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DEFORMATION ANALYSIS OF THE EDGERTON SLIDE

University — Université

U OF ALBERTA

Degree for which thesis was presented — Grade pour lequel cette thèse fut présentée

M. Sc.

Year this degree conferred — Année d'obtention de ce grade

1984

Name of Supervisor — Nom du directeur de thèse

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16 MARCH, 1984

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NL-91 (4/77)

THE UNIVERSITY OF ALBERTA

DEFORMATION ANALYSIS OF THE EDGERTON SLIDE

by

Fu-Sing CHU

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF Master of Science

Department of Civil Engineering

EDMONTON, ALBERTA

Spring, 1984

THE UNIVERSITY OF ALBERTA

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# Abstract

This dissertation is a documented case history in applying an analytical method to a field problem. First, the correct usage of the existing computer program ADINA (Automatic Dynamic Incremental Nonlinear Analysis) is presented. The application of ADINA to landslide problems is explored.

Strain-softening stress-strain behaviour and the resulting progressive type failure are common in many landslide problems, but the analyses available are empirical in nature. For this analysis, a load-transfer technique is developed for understanding the progressive failure process. The material model used results from a strain-weakening approach.

Examples are used to verify the load-transfer program. This approach is applied to a simple model to study the stress variation along the slip-surface (shear zone).

Finally, the approach is applied to the Edgerton Slide. The numerical results from the case study are compared with the available laboratory results and the field measurements. The Young's Modulus, used in the calculating displacements which are comparable to the field measurements, agrees with the value obtained from laboratory procedure.

The applications of this technique to other areas are discussed. The load-transfer program can apply to other material models such as strain-stiffening, creep and no-tension behaviour.

## Acknowledgments

I am very grateful for the encouragement from my fellow graduate students, who were always eager to help. Many aspects of the dissertation were discussed with the author's fellow students. In particular, the author would like to extend a special thanks to Mr. Dave Chan, who assisted with the computer aspects and the modelling of the load-transfer technique. The discussion with Mr. Andre Chan was appreciated. Many thanks to Mr. Tom Casey who instructed on the use of his many helpful computer programs.

The author of this thesis would also like to thank Dr. S. Thomson under whose supervision the research for this thesis was conducted. His assistance in correcting the grammar in this thesis is sincerely appreciated.

The author also wishes to thank Dr. John Hutchinson, visiting professor from Imperial College, for his comments and discussion during the initial stages of this research. The comments from Dr. Morgenstern during the final preparation stages are appreciated.

The author received help from Miss. D. Chernenko and Mr. P. Collins during the field surveying and from Department support staff, especially Mr. S. Gamble and Mr. R. Howells.

Finally, the author wishes to acknowledge, with many thanks, the continual encouragement and support from his parents, as without them, this thesis would not have been possible.

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# 1. Introduction : Scope Of This Study

## 1.1 Purpose Of The Research

The basic aim is to apply the analytical technique to gain an insight into landslide problems. The proposed technique is applied to an analysis of the Edgerton Slide. This case history has been studied by Robin Tweedie (1976) and Ronald Mokracki (1982). The present research program involves the use of the general purpose linear and nonlinear finite element analysis computer program ADINA (Automatic Dynamic Incremental Nonlinear Analysis) as a basic tool. The result will be a documented case history applying the analytical method to a field problem.

The analytical method is used to achieve the following goals:

1. Development of a suitable material model for the study of the shear zone behavior;
2. Development of an analytical procedure for the study of the Edgerton Slide;
3. Assessment of the analytical results (primarily comparing field data with the computer output);
4. Assessment of the general application of the proposed technique (i.e. load-transfer technique).

## 1.2 Organization Of The Thesis

The remaining part of this Chapter will deal with a literature review of the nature of progressive failure and some analytical work dealing with progressive failure.

Chapter 2 presents a description of the site and updated field data (to August, 1982).

The formulation of the analytical methods is given in Chapter 3, primarily to explore and understand the available functions of ADINA. The procedures developed will couple ADINA with the load-transfer program.

Chapter 4 presents the Finite Element analysis of the Edgerton Slide.

Chapter 5 offers the general application of the load-transfer technique and the conclusions of this thesis.

## 1.3 Review Of Nature Of Progressive Failure

Terzaghi (1936) suggested that the removal of lateral support in stiff fissured clays could cause opening of the fissures. Moisture ingress leads to a reduction in average strength and allows more deformation. This was supported by Cassel (1948) and Skempton (1948).

Terzaghi and Peck (1948) and Taylor (1948) pointed out that non-uniform straining of strain-softening material cannot obtain full peak strength. The soil along part of the sliding surface may be exerting its peak strength while that along the remainder may be exerting a smaller value. This hypothesis forms the basis of the definition of progressive failure.

Skempton (1964) in the Fourth Rankine Lecture introduced the phenomenon of residual strength which leads to the question in slope analyses : what strength parameters (i.e. peak strength or residual strength ) should be used in the design of slopes?

Bjerrum (1967) in the Third Terzaghi Lecture postulated a mechanism for progressive failure as a result of a large content of "recoverable strain energy" in overconsolidated, plastic clays. The conditions for this mechanism to occur were :

1. The material shows a large and rapid strength decrease after maximum strength is exceeded.
2. Local shear stresses tend to exceed the maximum strength.
3. Large movement due to the release of locked in strain energy.

Bishop (1967) suggested a mechanism based on local overstressing in terms of the shear stress (undrained condition) or the ratio of shear stress to the effective normal stress (drained condition). After the formation of a zone of plastic equilibrium at one point in the slope, the zone of failure would propagate along the potential slip surface.

Skempton and Petley (1967) suggested that large strains would require a pre-shearing of the clay and the forming of principal shear planes. Such movement would be obtained by processes such as landsliding, tectonic movements and glacial ice movements. Meantime, Peck (1967) pointed out that a major factor affecting our ability to predict whether or not a slide will occur is whether or not we are in an old slide area.

Yudhbir (1969) concluded that the release of horizontal stress (Ko effect) in these soils is a dominant factor. Bishop and Lovenbury (1969) showed that long term loading does not lead to substantial strength reductions, which suggests that there is no path to residual which by-passes the peak.

James (1971) pointed out that large displacements, often in the order of feet, are necessary to develop residual conditions on a continuous slip surface.

Morgenstern (1977) pointed out that there appear to be no well-documented case histories of first time slides in heavily overconsolidated soils to indicate that progressive failure plays a dominant role in governing stability.

The preceding discussion indicates that stiff-fissured clays and clayshales present difficult slope stability problems. These difficulties can be summarized as the stress-strain relationship of the soil, the effects of fissures and openings and the large horizontal stresses. However, case histories show that many slope failures in stiff fissured clays and clayshales cannot be explained in terms of peak strength values and equilibrium methods of analysis. Therefore, finite element method may help us to gain an insight on the failure mechanism of stiff fissured clay slopes.

**1.4 Review Of Analytical Work In Modelling The Problem Of Progressive Failure**

Prior to 1970, most of the analyses were based on limit equilibrium methods such as the Simplified Bishop, the Simplified Janbu, and the Morgenstern and Price methods.

However, in the area of research, Dingwall and Scrivener (1954) studied the stress distribution beneath slopes by the finite difference form coupled with relaxation procedures. They used the theory of elasticity to determine the shear and normal stresses for an uniform and a rigid boundary embankment.

Clough (1960 a,b) broadened the matrix method of structural analysis into the general finite element approach which can be applied to any structural mechanics from any field. The matrix method of structural analysis was later called the finite element method.

Peck (1967) pointed out that a definitive answer to the problem of progressive failure would require a finite element solution for a work-softening material.

Duncan and Dunlop (1969) and Dunlop and Duncan (1970), studied the distribution of stress in and beneath slopes by the finite element method. Their analyses were to determine differences in behaviour of slopes in materials with low and high initial horizontal stresses, representative of normally consolidated and heavily over-consolidated clay deposits. The material properties of the slope were represented by homogeneous, linear elastic, isotropic materials.

On the other hand, several analytical models were developed to evaluate the mechanistic approach. For example, Christian and Whitman (1969) approach the problem of progressive failure by developing the differential equation for displacement along the band from equilibrium of an infinitesimal element. Palmer and Rice (1973) considered the simple slope problem as an in-plane shear fracture.

Gibson (1974) in the Fourteenth Rankine Lecture stated that analytical methods draw attention to broad trends and help to distinguish between those factors that are of primary significance and those that are of secondary importance.

Simmons (1981) studied the behaviour of shear zones. The analyses of shearband yielding which involve non-weakening stress-strain behaviour are applied to two case histories. Therefore, the future study of the analytical method should handle the strain-softening behaviour asociated with soils which are vulnerable to progressive failure.

The present research studies the shear zone (or slip surface) behaviour by considering the strain-weakening stress-strain behaviour. The technique is applied to the Edgerton Slide.

# 2. Description Of The Site And Site Investigation

## 2.1 General

The Slides occurred about 48 kilometers northeast of Wainwright, Alberta. The three landslides are located adjacent to one another and are named the ' Edgerton Slides '. The first slide was termed the Edgerton-74 North Slide. The second and third slides were named the Edgerton-74 South Slide and Edgerton-80 Slide, respectively. A plan view of the Edgerton Slides is shown in Figure 2.1.

Since the first major movement took place in late August, 1974, Thomson and Bruce (1974), Tweedie (1976) and Mokracki (1982) have studied various aspects of the Edgerton Slides.

Thomson and Bruce (1974) reported the major features of the slide from a field reconnaissance. Tweedie (1976) did an extensive site investigation and laboratory testing program. Thomson and Tweedie (1978) published a summary of Tweedie's (1976) work. Mokracki (1982) summarized the survey data from 1975 to 1981.

## 2.2 Geology

Most of the findings in this section were obtained from Tweedie (1976) and Mokracki (1982). As Morgenstern (1977) pointed out since the geological conditions in heavily overconsolidated materials control failure geometry, some of the geological processes and materials will be re-emphasized in order to demonstrate a part of the reasoning behind the development of the analytical procedures in Chapter 3.

### 2.2.1 Geologic History

During the period from Upper Cretaceous to Lower Tertiary, the bedrock formations were deposited in a subsiding basin in Central Alberta. Vertical variation from marine shale at the base of the Upper Cretaceous to continental sandstone at the top was common ( Williams and Burke, 1964 ).

During late Mesozoic and early Tertiary time, the Columbian and Pacific orogeny transformed the Alberta basin from an area of subsidence and deposition to one of uplift and erosion. Rutherford (1928) estimated that about 600 meters of strata have been removed from the study area during Tertiary time. Large-scale downwasting and stagnation of the Keewatin ice-sheet, which advanced over the area during Pleistocene time, modified the late Tertiary landscape. Retreat of the Pleistocene glaciers about 10,000 years ago, lead to some topographic change due to the increase in river velocities and flow volumes. Landslide activity was started due to the steep walled, post-glacial valleys left by the rapid downcutting of rivers.

### 2.2.2 Surficial And Bedrock Geology

The glacial deposits of till are highly oxidized and columnar jointed. The average depth of till within the study area is about 5 meters. The composition of the till is : 50 percent sand, 30 percent silt, and 20 percent clay sizes ( Bayrock, 1967 ).

The bedrock of the study area consists of interbedded sandstone, siltstone and shale, and thin coal seams of late Cretaceous age ( Warren and Hume, 1939 ). The rocks are bentonitic and have a regional dip of a meter per kilometre to the southwest.

The area is underlain by the Bearpaw, Belly River and Lea Park Formations. The Bearpaw Formation has been completely eroded at the study site. Thus the landslide movements have occurred within the Belly River Formation.

Possible ice shoving processes have occurred and were observed over the full height of the scarp face. This contributed to the brecciated nature of the bedrock.

## 2.3 Description Of The Landslides

### 2.3.1 Observations

The first slide was discussed in detail by Thomson (1974) and Tweedie (1976). The third slide was reported in detail by Mokracki (1982). The second slide will be discussed fully in this section, and it will be used for later numerical analysis.

Airphotos of the study area show slump topography along both sides of the river valley, which indicates ancient landsliding. Groundwater discharge areas were found half way between the river and the local plain level. Toe erosion was noticed along the river valley.

In the fall of 1974, the scarp of the Edgerton-74 South Slide was between 0.45 and 0.6 meters in height. The slide profile of 1974 will be treated as pre-slide profile. In the summer of 1982, the scarp on the right flank was between 1.2 and 1.4 meters in height. At the same time, the toe of the south slide appears to have cropped out at the approximate location as predicted in 1974.

### 2.3.2 Climate

According to records of the Canada Department of Mines and Technical Services, 1957, the climate of the area is sub-humid continental. From the 19 years of continuous records, the average annual total precipitation is 39.75 centimeters of which 30.84 centimeters is rainfall and the rest is 89.15 centimeters snowfall. Unfortunately, the information of the rainfall period (either long or short) which is of more concern is not available.

## 2.4 Site Investigation

### 2.4.1 General

A review of the past site investigation is useful for evaluating the available information. Only part of the available information can be utilized as input data for numerical analysis. The procedural use of the data can affect the numerical model. This will be discussed in detail in Chapter 3.

### 2.4.2 Survey

The characteristic surface movements of the second slide have been monitored since early spring, 1975. The location of the profile is shown in the general plan of the landslides (Figure 2.2).

The location of these surface stakes were originally determined by the changes in the slope of the displaced mass or by local physical features along the survey line

The typical recurring topographic survey consists of determining the horizontal distances and vertical elevations of prescribed control points relative to a local datum. The datum point has been arbitrarily selected and assigned an elevation of 300 meters (local elevation). All horizontal distances to the control point are measured from this datum point.

### 2.4.3 Field Work And Laboratory's Results

Tweedie's (1976) field work consisted of subsurface exploration and instrumentation programs. Subsurface exploration included four boreholes and six toe trenches. Typical subsurface stratigraphy of the second slide is shown in Figure 2.3. Instrumentation programs consisted of four slope indicators and three piezometers. After a one year monitoring program, all the measuring devices had ceased to function due to continued slide movement.

Laboratory work consisted of direct shear tests. Detailed descriptions of the preparation of samples and the laboratory program are found in Tweedie (1976). The shear strength parameters which obtained from the laboratory program, are shown in Table 2.1.

**2.5 Section Summary**

The preceding discussion concluded that the recent slides are, in part, the reactivation of old landslides. Only surface movements have been monitored continuously since May, 1975.

Table 2.1 Summary Of Shear Strength Results From Laboratory Program (Modified After Tweedie 1976)

| MATERIAL | SHEAR STRENGTH PARAMETERS | | | |
|---|---|---|---|---|
| | PEAK | | RESIDUAL | |
| | C′ (kpa) | o′(degrees) | C′ (kpa) | o′(degrees) |
| UNWEATHERED SILTY CLAYSHALE | 160 | 41 | 0 | 16 - 19 |
| WEATHERED SILTY CLAYSHALE | 70 | 23 | 0 | 10 |
| REMOULDED BENTONITIC CLAYSHALE | - | - | 0 | 8 |

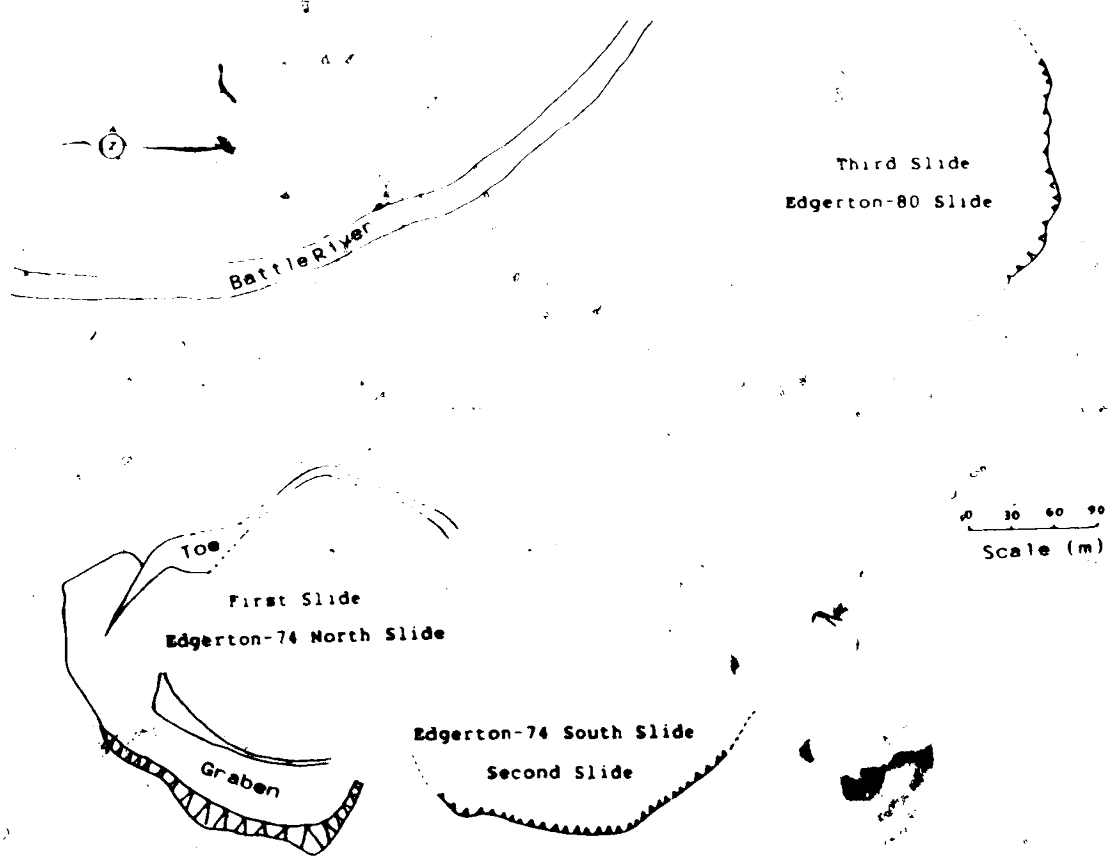



Figure 2.1 Plan View Of The Study Area With Slide Locations (Modified After Mokracki, 1982)

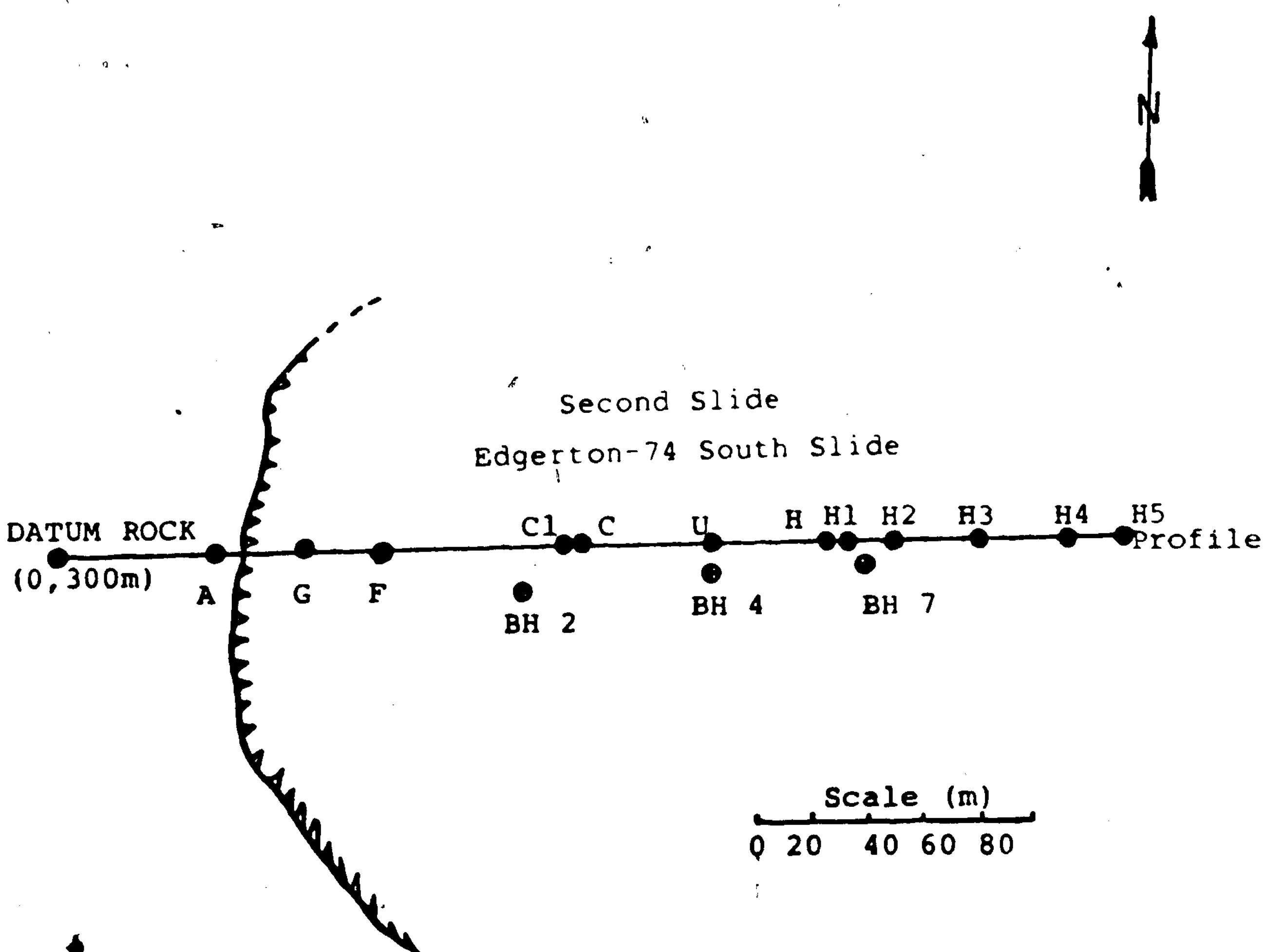



Figure 2.2 General Plan Of Second Landslide Showing Location Of Profiles And Boreholes (Modified After Mokracki, 1982)

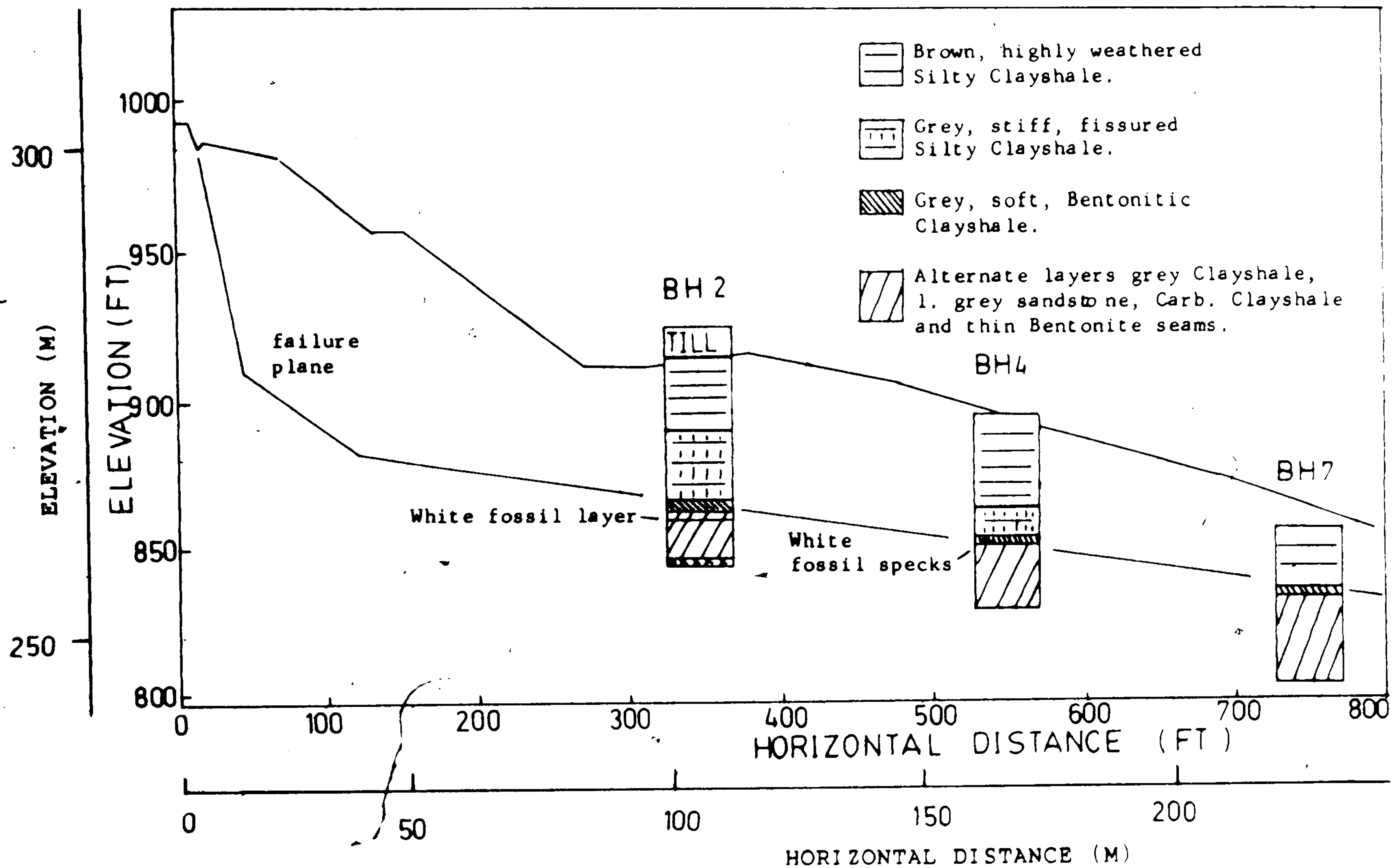



Figure 2.3 Stratigraphic Profile Of The Second Slide (After Tweedie, 1976)

# 3. FORMULATION OF ANALYTICAL PROCEDURE

## 3.1 Introduction

A major portion of this research work involved the understanding and the proper usage of the general purpose linear and non-linear finite element analysis computer program ADINA for simulating the failure mechanism of the landslide.

This chapter can be separated into two parts. The first part focuses on the exploration of the application of ADINA in the landslide problems. Some of the available options of ADINA will be utilized for specific purposes. For example, the option of element death will be used for producing a stress-free boundary after excavaton (or erosion). The three material models investigated are:

a. isotropic linear elastic

b. curve description with cracking

c. elastic-plastic (von Mises, isotropic hardening).

However, the analyses rely heavily on the isotropic linear elastic model. An attempt is made to model stain-softening material behaviour by utilizing the available options of the ADINA program. Ultimately, a load-transfer technique is developed to model the strain-softening behaviour.

The second part describes the set-up of the numerical model for finite element analysis. A simple model is used to study the boundary effect and the appearance of the softening zone. Finally, an analytical procedure is suggested for analysing the Edgerton landslide.

### 3.2 Application Of Finite Element Method (Program ADINA) In Modelling Excavation And Material Softening Behaviour

A three element model will be used to indicate the applicability of the various options of the ADINA program to the study of a natural slope. The available options of gravity loading, element death and material models are considered in the following sections.

#### 3.2.1 In-Situ Stresses

Stresses within gravity structures due to body forces are of importance and cannot be neglected. According to the ADINA user manual (Bathe 1976, section 2.3, 4, 6.1 and 32.4), the mass proportional loading vector is established from the density of the material and the concentrated mass input. However, the aim of this thesis is the study of a natural slope; there is no concentrated mass which derives from a man-made structure to be considered in the analysis. Hence, the gravity loading is directly proportional to the density of the material.

Some naturally occurring sediments are deposited in horizontal layers where no lateral yielding occurs. The ratio of lateral to vertical stresses is known as the coefficient of earth pressure at rest (Ko). For elastic isotropic material, under first loading, the value of Ko can be expressed directly in terms of Poisson's ratio ($\mu$); for example under plane strain condition,

$$Ko = \frac{\mu}{1 - \mu}$$

The in-situ stresses are derived from a switch on gravity approach; where the vertical stress is due to overburden load, and the lateral stress is equal to Ko times the vertical stress.

Dysli and Fontana (1982) used the ADINA program and stated that the initial stress state was created by the progressive application of gravity in about ten solution steps in one case and twenty-two solution steps in the other case. However, the ADINA program is very expensive to run, thus if one solution step yields the same results as that from many steps, instant gravity loading can be imposed on the structure. For any linearly elastic analysis, instant gravity loading can be applied without creating any discrepancy in finite element results. Therefore, linearly elastic analysis is favoured for the reasons of cost and checking by hand calculation for a simple problem.

### 3.2.2 Simulate Excavation Using ADINA

The process of erosion is similar to the process of excavation. The differences are the time scale and the boundary conditions. Erosion may take hundreds of years; while excavation will take only months or a year. The area of excavation is defined by the designer; the area of erosion will be of wide extent.

The element death option is used to simulate the process of excavation (or erosion). The excavated surface is considered to be stress-free (Desai and Christian,1977). The stress-free surface can be created by applying a set of equivalent forces at nodes on the surface in the direction opposite to the direction of stresses due to initial and subsequent loading conditions.

The option of element death will yield a stress-free boundary under the condition of no gravity loading. This is shown by a simple test which is illustrated in Figure 3.1. However, under gravity loading, the program yields a false stress-free boundary if the element death option is used. Kripakov (1983) realized that the stiffness of the element is eliminated at each time step specified, but that a portion of the weight of the element is not effected (i.e., not eliminated) if the gravity loading option is used to load the structure. However, this is critical to the analysis of a natural slope as gravity is the only loading mechanism which is imposed on the slope. Kripakov (1983) suggested the used of a reduced stiffness approach rather than the death option to simulate an excavation sequence.

Instead of modifying any portion of the ADINA program, or generating any complexity of the analysis; the problem can be resolved by using a thin layer of elements which is generated along the boundary of the excavation. Element 1 on the upper left of figure 3.2 can be divided into two elements, which leads to a three element model. This is shown on the upper right of Figure 3.2. The reasoning is shown in Figure 3.2. The error associated with this approach can be reduced by minimizing the thickness of the thin layer (element 1b in Figure 3.2). A numerical illustration of the magnitude of the error is shown in Table 3.1. Thus, using a thin layer element approach, a stress-free boundary can be obtained.

NOTE: the error, which is generated from using the death option under gravity loading, does not imply that the program itself is wrong. This is the standard finite element procedure of distributing the weight of an element to the surrounding nodes.

### 3.2.3 Material Models

Of the fourteen material models available in ADINA, only three are considered in this research. These are:

a. curve description with cracking

b. elastic-plastic (von Mises, isotropic hardening)

c. isotropic linear elastic.

Considering the curve description model, in the beginning some ADINA users thought that the curve description model can be used for modeling the post-failure behaviour of strain softening material. A decrease of strength occurs after shear straining past the peak value, which causes a progressive type of failure in stiff, fissured clay slopes. The stress-strain curve of a general strain-softening material is shown in Figure 3.3. If one uses the curve description model for modeling the post-failure behaviour of a strain-softening material, a matric inversion difficulty in ADINA is created. Both Kripakov (1983) and Dysli (1982) have already expressed their scepticism concerning this function of ADINA.

If the stresses exceed the yield stress, the results of an elastic-plastic analysis will indicate the location of the plastic zone. If the stresses are less than the yield stress, the results of such an analysis are identical to those from a linear elastic analysis. The elastic-plastic model cannot be used for a strain-softening material.

In usual engineering analysis, a non-linear analysis of a program is always preceded by a linear analysis. The advantages of an isotropic linear elastic analysis are:

1. the results can easily be checked by hand calculations for some cases.
2. the least number of input parameters are required for the analysis.
3. the strain-softening material behaviour cannot be modelled by any available material models. Therefore, there is no real advantage to using any sophisticated material model at this time.

Hence, material models of both elastic-plastic and curve description with cracking are not considered in the following analyses.

At first glance, material softening can be modelled by using both the options of element birth and element death simultaneously within the ADINA program. The approach is similar to the incremental elasticity approach in which the Young's Modulus (E) is decreasing. The stress-strain curve of the incremental elasticity approach is shown in Figure 3.4. The sequence of operation is that element 1b (refer to Figure 3.2) with E1 is killed and is replaced by a new element having E corresponding to E2 (refer to Figure 3.4). Process of replacing element 1b with an element of changing E according to Figure 3.4 is continued. As strain increases, E decreases. Hence, strain-softening can be modelled. On a theoretical basis, this approach is better for modelling the strain-softening behaviour than any available material models. However, if the death option is used in the first step, the stresses within the element will turn to zero. If the birth option is used in the second step, the element will carry no initial stresses. Therefore, the ADINA can be used to perform the incremental elasticity approach, but the stress output will be zero for the second step of the operation of death and birth sequence of the program used.

The previous discussions illustrate that the ADINA program should be used in conjunction with another program in order to model the strain-softening behaviour. Of course, the best approach is to modify the ADINA program so that this function is built into the ADINA program itself. The technique of the new approach will be discussed thoroughly in the next section, but the work of modifying the ADINA program to include the new material model is left for future research.

### 3.3 Load-Transfer technique

One of the necessary preconditions for progressive failure to occur is that the material of a slope must display strain-softening behaviour. The load-transfer technique, to be described in the following paragraphs, is actually a strain-weakening approach. While this is not the same as strain-softening, it is a better approach than most others for understanding the progressive failure process.

#### 3.3.1 Background Information

The technique is analogous to the stress transfer method (Zienkiewicz and et.al., 1968). The latter method is used for the stress analysis of a rock mass which cannot sustain tension. The load-transfer technique is used for reducing shear modulus of a shear zone material which cannot sustain excessive shear stresses. The assumed load redistribution approach is not capable of reproducing the true strain-softening behaviour, however, it does provide useful method for understanding the stability of a natural slope.

#### 3.3.2 Description

This technique utilizes the available program ADINA within the Department of Civil Engineering at the University of Alberta and the load redistribution program.

The essential steps of this approach can be described as follows:

1. analyse the problem as an isotropic elastic case using the ADINA program. The loading mechanism is gravitational force.
2. reduce the elastic modulus at certain locations (eg. shear zone or slip surface) which may not be capable of sustaining excessive shear stress.
3. the restraining forces are generated due to the reduction of shear strength in terms of elastic modulus. These forces are obtained from the load-transfer program. Total stress ($\sigma_t$) of the element can be separated into two components.

$$\left\{\sigma_t\right\} = \left\{\sigma_m\right\} + \left\{\sigma_n\right\}$$

$\sigma_m$ is the stress that can be sustained by the element with the reduced modulus.

$\sigma_n$ is the excess stress that cannot be sustained by the element due to softening which must be redistributed to other parts of the structure.

Therefore, the new stress for the element will be $\sigma_m$ and the equivalent load (R) from that must be applied to redistribute this excess stress.

$$\left\{R\right\} = \int_v \left[B\right]^T \left\{\sigma_n\right\} dV$$

More detail discussion of this procedure is given in

Appendix A.

4. the application of forces to relieve the restraining forces is used to maintain equilibrium. The ADINA program is re-utilized again. The results will be the incremental stresses.

5. the stresses are computed in such a way that the incremental stress ($\delta\sigma$) due to the applied load (R) will be superimposed on $\sigma_m$, but not on $\sigma_t$.

The flow chart is shown in Figure 3.5.

### 3.4 Examples Of The Load-Transfer Program

Two examples, namely pure shear and bending of a beam by uniform load, are used to verify the load-transfer program and to demonstrate the technique of reducing shear modulus. The data input instruction and the source code of the program is shown in the Appendix B.

#### 3.4.1 Pure Shear

The pure shear model and its finite element idealization, the material properties and the loading conditions are shown in Figure 3.6. If the shear modulus (G) is reduced from 385 kPa to 96 kPa, the generated loads due to excess shear stress have to be redistributed to the rest of the structure. Theoretically, the stresses and the strains from the analysis with the material property of 96 kPa should be the same as the summation of the stresses from that of 385 kPa and from excess shear stress.

Refer to Figure 3.6,

$$\sigma_{xyk} = \sigma_{xyi} + \delta\sigma_{xy}$$

$$\gamma_k = \gamma_i + \delta\gamma$$

where

$\sigma_{xyk}$ and $\gamma_k$ = final shear stress and shear strain respectively

$\delta\sigma_{xy}$ and $\delta\gamma$ = calculated incremental shear stress and shear strain respectively

$\sigma_{xyi}$ and $\gamma_i$ = initial shear stress and shear strain respectively

For the case 1:

-in step 1, the stresses and strains are calculated from the ADINA program with the Young's Modulus of 1000 kPa.

-in step 2, the excess shear stresses in terms of loads are calculated from the load-transfer program with the reduction of the Young's Modulus from 1000 kPa to 250 kPa.

-in step 3, the excess loads from step 2 are applied to the ADINA program where the incremental stresses and strains are calculated and are superimposed on those in step 2.

If the shear modulus is reduced to a quarter of its original magnitude, the strain has to increase in order to reach the same stress level. Figure 3.7

shows the stress path of the redistribution of stress due to the reduction of shear modulus. If the material cannot take the original stress for whatever reasons, the material properties (in terms of Young's Modulus or Poisson's Ratio) have to be reduced to a lower value. The excess shear stress will be developed due to the reduction of Young's Modulus. Therefore, the redistribution of the excess shear stress will generate an additional strain.

If the model is analysed by using the ADINA program with the Young's Modulus of 250 kPa, (i.e. case 2, refer to Figure 3.6) the results (both $\sigma_{xyk}$ and $\gamma_k$) of case 2 should be the same as those of case 3 in step 1. Therefore, it is concluded that the load-transfer program is acceptable.

### 3.4.2 Bending Of A Beam By Uniform Load

This example serves two purposes. These are:

a. apply the load-transfer program to a complex problem.

b. compare the result from the new technique with the closed form solution from the theoretical approach.

A cantilever beam subjected to uniform distributed load is being considered. The finite element idealization of the problem and the material properties are shown in Figure 3.8. The material property (primarily elastic modulus) is reduced, and the Poisson's ratio is kept constant.

The calculation of load-redistribution is based on the plane strain condition; however the theoretical solution for this problem is based on simple beam theory (plane stress). The equation which is used for calculating the deflection at node 35 (refer to Figure 3.8) is:

$$\delta = \frac{w\ l^4}{8EI}$$

where

w = uniformly distributed load

l = length of the beam

E = Young Modulus

I = moment of the inerita

$\delta$ = deflection

The result from the load redistribution approach follows closely the one from the theoretical solution. The results are presented in Figure 3.9.

### 3.5 Summary Of The Load-Transfer Technique

The new technique has been described and proven to be in the working stage. The technique is actually an additional material model for analysing any strain weakening problems. The last two examples demonstrate how the shear modulus was reduced for the entire structure.

The technique of reduction of the shear modulus for a part of the structure will depict the picture of non-homogeneous behaviour. Additionally, for a stability problem, the shear strength will be mobilized along the slip surface or a pre-existing weak zone (e.g. shear zone). The next section will study this question in detail and propose a method for analysing the Edgerton Slide.

### 3.6 Behaviour Of Simple Model

Usually, erosion of a landscape proceeds in a systematic way so that landforms evolve through a series of stages in which the ultimate landscape is reduced to a surface of low relief (Hamblin and Howard, 1975).

The simple model which is shown in Figure 3.10 undergoes the following steps:

1. assign material properties to the whole structure
2. switch-on gravity
3. excavate part of the structure
4. reduce the modulus in shear zone.

   The ADINA approach requires a pre-existing shear zone before the excavation process; while the present load-transfer approach allows the softening process after the excavation stage. (refer to Figure 3.10)
5. re-analyse the problem with new modulus using ADINA.

The aim of the simple model is to determine how much variation of the result will be arised from the modelling techniques.

### 3.6.1 Introduction

Theoretically, progressive failure indicates the spreading of the failure over the potential surface of sliding from a point or a line toward the boundaries of the surface or vice versa. Therefore, the strength properties across the shear zone will not be uniform. Two questions should be answered in order to understand the failure mechanism. These questions are:

1. how much reduction of shear modulus (in term of Young's Modulus) is required to induce the stress concentration?
2. where will the softening zone be first initiated and in what direction does this softening zone progress?

Several people have been investigating the second question for a long time, for example, Palmer and Rice (1973) and Chowdhury (1978).

The research of this thesis involves only the first question. Although the second question is studied, attempts have been made with no conclusive result which is required on fracture mechanics (J-integral) can be drawn from this study. The variation of the results should arise from the change of shear strength within the shear zone. Hence, it is important that the boundary effect not influence the result.

### 3.6.2 Boundary Effect

The aim of this section is to determine how much variation will be derived from changing the boundary conditions.

The nodes at A and B are assumed to be fixed. (refer to Figure 3.10) This is due to the stability of the structure. The nodes along the upslope and downslope side can be assumed to be on rollers because deformation will take place in a vertical direction under gravity loading. However, the nodes along the bottom boundary can be assumed to be either fixed or on rollers. If the boundary is set far enough away, the results from either fixed or roller bottom boundary should be approximately same.

It is assumed that the whole structure is uniform at this moment. The material properties are:

1) Young's Modulus = 137,900 kPa

2) Poisson Ratio = 0.42

The sequence of the analysis consists of switch-on gravity and then the excavation process. Therefore, the only variable for this problem comes from the geometry.

The accuracy can be increased by increasing the number of nodes and elements. However, there is a limit to increasing these two parameters because the computation time follows a power rule of the number of nodes and elements. An number of trials involving various meshes was done to accommodate the features of a stress-free boundary and shear zone. The final configuration of the mesh used in the analyses, consists of 280 nodes and 255 4-node elements (refer to Figure 3.11(b) ).

The results derived from roller bottom boundary differ from those derived from fixed bottom boundary. This difference reduces as the dimensions of B and H (refer to Figure 3.10) increase. However, the difference can be narrowed down to 10 to 20 percent without substantially increasing the dimension of B and H. Figure 3.11 shows two of the possibilities. Table 3.2 shows the percentage difference of the displacements obtained from both the fixed and roller bottom boundary.

When a relatively hard material is absent, it is necessary to establish finite boundaries within which the results will not be changed due to the boundary effect. It is assumed that the boundary effect of the configuration of Figure 3.11b is minimal.

### 3.6.3 Softening Zone

The schematic illustration of the simple model is shown in Figure 3.12. The simple model consists of four different layers. Their properties are shown in Table 3.3. The thinnest layer is assumed to be the shear zone (or slip surface). As the material within the shear zone will undergo non-uniform strength weakening, certain locations within the shear zone will have lower strength properties. Therefore, artifical reduction of Young's Modulus to either one or five percent of its original value is assigned for either location A or B. (refer to figure 3.12)

The purpose of this analysis is to determine how the stress variation will take place if the strength within the shear zone is reduced.

3.6.3.1 Procedures

The procedures are shown in Figure 3.10; namely, ADINA approach and load-transfer approach. However, the concepts of the two approaches are different. The former approach assumes that the softening zone exists before the excavation process. The latter approach assumes that the softening zone is generated after the excavation process.

The operational sequence of the simple numerical model consists of two steps. The boundary of the first step is MNYX as shown in Figure 3.11b which resembles the flat and horizontal landscape. The boundary of the second step is MABCYX as shown in Figure 3.11b, which portrays the eroded (or excavated) landscape. The only difference between these two approaches comes from the assigning of the material properties to each of the elements in the first step. For the ADINA approach, the material properties remain unchanged. On the other hand, the load-transfer approach assumes that the shear zone material is uniform in the first step. The material properties within a given area of the shear zone will be reduced after the second step. The detailed description has been mentioned in Section 3.3.2.

Although the final material properties derived from both approaches are same, the cost of a run using the ADINA approach is approximately half of that using the load-transfer approach. Will the results derived from these two approaches be significantly different?

Both approaches are used in analysing several trials. These trials are shown in Table 3.4. These trials may indicate how sensitive the result will be due to a reduction of the strength in terms of Young's Modulus.

3.6.3.2 Observation And Comparison Of Results

The output from any finite element program will be in terms of displacement (strain) and stress.

DISPLACEMENT

A typical pattern of the displacement arrows due to the excavation process (or valley erosion) is shown on Figure 3.13. The results of the surface displacements from the ADINA approach and the load-transfer approach are shown in Figure 3.14 for trial 1. (refer to Table 3.4) and in Figure 3.15 for trials 2 of the load-transfer approach and 3 of the ADINA approach. The phenomenon of valley rebound (Matheson, 1973) can be shown by the surface displacements.

From a comparison of the results of the analyses, the load-transfer approach tends to yield a lower surface rebound value than that of ADINA. The lower the Young's Modulus (eg. trial 1 refer to Table 3.4) yields a lower rebound value. Both the location and the size of the softening zone will affect the value of surface rebound value. However, the size of the softening zone is proportional to the value of the surface rebound.

STRESSES

The only loading mechanism of the first step in the analysis is the gravitational force. Since the simple model consists of uniform layers and no horizontal shear stress, therefore, the vertical stress is the minor principal stress (sign convention is that compression is negative). The horizontal stress should be the major principal stress. However, the analytical results show a small discrepancy (about 1 percent) with the closed-form solutions.

Four uniform layers as the case of step 1 of the load-transfer approach will yield zero horizontal shear stress within the structure. The horizontal, vertical and maximum shear stress contours will be a number of horizontal lines.

For the ADINA approach, the layer three (refer to Figure 3.12) is not uniform in step 1. Therefore, the concentration of the shear stress contours is primarily due to the non-homogeneous effect. This effect appears more if the softening zone is located at A (refer to Figure 3.12).

In the second step, a stress discontinuity will be obtained at the softening zone. The overall pattern of stress contours is almost identical from either one of the approaches. The softening zone at location A seems to have a large stress concentration area. This can be illustrated by several shear stress ($\tau_{xy}$) contours as shown on Figures 3.16 and 3.17.

However, if the results are studied in depth, one can conclude that the load-transfer approach yields a distinct stress discontinuity zone along the softening zone. Figure 3.18 illustrates the locations of the sections which will be used for showing the stress discontinuity.

By comparing both Figures 3.19 and 3.20, several observations can be made from the analyses. These are:

1. the area of stress concentration is only related to the area of the softening zone.
2. the load transfer approach yields uniform stress across the softening zone.
3. the ADINA approach yields a relative peak stress at the mid -section of the softening zone.

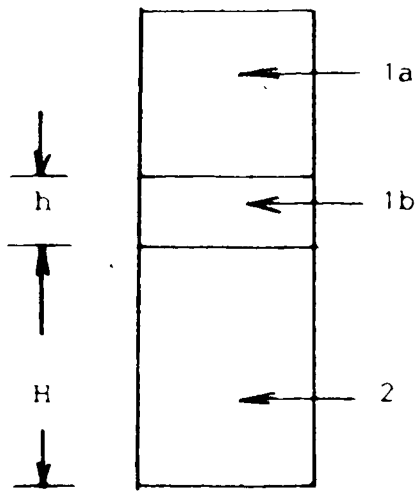
Additionally, it seems that a large reduction of elastic parameters is required to induce a stress concentration. From the experience of this analysis, the Young's Modulus has to be reduced to one to five percent of its original value to observe a noticeable stress concentration.

Although the results do not show any significant difference between the two approaches, it is important to recognize that the load-transfer approach matches the geological failure mechanism sequence of the Edgerton Slide. As noted previously, the load-transfer method does not assume a pre-existing weak zone. Therefore, the load-transfer approach will be used for the analysis in Chapter 4.

## 3.7 Chapter Summary

The preceding discussion summarized most of the required techniques which will be used for the deformation analysis of the Edgerton Slide. The correct usage of the computer program of ADINA was emphasized so that meaningful results can be obtained. The prime purpose of this chapter is to provide a conceptual feeling for the results so that fewer trials will be required in Chapter 4 and hence the computational cost will be reduced.

Table 3.1 Error Associated With The Thin Layer Approach




Assume

H = 10 m

$\rho$ = 2140 kg/cu. m

| h/H | VERTICAL STRESS (kPa) (at the centre of element 2) |
|---|---|
| 0.05 | 110.1030 |
| 0.01 | 105.9086 |
| 0.005 | 105.3843 |

These are compared to the

| | |
|---|---|
| Ideal Case where | 104.8600 |
| No Thin Layer (i.e. 2 element model) | 209.7200 |

Note :

1. h/H cannot equal zero.
2. h/H = .005 used in calculation of this comparison.
3. percentage error of h/H = .005 is 0.5 percent.

Table 3.2 Comparison Of Displacement Output

| CONDITION | | DISPLACEMENT OUTPUT FOR NODES | |
|---|---|---|---|
| | | 99 | 108 |
| FIGURE 3.11a | FREE | 0.0210 | 0.1409 |
| | FIXED | 0.0313 (33% higher) | 0.1169 (17% lower) |
| FIGURE 3.11b | FREE | 0.0714 | 0.2305 |
| | FIXED | 0.0728 (19% higher) | 0.2055 (17% lower) |

Table 3.3 Material Properties For Edgerton Slide

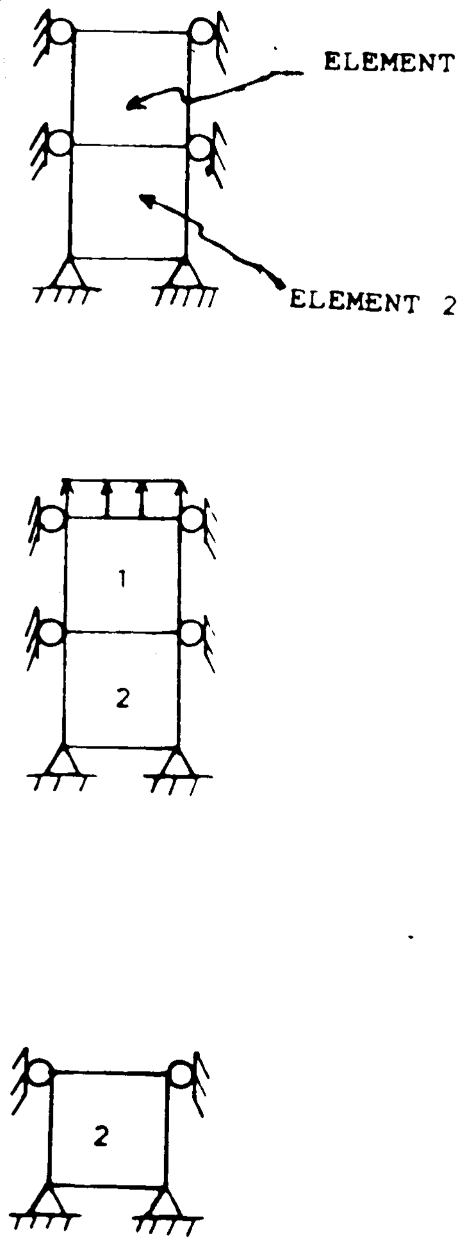
| LAYER | CORRESPONDING MATERIALS | POISSON RATIO | RELATIVE DENSITY (1000kg/cu m.) | YOUNG S MODULUS (kpa) |
|---|---|---|---|---|
| 1 | Columnar Jointed Till | 0 42 | 2 14 | 137,900 |
| 2 | Weathered Clayshale | 0 42 | 2 19 | 130,000 |
| 3 | Unweathered Clayshale | 0 4 | 2 19 | 140,000 |
| 4 | Shear Zone | 0.4 | 2 0 | 100,000 |
| 5 | Bedrock | 0 4 | 2.19 | 150,000 |

NOTE : Simple model combines layers 2 and 3 into a single layer

| | | | | |
|---|---|---|---|---|
| Combined Layer | | 0 41 | 2 19 | 135,000 |

Table 3.4 Location And Material Properties Of The Trials

| TRIAL | REDUCED YOUNG'S MODULUS (MPa) (original 100MPa) | LOCATION (refer to Figure 3.12) |
|---|---|---|
| 1 | 1 | A |
| 2 | 5 | A |
| 3 | 1 | B |
| 4 | 5 | B |




GIVEN THE STRUCTURE SHOWN

STEP 1

LOAD THE STRUCTURE AS SHOWN

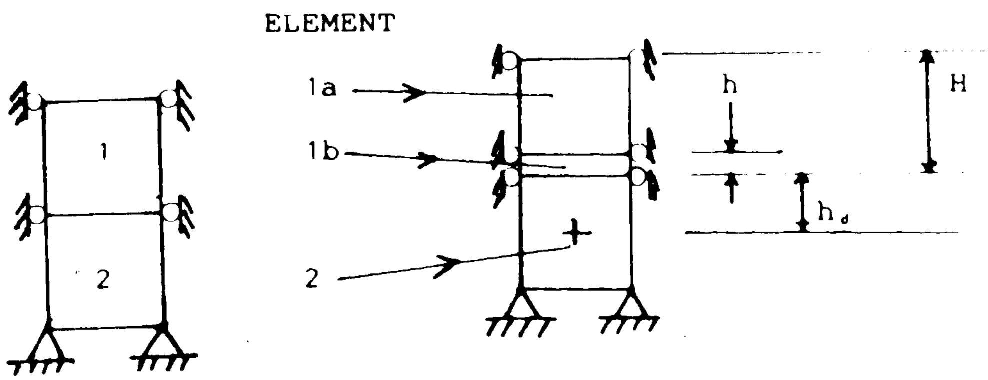
STEP 2

KILL (DESTROY) ELEMENT 1

CONCLUSION :

THE STRESS OF ELEMENT 2 SHOULD BE ZERO IF THE STRESS-FREE BOUNDARY IS VALID.

Figure 3.1 A Simple Test Of Element Death Option Without Gravity Loading (For Simplicity, Only Two Elements Are Used)




FROM FIGURE 3.1 MODIFIED STRUCTURE

STEP 1 LOAD THE RIGHT HAND STRUCTURE GRAVITY

STEP 2 KILL (DESTROY) ELEMENTS 1.a AND 1.b

The vertical stress of element 2 ($\sigma$) should be :

$$\sigma = \rho \, g \, h_d \quad \text{—— (1)}$$

However, the internal calculation of ADINA program would be :

$$\sigma = \rho \, g \, h_d + \rho \, g \frac{h}{2} \quad \text{—— (2)}$$

As h approaches zero, $\sigma(2)$ approaches $\sigma(1)$

Figure 3.2 A Simple Test Of Element Death Option With Gravity Loading

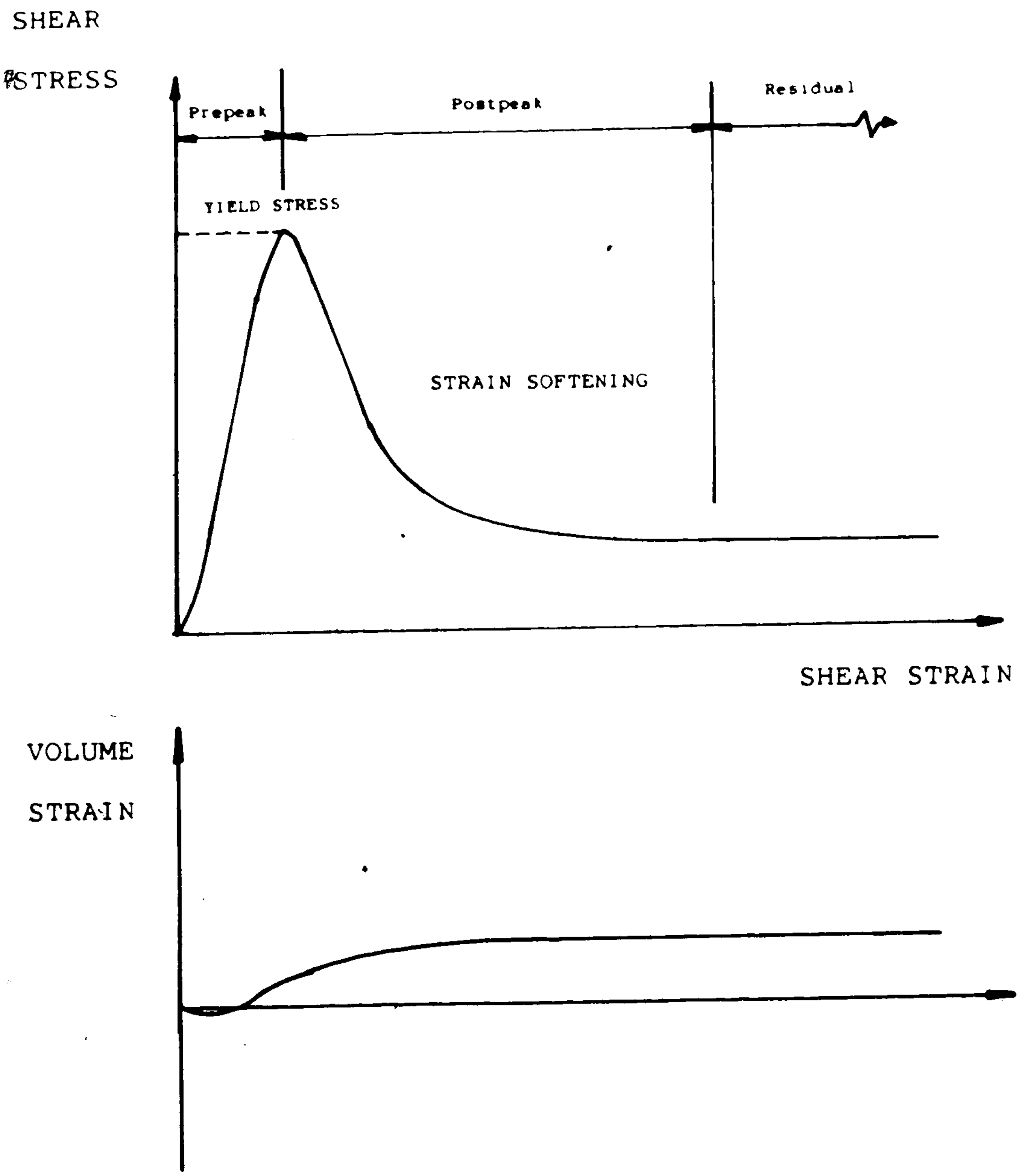
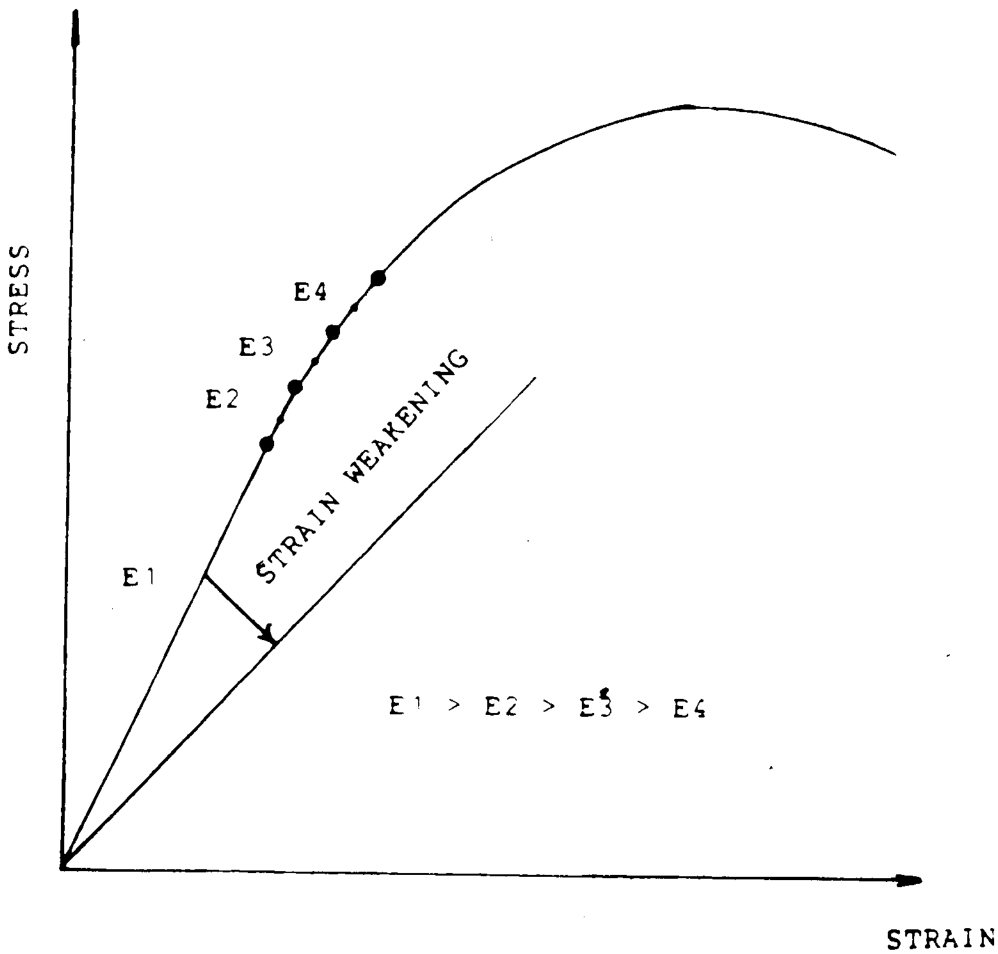



Figure 3.3 The Strain-Softening Model




NOTE : The value of E is the slope of the stress-strain curve.

Figure 3.4 The Incremental Elasticity Model

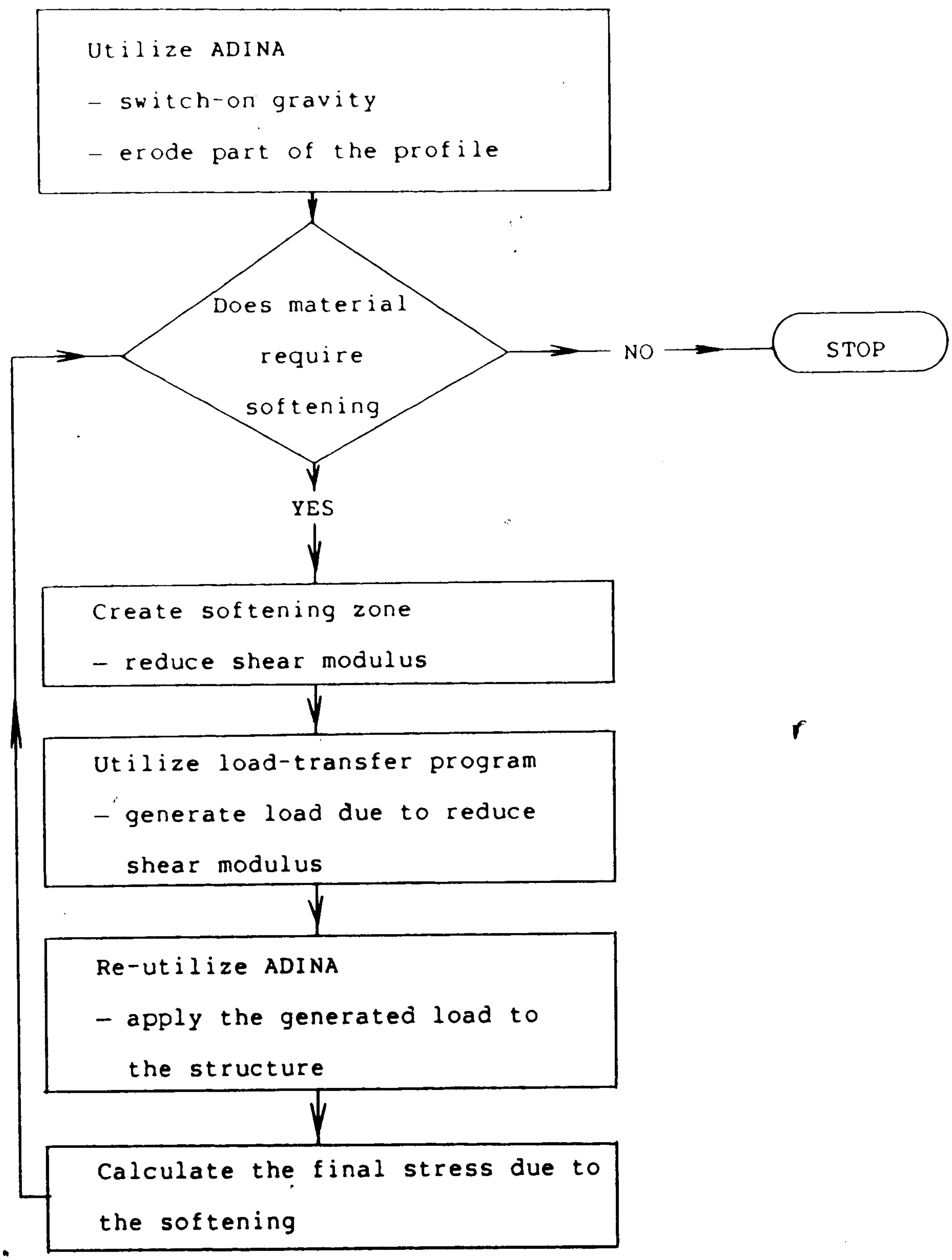
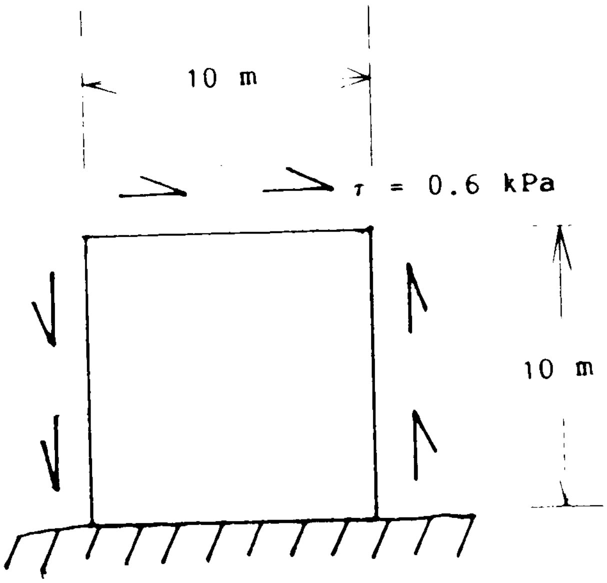
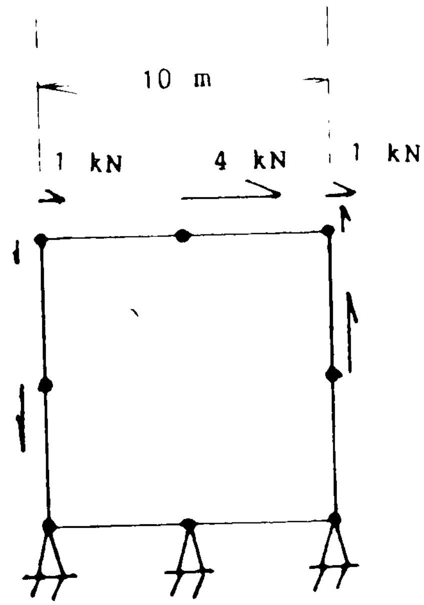



Figure 3.5 Flow Chart Of The Load-Transfer Technique




PURE SHEAR MODEL




FINITE ELEMENT IDEALIZATION

MATERIAL PROPERTIES :

Density ($\rho$) = 0 kg/cu.m.

Poisson's Ratio ($\mu$) = 0.3

Young's Modulus (E1) = 1000.0 kPa

Young's Modulus (E2) = 250.0 kPa

| CASE | STEP | E | G | $\delta G$ | $\sigma_{xyi}$ | $\delta\sigma_{xy}$ | $\sigma_{xyk}$ | $\gamma_i$ | $\delta\gamma$ | $\gamma_k$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | (kPa) | | | (kPa) | | | (X 0.001) | | |
| 1 | 1 | 1000 | 385 | – | 0.0 | 0.6 | 0.6 | 0.0 | 1.56 | 1.56 |
| | 2 | 250 | 96 | 289 | 0.6 | -0.45 | 0.15 | – | – | – |
| | 3 | 250 | 96 | 289 | 0.15 | 0.45 | <u>0.6</u> | 1.56 | 4.68 | <u>6.24</u> |
| 2 | 1 | 250 | 96 | – | 0.0 | 0.6 | <u>0.6</u> | 0.0 | 6.24 | <u>6.24</u> |

**Figure 3.6 Example Of Pure Shear**

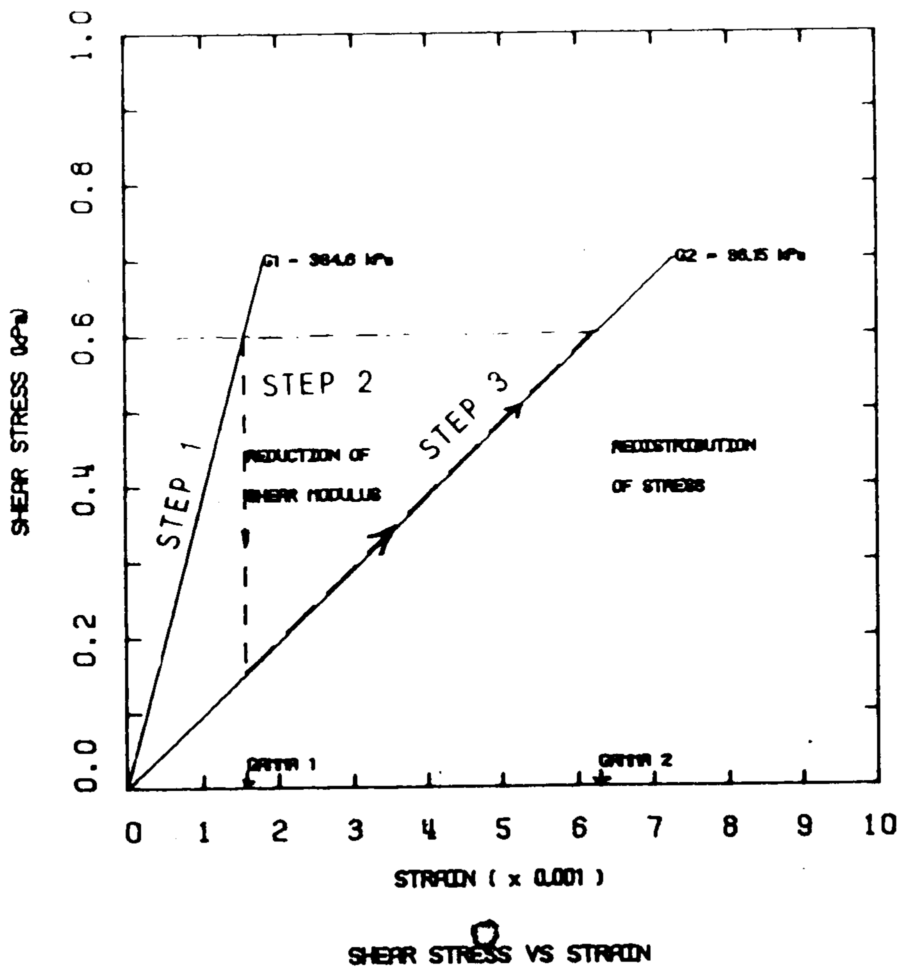
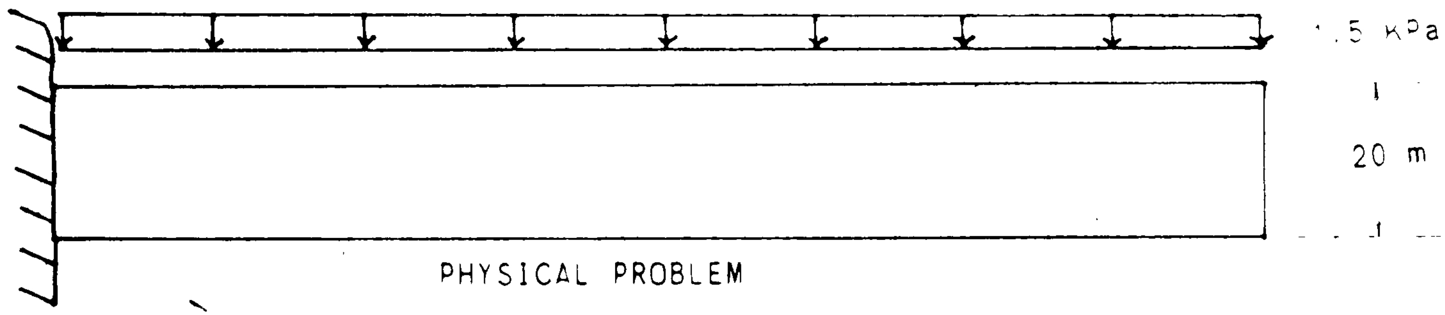
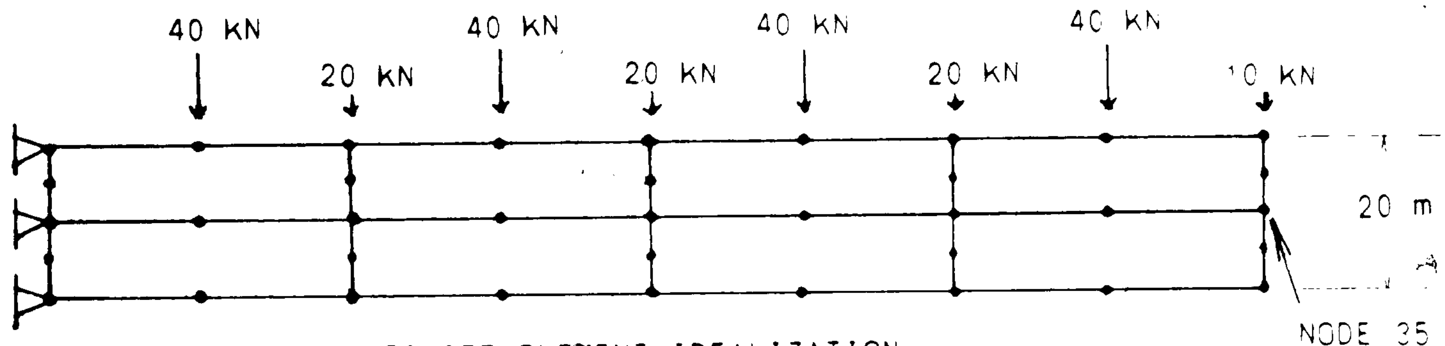



Figure 3.7 Stress Path For The Reduction Of Shear Modulus




PHYSICAL PROBLEM




FINITE ELEMENT IDEALIZATION

MATERIAL PROPERTIES :

Density ($\rho$) = 0 kg/cu.m.

Poisson's Ratio ($\mu$) = 0.3

Young's Modulus ranges from 1000 MPa to 250 MPa

Example 3.8 Example Of Bending Of A Beam By Uniform Load

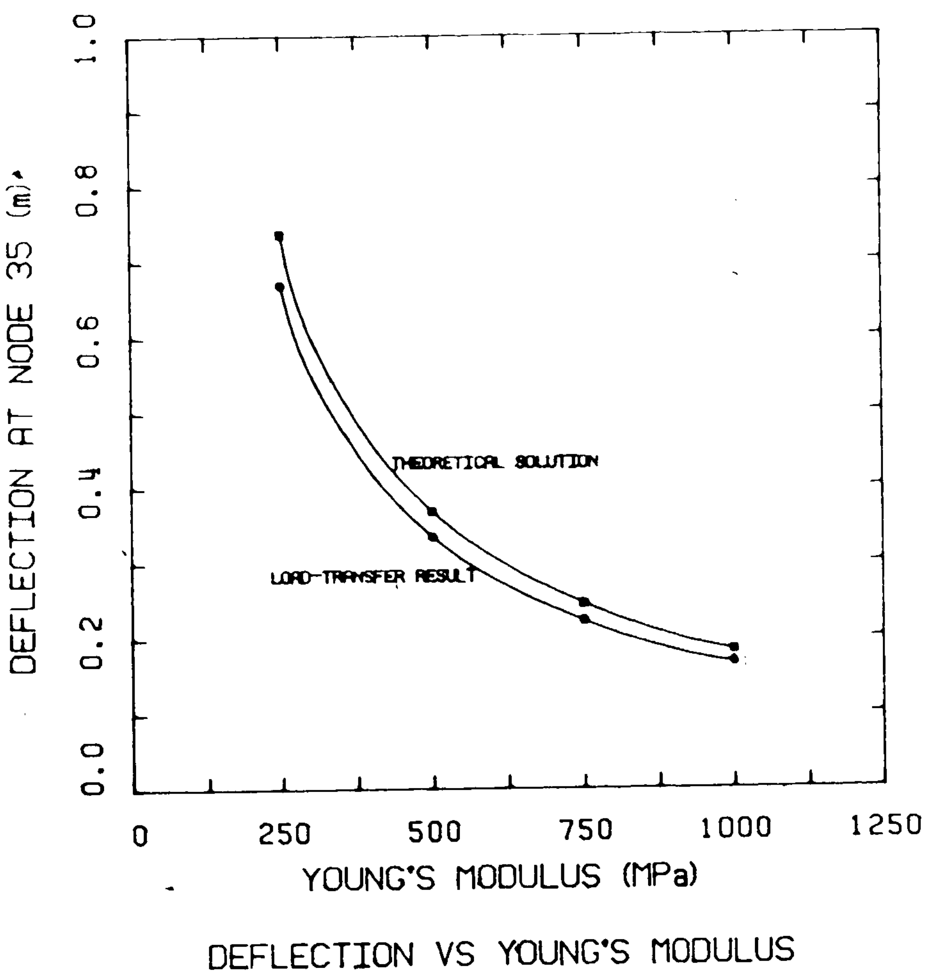



Figure 3.9 Deflection vs Young's Modulus For The Bending Of A Beam By Uniform Load

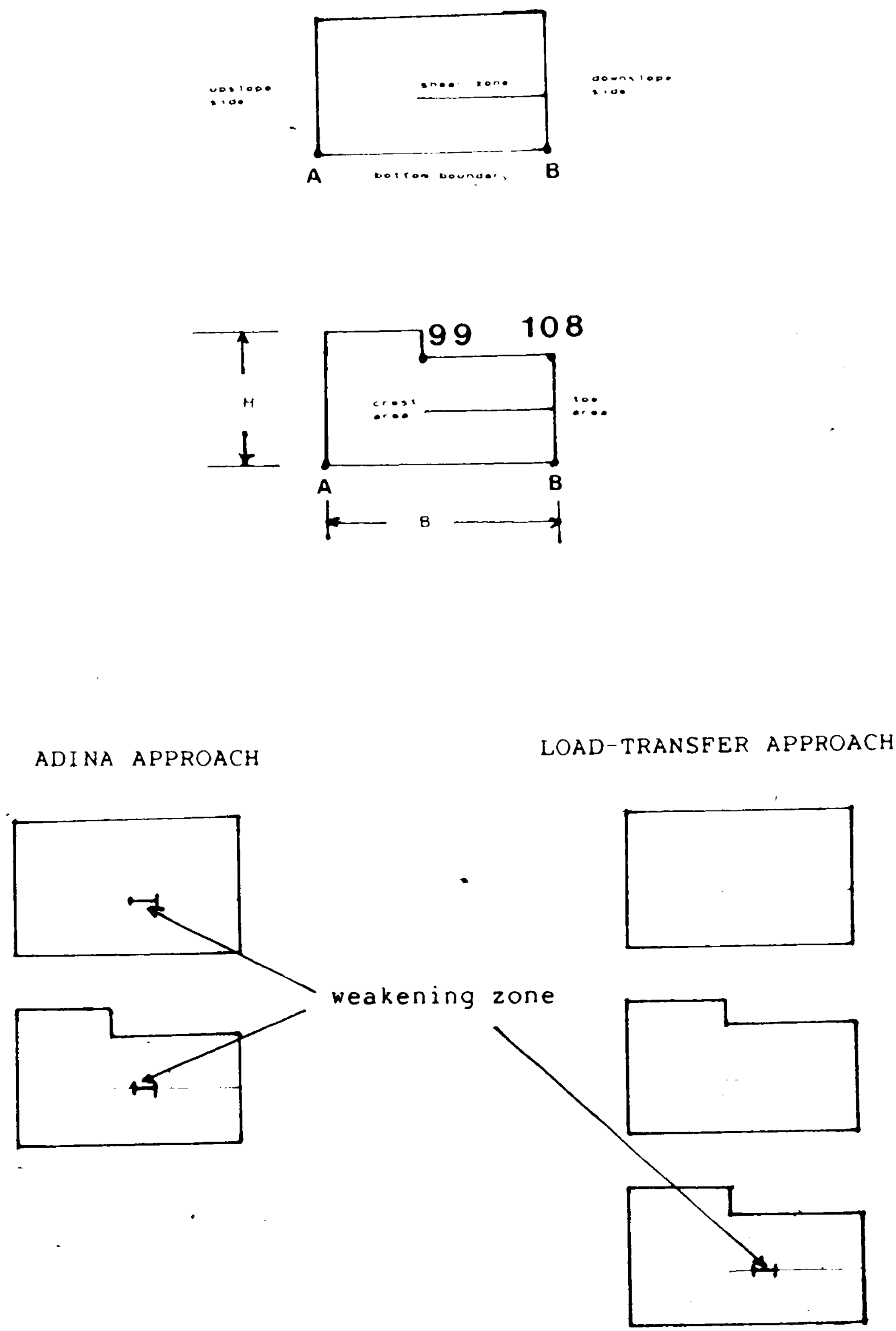



Figure 3.10 The Simple Model With 1. ADINA approach 2. Load-Transfer approach

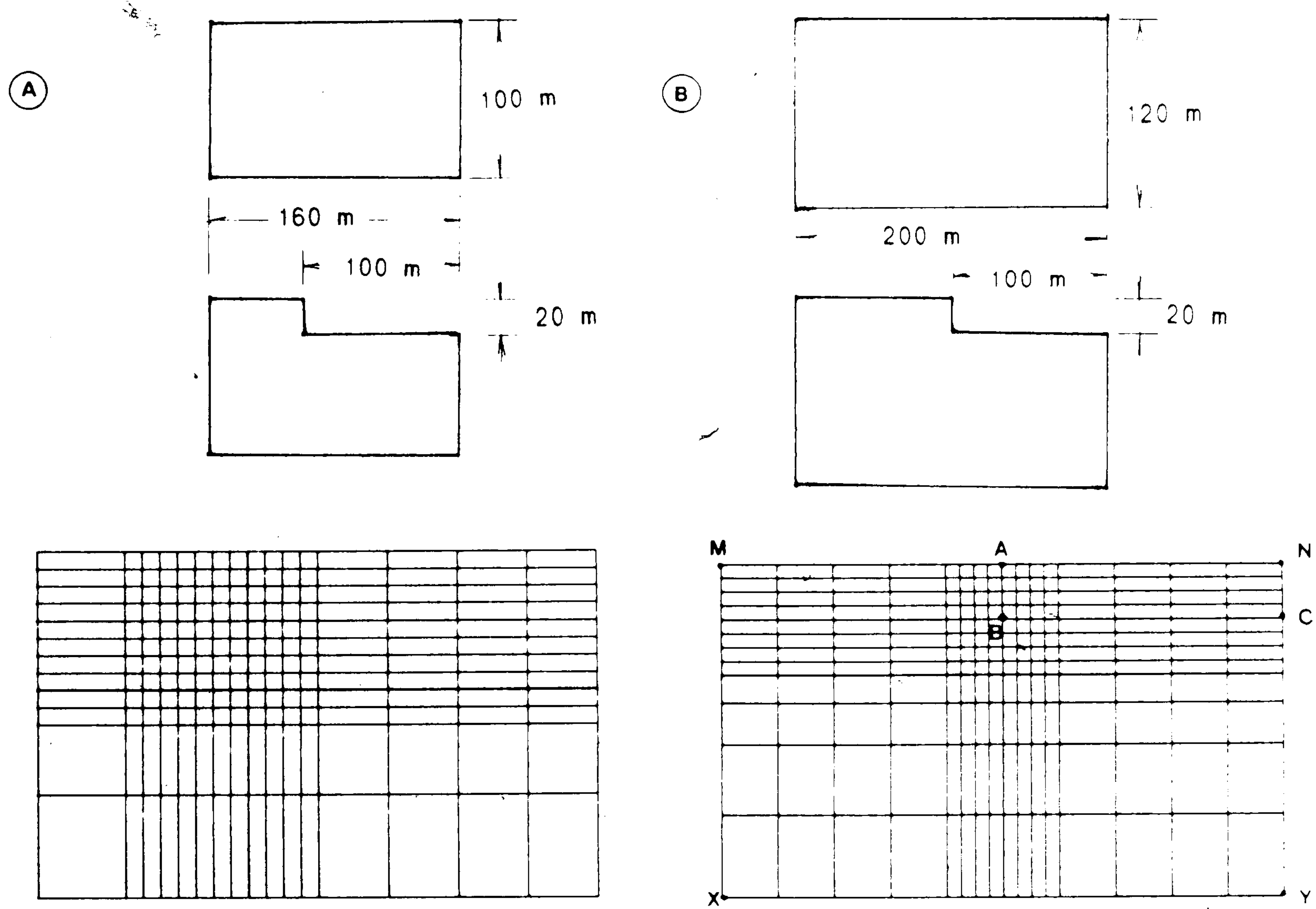
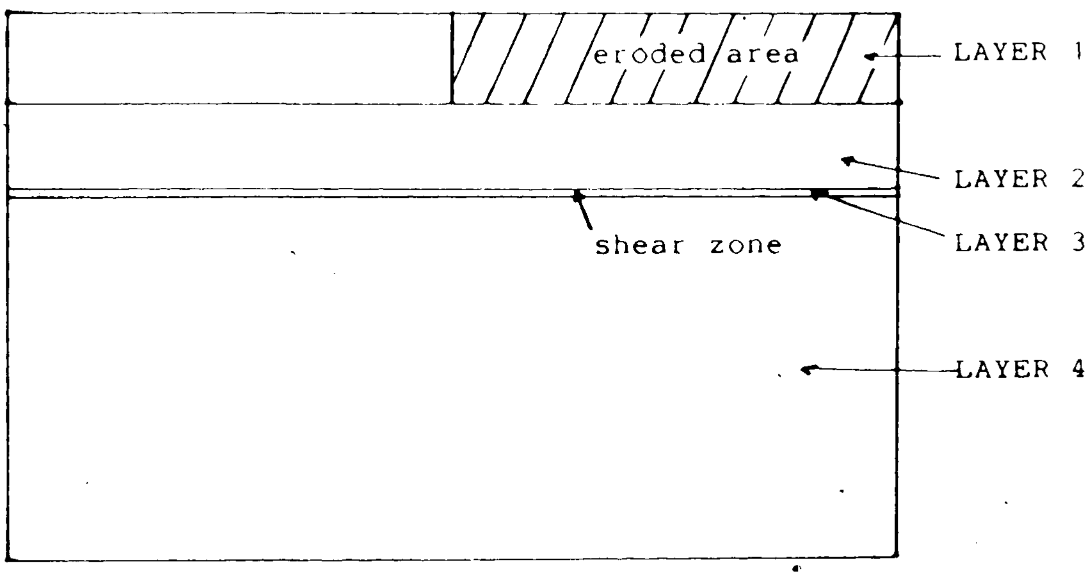
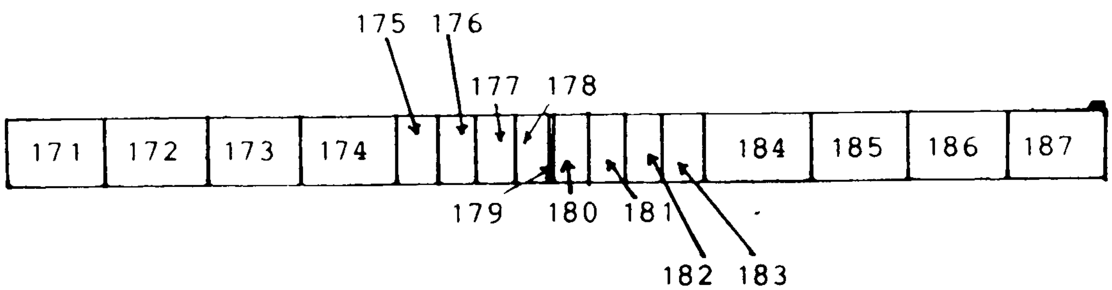



Figure 3.11 Meshes Of The Simple Model

SCHEMATIC DIAGRAM OF THE SIMPLE MODEL




MAGNIFICATION OF THE SHEAR ZONE (element numbering)




LOCATION OF A INCLUDES ELEMENTS 179, 180, 181 & 182

LOCATION OF B INCLUDES ELEMENTS 184, 185 & 186

Figure 3.12 Schematic Illustration Of Shear Zone

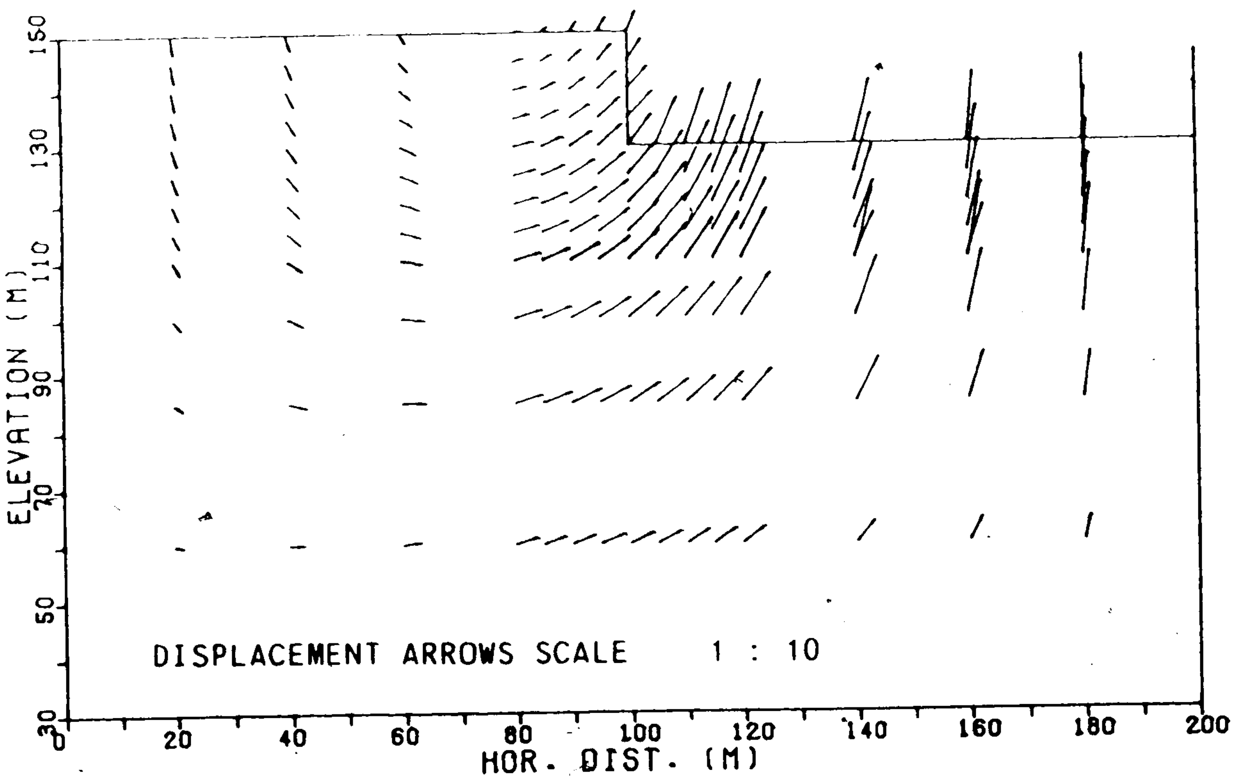



Figure 3.13 A Typical Pattern Of The Displacement Arrows Due To Excavation Process For Trial 3 (ADINA Approach)

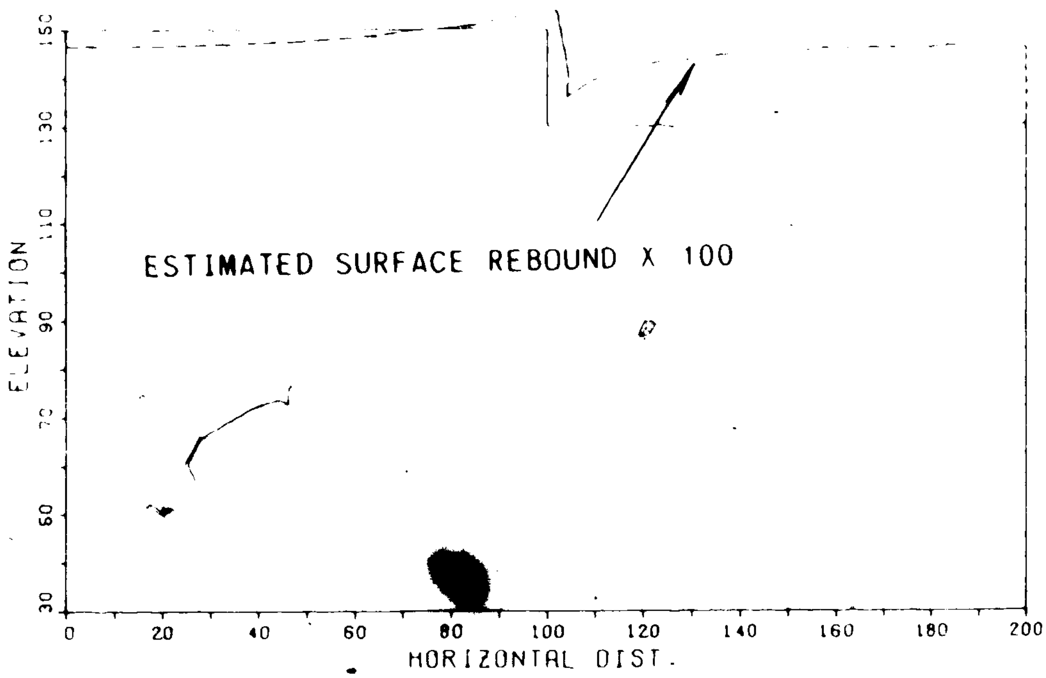



Figure 3.14 a) Surface Displacements For Trial 1 (ADINA Approach)

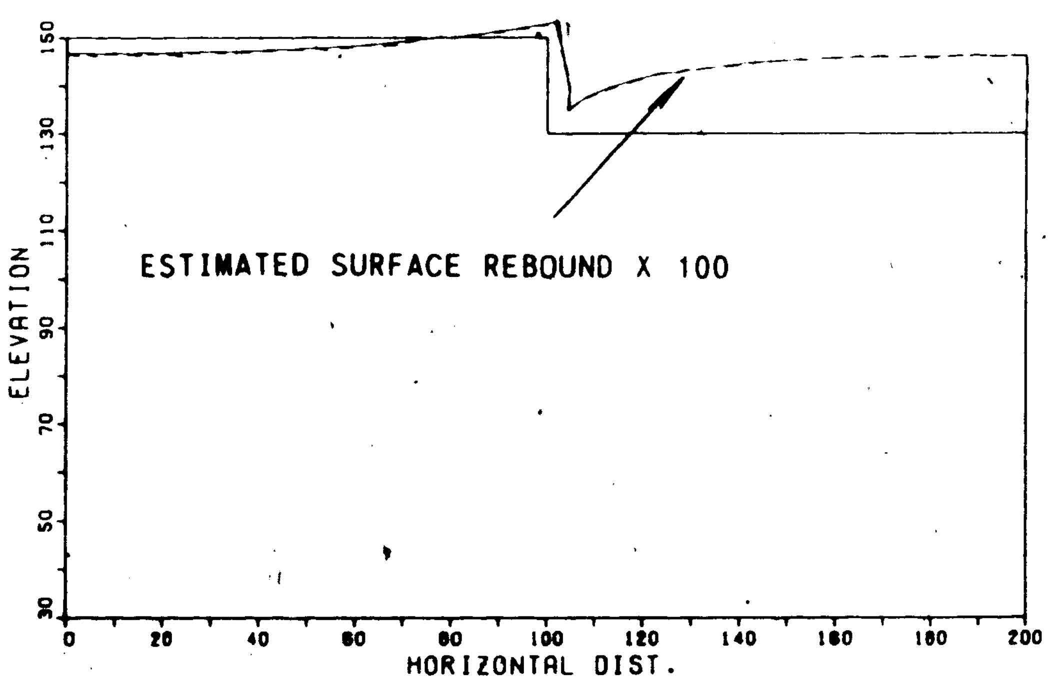



**Figure 3.14 b) Surface Displacements For Trial 1 (Load-Transfer Approach)**

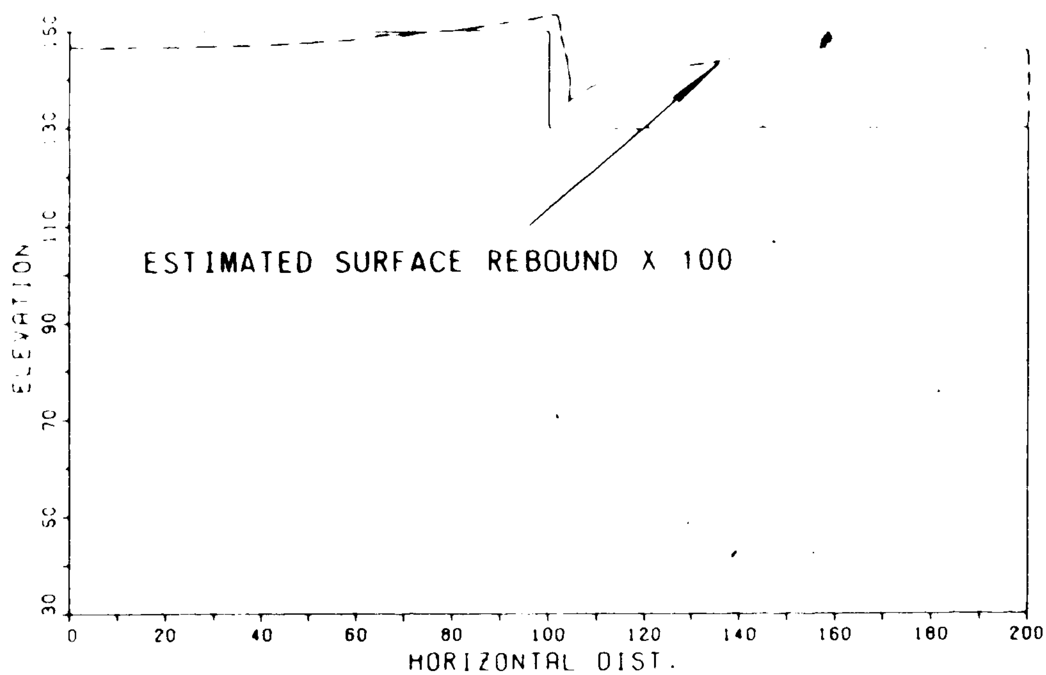



Figure 3.15 a) Surface Displacements For Trial 2 (Load-Transfer Approach)

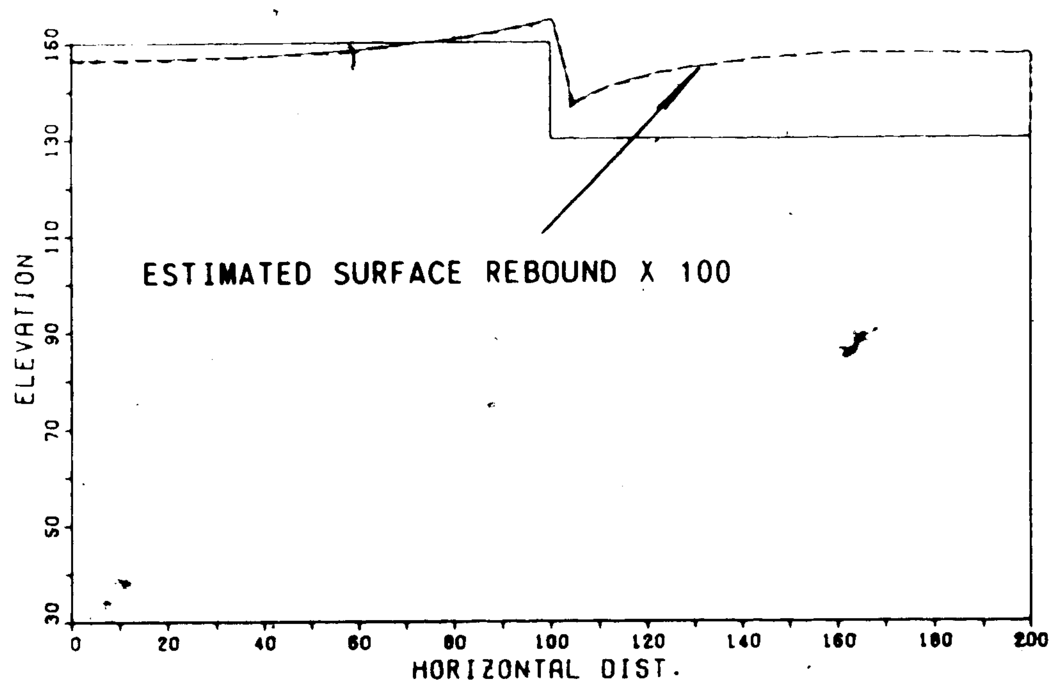



Figure 3.15 b) Surface Displacements For Trial 3 (ADINA Approach)

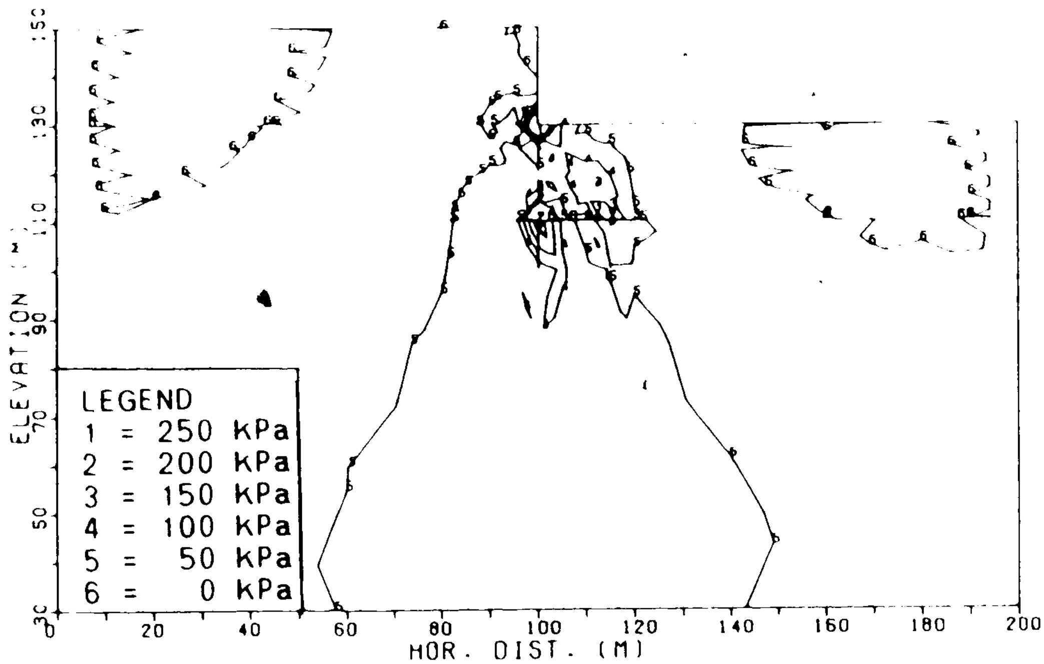



Figure 3.16 a) Shear Stress ($\tau_{xy}$) Contours For Trial 1 (ADINA Approach)

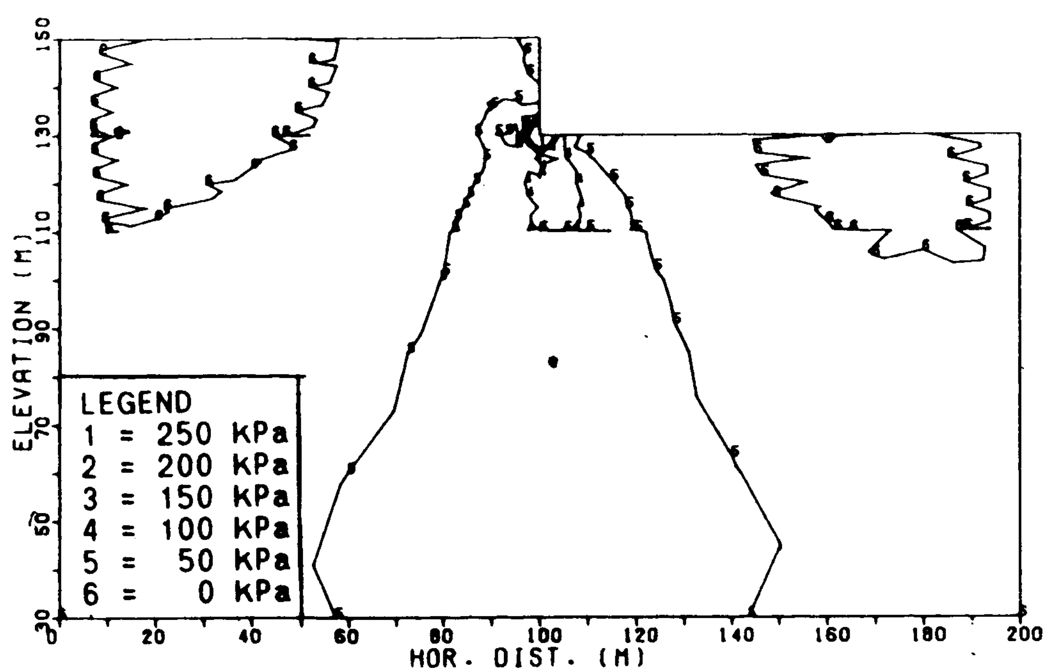



**Figure 3.16 b) Shear Stress ($\tau_{xy}$) Contours For Trial 1 (Load-Transfer Approach)**

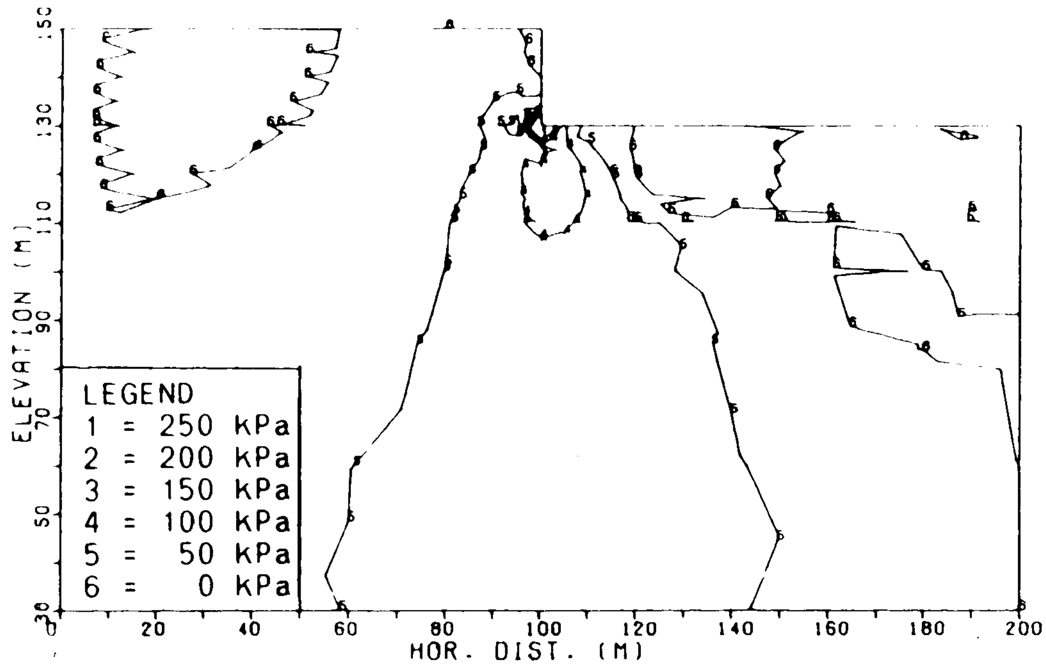



Figure 3.17 a) Shear Stress ($\tau_{xy}$) Contours For Trial 3 (ADINA Approach)

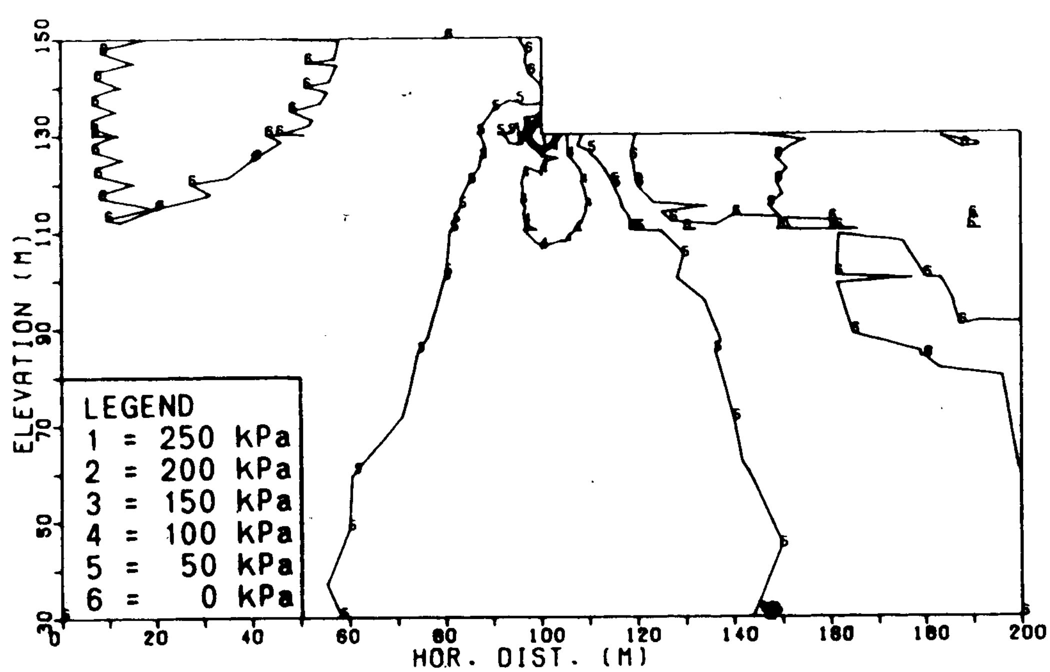



**Figure 3.17 b) Shear Stress ($\tau_{xy}$) Contours For Trial 3 (Load-Transfer Approach)**

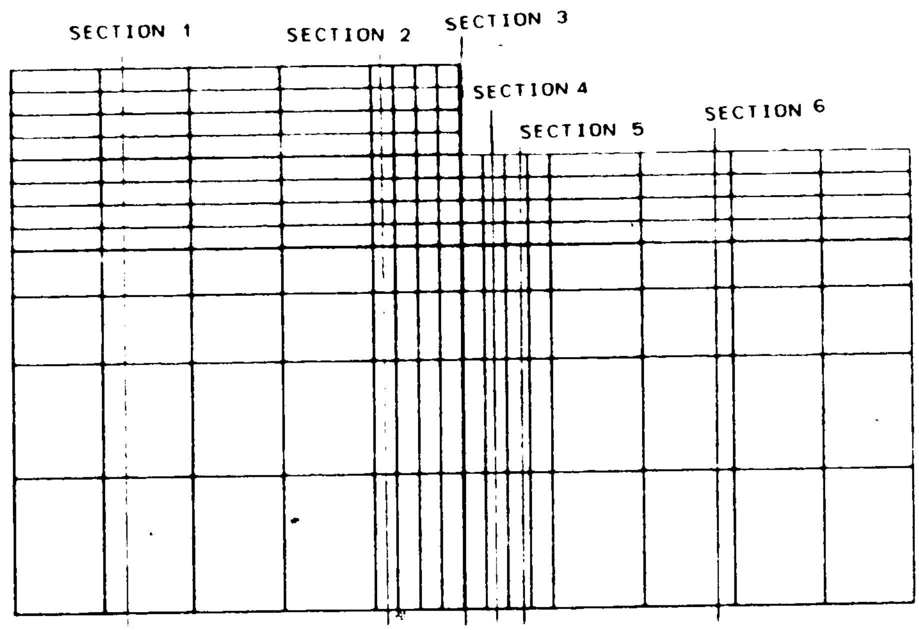



Figure 3.18 Illustration Of The Locations Of The Sections Used For Figures 3.19 and 3.20

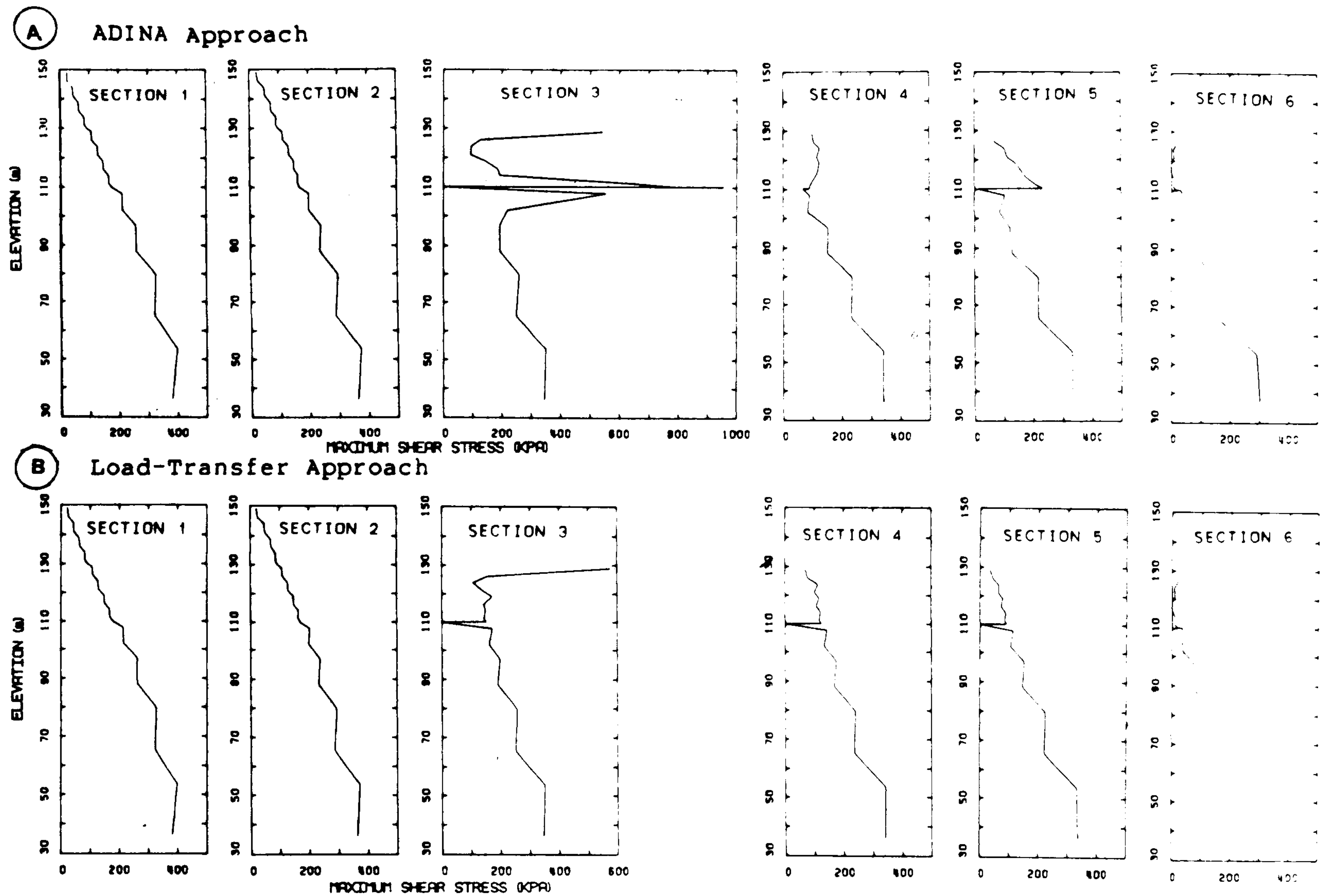
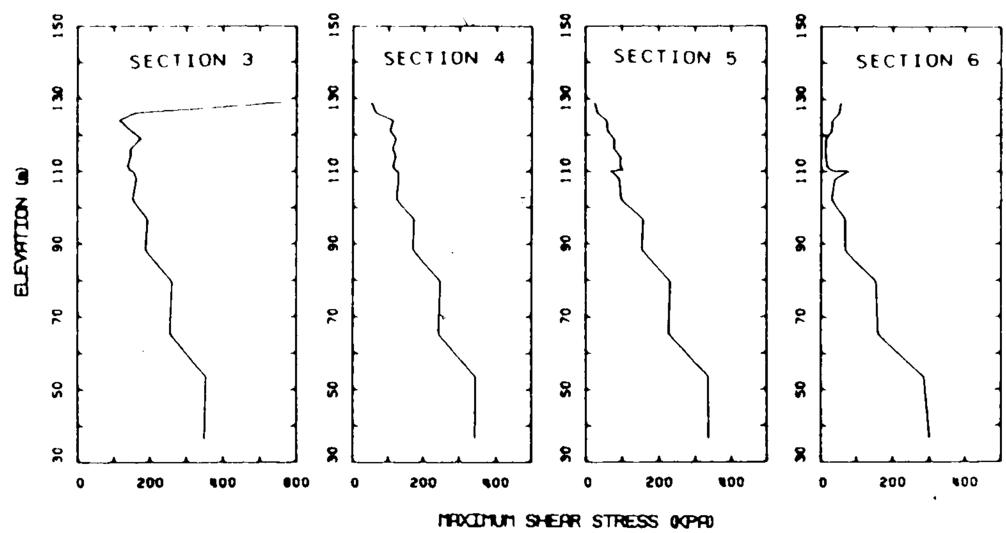



Figure 3.19 The Change Of The Maximum Shear Stress Across Several Sections For Trial 1

(A) ADINA Approach




(B) Load-Transfer Approach

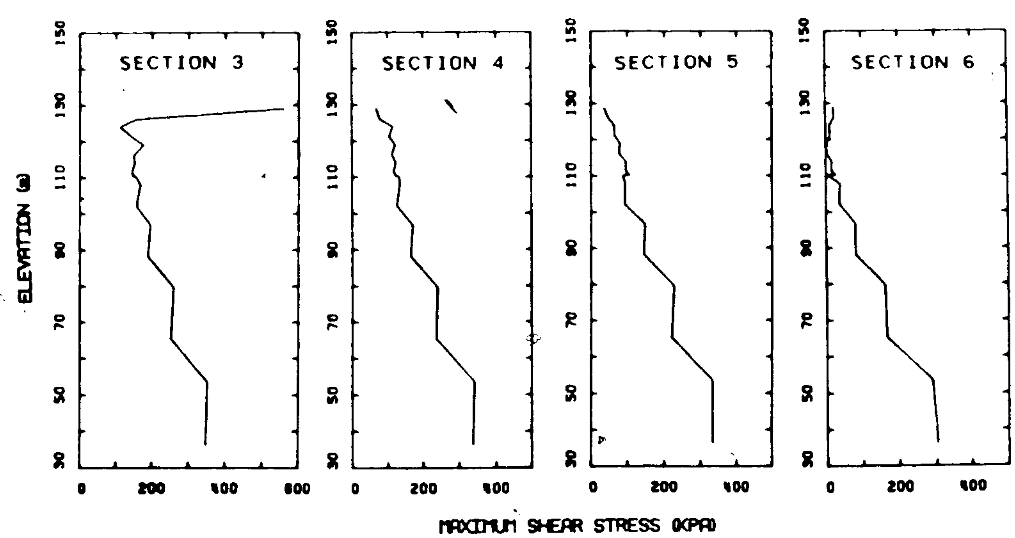



Figure 3.20 The Change Of The Maximum Shear Stress Across Several Sections For Trial 3

# 4. FINITE ELEMENT ANALYSIS OF EDGERTON SLIDE

## 4.1 Aim Of The Analysis

This chapter presents an example of the application of the analytical method to a field problem. A deformation analysis of the Edgerton Slide was carried out for this research.

The deformation analysis is able to clarify the nature of the failure mechanism as will be shown in this chapter. This has some important implications for stability design which is usually based on limit-equilibrium approach.

The relative importance of the following variables should be established before generalizing slopes on the basis of the geometry, scale and soil properties. These factors are quoted from Bishop (1971)

a. the relationship between post-peak drop off in strength and displacement.

b. the swelling characteristics of the soil.

c. the prepeak stress deformation characteristics of the soil under the appropriate conditions of stress change.

d. the value of the coefficient of earth pressure at rest before the formation of the slope.

e. the geometry and scale of the slope.

f. the long term flow pattern of the ground water.

All these factors will be considered except points (b) and (f); in an attempt to correlate the field data with the analytical results. The ultimate goal is to understand a failure mechanism and a secondary deformation pattern which will be used as a guide to assess the stability of a natural slope

The logic behind the analysis is of considerable importance in extrapolating from experience from the Edgerton slide to other similar slides.

The dilatancy which accompanies failure and post-failure behavior of over-consolidated clay, is localized in a very narrow zone around the failure surface. This behavior is inadequately represented in most of the numerical models recently proposed for describing the strain-softening and dilatancy of stiff clays.

The purpose of this chapter is to gain an insight into the landslide mechanism. The general purpose finite element program coupled with the load-transfer program is used to predict the stability of stiff-fissured clay slopes. The process of modelling the landslide from beginning to end involves large displacement finite element formulation. Additionally, the problems of rupture and cracking are very difficult to model. It seems that none of the existing programs can handle these problems. Therefore, the work of modelling the whole process of any landslide is left for future research.

### 4.2 Possible Mode Of Failure

Thomson and Tweedie (1977) postulated that the failure of the Edgerton Slide occurred due to a gradual loss of soil strength, manifested by a virtual disappearance of cohesion, with the final triggering mechanism being a spring time rise in the pore pressure within the slide mass. The existence of the pre-sheared failure plane is the result of several earlier stages of landslide activity.

The following analyses will rely heavily on the choice of the strength parameters. Both the time element and the piezometric level are not considered in the analysis. A weaker strength parameter is assigned to the material below the water table. Even though delayed pore pressure equalization is an important factor in an analysis of slope instability, it is most likely that the previous slide history or pore pressures are not known in detail.

### 4.3 Field Work Essential For An Analysis

The movement of the Edgerton Slide was carefully monitored by Tweedie (1975) using three slope indicators along the slide profile. The location of these indicators is shown in Figure 2.2. The slope indicator data are shown in Figures 4.1 to 4.3. Yearly surface movements had been documented by Mokracki (1982).

The slide profile has been monitored since 1975. From the interpretation of the yearly surface movements, the mass movement can be divided into four individual blocks.

Therefore, three probable rupture surfaces will likely be presented. These rupture surfaces will be considered in the design of the finite element mesh, which is shown on Figure 4.4.

From field observations, a consistent cracking pattern is found throughout the site; particularly the area close to the bulging and the toe. The hexagonal cracking pattern is observed in the field and is difficult to model by finite element analysis. However, the surficial cracking pattern may not lead to catastrophic failure. This cracking pattern may only indicate the area of tension.

### 4.4 Uncertainty

This portion of the work gained from personal discussions with Dr. John Hutchinson during the fall of 1982. As the major work of this thesis was to model the natural slope by the finite element method, a few possible conditions can occur as a result of limited field information. These are :

1. the relative displacements between the various layers are uncertain.

   The problem can be narrowed down to the shear band problem. Within the shear zone, the velocity gradient can be expected to be higher than those of the adjacent layers. From Figures 4.1 to 4.3, one can realize that the amount of deflection suddenly increases at the location of slip zone. This phenomenon is pronounced at the tiltmeter furthest downslope (BH7). However, the slope indicator data are available for only one year.

2. the sub-surface ground movements are uncertain.
   The information obtained from the field has one major disadvantage and that is that the movements of these stakes only represents the movement of the ground surface.
3. regional ground water flow has not been studied in detail.
   Local hydrology has not been studied in detail for the Edgerton area. The relationship between the change of moisture content from rainfall and snowfall and the change of pore pressure has not been established over a long time interval.

In addition to problems associated with the limited field data, there is a problem in the numerical formulation. The jointing of the stiff-fissured clay is similar to the discontinuity zones of rock. However, modelling the joint set is rather a difficult task.

## 4.5 Analysis

The entire set-up for the analysis is based on the data from field work coupled with the suggested mode of failure. The load-transfer technique discussed in Chapter 3 will be used to analyze the Edgerton Slide.

### 4.5.1 Mesh

The finite element mesh of the profile is shown in Figure 4.4. The positions of the nodes are governed by the locations of the slope indicators, the survey hubs and the stratigraphic profile of the slide. The use of a coarse mesh and of 4-node elements was dictated by the cost of computer runs.

Two thin layers of elements are used to model the shear zone material and to avoid the problem of a stress-free boundary (refer to Section 3.2.2). The element aspect ratio (which is defined as the length divided by the height of the element) ranges from 0.15 to 500.

The boundary effect is minimized by setting the boundaries according to the boundary studies by Desai and Christian (1977). The upslope boundary is approximately 300 meters away from the crest. The bottom boundary is approximately 100 meters below the shear zone. The downslope boundary is approximately 200 meters away from the central probable rupture surface.

Three probable rupture surfaces as shown in Figure 4.4 are considered in the design of the mesh.

### 4.5.2 Material Properties

Typical stratigraphy of the site is shown in Figure 2.3. All of the material parameters are based on either available data or on a reasonable estimate based on local experience.

Input parameters for numerical analyses depend on the type of analyses. However, one of the basic parameters, mass density, is required for all analyses. It is used in the calculation of the element mass matrix. Additional input parameters, for example, are Young's Modulus and Poisson's Ratio. These parameters are used to define the material properties for an isotropic linear elastic analysis.

At the outset, all the values of Young's Modulus and Poisson's Ratio were derived from Balanko (1981) and are shown in Table 3.3. The major reason is that both areas comprise similar material with similar properties and a similar stress history.

### 4.5.3 Approach

For the following numerical analyses, both the stratigraphic and piezometric profile were assumed to be unchanged from 1975.

The ratio of the width to the height of the Edgerton Slide is large, hence both the side and the end effects will be small. The results from the two-dimensional analysis will be adequate. Therefore, the two-dimensional analysis can be used to save computing time as well as provide a realistic, practical solution.

The time-dependent movement of the Edgerton landslide will be analysed by using a pseudo-elastic finite element model. It is assumed in the analysis that the Edgerton landslide movement is due to the shear strength reduction at the shear zone with time. By varying the shear modulus (G) in terms of the Young's Modulus, different surface displacements or horizontal displacement at the locations of slope indicator can be obtained from the finite element results. The predicted displacements are compared with the observed displacements in the field. The analytical approach has been applied to a simple model, as discussed in Chapter 3.

This analysis assumes that the surface displacement is predominantly caused by slippage at the shear zone. The properties of the other material (i.e. except shear zone material property) are assumed to be time-independent.

#### 4.5.4 Other Details For The Analysis

The original material properties along the slip surface elements are identical to those of the adjacent elements. Afterwards, the material properties along the slip surface are reduced uniformly to a lower value. The central probable rupture surface as shown on Figure 4.4 is used for joining the interbedded shear zone to the surface. This surface is favored over the other two by the field evidence of the cracking pattern and the uplift movements.

The results from an approximate 50 percent of Young's Modulus reduction do not agree in the order of magnitude of the field data (primarily displacements). Therefore, large reduction of Young's Modulus will be used. The following results will be derived from the reduction range of 10 to

0.5 percent of its original value.

**4.6 Results Of The Analyses**

The horizontal displacements from the analytical results along the locations of boreholes 2, 4 and 7 are plotted and are shown in Figures 4.1 to 4.3. The following points are of note:

a. Changes in Young's Modulus of slip surface have a reverse effect on the horizontal displacements.

b. From a comparison of Figures 4.1, 4.2 and 4.3, the largest horizontal displacement takes place in borehole number 2, which is the furthest upslope tiltmeter.

c. The areas within and/or adjacent to the shear zone (or slip surface) show larger movement than those below or above it. This phenomenon is particularly evident in borehole number 7, (Figure 4.3) which is the furthest downslope tiltmeter.

Three maximum shear stress contours are shown in Figures 4.5, 4.6 and 4.7. These contours represent three different stages; namely, prior to valley development, after valley development and after the development of the weakening zone.

The maximum shear stress contours prior to valley development are predominantly governed by the soil properties and the design of the mesh. (Figure 4.5) The maximum shear stress contours after the development of the weakening zone (Figure 4.7) show a stress concentration along the slip-surface; in particular, the portion furthest upslope.

Figures 4.8, 4.9 and 4.10 show the vertical, horizontal and shear stress contours respectively at the stage after the development of the weakening zone.

The surface displacements are plotted in Figure 4.11. The analytical result shows that the surface movements are primarily moving downhill.

## 4.7 Evaluation Of Young's Modulus From The Previous Laboratory Results

Tweedie (1976) conducted two direct shear tests for the remoulded bentonitic clayshale. The aim of this section is to derive the Young's Modulus from his laboratory results.

The procedure which was developed by Noonan and Nixon (1972) will be used to determine the Young's Modulus from the direct shear test. The following information together with the direct shear test results are required to determine the Young's Modulus:

a. The sample size is 5.08 centimeter (2 inches) square and 2,54 centimeter (1 inch) thich.

b. The gap between the upper and lower halves of the shear box is approximately 1 millimeter (by turning the screws one half to three quarter of a turn).

If the Poisson's Ratio is set to 0.42, the Young's Modulus will range from 4.0 MPa to 5 MPa. Therefore, the

Young's Modulus obtained from the laboratory procedure can be compared with that obtained from the analytical procedure.

### 4.8 Comparison And Discussion Of Results

Some of the analytical results can be compared with the available field data. Additionally, the input parameters for the analysis can be checked with the laboratory values. These comparisons are:

a. The horizontal displacements from the analytical results along the locations of boreholes 2, 4 and 7 can be compared with slope indicator readings.

b. The surface displacement from the analytical results can be compared with the field measurement.

c. The Young's Modulus which is used for the analysis to compare with the field data can be compared with the one obtained from the laboratory results.

A discussion of the stress contours is presented to explain the failure mechanism of the Edgerton Slide.

#### 4.8.1 Comparison With The Slope Indicator Readings

The analytical results of both boreholes 2 and 7 follow a similar trend as the slope indicator readings. However, the difference is that different material properties were used to match the field data in different locations. The Young's Modulus of 10 MPa and 1 MPa were used to match with the data of boreholes 2 and 7 respectively. These are shown in Figures 4.4 and 4.6. Perhaps, this is an indication that the strength properties along the shear zone are not uniform.

The analytical result of borehole 4 is similar to the slope indicator reading at the shear zone, but no comparison can be drawn from the movements above the shear zone. This may be due to another rupture suface co-existing with the central rupture suface.

### 4.8.2 Comparison With The Surface Displacement

It seems that the proposed analytical procedure does not adequately model the heaving portion located about the centre of the slide mass. This is indicated by a comparison of the two profiles on Figure 4.11. However, if the shear modulus above the shear zone is reduced and the bulk modulus is kept constant, then the analytical results may be comparable to the surface displacements from the field.

### 4.8.3 Comparison Of The Value Of Young's Modulus

The Young's Modulus ( $E_I$ ) which was used in the analysis ranges from 1 MPa to 10 MPa across the shear zone (or slip surface) elements. The results from the above range of $E_I$ approximately match the field data (primarily slope indicator readings).

On the other hand, the Young's Modulus ($E_J$) from the laboratory procedure (direct shear test) was derived from the remoulded bentonitic clayshale samples, which were taken from the core of borehole number 4 at the location of the failure plane. The Young's Modulus derived from the laboratory procedure is about 4 to 5 MPa.

Therefore, the Young's Modulus of the slip surface material can be derived from either the analytical approach or the laboratory approach. It is because $E_j$ falls in the range of $E_l$.

### 4.8.4 Discussion Of The Stress Contours

It is important to understand that the stress contours developed after the valley development are represented by a number of horizontal lines. This is due to the material properties being uniform within each layer.

A rapid stress change can be observed from the maximum shear stress contours after the development of the weakening zone (Figure 4.7); especially along the up-slope portion of the slip surface.

The interpretation from the vertical stress contours after the development of the weaking zone (Figure 4.8) is that a vertical stress transfer may be occurring along the up-slope portion of the slip surface.

It seems that there is little effect on the horizontal stresses after the development of the weakening zone. This indication arises from the fact that the horizontal stress contours (Figure 4.9) are essentially a number of horizontal lines.

The shear stress ($\tau_{xy}$) contours after the development of the weakening zone (Figure 4.10) indicate that a rotation of principal stresses may take place along the toe rupture surface and the up-slope portion of the slip surface. Note that the irregularity of the stress contours is primarily due to the discretization of the element stresses.

**4.9 Remarks And Summary**

If the slope indicator readings were available for more than one year, a further analytical reduction of the shear strength can be carried out in order to compare the analytical results with the field measurements observed over the longer time period.

The assumption of the shear modulus reduction at the shear zone with time seems to be appropriate in explaining the failure mechanism of the Edgerton Slide. From the results of Figures 4.7 to 4.10, it seems that the stress concentration is higher in the up-slope part of the slope than the down-slope portion. Therefore, a relatively large movement may first take place in the up-slope area of the slide mass. Hence, it is possible that movement is progressing from crest to toe.

One interesting conclusion may be drawn from this analysis. Since the Young's Modulus obtained from the laboratory procedure agrees with that obtained from the analytical approach, the laboratory value can be used as an input parameter in the analytical analysis to reduce the number of trials to predict the observed movement. Therefore, the laboratory value can be used in the analytical analysis during the design phase to predict future movement.

### 4.10 Area Required For Further Research

Improvements for representing and analyzing models more effectively than are already being analysed, are relatively important and are in demand by many practicing engineers. However, from a research point of view, the development of techniques for modelling new phenomena ,such as anisotropy, strain softening and dilatancy, is rather important and necessary. The formulation of the constitutive relations for most geological materials encounters difficulties and requires a great deal of rationalization. Yet, it appears to this writer that most researchers cannot distinguish which material model is the best for geological materials.

During the past decade, the development of non-linear finite element analytical techniques seemed to be very active. A result is the development of the computer programs such as ADINA and ADINAT. However, the development of non-linear finite element techniques requires research in various areas, as mentioned by Bathe (1980), approximation theory, numerical methods and computer program implementation. Because of this, the product of this general research derives from researchers specializing in different fields. Therefore, it is important to prove that the program does fullfil the original objectives.

It is the writer's opinion that the research in the verification and qualification of any non-linear analysis program is as important as the formulation of the program. After the clarification of these programs, the area of research may progress to another level.

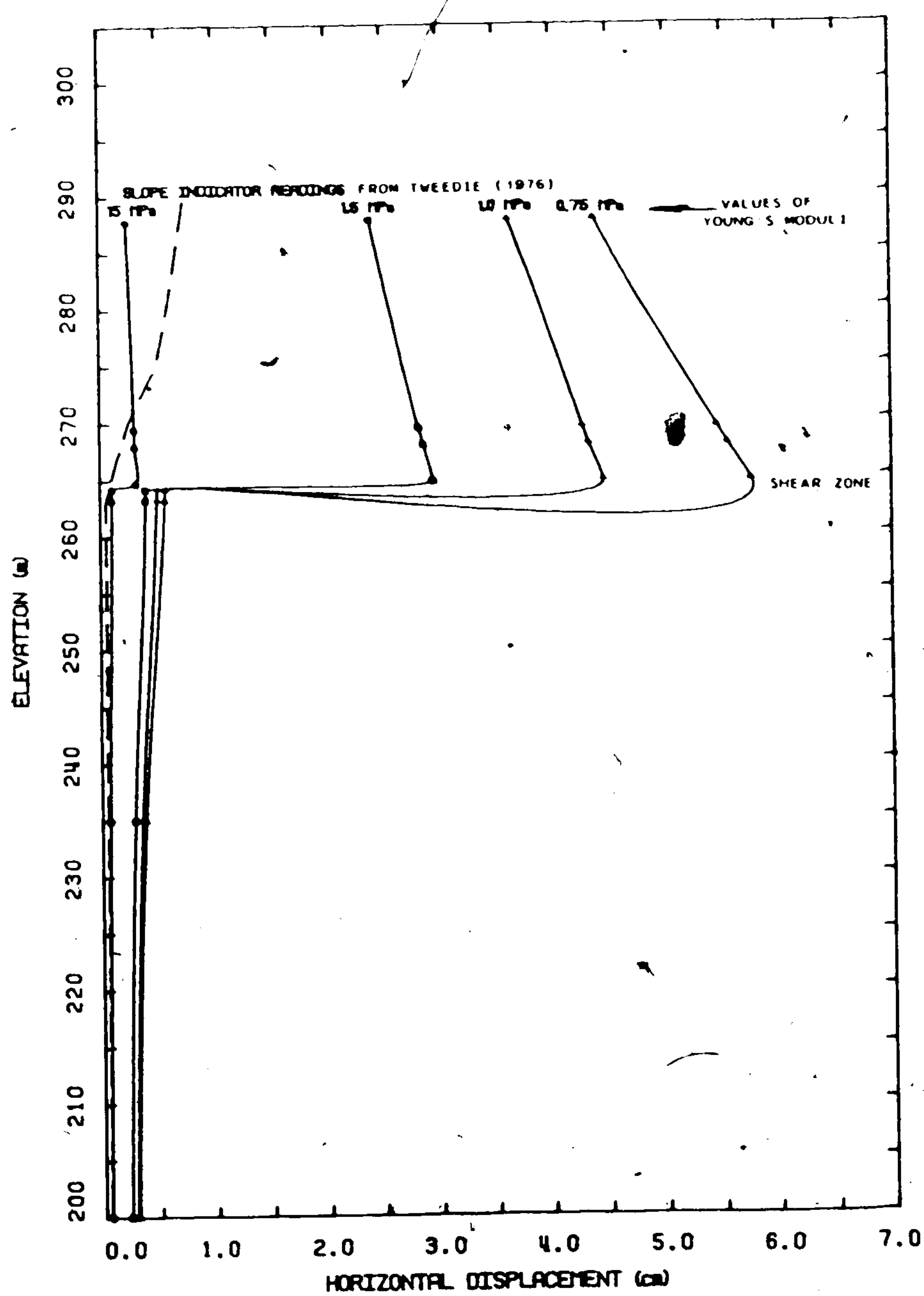



Figure 4.1 Horizontal Displacement Of Borehole 2

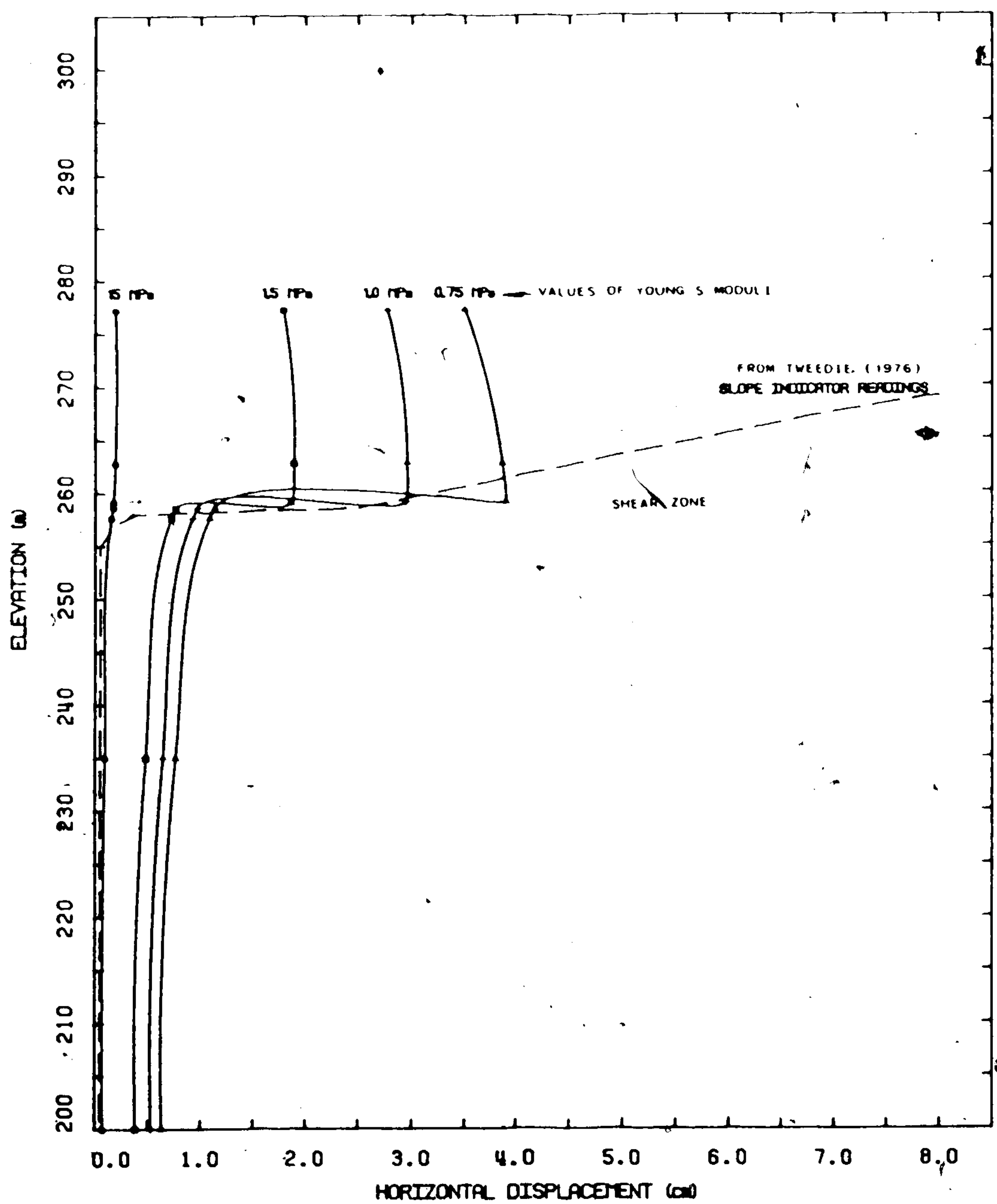



Figure 4.2 Horizontal Displacement Of Borehole 4

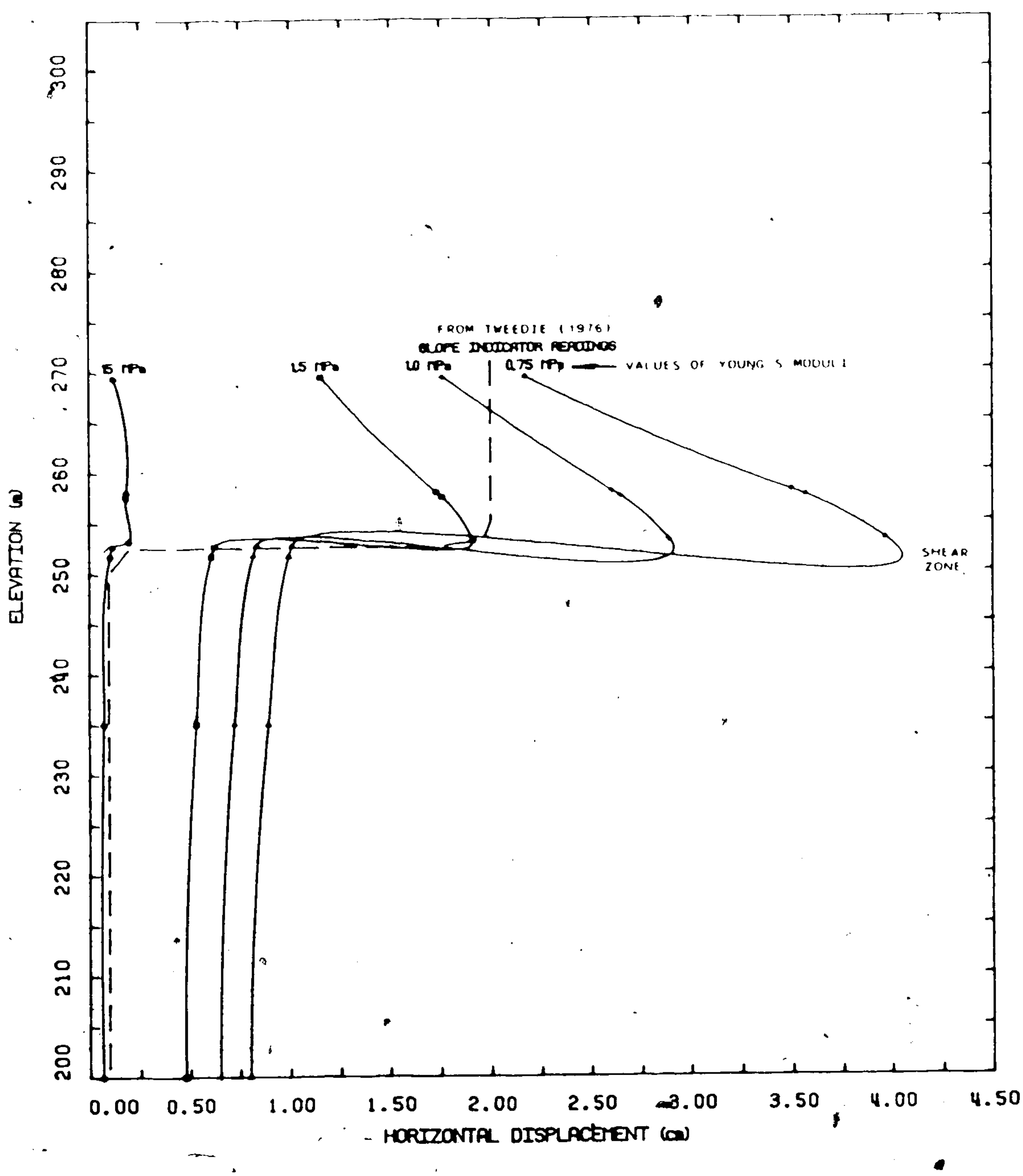



Figure 4.3 Horizontal Displacement Of Borehole 7

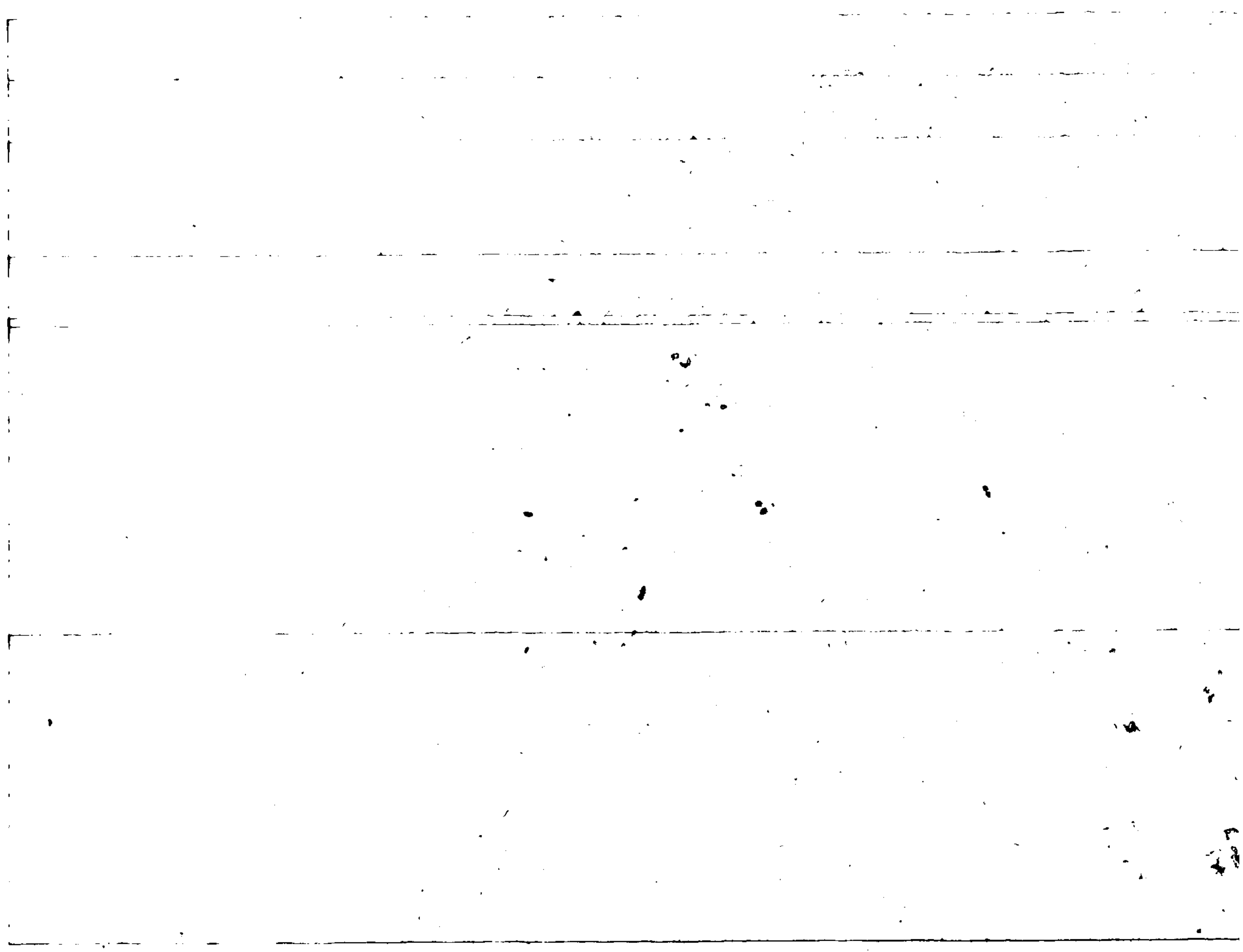


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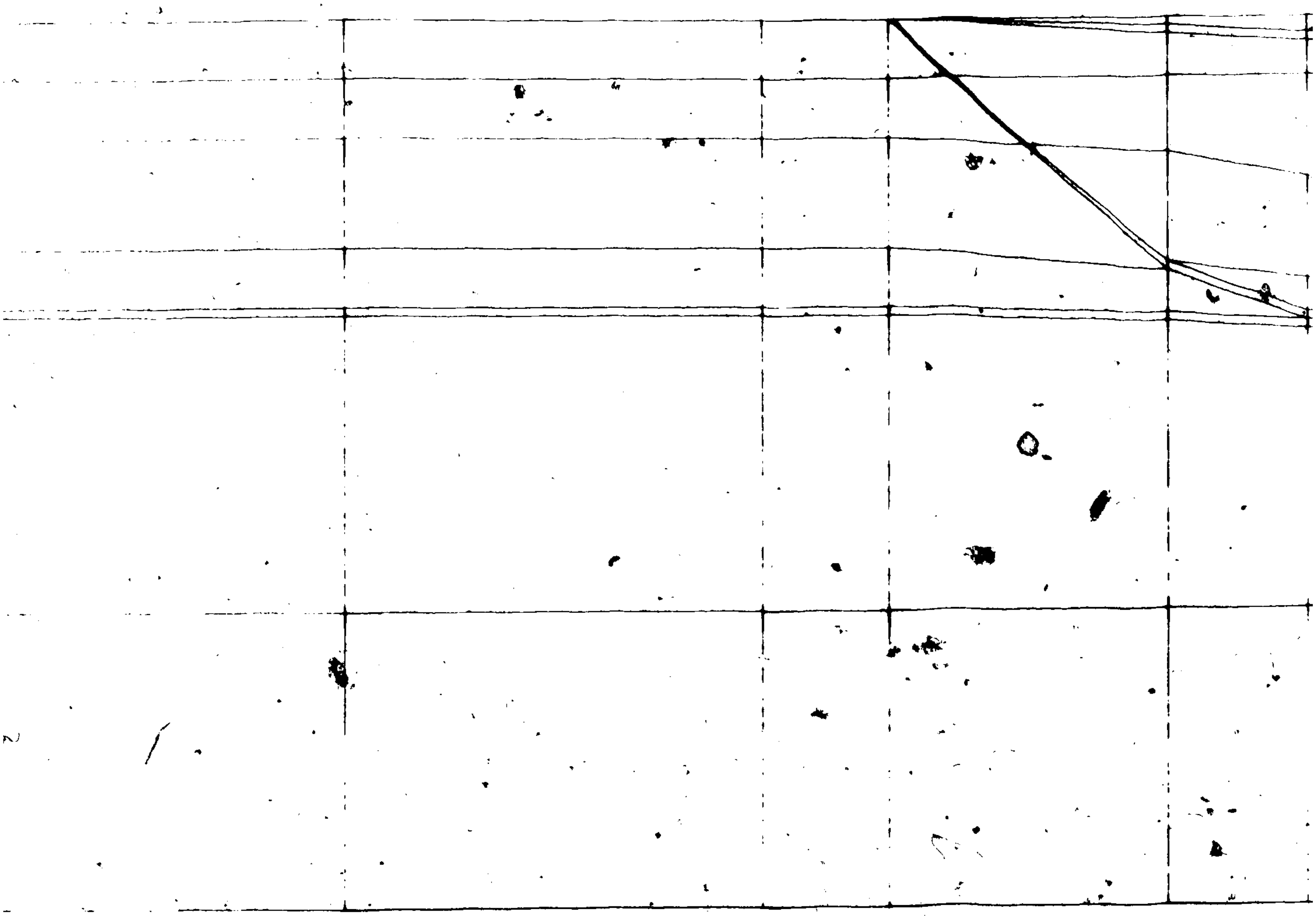
FIGURE 4.4 PROFILE AND MESH OF THE EDGERTON 74-SOUTH



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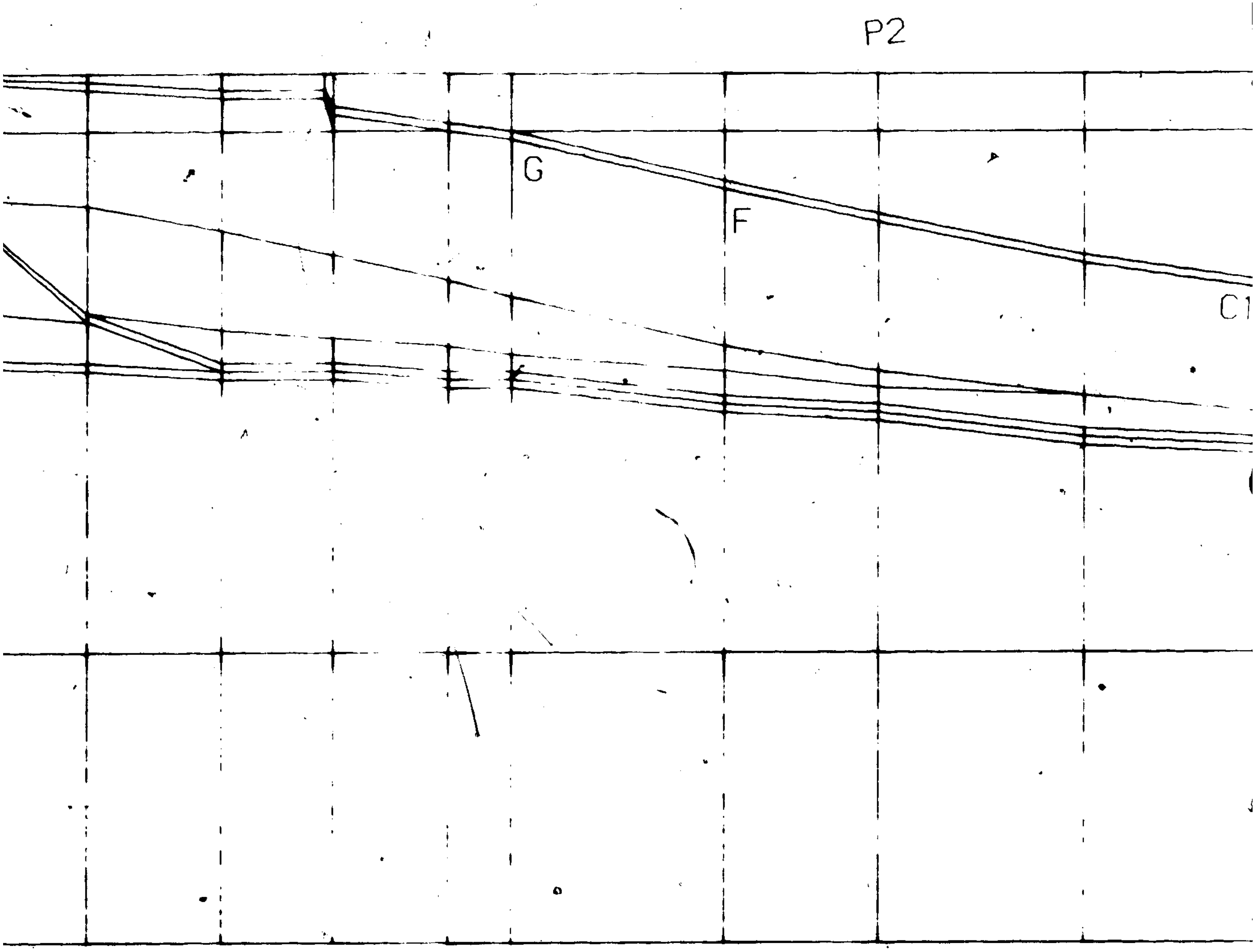


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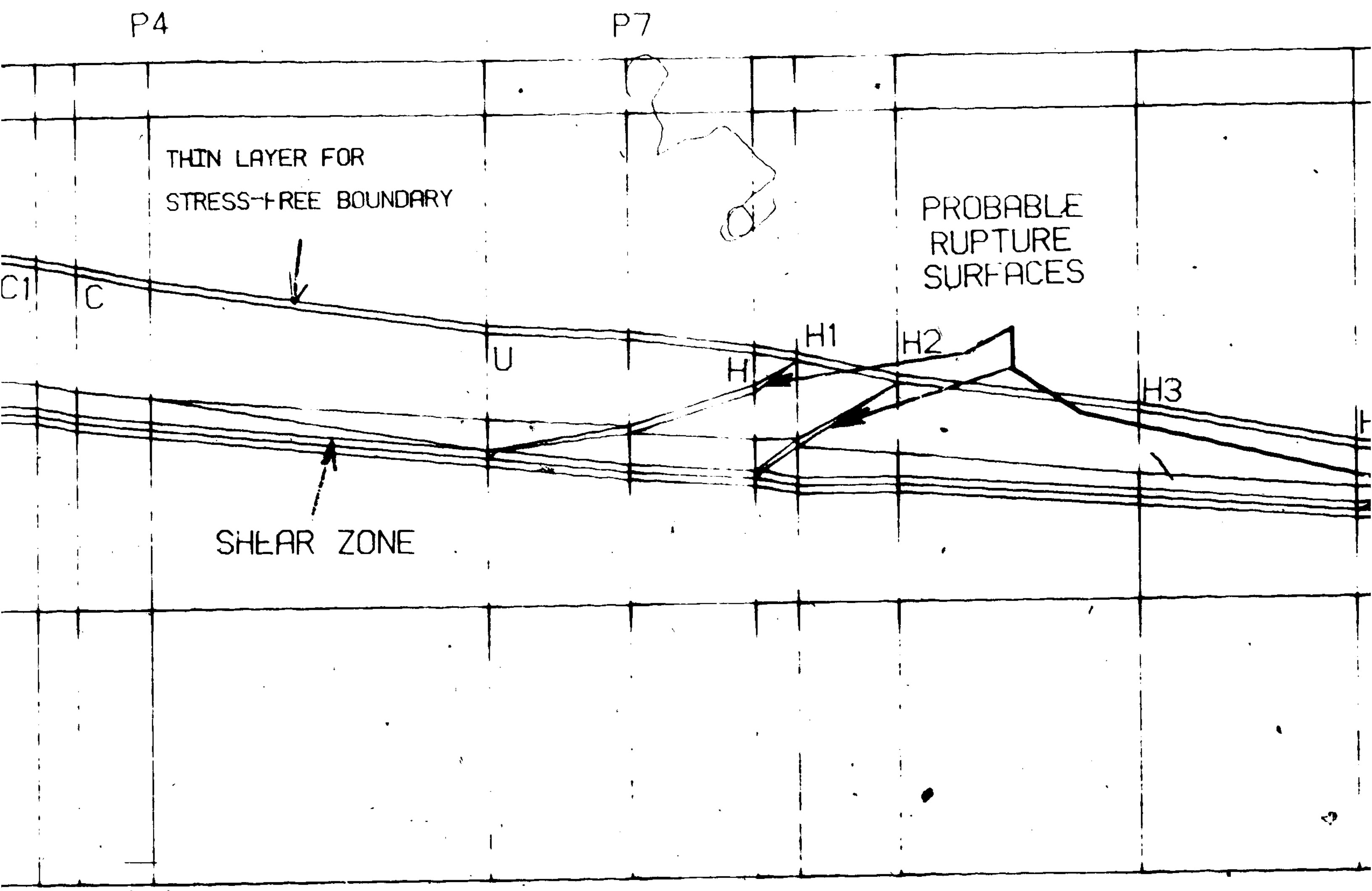
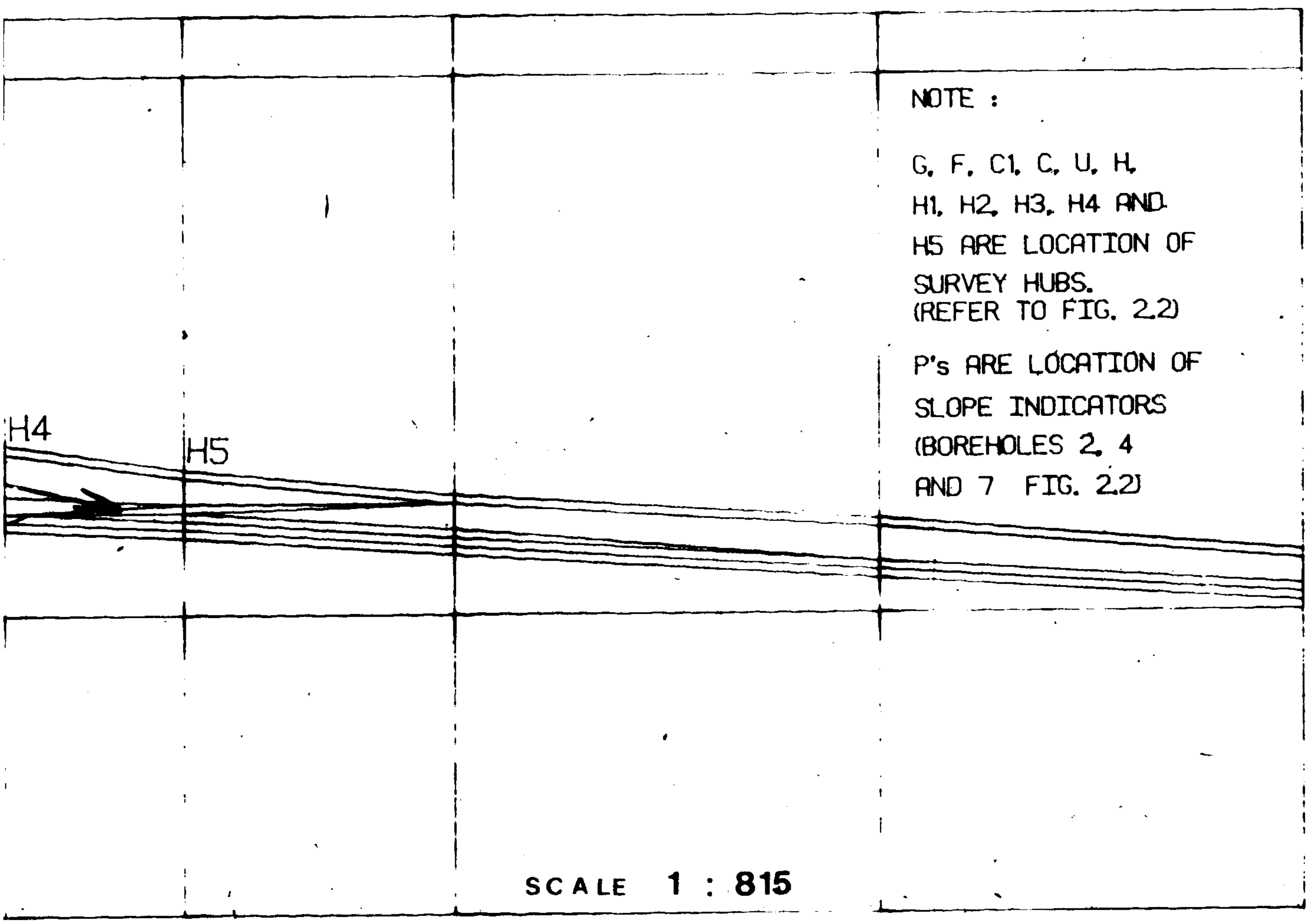

P4
P7
THIN LAYER FOR
STRESS-FREE BOUNDARY
PROBABLE
RUPTURE
SURFACES
C1
C
U
H
H1
H2
H3
SHEAR ZONE


H4
H5
NOTE :
G, F, C1, C, U, H,
H1, H2, H3, H4 AND
H5 ARE LOCATION OF
SURVEY HUBS.
(REFER TO FIG. 2.2)
P's ARE LOCATION OF
SLOPE INDICATORS
(BOREHOLES 2, 4
AND 7 FIG. 2.2)
SCALE 1 : 815

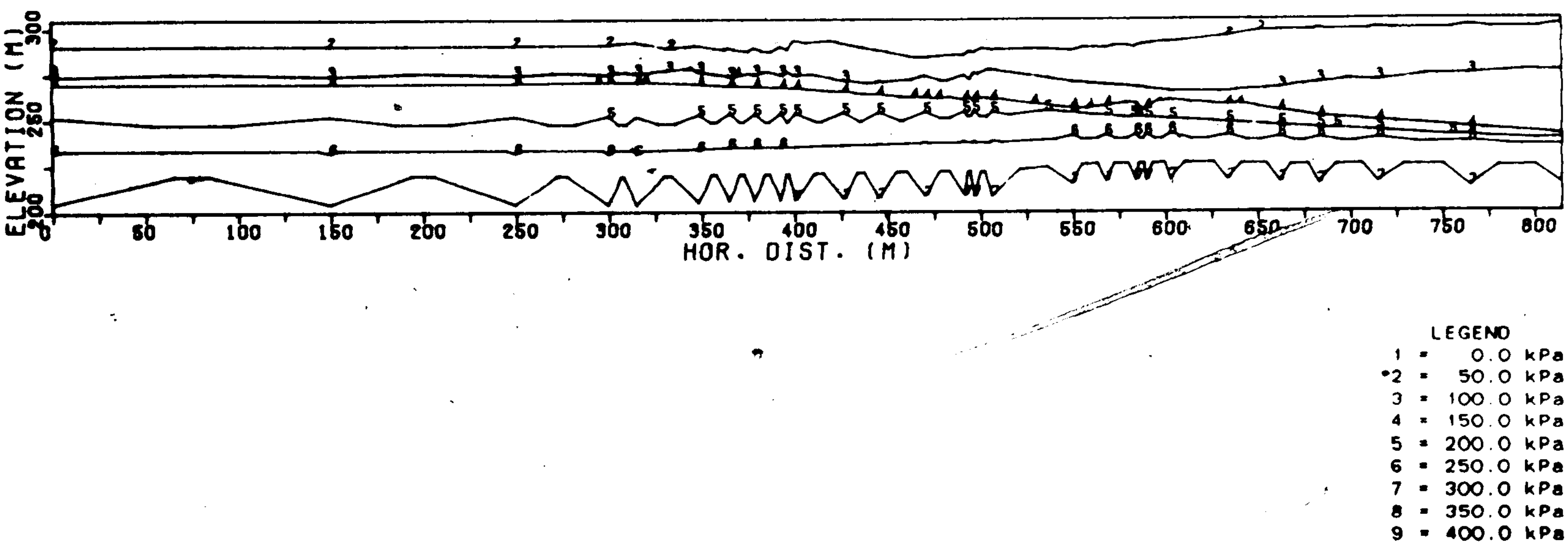



Figure 4.5 Maximum Shear Stress Contours Of The Edgerton Slide (Prior To Valley Development)

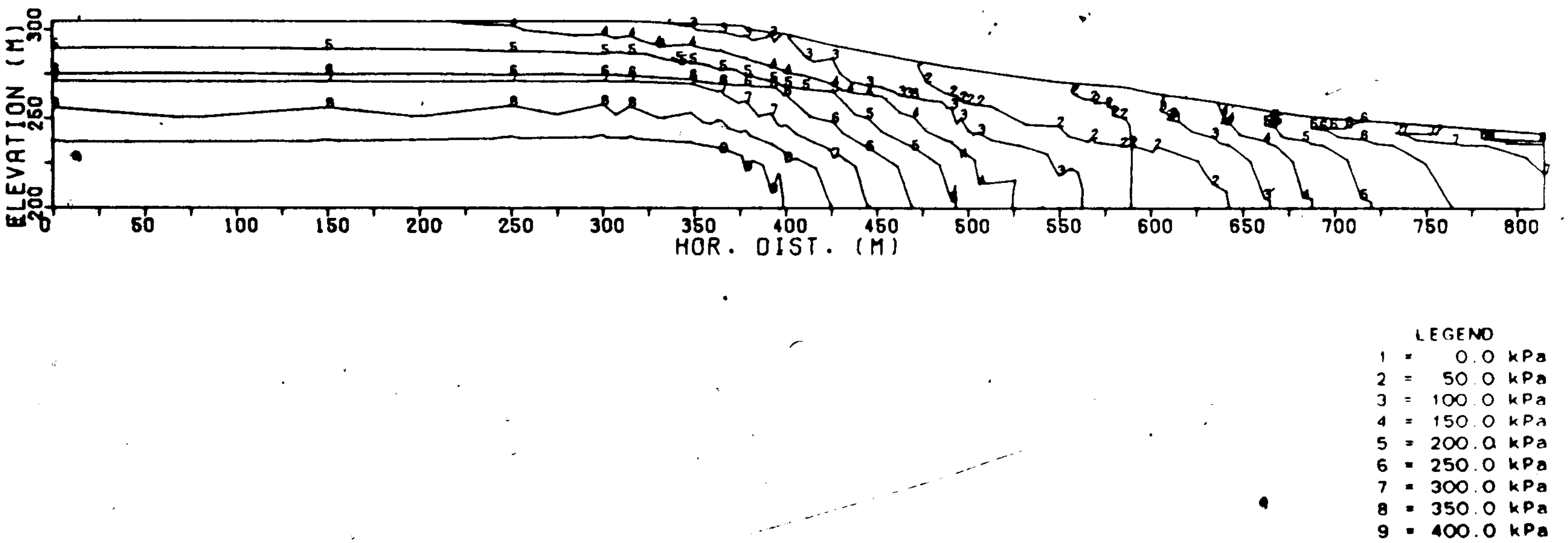



Figure 4.6 Maximum Shear Stress Contours Of The Edgerton Slide (After The Valley Development)

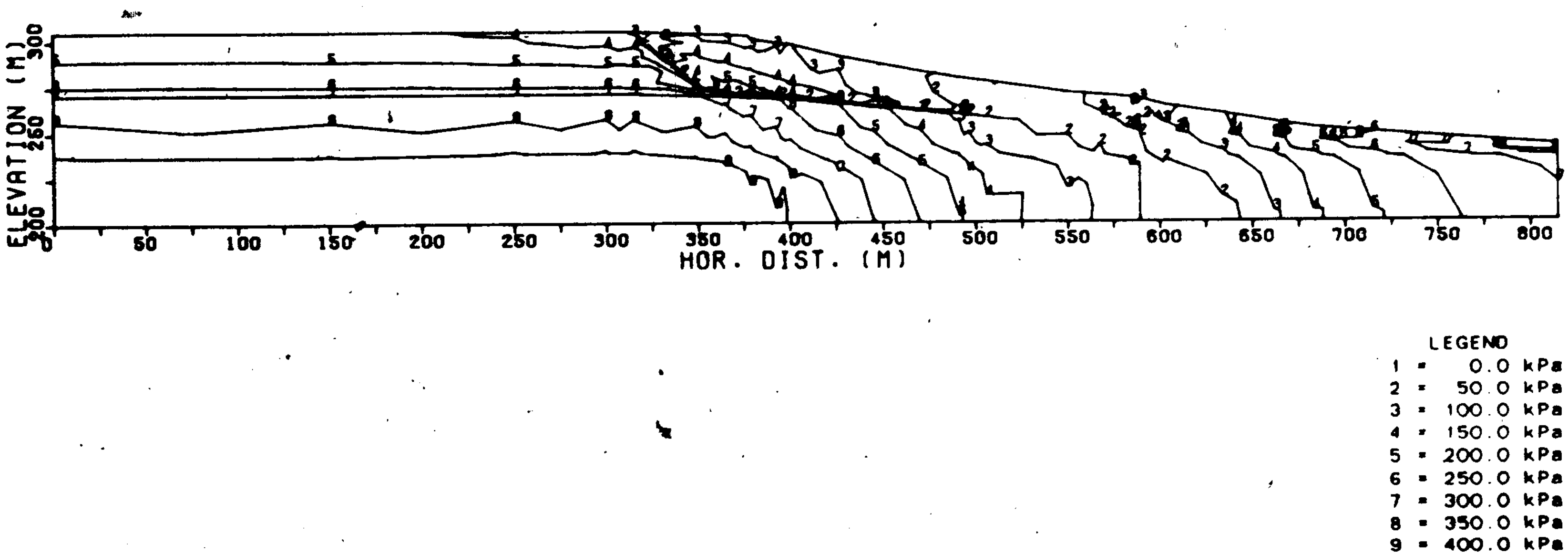



Figure 4.7 Maximum Shear Stress Contours Of The Edgerton Slide (After The Development Of The Weakening Zone With Young's Modulus Equal To 1.5 MPa.)

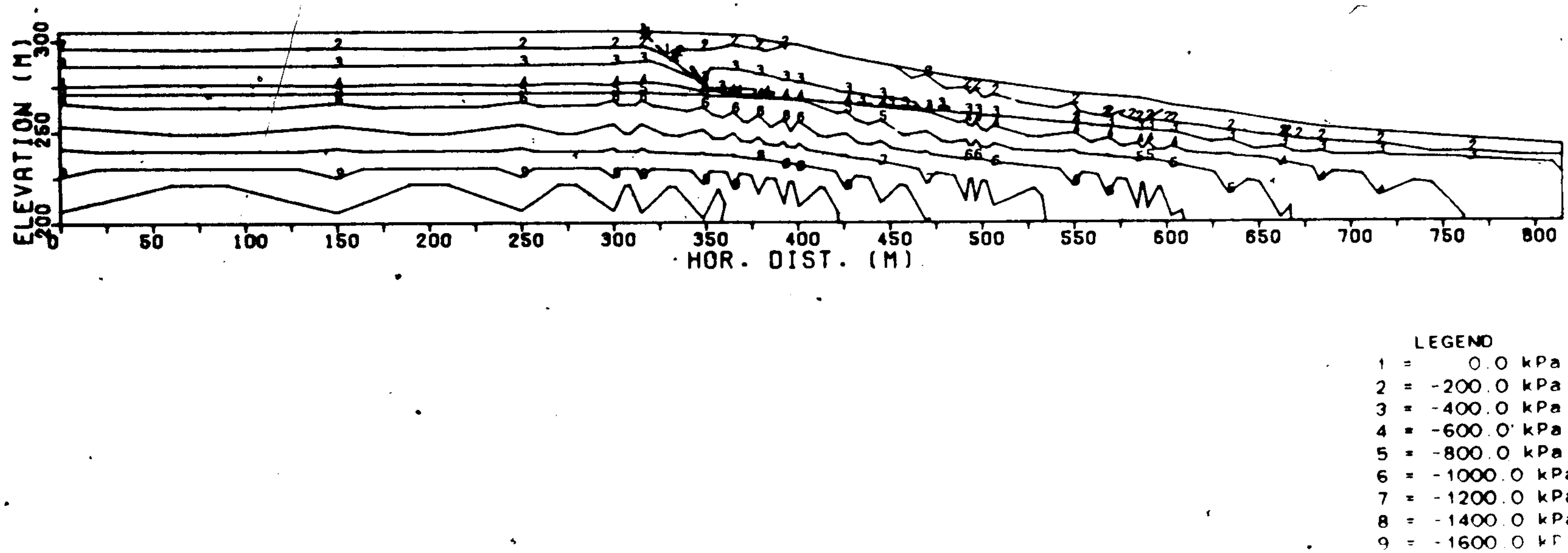



Figure 4.8 Vertical Stress Contours Of The Edgerton Slide (After The Development Of The Weakening Zone With Young's Modulus Equal To 1.5 MPa.)

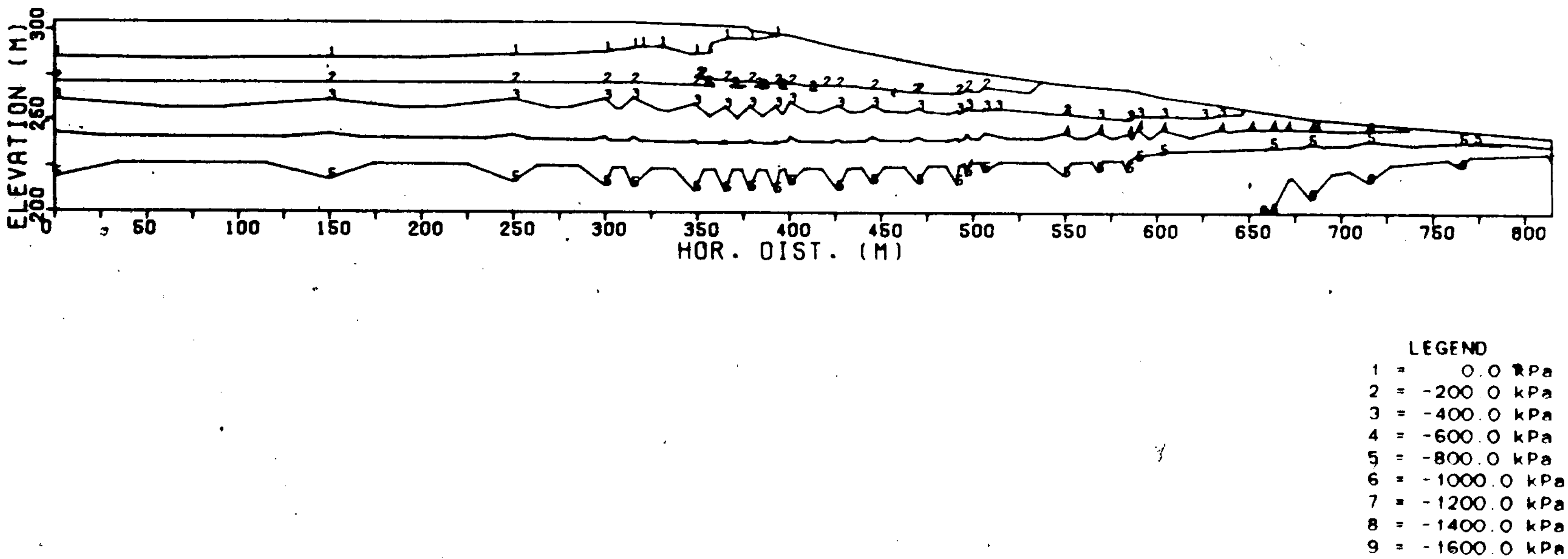



Figure 4.9 Horizontal Stress Contours Of The Edgerton Slide (After The Development Of The Weakening Zone With Young's Modulus Equal To 1.5 MPa.)

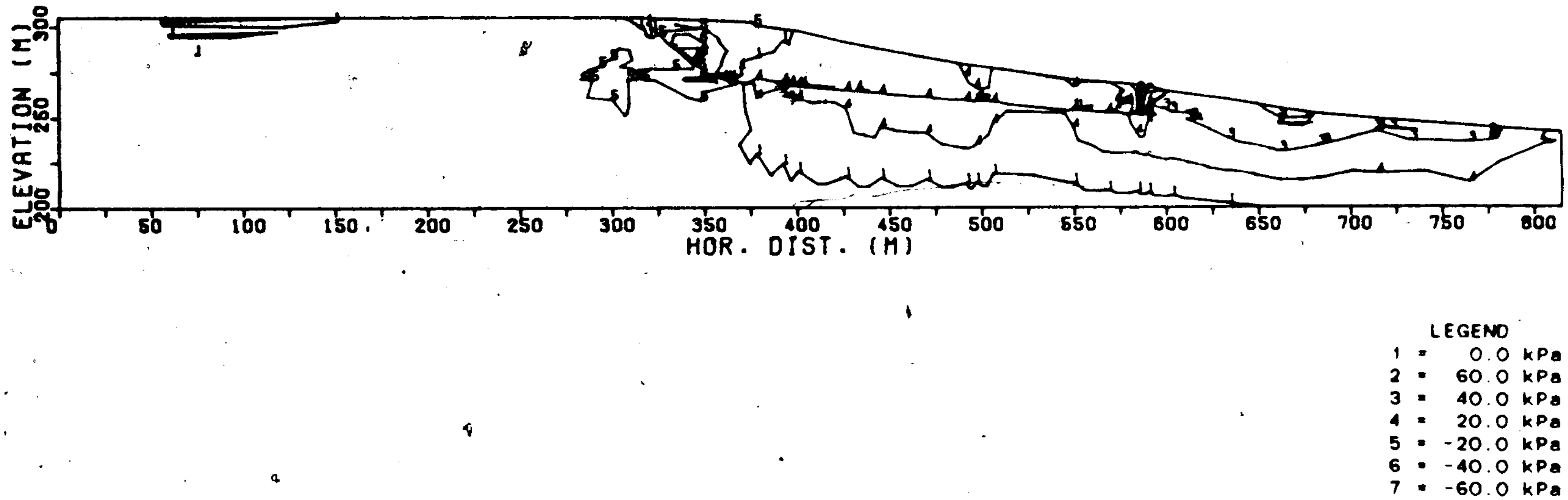



Figure 4.10 Shear Stress ($\tau_{xy}$) Contours Of The Edgerton Slide (After The Development Of The Weakening Zone With Young's Modulus Equal To 1.5 MPa.)

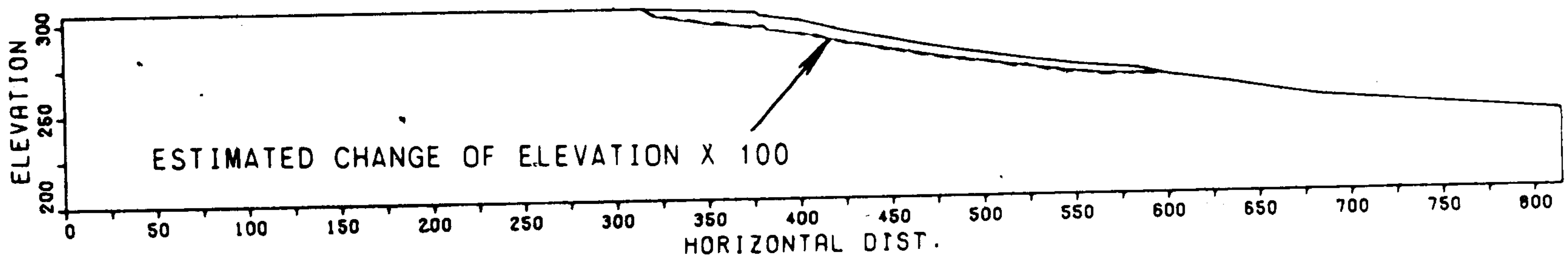



Figure 4.11 a) Surface Displacements Of The Edgerton Slide
(Analytical Approach With Young's Modulus Equal To 1.5 MPa.)

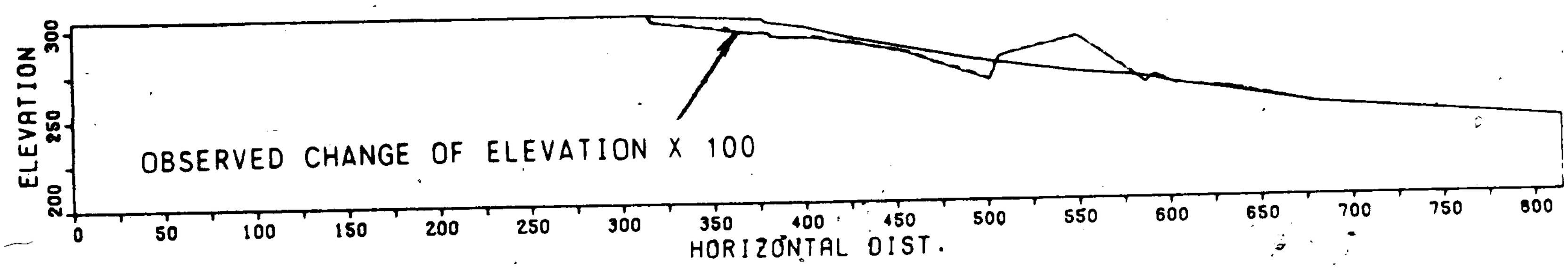



Figure 4.11 b) Surface Displacements Of The Edgerton Slide
(Field Measurement In 1976)

# 5. GENERAL APPLICATIONS OF THE LOAD-TRANSFER TECHNIQUE AND CONCLUSIONS

## 5.1 General Applications Of The Load-Transfer Technique

The load-transfer technique was developed to handle the strain-softening behaviour associated with soils which are vulnerable to progressive failure. The load-transfer program was shown in the working stage and was presented in Chapter 3. This approach was applied to the study of the Edgerton Slide and was presented in Chapter 4.

This program however can be used to solve other problems associated with strain-stiffening materials and materials with time-dependent effects. The concept can be used for stress analysis in a no-tension material as was first used by Zienkiewicz et.al. (1968).

The load-transfer technique is a pesudo-elastic analysis. The basic approach is to handle the excess shear stress due to whatever reason and the change in elastic properties ( eg. Young's Modulus (E), Poisson's Ratio ($\mu$), Shear Modulus (G), and Bulk Modulus (K) ) due to whatever reason.

### 5.1.1 The Strain-Stiffening Approach

This approach is direct opposite to the strain-weakening approach. However, the difference is that the former approach is used for increasing shear modulus of a material which can sustain excessive shear stresses. A schematic diagram is used to show the difference between these approaches and is shown in Figure 5.1. The strain-hardening approach may be applied to a study of the swelling behaviour of soils.

### 5.1.2 The Study Of Time-dependent effects

This approach is a pseudo-time dependent analysis. This can be illustrated by the following example.

If the slope indicator readings of the Edgerton Slide were available for more than one year, then a creep analysis could be done. The shear modulus (G) , used in calculating the displacements which is comparable to the field measurements of year 1, can be used to calculate the strain at year 1. Another reduction of shear modulus, due to whatever reasons, is done in order to compare the analytical results with the field measurements of year 2. Finally, the strain at year 2 is calculated. The process of calculation could be carried on for year 3 and so on. Ultimately, a strain-time curve (creep curve) can be plotted. Therefore, the calculated creep curve can be classified as one of the following stages :

a. primary creep

b. steady-state creep

c. tertiary creep

### 5.1.3 The Study Of No-Tension Materials

This approach can be used to study a rock mass in its natural state because it usually cannot sustain tension due to the presence of cracks and fissures.

However, the load-transfer program has to be modified such that the final stress ($\sigma_1$) (refer to Appendix A) is artificially reduced to zero. This technique actually forces the major principle stress (sign convention is that compression is negative) to zero, however the minor principal stress and the principal plane direction remain unchanged.

## 5.2 Summary Of This Thesis

This research is a documented case history in applying the analytical method to a field problem. The prime objective is to develop a procedure to analyze a natural slope.

The advantage of numerical analysis, as mentioned by Gibson (1974), is that this analysis can help to distinguish among those factors that are of primary significance and those that are of secondary importance.

Conclusions from this research concern four major issues:

a. The correct usage of any existing computer program is emphasized so that meaningful results can be obtained. Sometimes, the users have to run a simple test on the available function of any program in order to determine whether the program achieves this goal or not.

b. The load-transfer approach is developed to model the strain-weakening material, which is vulnerable to progressive failure.

c. The achievement arising from this research is that the variation of the Young's Modulus was used for matching the displacement history of the Slide mass. The analytical results show that the final results were independent of Poisson's Ratio and the initial Young's Modulus.

d. The load-transfer technique can be applied to other problems associated with strain-hardening material, creep material and no-tension material.

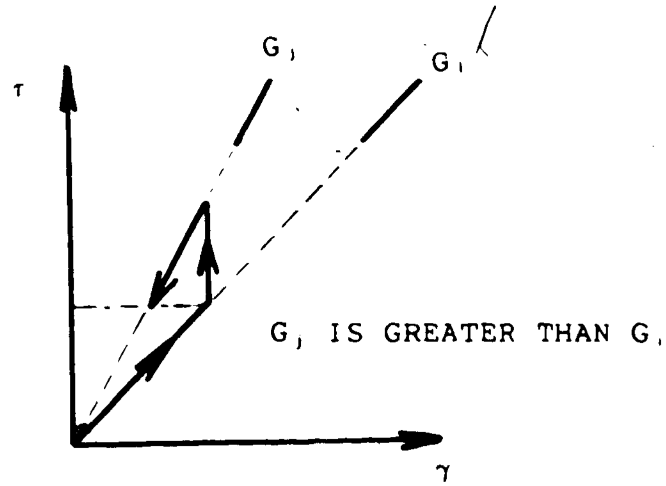
**5.3 Areas For Future Research**

A precise correspondence between predicted and the observed field measurements is rare. However, theoretical solutions aid in visualising possible failure mechanisms in different situations and in developing sound judgement concerning stability problems. The following areas require future research :

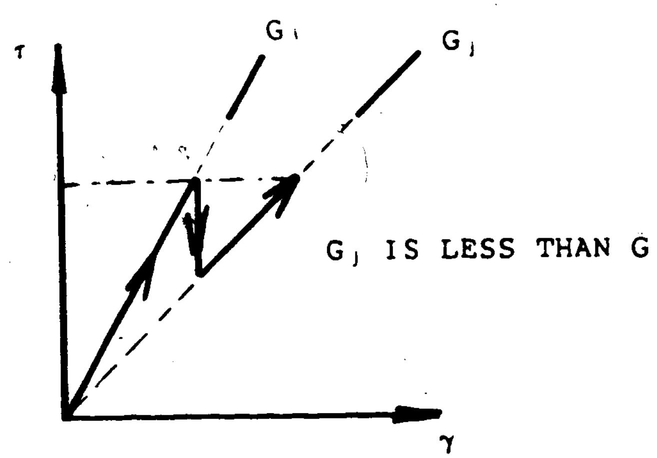
a. A more realistic stress-strain relationship should be developed in considering problems associated with the dilatancy, rupture and cracking, and the true strain-softening behaviour.

b. The load-transfer technique could be used to study other similar slides. From the results of the numerous case studies, a general approach can be established for the study of a natural slope.

c. The ADINA program could be modified so that the strain-weakening stress-strain behaviour will be one of the available material models. This can increase the efficiency of solving a problem and reduce the amount of work in preparing the input data file.




STRAIN-STIFFENING APPROACH




STRAIN-WEAKENING APPROACH

Figure 5.1 Schematic Diagram Of The Strain-Stiffening Approach And The Strain-Weakening Approach

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# Appendix A

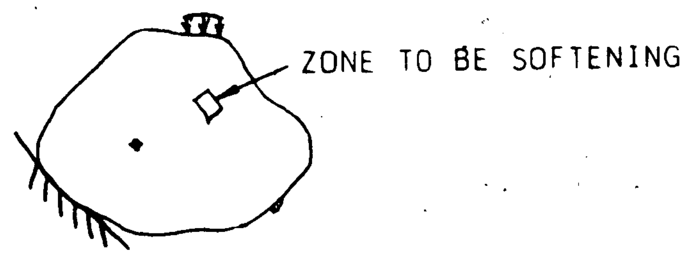
## The Finite Element Formulation Of The Incremental Loading Due To The Reduction Of Shear Modulus

# Appendix A

## Finite Element Formulation Of The Incremental Loading Due To The Reduction Of Shear Modulus

AT EQUILIBRIUM

At time equals $T_i$,




EQUILIBRIUM STATE

the incremental finite element equilibrium equation can be expressed in the following form by using Virtual Displacement Principle :

$$\int_v [B_i]^T \{\sigma_i\} dV = \{R_i\}$$

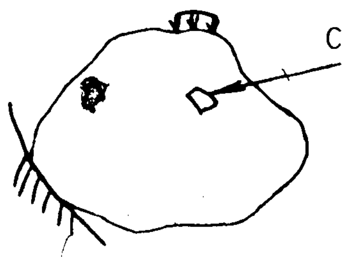
where

$[B_i]$ – strain displacement matrix at time $T_i$

$\{\sigma_i\}$ – internal stresses at time $T_i$

$\{R_i\}$ – external load at time $T_i$

At time equals $T_j$, where $T_j = T_i + \delta T$ ,




due to the change in elastic parameters at time $T_j$, there will be a corresponding changes in stress.

The strain ($\epsilon$) at time $T_i$ is the same as that at the beginning of time $T_j$.

Therefore, the amount of stress change is given by :

$$\left\{ \delta\sigma \right\} = \left\{ \sigma_j \right\} - \left\{ \sigma_i \right\}$$

$$= [C_j]\{\epsilon\} - [C_i]\{\epsilon\}$$

$$= \Big[ [C_j] - [C_i] \Big] \Big\{ \epsilon \Big\} \quad .$$

The equivalent nodal force due to this change in stresses is given by :

$$\int_v \Big[ B \Big]^T \Big\{ \delta\sigma \Big\} dV = \Big\{ \delta R \Big\}_i \quad .$$

This represent the portion of the stress that cannot be taken by the weaken zone. Hence, this stress must be redistributed to the other parts of the structure.

The new equilibrium equation becomes :

$$\int_v [B]^T \{\sigma_j\} \, dV = \int_v [B]^T \{\sigma_i + \delta\sigma\} \, dV$$

$$= \int_v [B]^T \{\sigma_i\} \, dV + \int_v [B]^T \{\delta\sigma\} \, dV$$

$$= \{R_i\} + \{\delta R\} \quad .$$

Therefore,

$$\int_v [B]^T \{\sigma_j\} \, dV = \{R_i\} + \{\delta R\} \quad ,$$

this equilibrium equation must be satisfied at the new stress state.

Appendix B

User's Manual and Source Code

Of

The Load-Transfer Program

# Appendix B

## USER'S MANUAL OF LOAD-TRANSFER PROGRAM

### Introduction

The load-transfer program consists of two parts; these are the main program and the library program. The following MTS commands can be used to run the load-transfer program :

```
$RUN CLOAD+CLIB 2=IN2 4=IN4 5=IN5 6=-OUT6 7=-OUT7
```

where

CLOAD = compiled version of the main program

CLIB = compiled version of the library program

IN2 = input file containing the modifying material parameters

IN4 = file containing the ADINA output; primarily displacements

IN5 = file containing ADINA input

-OUT6 = temporary output file echoing input information

-OUT7 = temporary output file of the new element stresses and the redistribution loads

## Detail Descriptions

### Data For IN2

Data consists of :

1. FORMAT(I5) – the number of elements to have changes made to their material properties.
2. FORMAT(I5,2G10.0) – a list of each element number, new Young's Modulus and new Poisson's Ratio.

### Data For IN4

Data consists of:

1. FORMAT(I7,31X,2G18.6) – the last step of the displacement output from ADINA.

### Data For IN5

The data format as described in the ADINA manual. However, the subroutine INDATA may have to be changed for each material model used other than linear-elastic.

### Output Of -OUT6

This file is primarily used for self-checking input information.

### Output Of -OUT7

Output consists of two parts :

1. the new element stresses are calculated and printed.
2. the redistribution loads are calculated and printed. These will be re-used as the input information for ADINA.

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C                          MAIN PROGRAM                              C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION BO(6,16),FL(16),X(8),Y(8),X8(3),W8(3),AN(9),ANS(9),
     1 ANR(9),ANT(9)
      DIMENSION ICO(10,1000),XX(1000),YY(1000),STRESS(4,4,1000),
     1 STRAIN(4,4,1000),UU(1000),VV(1000),ELMP(2,1000),
     2 EMPNEW(2,1000),P(1000),IX(2000),NEWEL(1000)
      DATA NICO,IN1,IN2,IN3,NGP,NS,NELMP/10,5,4,2,4,4,2/
      DATA IOUT1,IOUT2/6,7/
      DATA W8/0.55555555555556D0,0.88888888888889D0,
     1 0.55555555555556D0/
      DATA X8/-0.77459666924148D0,0.D0,0.77459666924148D0/
      NIP=2
      IF(NIP.GT.2) GO TO 30
      X8(1)=-0.5773502691896257D0
      X8(2)=-X8(1)
      W8(1)=1.D0
      W8(2)=1.D0
   30 CALL INDATA(ICO,XX,YY,NEL,NNOD,NICO,IN1,ELMP,NELMP,
     1 IOUT1,IX)
      CALL INSTR(ICO,XX,YY,NEL,NNOD,STRESS,STRAIN,UU,VV,NS,NGP,
     1 NICO,IN2,IOUT1)
      CALL MODIF(NEWEL,NNEWEL,EMPNEW,IN3,IOUT1,NELMP)
      CALL DLOAD(ICO,BO,FL,X,Y,X8,W8,AN,ANS,ANR,ANT,IOUT1,NEL,
     1 NNOD,NIP,UU,VV,ELMP,P,EMPNEW,NEWEL,NNEWEL,THICK,NICO,NELMP,
     2 XX,YY)
      CALL RESULT(NNOD,P,IOUT2)
      STOP
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C                READ ADINA MASTER INPUT FILE                        C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE INDATA(ICO,XX,YY,NEL,NNOD,NICO,IN1,ELMP,NELMP,
     1 IOUT1,IX)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION ICO(NICO,1),XX(1),YY(1),ELMP(NELMP,1),IX(1)
      READ(IN1,101) A
      READ(IN1,102) NNOD,NGRP
      CALL ISET(IX,2*NNOD)
      WRITE(IOUT1,201) NNOD,NGRP
      NI=13
      NNODEL=8
      DO 1 I=1,NI
    1 READ(IN1,101) A
      WRITE(IOUT1,202)
      DO 2 I=1,NNOD
      READ(IN1,103) I1,I2,XX(I),YY(I)
      IF(I1.EQ.0) IX(2*I-1)=1
      IF(I2.EQ.0) IX(2*I)=1
    2 WRITE(IOUT1,203) I,XX(I),YY(I),IX(2*I-1),IX(2*I)
      READ(IN1,101) A
      NEL=0
      IEL=0
      WRITE(IOUT1,204)
```

```
      DO 4 I=1,NGRP
      READ(IN1,104) NELGRP
      WRITE(IOUT1,205) I,NELGRP
      NEL=NEL+NELGRP
      READ(IN1,101) A
      READ(IN1,105) (ELMP(J,I),J=1,2)
      WRITE(IOUT1,206) (ELMP(J,I),J=1,2)
      DO 4 J=1,NELGRP
      IEL=IEL+1
      READ(IN1,106) ICO(9,IEL),(ICO(K,IEL),K=1,NNODEL)
C     DO 5 K=5,8
C   5 ICO(K,IEL)=0
      ICO(10,IEL)=I
    4 WRITE(IOUT1,207) IEL,(ICO(K,IEL),K=1,10)
      RETURN
  101 FORMAT(A4)
  102 FORMAT(I5,10X,I5)
  103 FORMAT(10X,2I5,25X,2G10.0)
  104 FORMAT(4X,I4)
  105 FORMAT(2G10.0)
  106 FORMAT(59X,I1,/,20I5)
  201 FORMAT(//,5X,'NO. OF NODES = ',I5,5X,'NO. OF ELEMENT GROUP =
     1 , I5)
  202 FORMAT(//,5X,'NODE',5X,'X-COORD.',5X,'Y-COORD',5X,'IX')
  203 FORMAT(5X,I5,5X,2F10.3,2I5)
  204 FORMAT(//,5X,'ELEMENT NUMBERING')
  205 FORMAT(//,5X,'NO. OF ELEMENTS IN GROUP ',I5,5X,'IS = ',I5)
  206 FORMAT(//,5X,'ELASTIC MOD. = ',E15.5,5X,'POISSON RATIO = ',
     1 E15.5)
  207 FORMAT(5X,I5,5X,10I5)
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C                SETTING MATRIX INTO ZERO                             C
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE ISET(M,N)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION M(1)
      DO 1 I=1,N
    1 M(I)=0
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C              READ ADINA OUTPUT FILE --- ONLY DISPLACEMENT           C
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE INSTR(ICO,XX,YY,NEL,NNOD,STRESS,STRAIN,U,V,NS,NGP,
     1 NICO,IN2,IOUT1)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION ICO(NICO,1),XX(1),YY(1),STRESS(NS,NGP,1),
     1 STRAIN(NS,NGP,1),U(1),V(1)
      WRITE(IOUT1,201)
      DO 1 J=1,NNOD
      READ(IN2,101) J,U(J),V(J)
    1 WRITE(IOUT1,202) J,U(J),V(J)
C     DO 2 I=1,NEL
C     IF(ICO(9,I).EQ.1) GO TO 2
C     READ(IN2,102) A
```

```
C       WRITE(IOUT1,203) I
C       DO 3 IGP=1,NGP
C       READ(IN2,103) (STRESS(K,IGP,I),K=1,NS)
C     3 WRITE(IOUT1,204) (STRESS(K,IGP,I),K=1,NS)
C     2 CONTINUE
        RETURN
  101 FORMAT(I7,31X,2G18.6)
  102 FORMAT(A4)
  103 FORMAT(18X,4G15.4)
  201 FORMAT(//,5X,'NODE',5X,'U-DISPL.',5X,'V-DISPL',//)
  202 FORMAT(5X,I5,5X,2E15.5)
  203 FORMAT(5X,'STRESSES OF ELEMENT ',I5)
  204 FORMAT(5X,4E15.5)
        END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                  C
C     READ ELEMENTS ASSOCIATED WITH CHANGING YOUNG'S MODULUS       C
C                                                                  C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
        SUBROUTINE MODIF(NEWEL,NNEWEL,EMPNEW,IN3,IOUT1,NELMP)
        IMPLICIT REAL*4(A-H,O-Z)
        DIMENSION NEWEL(1),EMPNEW(NELMP,1)
        READ(IN3,101) NNEWEL
        WRITE(IOUT1,201) NNEWEL
        DO 1 I=1,NNEWEL
        READ(IN3,102) NEWEL(I),(EMPNEW(J,I),J=1,NELMP)
      1 WRITE(IOUT1,202)  NEWEL(I),(EMPNEW(J,I),J=1,NELMP)
        RETURN
  101 FORMAT(I5)
  102 FORMAT(I5,2G10.0)
  201 FORMAT(//,5X,'NO. OF ELEMENT WITH NEW STIFFNESS = ',I5)
  202 FORMAT(/,5X,'ELEMENT =',I5,5X,'NEW ELASTIC MOD. =',E15.5,
     1 5X,'NEW POISSON RATIO =',E15.5)
        END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                  C
C           CALCULATION OF DELTA SIGMA                             C
C                                                                  C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
        SUBROUTINE DLOAD(ICO,BO,FL,X,Y,X8,W8,AN,ANS,ANR,ANT,IOUT1,
     1 NEL,NNOD,NIP,UU,VV,ELMP,P,EMPNEW,NEWEL,NNEWEL,
     2 THICK,NICO,NELMP,XX,YY)
        IMPLICIT REAL*4(A-H,O-Z)
        DIMENSION ICO(NICO,1),BO(6,1),FL(1),X(1),Y(1),X8(1),W8(1),
     1 AN(1),ANS(1),ANT(1),ANR(1),U(16),EMP(2),EMPN(2),XX(1),YY(1),
     2 UU(1),VV(1),ELMP(NELMP,1),P(1),NEWEL(1),EMPNEW(NELMP,1)
        CALL PSET(P,2*NNOD)
        THICK=1.D0
        DO 1 IN=1,NNEWEL
        IEL=NEWEL(IN)
        CALL PSET(U,16)
        CALL PSET(X,8)
        CALL PSET(Y,8)
        DO 2 I1=1,8
        IICO=ICO(I1,IEL)
        IF(IICO.EQ.0) GO TO 2
        X(I1)=XX(IICO)
        Y(I1)=YY(IICO)
        U(2*I1-1)=UU(IICO)
        U(2*I1)=VV(IICO)
```

```
    2 CONTINUE
      DO 3 I1=1,2
      IICO=ICO(10,IEL)
      EMP(I1)=ELMP(I1,IICO)
    3 EMPN(I1)=EMPNEW(I1,IN)
C     WRITE(6,1201) (X(II),II=1,8)
 1201 FORMAT(/,5X,'X = ',8F10.3)
C     WRITE(6,1202) (Y(II),II=1,8)
 1202 FORMAT(/,5X,'Y = ',8F10.3)
C     WRITE(6,1203) (U(II),II=1,16)
 1203 FORMAT(/,5X,'U = ',2(8E12.5,/,9X))
C     WRITE(6,1204) (EMP(II),II=1,2)
 1204 FORMAT(/,5X,'EMP = ',8F10.3)
C     WRITE(6,1205) (EMPN(II),II=1,2)
 1205 FORMAT(/,5X,'EMPN = ',8F10.3)
      CALL LOAD(IEL,BO,FL,X,Y,ICO,NEL,X8,W8,NIP,AN,ANS,ANT,
     1 IOUT1,THICK,EMP,EMPN,NICO,ANR,U)
      CALL SETUP(P,FL,ICO,NICO,IEL)
    1 CONTINUE
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                  C
C            WRITING THE REDISTRIBUTION LOADS                      C
C                                                                  C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE RESULT(NNOD,P,IOUT2)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION P(1)
      DO 1 INOD=1,NNOD
      I1=2*INOD-1
      I2=2*INOD
      K1=1
      K2=2
      K3=3
      A1=0.00
      WRITE(IOUT2,201) INOD,K2,K1,P(I1),A1
    1 WRITE(IOUT2,201) INOD,K3,K1,P(I2),A1
      RETURN
  201 FORMAT(3I5,2E10.3)
      END
End of file
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CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C                  LIBRARY FOR LOAD-TRANSFER PROGRAM                 C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C             CALCULATION OF EXTERNAL LOAD FOR EACH ELEMENT          C
C                        ( R MATRIX )                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE LOAD(IEL,BL,FL,X,Y,ICO,NEL,
     1 X8,W8,NIP,AN,ANS,ANT,IOUT1,THICK,EMP,EMPN,NICO,ANR,U)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION BL(6,1),FL(1),X(1),Y(1),Z(1),EMP(1),EMPN(1),
     1 ICO(NICO,1),X8(1),W8(1),AN(1),ANS(1),ANT(1),
     2 AJ(3,3),AI(3,3),ANR(1),CE(3,3),STR(3),FO(3),F1(3),
     3 SIG(6),F(16),U(1)
      CALL PRESET(BL,6,16)
      CALL PSET(SIG,6)
      CALL PSET(FL,16)
      IGP=1
      DO 26 I=1,NIP
      R=X8(I)
      DO 26 J=1,NIP
      S=X8(J)
      CALL SHAPE(ANR,ANS,R,S,IEL,ICO,NEL,8,AN,NICO,2)
      CALL BOMAT(AN,ANR,ANS,ANT,X,Y,BL,AJ,AI,DET,IEL,IOUT1,
     1 8,2,Z,2,RAD)
      CALL MULT3(BL,6,U,STR,3,16,1)
C     WRITE(IOUT1,1202) (STR(II),II=1,3),DET
 1202 FORMAT(/,5X,'STR = ',3E12.5,5X,'DET =',E15.5)
      CALL ELASTC(CE,EMP,IEL)
      CALL MULT3(CE,3,STR,FO,3,3,1)
C     WRITE(IOUT1,1203) (FO(II),II=1,3)
 1203 FORMAT(/,5X,'FO = ',3E12.5)
      CALL ELASTC(CE,EMPN,IEL)
      CALL MULT3(CE,3,STR,F1,3,3,1)
C     WRITE(6,1201) ((BL(I,J),I=1,6),J=1,16),(FO(I),I=1,3)
 1201 FORMAT(/,5X,'BL IN LOAD = ',16(/,5X,6E12.5),/,5X,'SIG IN LOAD
     1 ,/,5X,3E12.5)
      WRITE(IOUT1,1204) (F1(II),II=1,3)
 1204 FORMAT(/,5X,'F1 = ',3E12.5)
      DO 3 K=1,3
    3 SIG(K)=FO(K)-F1(K)
      CALL MULT3(BL,6,SIG,F,3,16,2)
      DO 4 K=1,16
    4 FL(K)=FL(K)+DET*W8(I)*W8(J)*F(K)*THICK
   26 IGP=IGP+1
C     WRITE(IOUT1,201) (FL(I),I=1,16)
  201 FORMAT(/,5X,'EQUIVALENT LOAD VECTOR IS ',2(/,5X,8E15.6))
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C     CALCULATION OF SHAPE FUNCTIONS                                 C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

```
      SUBROUTINE SHAPE(ANR,ANS,R,S,IEL,ICO,NEL,NODE,AN,NICO,IPS)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION ANR(1),ANS(1),ICO(NICO,1),AN(1)
      ANR(9)=-2.D0*R*(1.D0-S*S)
      ANS(9)=-2.D0*S*(1.D0-R*R)
      AN(9)=(1.D0-R*R)*(1.D0-S*S)
      IF(NODE.EQ.9) GO TO 4
      ANR(9)=0.D0
      ANS(9)=0.D0
      AN(9)=0.D0
    4 ANR(7)=-R*(1.D0+S)-ANR(9)/2.D0
      ANR(8)=-(1.D0-S*S)/2.D0-ANR(9)/2.D0
      ANR(5)=-R*(1.D0-S)-ANR(9)/2.D0
      ANR(6)=(1.D0-S*S)/2.D0-ANR(9)/2.D0
      ANS(7)=(1.D0-R*R)/2.D0-ANS(9)/2.D0
      ANS(8)=-S*(1.D0-R)-ANS(9)/2.D0
      ANS(5)=-(1.D0-R*R)/2.D0-ANS(9)/2.D0
      ANS(6)=-S*(1.D0+R)-ANS(9)/2.D0
      AN(7)=(1.D0-R*R)*(1.D0+S)/2.D0-AN(9)/2.D0
      AN(8)=(1.D0-S*S)*(1.D0-R)/2.D0-AN(9)/2.D0
      AN(5)=(1.D0-R*R)*(1.D0-S)/2.D0-AN(9)/2.D0
      AN(6)=(1.D0-S*S)*(1.D0+R)/2.D0-AN(9)/2.D0
      DO 2 I=5,8
      IF(ICO(I,IEL)) 1,1,2
    1 ANR(I)=0.D0
      ANS(I)=0.D0
      AN(I)=0.D0
    2 CONTINUE
      ANR(3)=(1.D0+S)/4.D0-(ANR(6)+ANR(7))/2.D0-ANR(9)/4.D0
      ANR(4)=-(1.D0+S)/4.D0-(ANR(7)+ANR(8))/2.D0-ANR(9)/4.D0
      ANR(1)=-(1.D0-S)/4.D0-(ANR(5)+ANR(8))/2.D0-ANR(9)/4.D0
      ANR(2)=(1.D0-S)/4.D0-(ANR(5)+ANR(6))/2.D0-ANR(9)/4.D0
      ANS(3)=(1.D0+R)/4.D0-(ANS(6)+ANS(7))/2.D0-ANS(9)/4.D0
      ANS(4)=(1.D0-R)/4.D0-(ANS(7)+ANS(8))/2.D0-ANS(9)/4.D0
      ANS(1)=-(1.D0-R)/4.D0-(ANS(5)+ANS(8))/2.D0-ANS(9)/4.D0
      ANS(2)=-(1.D0+R)/4.D0-(ANS(5)+ANS(6))/2.D0-ANS(9)/4.D0
      AN(3)=(1.D0+R)*(1.D0+S)/4.D0-(AN(6)+AN(7))/2.D0-AN(9)/4.D0
      AN(4)=(1.D0-R)*(1.D0+S)/4.D0-(AN(7)+AN(8))/2.D0-AN(9)/4.D0
      AN(1)=(1.D0-R)*(1.D0-S)/4.D0-(AN(5)+AN(8))/2.D0-AN(9)/4.D0
      AN(2)=(1.D0+R)*(1.D0-S)/4.D0-(AN(5)+AN(6))/2.D0-AN(9)/4.D0
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C        CALCULATION OF B MATRIX --- STRAIN-DISPLACEMENT              C
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE BOMAT(AN,ANR,ANS,ANT,X,Y,BL,AJ,AI,DET,IEL,IOUT1,
     1 NODE,IPS,Z,NVAR,R)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION ANS(1),ANT(1),X(1),Y(1),BL(6,1),AJ(3,1),AI(3,1),
     1 Z(1),ANR(1),AN(1)
      CALL PRESET(AJ,3,3)
      CALL PRESET(AI,3,3)
      R=0.D0
      DO 1 K=1,NODE
      IF(IPS.EQ.3) R=R+AN(K)*X(K)
      AJ(1,1)=AJ(1,1)+ANR(K)*X(K)
      AJ(1,2)=AJ(1,2)+ANR(K)*Y(K)
      AJ(2,1)=AJ(2,1)+ANS(K)*X(K)
```

```
      AJ(2,2)=AJ(2,2)+ANS(K)*Y(K)
    1 CONTINUE
    3 DET=AJ(1,1)*AJ(2,2)-AJ(1,2)*AJ(2,1)
      IF(DET.LE.0.D0) GO TO 999
      AI(1,1)=AJ(2,2)/DET
      AI(1,2)=-AJ(1,2)/DET
      AI(2,1)=-AJ(2,1)/DET
      AI(2,2)=AJ(1,1)/DET
    4 DO 2 K=1,NODE
      K1=NVAR*(K-1)+1
      DX=AI(1,1)*ANR(K)+AI(1,2)*ANS(K)+AI(1,3)*ANT(K)
      DY=AI(2,1)*ANR(K)+AI(2,2)*ANS(K)+AI(2,3)*ANT(K)
      BL(1,K1)=DX
      BL(2,K1+1)=DY
      BL(3,K1)=DY
      BL(3,K1+1)=DX
      IF(IPS-3) 2,5,6
    5 BL(4,K1)=AN(K)/R
      GO TO 2
    6 DZ=AI(3,1)*ANR(K)+AI(3,2)*ANS(K)+AI(3,3)*ANT(K)
      BL(4,K1+2)=DZ
      BL(5,K1+1)=DZ
      BL(5,K1+2)=DY
      BL(6,K1)=DZ
      BL(6,K1+2)=DX
    2 CONTINUE
      RETURN
  999 WRITE(IOUT1,201) IEL
  201 FORMAT(5X,'*****ERROR***** DET. OF JACOBIAN ',
     1 'MATRIX IS ZERO ELEMENT NUMBER = ',I5)
      STOP
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                C
C              CHECKING THE NODAL FORCE EQUILIBRIUM              C
C                                                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE SETUP(B,FL,ICO,NICO,IEL)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION B(1),FL(1),ICO(NICO,1)
      DO 1 I=1,8
      IICO=ICO(I,IEL)
      IF(IICO.EQ.0) GO TO 1
      B(2*IICO-1)=B(2*IICO-1)+FL(2*I-1)
      B(2*IICO)=B(2*IICO)+FL(2*I)
    1 CONTINUE
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                C
C             MATRIX MULTIPLICATION                              C
C                                                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE MULT3(X,NXX,Y,Z,NX,NY,ICODE)
C   MULTIPLY A 2-D MATRIX X(NX,NY) , BY A VECTOR Y(1)
C   IF ICODE=1  X*Y
C   IF ICODE=2  Y(TRANSPOSE)*X OR X(TRANSPOSE)*Y
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION X(NXX,1),Y(1),Z(1)
      IF(ICODE.NE.1) GO TO 3
```

```
    9 DO 1 IX=1,NX
      XX=0.D0
      DO 2 IY=1,NY
    2 XX=XX+X(IX,IY)*Y(IY)
    1 Z(IX)=XX
      RETURN
    3 DO 4 IX=1,NY
      XX=0.D0
      DO 5 IY=1,NX
    5 XX=XX+X(IY,IX)*Y(IY)
    4 Z(IX)=XX
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                   C
C         BOTH PRESET AND PSET ARE USED TO SET ZERO MATRIX          C
C                                                                   C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE PRESET(A,M,N)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION A(M,1)
      DO 1 I=1,M
      DO 2 J=1,N
    2 A(I,J)=0.D0
    1 CONTINUE
      RETURN
      END
      SUBROUTINE PSET(A,N)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION A(1)
      DO 2 J=1,N
    2 A(J)=0.D0
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                   C
C       CONSTITUTIVE MATRIX                                         C
C                                                                   C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE ELASTC(CE,EMP,IEL)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION CE(3,1),EMP(1)
C     WRITE(6,1201) IEL,IM,NELMP,(ELMP(I,IM),I=1,NELMP)
 1201 FORMAT(//,5X,'IEL,IM,NELMP,ELMP',/,5X,3I5,/,5X,6E12.5)
      CALL PRESET(CE,3,3)
      E=EMP(1)
      V=EMP(2)
      C=E/((1.D0+V)*(1.D0-2.D0*V))
      C1=(1.D0-V)*C
      C2=V*C
      C3=(1.D0-2.D0*V)*C/2.D0
      DO 4 I=1,2
      DO 3 J=1,2
    3 CE(I,J)=C2
      CE(I,I)=C1
    4 CONTINUE
      CE(3,3)=C3
      RETURN
      END
```

End of file