result supports the observation that farm size is increasing and farm employment is decreasing, which is generally accredited to the farm mechanization.

Intermediate goods and land and building structures are substitutes but the degree of substitutability is not very large, since the estimate of the elasticity of substitution is approximately .5.

The rest of the elasticities of substitution are not statistically different from zero. The conclusion that capital and labor are not substitutes for each other is a surprising result, since earlier studies, Adamowicz (1986) and Lopez (1980) concluded that capital and labor were substitutes. However, if either a point estimate or standard error of coefficient (Π_{ij}) were used in this study to obtain significance levels, then it would be concluded that capital and labor are substitutes since the estimate of elasticity of substitution between labor and capital is 1.3511. Furthermore, following the same line of thought, capital and land would be viewed as complements. Hence, conclusions based on point estimates alone can be misleading.

The rest of the estimates of the elasticities of substitution are not statistically significant from zero and, therefore, they are taken to be independent of each other. Independence here means that these factors cannot be substituted in response to changes in their relative prices. These inputs are therefore used in fixed proportions.

Table 3.3.1: Elasticities of Substitution for Canada

	ESTIMATED ELAST.	SIMULATED		CONFIDENCE-INTERVALS			
		MEAN ELAST	S.E. ELAST.	— 95% —		— 90% —	
				LOW	HIGH	LOW	HIGH
Sample for the Years 1959-68							
CAP-INT	0.4706	0.4956	0.3678	-0.2252	1.2164	-0.1094	1.1005
CAP-LAB	1.3304	1.3433	1.6756	-1.9409	4.6274	-1.4131	4.0996
CAP-LND	-2.0648	-2.0775	1.6963	-5.4022	1.2472	-4.8679	0.7129
INT-LAB	0.0985	0.1023	0.1745	-0.2397	0.4442	-0.1847	0.3893
INT-LND †	0.5041	0.5131	0.1658	0.1880	0.8381	0.2403	0.7859
LAB-LND †	2.4616	2.4950	0.6589	1.2036	3.7865	1.4111	3.5789
Sample for the Years 1969-78							
CAP-INT	0.4913	0.5208	0.3501	-0.1654	1.2070	-0.0551	1.0968
CAP-LAB	1.3420	1.2905	1.7989	-2.2354	4.8165	-1.6687	4.2498
CAP-LND	-1.8550	-1.7489	1.4421	-4.5754	1.0777	-4.1211	0.6234
INT-LAB	0.0652	0.0677	0.1675	-0.2606	0.3960	-0.2079	0.3432
INT-LND †	0.5373	0.5472	0.1515	0.2502	0.8443	0.2980	0.7965
LAB-LND †	2.4695	2.5158	0.6871	1.1690	3.8626	1.3854	3.6462
Sample for the Years 1979-88							
CAP-INT ‡	0.5805	0.5788	0.3229	-0.0541	1.2118	0.0476	1.1100
CAP-LAB	1.4264	1.5296	2.1739	-2.7313	5.7904	-2.0465	5.1056
CAP-LND	-2.4119	-2.6640	2.3053	-7.1824	1.8544	-6.4563	1.1282
INT-LAB	-0.0574	-0.0652	0.2547	-0.5644	0.4341	-0.4841	0.3538
INT-LND †	0.4983	0.5023	0.1825	0.1445	0.8600	0.2020	0.8025
LAB-LND †	3.4087	3.4556	1.0631	1.3720	5.5392	1.7069	5.2044
Sample for the Years 1926-88							
CAP-INT	0.4156	0.4396	0.4109	-0.3658	1.2450	-0.2364	1.1155
CAP-LAB	1.3511	1.3558	1.7839	-2.1406	4.8523	-1.5787	4.2904
CAP-LND	-2.2602	-2.2644	1.7919	-5.7765	1.2478	-5.2121	0.6834
INT-LAB	0.1167	0.1199	0.1737	-0.2204	0.4603	-0.1657	0.4056
INT-LND †	0.5137	0.5222	0.1634	0.2019	0.8425	0.2534	0.7910
LAB-LND †	2.3800	2.4106	0.6201	1.1951	3.6260	1.3904	3.4307

¹ Symbol † indicates the elasticity is significant at 5% level of significance i.e., zero does not belong to the 95% confidence interval.

² Symbol ‡ indicates the elasticity is significant at 10% level of significance i.e., zero does not belong to the 90% confidence interval.

3.3.2 OWN-PRICE ELASTICITIES

The signs of the estimates of own-price elasticities are negative as expected. The estimates of own-price elasticities for intermediate goods and land and building structures are significant at 5% level of significance. Factor price elasticity of land is largest and is approximately equal to $-.84$. Own price elasticities of capital and labor are not significant at 10% level of significance. However, all of the factor price elasticities are negative and greater than minus one implying that all four of the factors of production are inelastic to their own prices.

Table 3.3.2: Own-Price Elasticities for Canada

	ESTIMATED ELAST.	SIMULATED		CONFIDENCE-INTERVALS			
		MEAN ELAST	S.E. ELAST.	— 95% — LOW HIGH		— 90% — LOW HIGH	
Sample for the Years 1959-68							
CAP	-0.4658	-0.4640	0.3510	-1.1521	0.2240	-1.0415	0.1135
INT †	-0.1143	-0.1164	0.0354	-0.1857	-0.0470	-0.1746	-0.0582
LAB	-0.3830	-0.3989	0.2582	-0.9049	0.1071	-0.8236	0.0257
LND †	-0.8449	-0.8473	0.1037	-1.0504	-0.6441	-1.0178	-0.6768
Sample for the Years 1969-78							
CAP	-0.4734	-0.4395	0.3969	-1.2175	0.3384	-1.0925	0.2134
INT †	-0.1111	-0.1129	0.0339	-0.1794	-0.0464	-0.1687	-0.0571
LAB	-0.3817	-0.4090	0.2765	-0.9509	0.1329	-0.8638	0.0458
LND †	-0.8434	-0.8445	0.0987	-1.0379	-0.6510	-1.0068	-0.6821
Sample for the Years 1979-88							
CAP	-0.4899	-0.5117	0.2990	-1.0978	0.0743	-1.0037	-0.0198
INT †	-0.0763	-0.0784	0.0321	-0.1412	-0.0155	-0.1311	-0.0256
LAB	-0.3484	-0.3459	0.3655	-1.0623	0.3705	-0.9471	0.2554
LND †	-0.8469	-0.8511	0.1234	-1.0929	-0.6093	-1.0540	-0.6482
Sample for the Years 1926-88							
CAP	-0.4303	-0.4240	0.3912	-1.1907	0.3427	-1.0675	0.2194
INT †	-0.1157	-0.1178	0.0359	-0.1882	-0.0474	-0.1769	-0.0587
LAB	-0.3829	-0.3980	0.2511	-0.8902	0.0942	-0.8111	0.0151
LND †	-0.8441	-0.8468	0.1010	-1.0447	-0.6489	-1.0129	-0.6807

¹ Symbol † indicates the elasticity is significant at 5% level of significance i.e., zero does not belong to the 95% confidence interval.

4. ASSESSMENT AND COMPARISON OF EMPIRICAL RESULTS

In this chapter, a comparison is made between the empirical results obtained from this study and the earlier studies of this nature on the Canadian agricultural sector by Lopez (1980), Islam and Veeman (1980) and Adamowicz (1986).

4.1 TECHNICAL CHANGE

Technical change in this study is found to be factor biased for the Canadian agricultural sector. The bias of technical change is land and building structures saving and there is some evidence that it is intermediate goods using. Technical change is neutral to capital and labor.

In Lopez (1980), the hypothesis of factor neutral technical change is not rejected. Therefore, it was concluded that technical change is factor neutral for the Canadian agricultural sector. However, it was pointed out in that study that, when homotheticity was imposed, technical change was found to be factor biased. Lopez (1980) observed that, "The assumption of a homothetic production function is a crucial assumption when testing for technical progress. When homotheticity is imposed, the output expansion effects on input shares are incorrectly attributed to biased technical change". Islam and Veeman (1980) concluded in their study that technical change is factor biased. However, it should be noted that they imposed homotheticity without testing and the hypothesis of factor neutral technical change was not tested. Their conclusions are based on the significance of the estimated values of the coefficients of time in the share equations alone. Following Lopez (1980), it can be concluded that biases may have been incorrectly attributed to the factors.

Adamowicz (1986) tested and rejected the hypothesis of factor neutral technical change. It was found that technical change was capital and material (intermediate goods) using and land and labor saving. However, the results of this study indicate that technical change is neutral to capital and labor and land saving and there is weak evidence that it is intermediate goods using.

4.2 SCALE EFFECTS

In this study, the hypothesis of homotheticity is tested and rejected, while scale of production is intermediate goods using and labor saving.

Lopez (1980) also rejected the hypothesis of a homothetic production function, but found that scale was saving for all factors. It was concluded, "as scale of production is expanded efficiency in the use of factors increases". Lopez (1980) results are surprising, since he both rejected homotheticity and found that scale is factor saving for all factors. In general, for non-homothetic production functions, one would expect that scale be factor using or neutral for at least one factor.

Islam and Veeman (1980) imposed homotheticity without testing and so no comparisons can be made.

Adamowicz (1986) also rejected the hypothesis of homotheticity. Adamowicz found that scale of production was labor saving and neutral

for capital, which is consistent with the results of this study. However, he also found that scale was land using and neutral with respect to material (intermediate goods); which were results not found in this study.

4.3 ELASTICITY OF SUBSTITUTION

In this section, estimates of the elasticities of substitution from the ARX(1,1) model are compared to estimates from the static model (see Table 4.1.) with the same economic restrictions imposed as for the ARX(1,1) model.

It should be first noted that the elasticities of substitution are significantly different from zero for more pairs of inputs for the static model as compared to those for the ARX(1,1) model. Secondly, the elasticities are smaller in magnitude for the static model as compared to that for the ARX(1,1) model. The exceptions are those for the intermediate goods and land and building structures, which are almost equal in magnitude. Therefore, there are two general conclusions that can be drawn. Firstly, long-run elasticities from the ARX(1,1) model are larger than the elasticities from the static model i.e., the static model tends to under estimate the elasticities. Secondly, the elasticities from the static model appear to be statistically significant from zero for more factor input pairs as compared to the long-run elasticity estimates from the ARX(1,1) model. These results indicate that the elasticities of substitution from the static model are sensitive to changes in relative prices but the degree of responsiveness is limited. This infers that the estimates of elasticities may also contain some short-run responses.

It is difficult to directly compare the elasticities estimated in different studies, since these studies were undertaken for different time periods. Furthermore, different aggregations of the data are used. The only results that can be adequately compared, are those which are statistically significant.

Lopez (1980) concluded that all factor inputs are substitutes and capital and labor, capital and intermediate are inputs with highest degree of substitutability. It should be pointed out that these results can be completely misleading since it was never tested whether these estimates were statistically different from zero. As can be seen in this study, from the estimates of elasticity of substitution of capital and labor which is greater than one for all periods, it may be concluded that capital and labor are substitutes. However, the hypothesis that elasticity of substitution between capital and labor is zero is not rejected. Therefore, if statistical significance of the estimates is not tested, it can incorrectly be concluded that capital and labor are substitutes.

The estimates of elasticity of substitution for intermediate goods and land and building structures are significant for this study, in both the static and dynamic models. It was also found to be significant by Adamowicz (1986). The magnitude of the estimates of elasticities of substitution also appear to be very close in all three models (ARX(1,1), static, and Adamowicz).

The estimates of the elasticity of substitution for labor and land and building structures are large in magnitude and significantly different from zero for the ARX(1,1) and the static model estimated in

this study, but are very small in Lopez (1980) and Adamowicz (1986). Lopez (1980) did not provide any significance level calculations for the estimates of the elasticities and therefore, comparing the results to this study is difficult.

Table 4.1: Comparison of Elasticities of Substitution for Canada

	ARX(1,1) ESTIMATED ELAST.	STATIC ESTIMATED ELAST	Lopez	Adamowicz	Islam and Veeman
1926-88			1946-77	1940-81	1961-78
CAP-INT	0.4156	0.2170‡	1.555	0.0889	
CAP-LAB	1.3511	0.3209‡	1.779	0.5535*	
CAP-LND	-2.2602	-0.5549	0.234	-0.4003	
INT-LAB	0.1167	-0.0476	0.875	1.3925*	
INT-LND	0.5137†	0.6491†	0.991	0.4420*	
LAB-LND	2.3800†	1.8888†	0.113	-0.1383	0.3176
1959-68					
CAP-INT	0.4706	0.3013†			
CAP-LAB	1.3304	0.3656†			
CAP-LND	-2.0648	-0.4375			
INT-LAB	0.0985	-0.0722			
INT-LND	0.5041†	0.6446†			
LAB-LND	2.4616†	1.9426†			
1969-78					
CAP-INT	0.4913	0.2923†			
CAP-LAB	1.3420	0.3451†			
CAP-LND	-1.8550	-0.4258			
INT-LAB	0.0652	-0.0794			
INT-LND	0.5373†	0.6562†			
LAB-LND	2.4695†	1.9292†			
1979-88					
CAP-INT	0.5805‡	0.4859†			
CAP-LAB	1.4264	0.1983			
CAP-LND	-2.4119	-0.4266			
INT-LAB	-0.0574	-0.2988‡			
INT-LND	0.4983†	0.6618†			
LAB-LND	3.4087†	2.5400†			

¹ Symbols † and ‡ denote that the estimated elasticities are significant at the 5% and 10% levels of significance. The significance of the estimates are determined on the basis of confidence intervals constructed using simulated results. For the confidence of simulated results of static model see Table A4.1 Appendix 4.

² Symbol * for the Adamowicz (1986) model indicates estimates are greater than twice their standard error. The standard errors of the estimates are approximated with the standard errors of the π_{ij} 's parameters.

4.4 OWN-PRICE ELASTICITIES

Most of the estimates of own-price elasticities from the ARX(1,1) model are larger, almost twice the size, in absolute value as compared to those estimated from the static model (see Table 4.2). The exception is for the estimate of own-price elasticity for land and building structures, which are fairly close. This type of relationship between own-price elasticity estimates from the two models is expected (Nakamura, 1985). Estimates of own-price elasticities from the ARX(1,1) model are long-run estimates, whereas the estimates from the static model may include short-run responses. However, as in the case with elasticities of substitution, own-price elasticities are statistically significant for more inputs from the static model as compared to those from ARX(1,1) model. It should also be noted, that the own-price elasticity estimates for land and building structures are almost equal for both the static and ARX(1,1) models in this study. This implies that there may not be significant short-run responses to changes in the prices of land and building structures.

As stated previously, it is difficult to compare the estimates of own-price elasticities from this study to those from earlier studies by Lopez (1980), Islam and Veeman (1980) and Adamowicz (1986). Except for signs which are negative as expected, there does not appear to be much similarity among the estimates of this study and the earlier ones.

Table 4.2: Comparison of Own-Price Elasticities for Canada

	ARX(1,1) ESTIMATED ELAST.	STATIC ESTIMATED ELAST	Lopez	Adamowicz	Islam and Veeman
1926-88			1946-77	1940-81	1961-78
CAP	-0.4303	-0.1666†	-0.347	-0.1680	
INT	-0.1157†	-0.0581†	-0.410	-0.2396	
LAB	-0.3829	-0.1689†	-0.517	-0.3441	-0.3315
LND	-0.8441†	-0.9170†	-0.422	-0.0938	-0.0626
1959-68					
CAP	-0.4658	-0.2332†			
INT	-0.1143†	-0.0572†			
LAB	-0.3830	-0.1626†			
LND	-0.8449†	-0.9194†			
69-78					
CAP	-0.4734†	-0.2215†			
INT	-0.1111	-0.0567†			
LAB	-0.3817	-0.1601†			
LND	-0.8434†	-0.9168†			
1979-88					
CAP	-0.4899	-0.3073†			
INT	-0.0763†	-0.0286			
LAB	-0.3484	-0.0328			
LND	-0.8469†	-0.9306†			

¹ Symbols † and ‡ denote that the estimated elasticities are significant at the 5% and 10% levels of significance. The significance of the estimates are determined on the basis of confidence intervals constructed using simulated results. For the confidence of simulated results of static model see Table A4.2 Appendix 4.

² Symbol * for the Adamowicz (1986) model indicates estimates are greater than twice their standard error. The standard errors of the estimates are approximated with the standard errors of the π_{ij} 's parameters.

5. SUMMARY

A multivariate autoregressive process ARX(1,1) model of a system of share equations of the translog functional form is found to fit the data on the Canadian agricultural sector adequately for the period 1926-88. Other models which are tried and rejected are; static (long-run equilibrium) model, a partial adjustment, autoregressive process of order one for disturbances model. Selection criterion is that model should satisfy the economic restriction and pass the statistical test of goodness of fit. Adamowicz (1986) and Islam and Veeman (1980) did not perform these tests on their models. While Lopez (1980) tested for the economic restriction, but did not perform statistical goodness fit tests. If these tests were performed on the models of earlier studies, the static models may not have been selected. Further the underlying assumption for the static model, that Canadian agriculture is in equilibrium state and adjusts instantaneously to the changes in relative factor price, is very unrealistic.

Therefore, a correct model for time series analysis would be a dynamic model as is the case in this study. Further, the conclusions based on an estimates static model will be misleading. As shown in this study, estimates of elasticities from a static model which will include short run responses which may not correspond to the long run process. It is found that long run estimates of elasticities obtained from dynamic model are statistically significant for fewer factor pairs as compared to those from the static model. However, the estimates of elasticities for the dynamic model are larger in magnitude. Therefore, if only static function was fitted in this study, then conclusions would be very misleading about the substitution behavior among the factor inputs.

One significant contribution of this study is that standard errors and confidence intervals of estimates of the elasticities are obtained using a statistical simulation procedure. Earlier studies either did not calculate standard errors of estimates of elasticities or utilized standard errors of estimates of coefficients (π_{ij} 's), which are crude approximations. Further, in this study asymptotic 95% and 90% confidence intervals for elasticities are constructed using the simulated results.

In summary, this study presents an improved model of the Canadian agricultural production sector. Therefore a more accurate analysis of biases of technical change and scale effects is performed. Further, unlike earlier studies, conclusions regarding response of factor inputs to the changes in their own price and other factor input prices is correctly made by calculating the confidence intervals of the respective elasticities.

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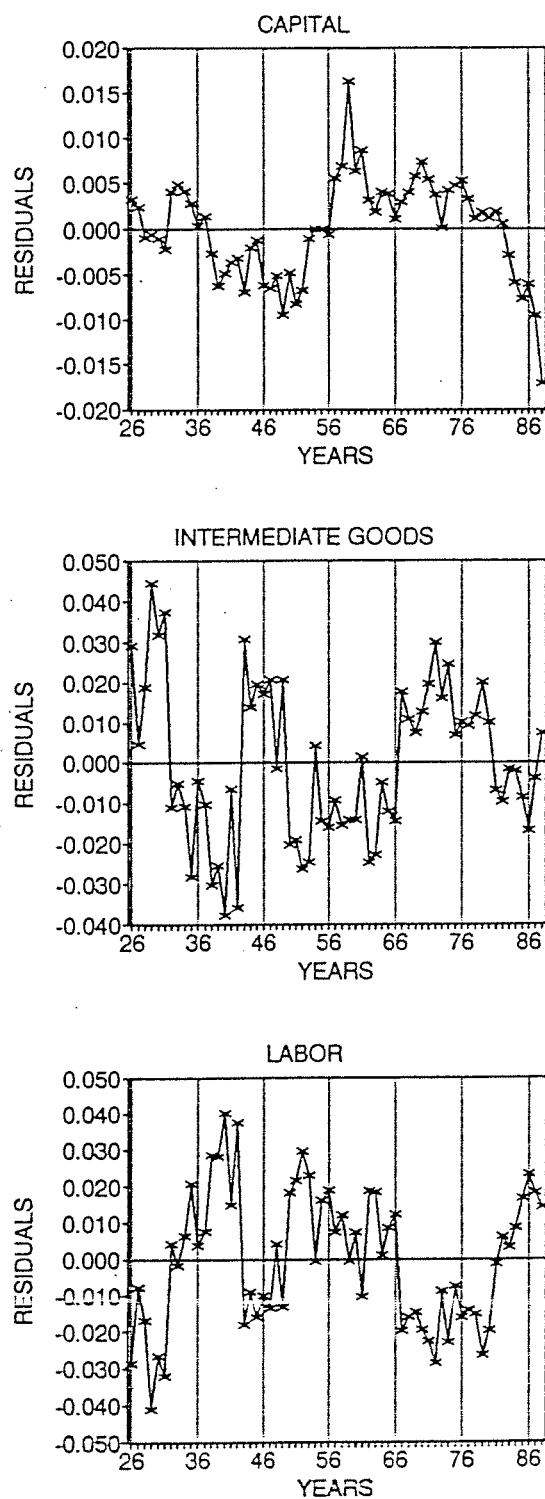
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APPENDIX 1
RESIDUAL ANALYSIS PLOTS FOR CANADA

The following abbreviations are used to represent the model of the system of share equations:

STCSY	Symmetric Static Model.
PADSY	Symmetric Partial Adjustment Model.
EARSY	Symmetric Autoregressive Process for Disturbances Model.
ARXSY	Symmetric Multivariate Autoregressive Process Model.

Figure 1: Residual Plots of STCSY for Canada



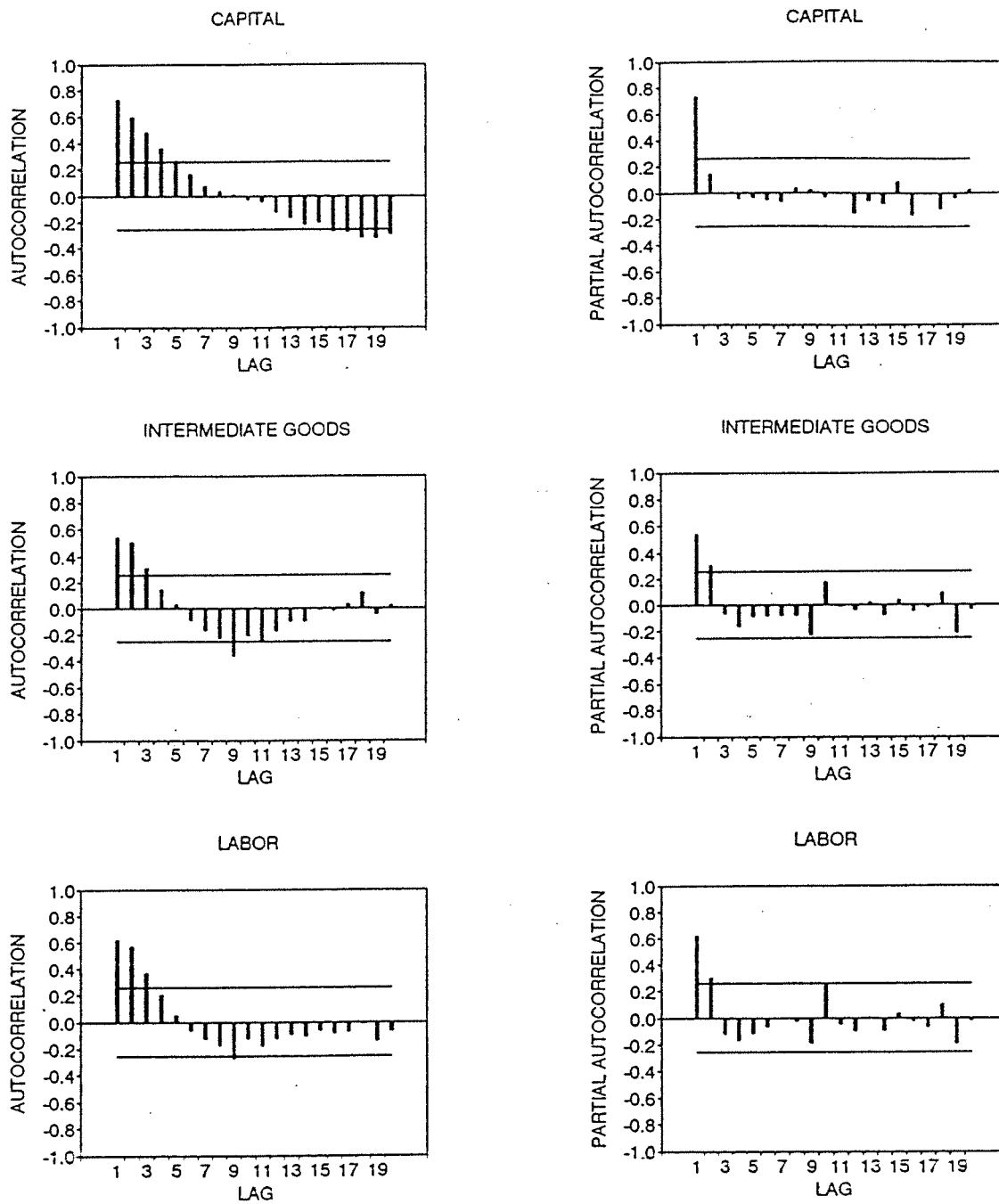


Figure 3: Normal Probability Plots of STCSY for Canada.

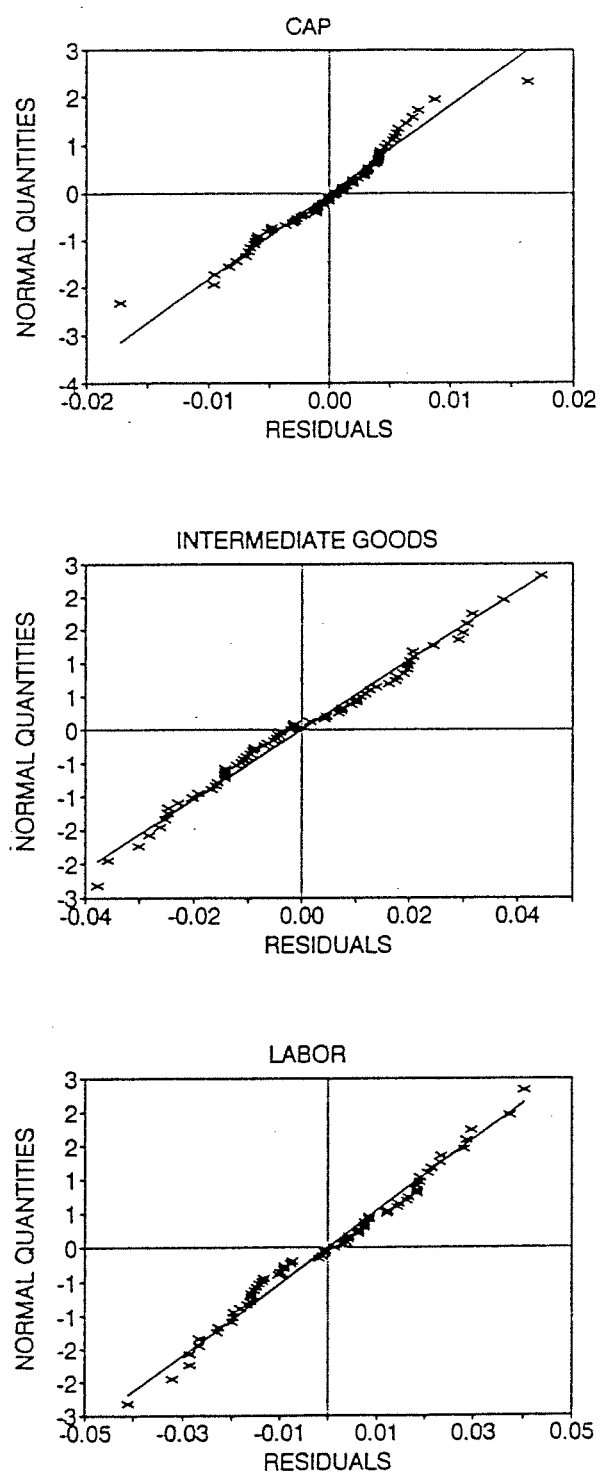


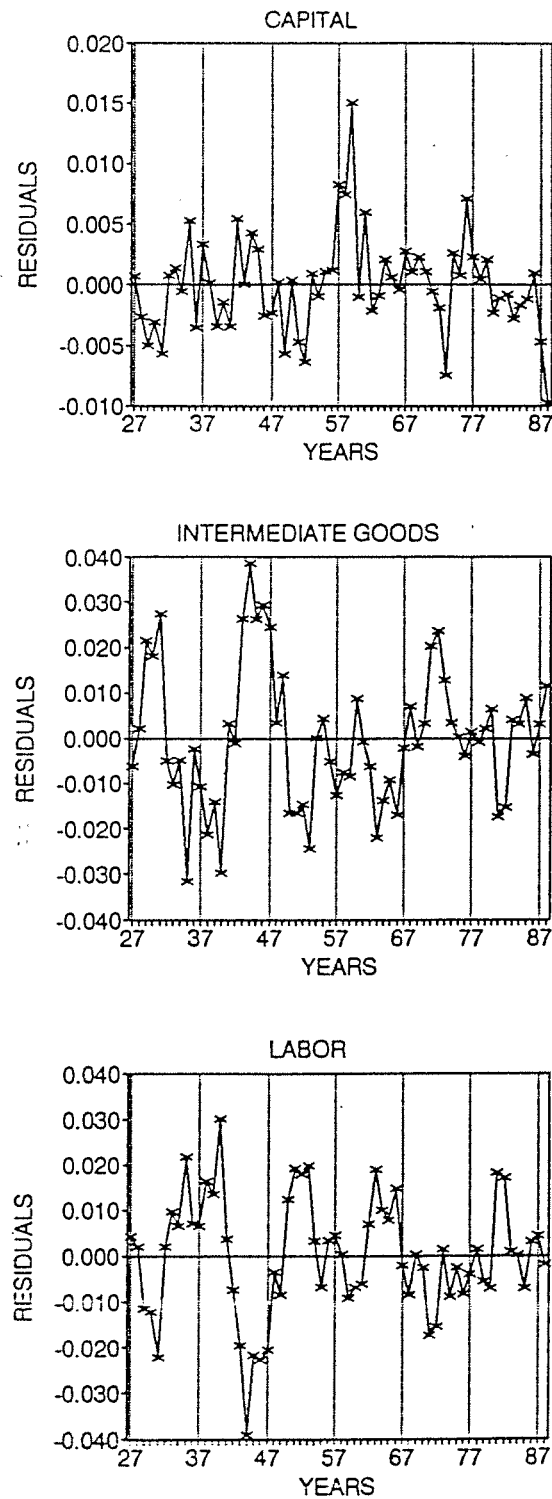
Figure 4: Residual Plots of PADS_Y for Canada

Figure 5: Residual Autocorrelation and Partial Autocorrelation Function of PADSY for Canada.

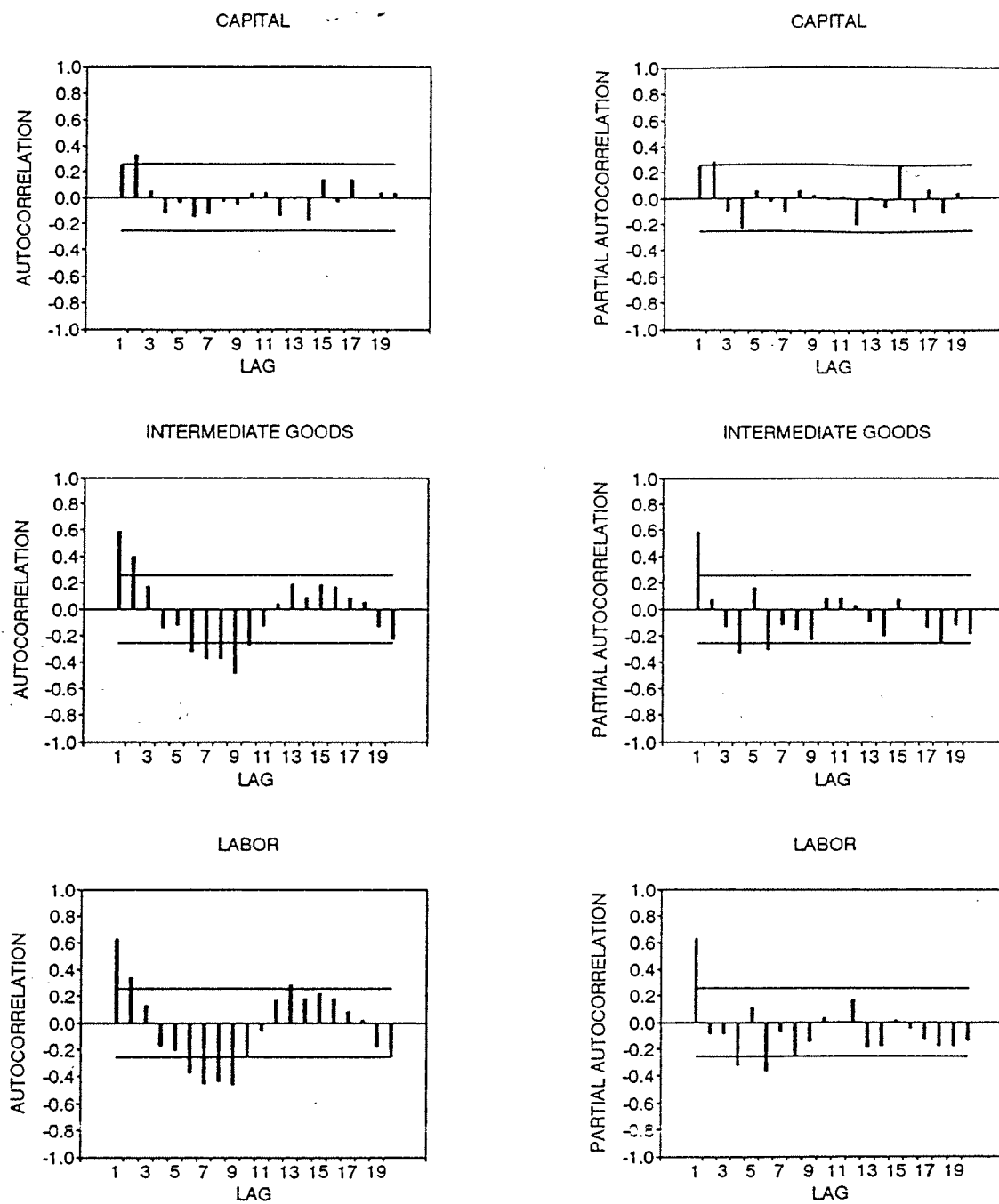


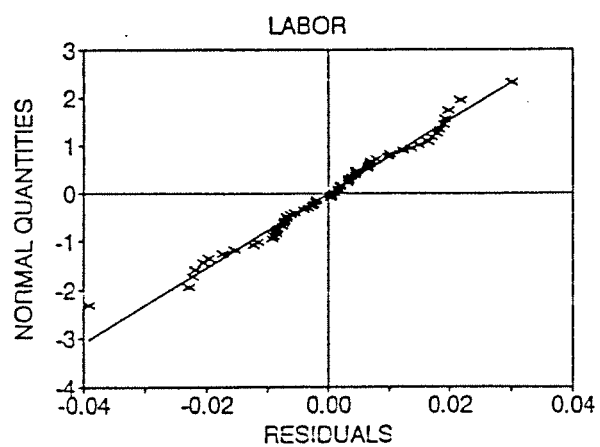
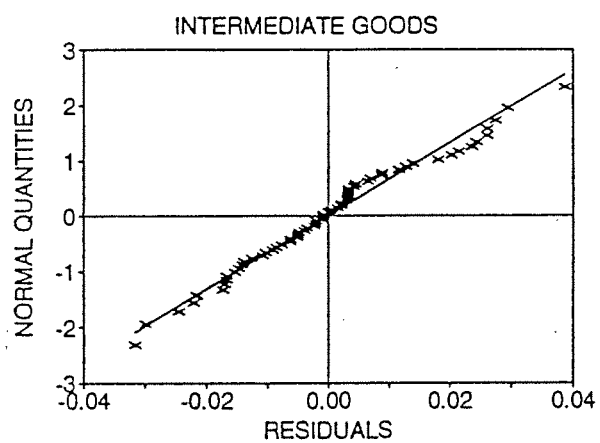
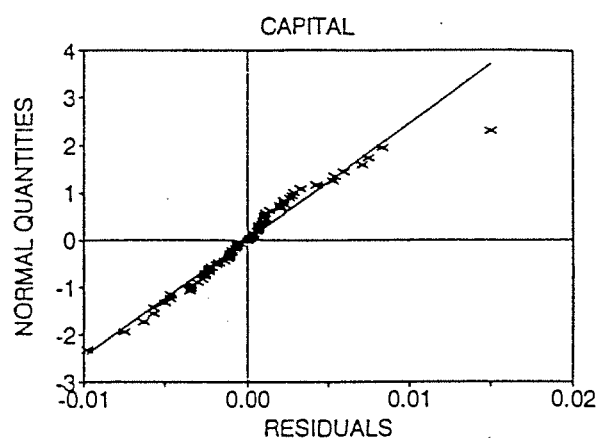
Figure 6: Normal Probability Plots of PADS_Y for Canada.

Figure 7: Residual Plots of EARSY for Canada

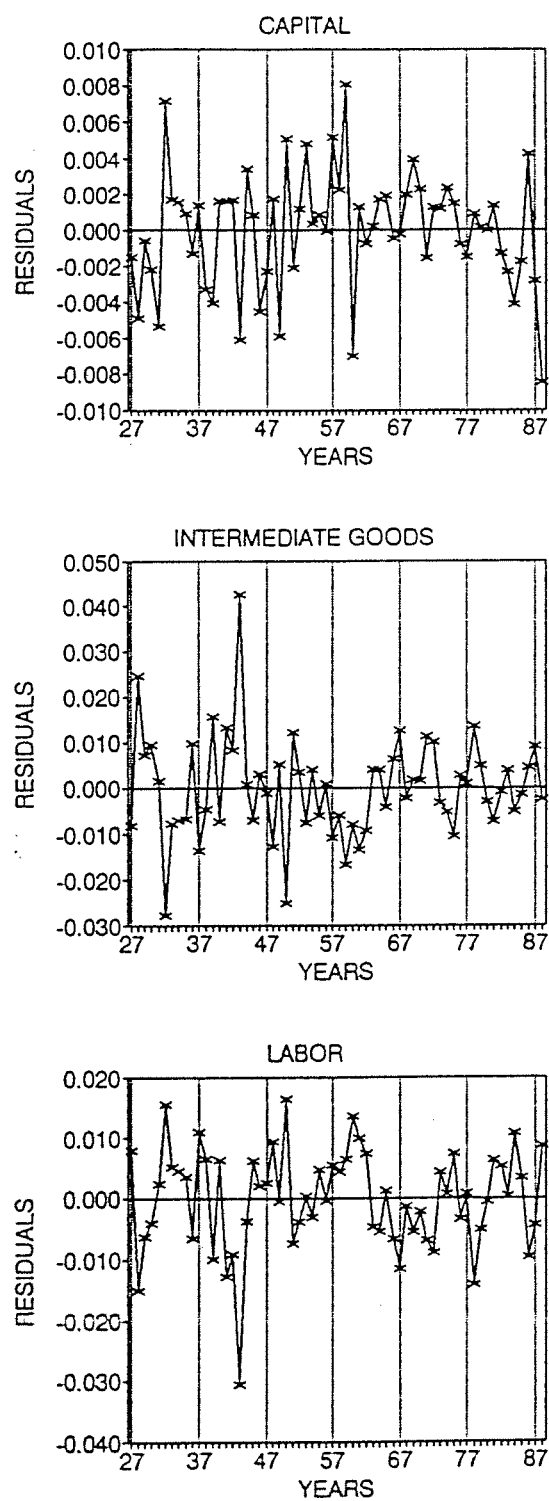


Figure 8: Residual Autocorrelation and Partial Autocorrelation Function of EARSY for Canada.

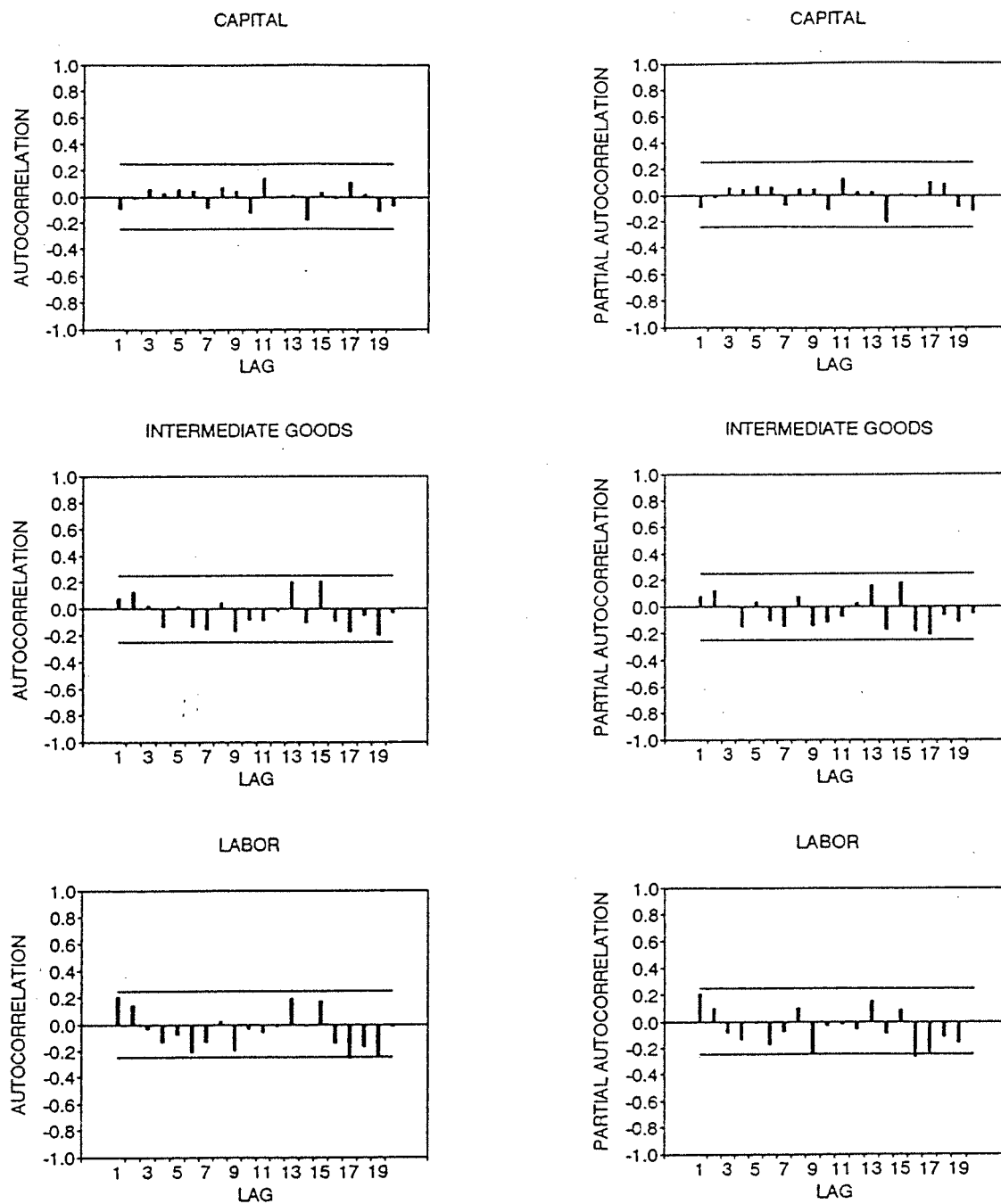


Figure 9: Normal Probability Plots of EARSY for Canada.

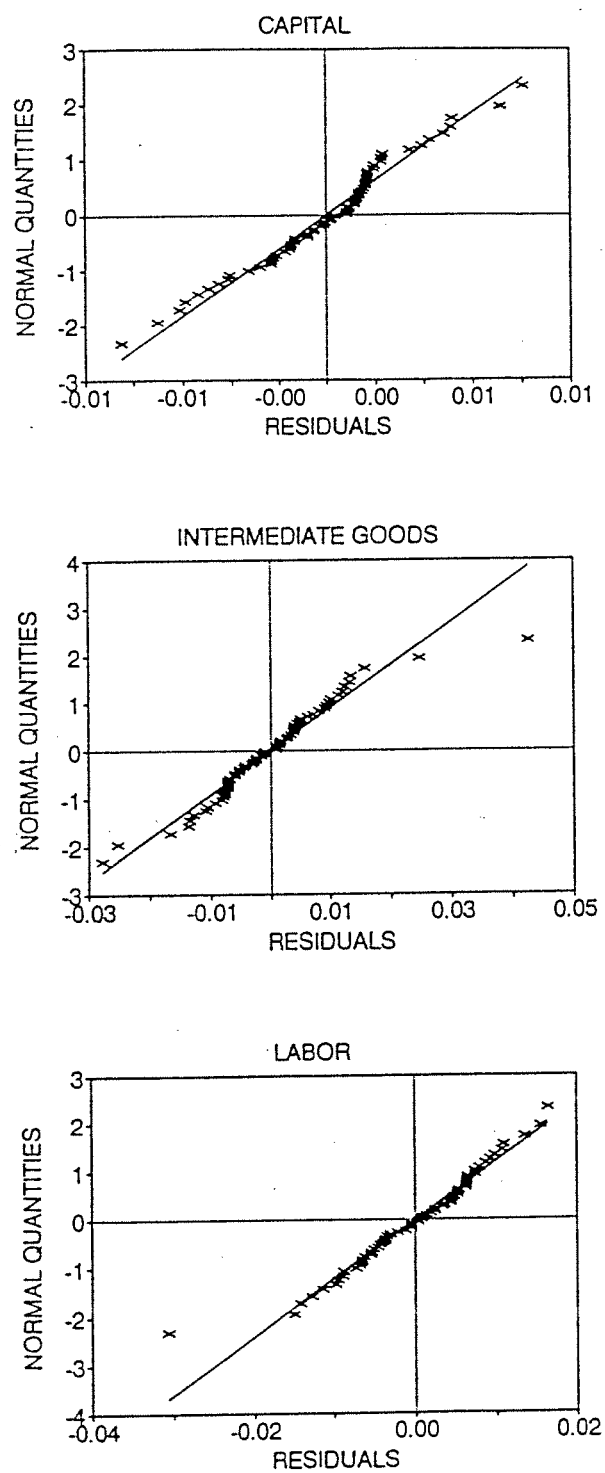


Figure 10: Residual Plots of ARXSY for Canada

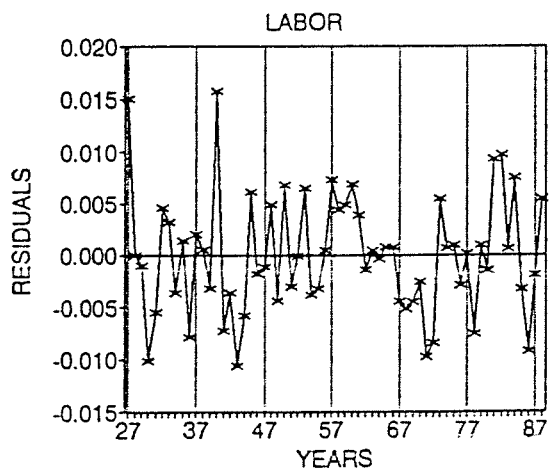
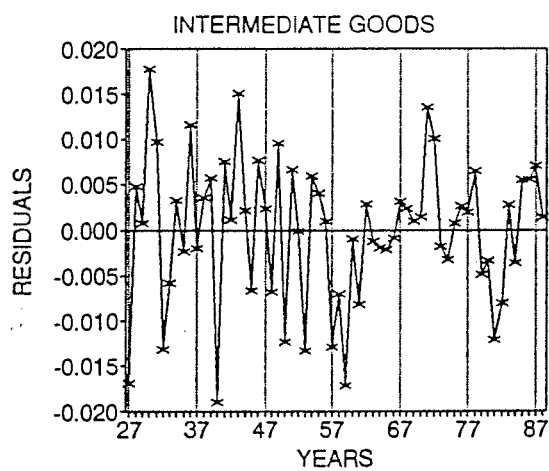
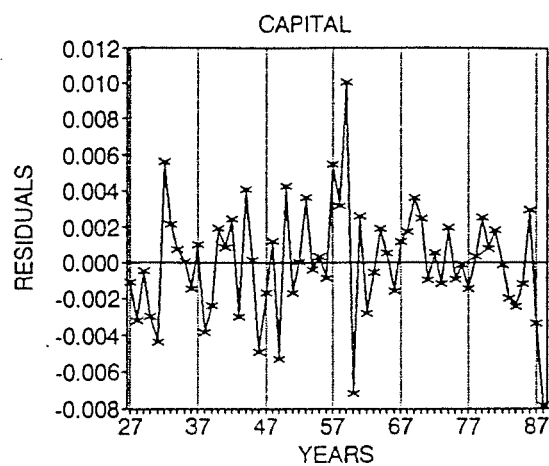


Figure 11: Residual Autocorrelation and Partial Autocorrelation Function of ARXSY for Canada.

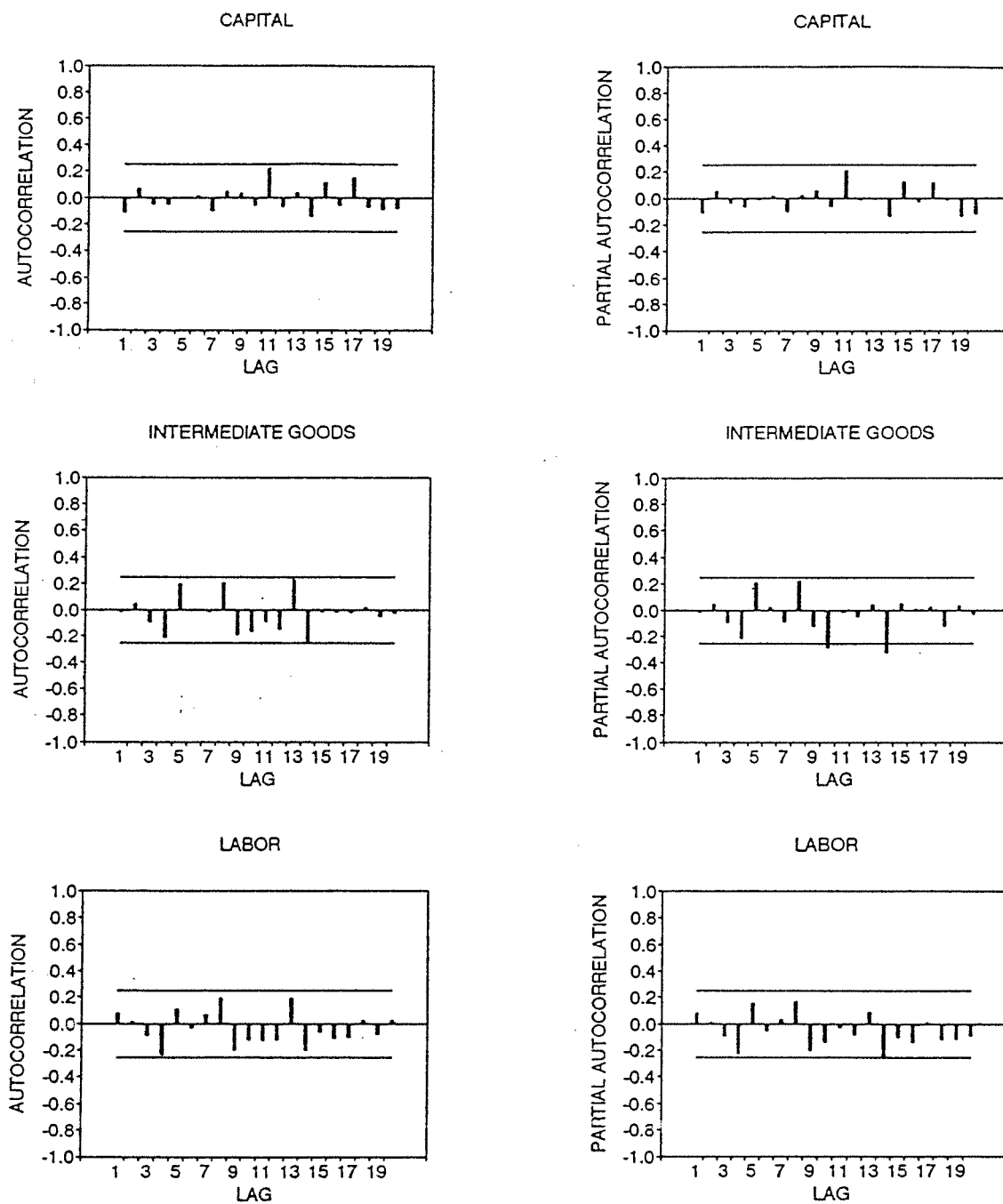


Figure 12: Normal Probability Plots of ARXSY for Canada.

