Proceedings of the Canadian Society for Mechanical Engineering International Congress 2022
CSME Congress 2022
June 5-8, 2022, Edmonton, AB, Canada

Comparing Artificial Dissipation based Shock Capturing Schemes for Discontinuous Galerkin Methods in the presence of Curved Shocks

Pranshul Thakur[1], Siva Nadarajah[1]
[1]Department of Mechanical Engineering, McGill University, Montreal, Canada

ABSTRACT

Discontinuous Galerkin numerical methods are widely used higher order schemes to solve partial differential equations which can achieve high orders of accuracy. However, in these methods, the residual usually diverges in the presence of shocks, unless stabilization; notably a dissipation term is added. This paper focuses on applying different artificial dissipation schemes to the test case of capturing a curved shock resulting from transonic flow over an Euler gaussian bump and comparing the convergence of error in total enthalpy. Three artificial dissipation schemes are applied: 1. simple Laplacian dissipation, 2. physical artificial dissipation (with the dissipation term being the viscous flux of the Navier-Stokes equation) and 3. a modified Laplacian in which the energy term is replaced by total enthalpy. These viscous fluxes were applied using viscosity determined from the sub-cell shock capturing method introduced by Persson and Peraire, in which the viscosity is added only in the vicinity of the shock and is zero elsewhere. Discontinuity/shock was detected by projecting the solution to a higher polynomial order and the viscosity was varied between a maximum value to zero sinusoidally (with viscosity being constant in each cell), depending on the discontinuity in the cell. It was shown in previous studies that the sub-cell shock capturing method with physical dissipation conserved enthalpy for 1D Euler equations. However, whether it conserves enthalpy in the case of 2D Euler Equations with curved shocks is yet to be investigated. This is one of the goals of the current work. A grid convergence study was performed with varying polynomial orders. The L_2 norm of the error in enthalpy was obtained from all three methods and it was found that the modified Laplacian (with energy replaced by enthalpy) resulted in higher convergence orders. In the remaining two dissipation schemes, the error in enthalpy approached a plateau. While the modified Laplacian scheme was better at conserving enthalpy, all the three dissipation schemes exhibited convergence orders around or lower than 1, which is to be expected from the discontinuity introduced by the shock. However, the shock was well captured by the three schemes. This work presents the convergence tables of error in enthalpy for these different schemes and highlights the curved shock captured by the schemes. Distribution of artificial viscosity in the domain is also shown. This study, on the whole, compares the different schemes in terms of their ability to capture curved shocks over curved surfaces and conserve enthalpy.