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Massive Neutrinos, R-Parity Violation and LSP Decay

By

Rouzbeh Allahverdi ©

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "Massive Neutrinos, R-Parity Violation and LSP Decay " submitted by Rouzbeh Allahverdi in partial fulfilment of the requirements for the degree of Master of Science in Theoretical Physics

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to

my Parents.

Abstract

In the minimal supersymmetric standard model (MSSM) there is a natural candidate for dark matter, the lightest supersymmetric particle (LSP). It is stable because model has a discrete symmetry, R-parity, which discriminates between standard model particles and their supersymmetric partners. However, if R-parity is violated the LSP is not stable anymore and decays to standard model particles. R-parity violation can happen through violation of lepton or baryon number by an odd number of units. We consider a minimal extension of MSSM which includes (heavy) right-handed (s)neutrinos in three generations to generate neutrino masses via the see-saw mechanism, and violates R-parity as well as lepton number in the heavy field sector. Nevertheless, R-parity violation affects the light field sector, most notably through LSP decay. We compute the rates for the resulting LSP decay, depending on the particular superpotential couplings responsible for the violation of R-parity. We then compare to cosmological constraints on the decay of massive particles and lepton number violating interactions. Interestingly enough, we find that for some superpotential couplings LSP decays too fast, almost independent of how large the mass of the right-handed (s)neutrino is.

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Chapter 1

Introduction

Everything we can calculate within the standard model of elementary particles seems to agree with experiment. This, however wonderful it might be for the model, is boring for the theoretical physicist. The standard model is not a real unified theory; it's gauge group $SU(3) \times SU(2) \times U(1)$ has three different gauge couplings. It has too many parameters, all put in by hand, and leaves unanswered some fundamental questions. Perhaps most importantly, it is not natural. If the standard model is not the ultimate theory, it is a low-energy effective theory of a more fundamental theory. There are two ways to get information about such a fundamental theory from low-energy data. One is to try to build a more fundamental theory that includes the standard model as the leading term of its low-energy effective theory and then compare the effects of other terms (that are beyond the standard model) with experiment. Another way is the opposite: add new interactions to the standard model and find limits to their scale from comparison with experiment. This way, hopefully, we could understand some features of the underlying high-energy theory. We note that as long as we don't know the ultimate theory, every theory we study could be useful at some level.

An interesting fact is that cosmology can play a unique role in probing the high-energy theory from low-energy observations. Just after the birth of universe high energy theory was governing all processes, not like the low-energy world which only gets tiny contributions from high-energy theory. There might have been new phenomena in the early univerese, ruled by the high-energy theory, that are completely beyond the predictions of the standard model. Then, when the universe cooled down gradually remnants of high-energy processes were transferred to the low-energy world. Topological defects, dark matter and the baryon asymmetry of the universe could be examples of such remnants.

The neutrino allows us to probe new physics beyond the standard model. Neutrinos are massless in the standard model, so, neutrino mass means physics beyond the standard model. It is possible to have massive neutrinos coming from a grand unified theory. On the other hand we can give mass to neutrinos by adding new interactions to the standard model. The cosmological signature of a massive neutrino could be the solution to the dark matter problem and/or baryon asymmetry of the universe.

We will consider an extension of the standard model that includes massive neutrinos via the see-saw mechanism, in the context of supersymmetry. In the chapter 2 we review the standard model and some of its minimal extensions very briefly. We will have a quick look at the cosmological importance of neutrinos as well as some constraints in chapter 3. In chapter 4 (and the Appendix A) we introduce some necessary tools for supersymmetric model building. We also see the simplest supersymmetric version of the standard model and some possibilities that supersymmetry brings to cosmology. In the final chapter (which is research material of the thesis) we consider a minimal extension of the supersymmetric standard model that includes massive neutrinos, and compare an important prediction of that extension with cosmological constraints.

Chapter 2

Neutrinos and The Standard Model

2.1 The Electroweak Standard Model

The standard model of elementary particles is in agreement with precision tests, so far. In this model the fundamental constituents of matter are spin-1/2 fermions divided into leptons and quarks. There are spin-1 bosons which mediate fundamental interactions between the fermions and there are spin-0 bosons that generate masses in theory and haven't been observed yet.

Leptons and quarks come in three generations. Leptons have only electroweak interactions represented by the gauge group $SU(2) \times U_Y(1)$. Left-handed leptons are $SU(2)$ doublets (one lepton and it associated neutrino) while right-handed leptons are singlets of $SU(2)$. Quarks, in addition, participate in strong interactions with $SU_c(3)$ gauge group. Like leptons, left-handed quarks are $SU(2)$ doublets and right-handed quarks are $SU(2)$ singlets. Both leptons and quarks carry weak hypercharge Y that obeys the relation $Q = e(T_3 + \frac{Y}{2})$, where Q is electric charge and T_3 is the weak isospin of particle. For a complete list of $SU(2) \times U_Y(1)$ assignments of leptons and quarks see, e.g. [1]. Each quark is also in a fundamental representation of $SU_c(3)$, something which makes anomaly cancellation possible and, therefore, the theory renormalizable. Focusing on the electroweak part of the theory, the Lagrangian consists of a kinetic part and a gauge field part. The kinetic part is:

$$L^{kin} = \sum_i (\bar{l_L}^i \gamma^\mu D_\mu l_L{}^i + \bar{e_R}^i \gamma^\mu D_\mu e_R{}^i + \bar{q_L}^i \gamma^\mu D_\mu q_L{}^i$$
$$+ \bar{u_R}^i \gamma^\mu D_\mu u_R{}^i + \bar{d_R}^i \gamma^\mu D_\mu d_R{}^i) \tag{2.1}$$

where $D_\mu = \partial_\mu - ig_2 W_{\mu,a} \frac{\sigma_a}{2} - ig_1 \frac{Y}{2} B_\mu$ for doublets and for singlets $D_\mu = \partial_\mu - ig_1 \frac{Y}{2} B_\mu$. $\vec{W}_\mu$ are the triplet of $SU(2)$ gauge bosons, B_μ is the gauge boson for hypercharge, and g_1, g_2 are $U_Y(1)$ and $SU(2)$ couplings respectively. The gauge field part is

$$L^G = -\frac{1}{4} W_{\mu\nu}{}^a W^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{2.2}$$

where

$$W_{\mu\nu}{}^a = \partial_\mu W_\nu{}^a - \partial_\nu W_\mu{}^a + g_2 \epsilon^{abc} W_\mu{}^b W_\nu{}^c$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.3}$$

So far there is no mass term for fermions and the gauge bosons. We know that the electron is massive and the gauge bosons that mediate electroweak interaction must be massive because it is a short range interaction. A mass term for fermions couples a left-handed spinor to a right-handed one. Such a term is not gauge invariant in the standard model because the $SU(2)$ symmetry is not vectorial. Also we can't construct a gauge invariant mass term for gauge fields in four dimensions (though it is possible in three dimensions). Gauge invariance is necessary for renormalizability of gauge theories. In order not to destroy it, explicit mass terms are forbidden in the standard model. Here the Higgs boson comes to rescue. We can couple an $SU(2)$ doublet of spin-0 bosons with hypercharge $Y = 1$ to fermions (Yukawa couplings) to have a gauge invariant term. A non-zero vev (vacuum expectation value) for the Higgs results in a mass term for the fermions. We exploit the spontaneous symmetry breaking mechanism to get a non-zero vev for Higgs. It can be done by introducing the following Higgs lagrangian:

$$L^H = D_\mu H^\dagger D^\mu H + \frac{1}{2}M^2 H^\dagger H - \frac{1}{4}\lambda(H^\dagger H)^2 \tag{2.4}$$

with $M^2 > 0$ it is easy to see that the vacuum is degenerate. Rotating the Higgs doublet $\begin{bmatrix} h^+ \\ h^0 \end{bmatrix}$ to a suitable state we choose

$$< h^0 >= \frac{M}{\sqrt{\lambda}} \quad ; \quad < h^+ >= 0 \tag{2.5}$$

The Charge operator $Q = e(T_3 + \frac{Y}{2})$ gives zero when acting on the vacuum while the other three generators of $SU(2) \times U_Y(1)$ do not. This leads to the existence of three Goldstone bosons. These Goldstone bosons are eaten by the gauge fields $W^\pm$ and Z

$$W_\mu{}^\pm = \frac{W_\mu{}^1 \pm i W_\mu{}^2}{\sqrt{2}} \quad ; \quad Z_\mu = -B_\mu \sin\theta_w + W_{\mu,3}\cos\theta_w \qquad \tan\theta_w = \frac{g_1}{g_2} \tag{2.6}$$

giving mass to them and breaking the $SU(2) \times U_Y(1)$ symmetry group to the $U_Q(1)$ group where its gauge field, the photon:

$$A_\mu = B_\mu \cos\theta_w + W_{\mu,3}\sin\theta_w \tag{2.7}$$

remains massless. Afterwards, there is one real Higgs field with mass M. The most general gauge invariant form for Yukawa couplings is:

$$L^{Yuk} = h_{ij}{}^e \bar{l_L}{}^i H e_R{}^j + h_{ij}{}^d \bar{q_L}{}^i H d_R{}^j + h_{ij}{}^u \bar{q_L}{}^i \hat{H} u_R{}^j \tag{2.8}$$

where $\hat{H} = i\tau_2 H^*$ is the charge conjugate Higgs. The mass matrices $h_{ij}\frac{M}{\sqrt{\lambda}}$ can be ·:agonalized by unitary transformations acting on quark and lepton representations.

2.2 Neutrino Mass Problem

A look at the Yukawa couplings in the standard model shows that neutrinos are massless. To see why this happens and how we could go to extended versions of the standard model where neutrinos are massive, we consider the construction of mass terms for fermions in more detail. The building blocks for representations of the Lorentz group $SO(3,1)$ are left- and right-handed spin-1/2 representations, called Weyl spinors and shown as $(\frac{1}{2},0)$, $(0,\frac{1}{2})$ respectively, that transform independently under the action of the Lorentz group [2]. All higher spin representations can be constructed from Weyl spinors. When parity is relevant we consider the $(\frac{1}{2},0)+(0,\frac{1}{2})$ representation, called a Dirac spinor. A Weyl spinor has two degrees of freedom: a left (right) handed particle and a right (left) handed antiparticle. A Dirac spinor, however, has four degrees of freedom: left-handed particle and right-handed antiparticle, right-handed particle and left-handed antiparticle.

Having a left- (right-) handed spinor, we can construct a right- (left-) handed spinor out of it through a CP transformation

$$\psi_L \rightarrow (\psi_L)^c = (\psi^c)_R \tag{2.9}$$

A mass term couples a left-handed particle to a right-handed one. For a Dirac fermion we can couple a left-handed particle either to a right-handed particle (Dirac mass term) or to a right-handed antiparticle (Majorana mass term) . For a Weyl fermion, however, the only possibility is coupling of the left (right) handed particle to the right (left) handed antiparticle. As far as Lorentz invariance is concerned both of these terms are good. When other symmetries like gauge or global symmetries exist, a Majorana mass term is not always allowed. For example, in the standard model a Majorana mass term violates the $SU(2) \times U_Y(1)$ symmetry.

Now we come back to the neutrinos. There is no right-handed neutrino in the standard model so a Dirac mass term can't be constructed. An explicit Majorana mass term is also forbidden because of the gauge symmetry considerations. The question is if we can construct a Majorana mass term through Yukawa couplings. In the standard model the answer is no. A triplet or singlet Higgs with $Y = 2$ is needed to derive a Majorana mass term for left-handed neutrinos. This means that in the standard model, with its gauge group and particle content, there is no room for massive neutrinos. It is important to notice that this does not have anything to do with the existence of only left-handed neutrinos. With a Majorana mass term the physical state would be a four-spinor that is invariant under CP transformation, a Majorana fermion:

$$\psi_M = \begin{bmatrix} \psi_L \\ (\psi_L)^c \end{bmatrix} \tag{2.10}$$

In order to have massive neutrino we must go beyond the standard model, in the minimal extensions either by introducing a right-handed neutrino or new Higgs multiplets. One may ask what is the benefit of a massive neutrino. It could possibly solve several problems we are confronting today:

1- The famous solar (and atmospheric) neutrino puzzle. There is a large deficit in the flux of neutrinos coming from the sun, something between 25 and 40 percent of what standard solar model predicts. Perhaps something is incomplete about our knowledge of solar physics or astrophysics. However, massive neutrinos could solve this puzzle through the vacuum oscillation and MSW scenario [3].

2- The light neutrino is a good candidate for hot dark matter. It is seriously speculated that most of the matter in the world is not observable and in the form of which we all have been made.

3- Heavy neutrinos suggest a simple mechanism of baryogenesis. Their decay could create an excess of leptons over antileptons which were partially converted to baryons later [4].

2.3 Neutrino Mass in Minimal Extensions of Standard Model

If the right-handed neutrino exists, it is an $SU(2) \times U_Y(1)$ singlet. It is a singlet of $SU(2)$ like the other right-handed fermions and $Q = e(T_3 + \frac{Y}{2})$ gives $Y = 0$ because it is neutral. So a Majorana mass term (even without Yukawa coupling) does not violate gauge symmetry and is as good as a Dirac mass term. We notice that the right-handed neutrino is added by hand. However, in GUTs (grand unified theories) right-handed neutrinos may arise naturally from group theory considerations. For example, there is a natural candidate for the right-handed neutrino in the $SO(10)$ GUT [1].

Without the right-handed neutrinos new Higgs multiplets are needed to give mass to neutrinos. If we want the lepton number to be unbroken the new Higgs must carry lepton number. Then, through a non-zero vev for the new Higgs the lepton number is spontaneously broken and a Majorana mass term appears. Models built using this mechanism are called Majoron models. There exists a Goldstone boson, the majoron, in these models coming from spontaneous breakdown of lepton number. In the GR model [5] a Higgs triplet that carries two units of lepton number is introduced to generate a Majorana mass term for the left-handed neutrino. This model, however, is already ruled out by LEP experiments. In the BS model [6] one Higgs doublet and one singlet both carrying two units of lepton number are introduced and Majorana mass for left-handed neutrino appears at the one-loop level. This model is also ruled by LEP results. The only majoron model that is not ruled out by laboratory experiments is the CMP model [7]. This model contains a right-handed neutrino and has both Dirac and Majorana mass terms, the latter is accomplished by introducing a Higgs singlet that carries two units of lepton number and has a non-zero vev. In this model the coupling of the majoron to the standard model particles is too weak to give observable consequences. Although this model may have astrophysical implications: a spontaneous lepton number violation happens in it.

For the right-handed neutrino an explicit Majorana mass term is allowed because such a term is a gauge singlet. However, lepton number will be explicitly violated in this case. Lepton number violation has important consequences like neutrinoless double beta decay and magnetic dipole moment for neutrinos [8]. Because the Majorana mass term for the right-handed neutrino doesn't violate any symmetry it could be as large as possible, perhaps up to the Planck scale. This offers an interesting mechanism for generating neutrino masses, the see-saw mechanism [9]. Consider both Majorana and Dirac mass terms:

$$m{\nu_L}^\dagger \nu_R + M({\nu_R}^c)^\dagger \nu_R + h.c. = \begin{bmatrix} {\nu_L}^\dagger & ({\nu_R}^c)^\dagger \end{bmatrix} \begin{bmatrix} 0 & m \\ m & M \end{bmatrix} \begin{bmatrix} ({\nu_L}^c)^\dagger \\ \nu_L \end{bmatrix} + h.c. \qquad (2.11)$$

for $M \gg m$ eigenvalues of the mass matrix are

$$m_1 \simeq -\frac{m^2}{M} \quad ; \quad m_2 \simeq M \qquad (2.12)$$

and mass eigenstates are

$$(\nu_1)_L \simeq \nu_L + \frac{m}{M}{\nu_R}^c \quad ; \quad (\nu_2)_R \simeq -\frac{m}{M}{\nu_L}^c + \nu_R \qquad (2.13)$$

the physical light neutrino $(\nu_1)_L$ mainly consists of ν_L . For $\frac{m}{M} \ll 1$ lepton number violation and other effects caused by the Majorana mass term (like additional CP-violating phases in the lepton sector [10]) are very slight at low energies. We see that the see-saw mechanism can explain why light neutrinos have such a small mass, if any at all.

2.4 Laboratory Bounds on Neutrino Mass and Number of Generations

Bounds on the mass of neutrinos of different generations come from different experiments. For ν_τ the decay channel

$$\tau \rightarrow \nu_\tau + 5\pi \tag{2.14}$$

is studied. The current upper limit to the mass of the tau neutrino is just above 30 MeV [11] . For ν_μ the best limit comes from studying the two-body π-decay

$$\pi \rightarrow \mu + \nu_\mu \tag{2.15}$$

with an upper bound around 250 KeV. For ν_e the best limit comes from the measurement of endpoint of the spectrum of tritium beta decay

$$^3H \rightarrow ^3He + e + \bar{\nu}_e \tag{2.16}$$

but the location of the endpoint is very sensitive to systematic errors. At present the best upper bound is 4.5 eV.

It is notable that the see-saw mechanism with suitable Majorana and Dirac masses can approximately generate these upper bounds [12].

There is also a bound on number of generations from LEP experiments. Measurements on the Z width show that number of light neutrinos is at most three. By light we mean any neutrino with a mass less than $\frac{m_Z}{2}$ whether or not it is stable. This is specially important when compared with cosmological bounds on number of light neutrinos.

Chapter 3

Neutrinos and Cosmology

The universe, though old and large at present, is believed to have been very small at its early stages. This suggests that particle physics has played a role in the dynamics of the universe since the very beginning. Actually there is a strong interplay between particle physics and cosmology: particle theory could explain some of the astrophysical phenomena, and cosmology strongly restricts particle models like standard model extensions. The early universe, because it is in thermal equilibrium, is a laboratory for testing new physics at energies unreachable on the earth.

3.1 Big Bang Cosmology

The standard big bang cosmology is a successful model that explains some of the observational aspects of the universe very beautifully. The big bang model is based on three theoretical pillars: the Einstein equations, the cosmological principle and the perfect fluid description of matter. These together give the following metric:

$$ds^2 = a^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)] \tag{3.1}$$

for the universe when using spherical coordinates. The constant k is +1, 0 or -1 for a closed, flat or open universe respectively. $a(t)$ is the scale factor of the universe and its evolution depends on the phase of the perfect fluid (matter, radiation, ...) . The universe is matter dominated today but at early stages most of its energy was in the form of radiation. There are three observational pillars for the big bang model: the redshift of galaxies, the cosmic microwave background radiation and the light element abundances, all in very good agreement with theory [13]. There are, however, some shortcomings in big bang cosmology that are not in contradiction with theory but need fine tuning and very special initial conditions. The most famous of them are the flatness-entropy and horizon problems, and depending on the particle model we use there could be monopole and gravitino problems as well [14]. One way to escape these problems is inflation [14]. In inflationary models there is a period in which the universe is expanding very fast and this can resolve the above-mentioned problems. Though there are other suggestions to solve these, inflation looks the only candidate with the ability to solve all [15], even predicting interesting results far beyond the big bang cosmology [16]. Unfortunately there is no realistic model for inflation from the particle perspective so far.

Predictions of BBN (Big Bang Nucleosynthesis) are perhaps the strongest observational support for the big bang model. The primordial abundance of ^4He depends on three factors: neutron lifetime, number of light neutrinos (light enough to be relativistic at T=1 MeV, when weak interactions freeze) , and the ratio of the density of baryons to the density of photons [17]. The ^4He abundance gives the bound $N_\nu < 4$, there is room for only one more light neutrino. A recent result based on extrapolation from observations of helium-4, nitrogen and oxygen from some four dozen, low metalicity extra-galactic HII regions gives the bound $N_\nu < 2$ [18]. This limit is below the one from LEP experiments but there is no contradiction. This limit applies for those neutrinos with a mass less than 1 MeV, but the LEP experiment bounds number of neutrinos with a mass less than $\frac{m_Z}{2}$. Also, we must stress that it is possible to have a large number of massless species, much larger than three, if they decouple very early. For example, up to 30 Goldstone bosons species are allowed if they decouple at temperatures higher than 250 GeV [19].

3.2 Baryogenesis

All the matter we observe today is made out of baryons, almost all of it nucleons. However, in quantum field theory creation and annihilation operators of particles and antiparticles come in pairs. The question is what causes such an asymmetry between baryons and antibaryons. It could be an initial condition but then we need a fine tuning which is not desirable. Also with a period of inflation, such a baryon asymmetry will be inflated away. Another possibility is that after freeze-out of nucleon-antinucleon annihilation they were separated somehow, so there are parts of the universe which fully consist of antibaryons like our part consists of baryons. This suggestion has serious problems, it does not give the right amount of baryons we observe today and apparently violates causality [20].

It was Sakharov [21] who first pointed out the necessary ingredients to produce a baryon asymmetry from a symmetric initial condition: baryon number violating interactions, C and CP violation, and out of thermal equilibrium conditions.

It was figured out later that GUT's could provide these conditions. In GUT's there are heavy gauge and Higgs bosons that carry baryon number [1], CP violation is provided through phases in fermion representations and the out of equilibrium condition is satisfied when the temperature of the early universe drops below the gauge (Higgs) boson mass. Lower bounds on the mass of heavy gauge bosons can be found from laboratory bounds on baryon number violating interactions like proton decay. However, in the simplest GUT's (e.g. $SU(5)$) which preserve $B-L$ it is easy to show that if all interactions are in thermal equilibrium, any baryon asymmetry is eventually washed out [20]. There are other suggestions to produce a BAU (Baryon Asymmetry of the Universe). It can be shown [22] that with some conserved number and asymmetry among generations a net baryon number is generated after fermions get mass, even starting initially with $B-L=0$. It was also suggested [23] that an asymmetry would be preserved if some of it was carried by right-handed fermions, as long as some of the Yukawa couplings were out of equilibrium.

The out of equilibrium gauge boson decay scenario does not seem viable in the presence of inflation. The reheat temperature in inflationary models is generically of the order $10^{10} - 10^{11}$ GeV. In $SU(5)$,for example, the mass of the heavy gauge boson is of the order 10^{15} GeV. So there would not be enough gauge bosons after reheating to produce the observed BAU. The situation for the the heavy Higgs is better; their mass is typically four order of magnitude smaller than the mass of heavy gauge boson [20].

There are other mechanisms to generate a baryon asymmetry without GUT's. One uses the chiral current anomaly of the standard model as its basic ingredient. The $SU(2)$ vacuum is not unique, it consists of topologically distinct sectors with their assigned Chern indices [2]. Going from one sector to another changes both the baryon and lepton number, but preserves their difference $B - L$. At zero temperature the only way for the transition to occur is through tunneling, via the so-called instanton solution. At finite temperature sphalerons make the transition occur and above the electroweak breaking scale the transition rate is very fast. Sphalerons, CP-violating phases in the CKM matrix in the standard model and the electroweak phase transition (if first order) provide all ingredients pointed out by Sakharov and could result in the baryon asymmetry we observe today [24]. However, it seems that in the context of the standard model this mechanism cannot lead to the observed BAU [25].

There is another alternative which uses sphaleron effects. We can get a lepton number asymmetry when lepton number violating interactions that violate C and CP become out of equilibrium. In a simple model [4] three heavy right-handed neutrinos with both Dirac and Majorana mass terms are added to the standard model. Lepton number is violated by the Majorana mass term and CP violating phases can appear in the Yukawa couplings. Out of equilibrium decay of heavy neutrinos generates a net lepton number in the light sector that is partially converted to baryon number through sphalerons. This is what we pointed out earlier in chapter 1: the existence of heavy right-handed neutrinos in addition to the see-saw mechanism gives a nice way to produce the BAU without any need to go to GUT's. It is shown that a wide range of heavy neutrino masses (1TeV-10^{19} GeV, depending on the details of model) could give the observed BAU [26].

Baryon and lepton number violating operators, although good for our purpose, could be dangerous too [27]. They can erase any baryon (lepton) asymmetry if they are in thermal equilibrium. For example, in the model we discussed above there is the effective operator $LLHH$ (L the lepton doublet, H the Higgs field) induced by exchange of a heavy neutrino N. This is a dimension 5 operator and is accompanied by a coefficient $\frac{1}{M}$ where M is some mass [1]. Assume T is much higher than the scale of the light sector, then the energy of particles in the light sector is effectively T. So the rate of lepton number violation through $LLHH$ operator is proportional to $\frac{T^3}{M^2}$. If this is bigger than the expansion rate of the universe $H \sim \frac{T^2}{m_{Pl}}$ any lepton asymmetry is washed out very rapidly. Generally, for a dimension n operator out of the thermal equilibrium condition is:

$$\frac{T^{2n-7}}{M^{2n-8}} < \frac{T^2}{m_{Pl}} \Rightarrow T^{2n-9} < \frac{M^{2n-8}}{m_{Pl}} \tag{3.2}$$

We notice that for $n > 4$ the situation is somehow controllable. These operators are out of equilibrium below

$$T_0 = (\frac{M^{2n-8}}{m_{Pl}})^{\frac{1}{2n-9}} \tag{3.3}$$

[1] For couplings of order one, M is approximately mass of the heavy neutrino.

but for $n \leq 4$ (renormalizable and super-renormalizable terms) the operator is in equilibrium below:

$$T_0 = (M^{8-2n} m_{Pl})^{\frac{1}{9-2n}} \tag{3.4}$$

so these terms are total disaster. These operators would erase any lepton (baryon) asymmetry at low temperatures. There is one way out of this undesirable situation. Sphalerons are in equilibrium in the (approximate) range:

$$100\text{GeV} < T < 10^{12}\text{GeV} \tag{3.5}$$

so any lepton asymmetry is partially converted to baryon asymmetry (and vice versa) in this range. Lepton number violating operators that are in thermal equilibrium at this range can wash out a baryon asymmetry as well as a lepton asymmetry. But those dimension four (or less) lepton number violating operators that are in equilibrium below 100 GeV can erase only the lepton asymmetry, the baryon asymmetry is safe. So it is possible to take care of even (super-) renormalizable operators and keep the BAU intact.

3.3 Dark Matter

Based upon BBN, the concordance of light element primordial abundances requires [17]

$$4(3) \times 10^{-10} < \eta < 7(10) \times \times 10^{-10} \tag{3.6}$$

where $\eta = \frac{n_B}{n_\gamma}$. This means

$$0.015(0.011) \leq \Omega_B h^2 \leq 0.026(0.037) \tag{3.7}$$

where $H = 100h$ and $\Omega_B = \frac{8\pi G}{3}\frac{\rho_B}{H^2}$. For a generous range in the Hubble constant

$$0.015(0.011) \leq \Omega_B \leq 0.16(0.21) \tag{3.8}$$

The Einstein equations give (neglecting the cosmological constant)

$$H_0{}^2 = \frac{3}{8}\pi G\rho_0 - \frac{k}{a^2} \tag{3.9}$$

we define $\rho_c = \frac{8}{3\pi G}H^2$ as the critical density and $\Omega_0 = \frac{\rho_0}{\rho_c}$. We notice that for a flat ($k = 0$) universe $\Omega = 1$. Measurements based upon cluster dynamics, based upon the ratio of total mass to baryonic mass in clusters, and those based upon relating the peculiar motions of galaxies to the observed distribution of matter all strongly favor a value for Ω_0 that is much larger than 0.1 [28]. The inflationary scenario also predicts $\Omega_0 = 1$ at the present epoch,otherwise fine tuning will be needed.

That part of matter we observe in our galaxy (all baryonic) accounts just for $\Omega \simeq 0.01$. Flatness of the rotational curve of the galaxy beyond its disk suggests that $\Omega \simeq 0.1$ (at least) for the galaxy. These all provide evidence that most (almost all) of the matter in the universe could be dark and non-baryonic. There are many candidates for dark matter in particle theories. For the baryonic case see [29]. Three major candidates for non-baryonic dark matter are:

1- Neutrinos: (a)- A light, stable neutrino (hot [2] dark matter) provides up to the closure density ρ_c of the universe if $m_\nu < 91.5$eV [17]. It is referred to as the Cowsik-McCelleland bound. (b)- A heavy, stable neutrino could account for cold dark

[2]Dark matter can be hot or cold, depending on whether it is relativistic or non-relativistic, respectively, at the time of decoupling.

matter if $m_\nu > 2$GeV for a Dirac neutrino and $m_\nu > 5$GeV for a Majorana neutrino [17]. It is often called the Lee-Weinberg bound. (c)- An unstable neutrino generates entropy during its decay. This can change BBN predictions or result in diffuse photon background. To avoid these problems some regions of the mass-lifetime plane are allowed [17].

2- Axions (cold dark matter): this is a pseudo-Goldstone boson that appears in the PQ (Peccei-Quinn) solution to the strong CP problem. An axion with the mass 10^{-5}eV could overclose universe. We don't get in to the axion subject here. For a good review see, for example [30].

3- LSP (Lightest Supersymmetric Particle): a cold dark matter candidate in supersymmetric models. We will consider it in detail in the next chapter.

There are many astrophysical aspects of dark matter, such as what model for dark matter (hot, cold or mixed) could provide enough power at galaxy scales and whether the seed for structure formation is quantum fluctuations generated during inflation or topological defects generated subsequently [13]. It is noticeable that we used the BBN bound on η to derive Ω_B that is significantly less than 1. ^2D and ^3He abundances decrease sharply with an increase in η but ^4He abundance increases smoothly (logaritmically) with η [17]. This means any increase in Ω_B (so in η) is in direct violation of the observational facts about ^2D and ^3He . However we should not forget that these results are in the context of standard model. It has recently been suggested [31] that a τ neutrino of mass 20 MeV to 30 MeV with a lifetime of order a few hundred seconds and whose decay products include electron neutrinos (here we go beyond the standard model) could keep ^2D and ^3He abundances unchanged while η changes drastically. However, COBE provides indirect evidence that dark matter is non-baryonic. In a baryon dominated universe, fluctuations can grow only after recombination and can't be non-linear even now [32].

Chapter 4

Supersymmetry

Although the Standard model is in very good agreement with experiment, it has curiousities from the viewpoint of a theoretical physicist. One problem is the large number of parameters: 19 to give mass to fermions and gauge bosons, mixing angles and CP violation in the electroweak and, also, the strong sector. There is also no explanation for the number of generations and charge quantization. GUT's could help to solve some of these problems [1] but a serious problem, the hierarchy problem, remains. In the standard model the radiative correction to the Higgs mass squared is quadratically divergent. This means that no matter how small the Higgs mass is at the tree level, it grows uncontrollably through loop orders even to the GUT scale. A Higgs mass larger than 1 Tev, however, makes the model ill-defined perturbatively [1]. In GUT's we have a similar situation. To avoid a rapid decay rate for the proton those Higgs fields that carry baryon number must be much heavier than the usual Higgs doublet. Even achieving this at tree-level through fine tuning, radiative corrections will destroy it. In general many orders of perturbation theory have to be computed for consistent fine tuning. The hierarchy problem arises in every theory that has fundamental scalars and two scales differing by a large number of orders of magnitude (like GUT and electroweak symmetry breaking scale). This is related to another problem, the naturalness problem. Naturalness states that if there is a parameter in the theory whose absence restores a symmetry, then perturbative corrections to that parameter are no larger than the physical value. As an example consider the electron mass in the standard model. Chiral symmetry is unbroken when the electron mass is zero. This translates to the fact that the radiative corrections to the electron mass are only logarithmic, and not too large when compared with the mass itself.

For the Higgs particle the situation is totally different. The standard model with a massless Higgs has the same symmetries as with a massive one. The Higgs mass is not natural in this sense. There are a couple of ways to get out of this trouble. One is the technicolor model, that assumes Higgs is a fermion-antifermion condensate below 1 Tev rather than a fundamental scalar. Technicolor has its own problems and it is not our aim to go into this subject here. Another suggestion is supersymmetry, a symmetry which unites fermions and bosons. In supersymmetry there is a fermionic degree of freedom for each bosonic degree of freedom (and vice-versa) both with the same mass. It is easy to see why the Higgs mass is natural in supersymmetric models. With a chiral symmetry (not vectorial) fermion masses are natural. Supersymmetry ensures massless bosons when fermions are massless. In other words, fermions protect bosons from radiative generation of mass through supersymmetry. In the language of Feynman diagrams this is the miraculous cancellation of quadratic divergences in supersymmetry. Roughly speaking, for every bosonic loop there is a fermionic loop with opposite sign that cancels it. We will come back to these statements in more detail and precision.

4.1 Supersymmetry Algebra

Of all extensions of Lie algebras, only supersymmetry algebras generate symmetries of the S-matrix consistent with relativistic quantum field theories [33]. The proof of this statement is based on the Coleman-Mandula theorem, the most powerful in a series of no-go theorems about the possible symmetries of the S-matrix. Using this theorem, with a couple of additional assumptions the supersymmetry algebra is found to be [33] :

$$[Q_\alpha, \bar{Q}_{\dot\beta}]_+ = 2\sigma_{\alpha\dot\beta}{}^m P_m \quad ; \quad [P_m, Q_\alpha] = [P_m, \bar{Q}_{\dot\beta}] = 0$$
$$[Q_\alpha, Q_\beta] = [\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}] = 0 \tag{4.1}$$

where Q_α and $\bar{Q}_{\dot\beta}$ [1] are fermionic generators which transform like $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ spinors, respectively, under the Lorentz group action.

This is the algebra of the so called $N = 1$ supersymmetry that has only one pair of fermionic generators Q and $\bar{Q}$. In general we can have any number of fermionic generators. Our focus here is on $N = 1$ supersymmetry.

An appropriate language to formulate supersymmetry is the superfield language which is formulated in superspace, with a set of coordinates $(x_\mu, \theta_\alpha, \bar\theta_{\dot\beta})$ where θ_α and $\bar\theta_{\dot\beta}$ are Grassmann variables that satisfy anti-commutation relations

$$[\theta_\alpha, \theta_\beta]_+ = [\bar\theta_{\dot\alpha}, \bar\theta_{\dot\beta}]_+ = [\theta_\alpha, \bar\theta_{\dot\beta}]_+ = 0 \tag{4.2}$$

the supersymmetry algebra can then be rewritten as:

$$[\theta Q, \bar{Q}\bar\theta] = 2\theta\sigma_\mu\bar\theta P^\mu$$
$$[\theta Q, \theta Q] = [\bar{Q}\bar\theta, \bar{Q}\bar\theta] = 0 \tag{4.3}$$

with spinor indices suppressed for simplicity, and a supersymmetry transformation written as:

$$S(x, \theta, \bar\theta) = exp[i(\theta Q + \bar{Q}\bar\theta - x_\mu P^\mu)] \tag{4.4}$$

It is shown [35] that a representation of the supersymmetry algebra in terms of differential operators

[1] for a review on spinor notation and conventions see, e.g. [34].

$$P_\mu = i\partial_\mu \quad ; \quad Q = \partial_\theta - i\sigma^\mu\bar{\theta}\partial_\mu \quad ; \quad \bar{Q} = -\partial_{\bar{\theta}} + i\theta\sigma^\mu\partial_\mu \tag{4.5}$$

can be derived and covariant derivatives that anticommute by infinitesimal supersymmetry transformations are

$$D = \partial_\theta + i\sigma^\mu\bar{\theta}\partial_\mu; \quad \bar{D} = -\partial_{\bar{\theta}} - i\theta\sigma^\mu\partial_\mu \tag{4.6}$$

There are also left- and right-representations of the supersymmetry algebra [35] where

$$\begin{aligned} Q_L = \partial_\theta \quad &; \quad \bar{Q}_L = -\partial_{\bar{\theta}} + 2i\theta\sigma^\mu\partial_\mu \\ Q_R = \partial_\theta - 2i\sigma^\mu\bar{\theta} \quad &; \quad \bar{Q}_R = -\partial_{\bar{\theta}} \end{aligned} \tag{4.7}$$

with the corresponding covariant derivatives

$$\begin{aligned} D_L = \partial_\theta + 2i\sigma^\mu\bar{\theta}\partial_\mu \quad &; \quad \bar{D}_L = -\partial_{\bar{\theta}} \\ D_R = \partial_\theta \quad &; \quad \bar{D}_R = -\partial_{\bar{\theta}} - 2i\theta\sigma^\mu\partial_\mu \end{aligned} \tag{4.8}$$

For a brief review on the notion of superfields see the Appendix.

4.2 Supersymmetric Lagrangin

As mentioned in the Appendix A the variation of F- and D-terms under supersymmetry transformations is a total derivative. This gives us a clue for supersymmetry model building. Integration of a chiral (vector) superfield over superspace coordinates leaves us with the integral of F- (D-)term over spacetime which is invariant not only under Lorentz transformation but also under supersymmetry transformations.

As a simple example we consider the supersymmetric version of the $\lambda\phi^4$ theory. With a left-handed chiral superfield:

$$\Phi(x) = \phi(x) + \theta\psi(x) + \theta\theta F(x) \tag{4.9}$$

mass terms and interactions are derived from the F-term (superpotential):

$$L_F = m\Phi^2 + \lambda\Phi^3 \tag{4.10}$$

and kinetic terms are added through the D-term:

$$L_D = \Phi^\dagger\Phi \tag{4.11}$$

and lagrangian density of the theory is:

$$L = \int d^2\theta d^2\bar{\theta} L_D + \int d^2\theta L_F + h.c. \tag{4.12}$$

The F field is an auxiliary field and can be eliminated through the equations of motion:

$$F = -2m\phi^* - 3\lambda\phi^{*2} \tag{4.13}$$

In general given a superpotential $W(\Phi_i)$, we can find the scalar potential arising from it:

$$V = \sum_i |\frac{\partial W(\phi_i)}{\partial \phi_i}|^2 \tag{4.14}$$

The Supersymmetric version of a gauge field strength is derived from an F-term. With the vector superfield V (not to be confused with scalar potential):

$$V = -\theta\sigma_\mu\bar{\theta}V^\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D \tag{4.15}$$

in Wess-Zumino gauge [35] (The Weyl fermion λ that accompanies the gauge field V_μ is called a gaugino) it comes out that $\frac{1}{32}(W^\alpha W_\alpha)$ gives us the desired term, where for the abelian case

$$W_\alpha = \bar{D}\bar{D}D_\alpha V \tag{4.16}$$

and for the non-abelian case:

$$W_\alpha = \bar{D}\bar{D}[exp(-gV)D_\alpha exp(gV)] \tag{4.17}$$

g is the gauge coupling, $V_\mu = V_\mu{}^a T^a$, and T^a are generators of the non-abelian gauge group. The D field is also an auxiliary field which can be eliminated through the equations of motion.

Finally, coupling of the gauge field to matter is through the D-term:

$$[\Phi^\dagger exp(2gV)\Phi]_D \tag{4.18}$$

where Φ is in some representation of the gauge group [33]. For a detailed look at the lagrangian of a supersymmetric gauge theory see, e.g. [34].

It is instructive at this point to restate that any superfield contains an equal number of bosonic and fermionic degrees of freedom. For a chiral superfield Φ two bosonic degrees of freedom are provided by ϕ and two fermionic degrees of freedom are provided by ψ (a massless Weyl fermion or a massive Majorana fermion). The F field is an auxiliary field and does not add any degrees of freedom. For a vector superfield two fermionic degrees of freedom are provided by λ and two bosonic degrees of freedom are provided by the massless gauge field (transverse polarizations), while the D field adds no degrees of freedom.

4.3 Supersymmetry Breaking

As long as supersymmetry is exact there is no renormalization of the superpotential, neither finite or infinite, in perturbation theory. So if, for some reason, a fine tuning happens at tree-level, it will be preserved to any order of perturbation theory. This is the famous non-renormalization theorem for supersymmetry. It ensures that if supersymmetry is unbroken at tree-level, it is unbroken at any order of perturbation theory. So there is no analogue to the Coleman-Weinberg mechanism for supersymmetry breaking. In exact supersymmetry all renormalization is in D-terms and divergences are absorbed in wavefunctions and gauge couplings. Furthermore, there are no quadratic divergences, only logarithmic ones. All the statements we made can be proved by using the superfield language straightforwardly [36]. This is the reason behind our hope to make the standard model into a natural theory by making it supersymmetric.

But if supersymmetry has anything to do with particle physics, it must be broken at low energies. We do not observe superpartners of the standard model particles with the same mass. Supersymmetry breaking can be either spontaneous or explicit. In the case of spontaneous breaking the vacuum is not invariant under supersymmetry transformations. We recall the variation of a chiral superfield under such a transformation

$$\delta\phi = \sqrt{2}\alpha\psi \quad ; \quad \delta\psi = \sqrt{2}\alpha F + i\sqrt{2}\sigma^\mu\bar{\alpha}\partial_\mu\phi \quad ; \quad \delta F = -i\sqrt{2}\partial_\mu\psi\sigma^\mu\bar{\alpha} \qquad (4.19)$$

We want Lorentz transformation to be unbroken so the vev of ψ must be zero. Also $\partial_\mu\phi = 0$ at the ground state and consequently $\delta\phi = \delta F = 0$. $\delta\psi$ is non-zero only if $< F > \neq 0$. A massless fermion, the Goldstino, arises in the spontaneous breakdown of supersymmetry through a non-zero vev for the F-term, the F-type breaking. It is also possible that supersymmetry breaking happens through a non-zero vev for a D-term, D-type breaking, and there again exists a Goldstino.

There is no quadratic divergence and only finite renormalization of the superpotential for F-type (and upon satisfying some conditions for D-type) breaking [36]. However there are some phenomenological problems in building realistic particle physics models based on the F-type breaking (O'Rafeartaigh model) and D-type breaking (Fayet-Illiopoulos model) [35]. Today the most common scenario for producing low-energy supersymmetry is the hidden-sector scenario. In this scenario local supersymmetry (supergravity) breaks in a hidden sector (that interacts only through gravity with ordinary matter) at a high scale (e.g. 10^{11} GeV) and supersymmetry breaking manifests itself in explicit supersymmetry breaking terms which are continued through renormalization group equations to low energies. Our choice of explicit breaking terms is limited if we want to keep the advantage of having no quadratic divergences. These are called soft supersymmetry breaking terms and have been categorized [37]. They are displayed below

$$\tilde{M}_1 Re(\phi^2) + \tilde{M}_2 Im(\phi^2) + C(\phi^3 + h.c.) + \tilde{M}_3(\lambda^a\lambda^a + \bar{\lambda}^a\bar{\lambda}^a) \tag{4.20}$$

where ϕ is a scalar field and λ^a is a gaugino (notice that all terms are gauge singlet combinations). These terms lead to logarithmic divergences and all divergences are absorbed in wavefunctions, gauge couplings and soft supersymmetry breaking terms. The non-renormalization theorem (and its slight modification in the presence of soft supersymmetry breaking terms) is proved in the context of perturbation theory, by using Feynman supergraphs. When supersymmetry breaks non-perturbatively (as generally believed) it is no longer valid. However, there is a new non-renormalization theorem in this case, valid under some assumptions [38].

4.4 Supersymmetry and Particle Physics

Supersymmetry, because of its potential to solve the hierarchy problem, can play an important role in particle model building. Besides that, there are other hints that tell us maybe supersymmetry is relevant. Among them we can name the natural emergence of gravity from local supersymmetry and the viability of the unification of coupling constants of the standard model in the context of supersymmetry. For a good review of what supersymmetry can do and what a realistic supersymmetric model must accomplish see [39]. The most important problem remaining to be solved is the mechanism of supersymmetry breaking and its manifestation at low energies. Here we consider the supersymmetric version of the standard model, the MSSM (Minimal supersymmetric Standard Model) which contains no new particles besides superpartners of the standard model particles and a new Higgs superfield which is needed for technical reasons [2] Superpartners and the standard model particles are in the same representations and have the same quantum numbers. The fermionic partner of the standard model Higgs is a doublet with $Y = 1$. For anomaly cancellation another fermionic doublet (and its bosonic partner) with $Y = -1$ is needed. This is the origin of the new Higgs superfield. This new Higgs is also needed to give mass to the charge $\frac{2}{3}$ quarks. In supersymmetry, Yukawa couplings come from a superpotential and the superpotential is an analytic function of superfields. So we can't use the charge conjugate of the standard model Higgs to give mass to the charge $\frac{2}{3}$ quarks, we have to introduce a new Higgs with the quantum numbers of the charge conjugate of the standard model Higgs.

The superpotential of the MSSM can be written as the sum of Yukawa couplings (with generation indices suppressed)

[2]It is easy to see that none of the superpartners of the standard model particles are contained in the standard model [35]

$$F_Y = h_u H_1 Q u^c + h_d H_2 Q d^c + h_e H_2 L e^c \qquad (4.21)$$

where H_1 and H_2 are the two Higgs doublets, Q and L the doublet of left-handed quarks and leptons respectively, u the right-handed up quark, d the right-handed down quark and e the right-handed electron, all superfields. It is interesting to see what global phase symmetries this superpotential can have. There are two types of such symmetries: those which commute with supersymmetry and those which do not. In the first type all component fields of a superfield transform the same way under the action of the $U(1)$ symmetry group. It is easy to show that there are four symmetries of this type for F_Y : lepton number, baryon number, hypercharge and a PQ symmetry [40], where all but the hypercharge are anomalous.

The second type of global phase symmetries are called R-symmetries and under them θ coordinates also transform. This means that the component fields of a superfield transform differently under the action of the $U(1)$ group. F_Y has also an R-symmetry [40]. R-symmetry does not seem to be a symmetry of nature so, if not explicit, must be broken spontaneously. R-symmetry is anomalous and this leads to the emergence of a pseudo-Goldstone boson, the R-axion in the case of spontaneous breaking. This is problematic from experimental point of view. R-symmetry also forbids gauginos from having mass, which is not viable.

If we add the gauge singlet term $\mu H_1 H_2$ to F_Y , both the PQ and the R-symmetry break explicitly. However the linear combination $\frac{1}{3}$PQ+R is still preserved and anomalous, so the same problem arises. By adding soft supersymmetry breaking terms

$$m_1{}^2 \mid H_1 \mid^2 + m_2{}^2 \mid H_2 \mid^2 + m_u h_u H_1 \tilde{Q} \tilde{u}^c + m_d H_2 \tilde{Q} \tilde{d}^c + m_e H_2 \tilde{L} \tilde{e}^c + m_6 \mu H_1 H_2 \quad (4.22)$$

this symmetry breaks to a Z_2 discrete symmetry called R-parity. R-parity assigns the factor $(-1)^{L+3B+F}$ to each particle where L is lepton number, B baryon number and F fermion number. From now on we consider the following superpotential

$$F = F_Y + F_H = h_u H_1 Q u^c + h_d H_2 Q d^c + h_e H_2 L e^c + \mu H_1 H_2 \tag{4.23}$$

along with the above soft supersymmetry breaking terms for the MSSM.

Soft supersymmetry breaking terms also provide the electroweak symmetry breaking. After symmetry breaking of the minimal supergravity model the effective theory at the Planck scale is a globally supersymmetric model broken by soft supersymmetry breaking terms of order 1 Tev. The low-energy consequences of such a theory are revealed by using renormalization group equations with Planck scale boundary condition. One of the Higgs scalar squared-masses which is positive at the Planck scale, is driven negative due to effects of the large top-quark Yukawa coupling; this triggers electroweak symmetry breaking at the required scale. Because the renormalization group equation is logarithmic in nature an exponential hierarchy between the electroweak and the Planck scale can develop [41].

There are several points worth mentioning here. First, because of the mixing between H_1 and H_2 which is caused by soft supersmmetry breaking terms it is not so clear that we can rotate both Higgs vev's to their neutral components such that the charge operator remains unbroken. There is no problem in the standard model because we have only one doublet and as long as there is no mixing between doublets they can be rotated independently. It can be shown that there is no difficulty in the MSSM because of the structure of the potential in supersymmetry [42]. The mixing term is also needed to prevent an unlikely consequence. Without it there is a massless boson which is a linear combination of the H_1 and H_2 neutral components, something problematic experimentally. Finally, R-parity is not the only discrete symmetry of the MSSM. There is just one other example [43], a Z_3 symmetry called $GBPR_3L_3$.

4.5 Supersymmetry and Cosmology

Supersymmetry brings both hopes and problems to cosmology. It could possibly answer some basic questions like the origin of dark matter and the BAU; at the same time it could produce new problems. As an example consider inflation in the context of supersymmetry. As mentioned earlier, the superpotential gets only finite renormalization from radiative corrections when supersymmetry is softly broken. That part of the potential which comes from D-terms:

$$V = \frac{1}{2}D^a D^a \quad ; \quad D^a = g\phi_i{}^* T^a{}_{ij}\phi_j \tag{4.24}$$

has logarithmic divergences but it is zero when ϕ is a gauge singlet. It suggests that for a singlet inflaton (Scalar field that drives inflation) we can construct a flat potential at tree-level without worrying about radiative corrections. On the other hand the reheat temperature in supersymmetric models must be lower than 10^8 GeV. The reason is the presence of gravitinos, superpartners of gravitons. They are so weakly coupled to other particles that if they are produced at temperatures higher than 10^8 GeV, they can't decay rapidly enough and eventually dominate energy density of the universe, destroying predictions of BBN [14].

The MSSM also introduces a natural candidate for dark matter as a result of its discrete symmetry, R-parity. Under R-parity all standard model particles are assigned 1, while all their superpartners are assigned -1. R-parity conservation means that the decay products of a supersymmetric particle must include an odd number of supersymmetric particles. So the lightest supersymmetric particle (LSP) is stable as long as R-parity is conserved. The question is: what is the LSP? It is likely to be electrically neutral and have only weak interactions. If it had either electric charge or strong interactions, it would presumably have lost energy and condensed into the galactic disk along with ordinary matter. In this case, it would be detectable in anomalous heavy isotopes of ordinary matter, in conflict with the experimental limits. It also occurs rarely in models to have a coloured or charged LSP, naively because mass of such a LSP is more sensitive to radiative corrections.

The neutral and colourless candidates in MSSM are gravitino, sneutrino, two neutral Higgs fermions (Higgsinos) and two neutral gauge fermions (gauginos). It is possible that the gravitino is the LSP, but its cosmological relevance requires inflation with just the right amount of reheating. Sneutrinos are cosmologically acceptable, but there are very strong limits on sneutrinos as dark matter based on both direct detection [44] and the absence of very high energy neutrinos from the sun [45]. The remaining candidates are neutral gauginos ($\tilde{W}^3$, $\tilde{B}$) and Higgsinos ($\tilde{H}_1^0$, $\tilde{H}_2^0$), known as neutralinos. The LSP is the linear combination of them that has the lowest mass. There are allowed regions of parameter space of the MSSM for an LSP that are specified by dark matter considerations [46]. In special cases the LSP is essentially a pure photino, bino or some combination of Higgsinos [29].

Supersymmetry also provides an interesting alternative for baryogenesis, the Affleck-Dine mechanism [47]. In the limit of unbroken supersymmetry, the ground state of the theory has many flat directions. Soft supersymmetry breaking terms slightly lift this large vacuum degeneracy. If there are effective terms in the potential that violate baryon number (these terms could come from a GUT) plus CP-violating phases, an initial value of the scalar field along a flat direction [3] can evolve to a net baryon number that is conserved in sfermions during the expansion of early universe [47]. After reheating and decay of the flat direction this baryon number is carried both by fermions and sfermions and eventually leads to the observed BAU.

In the original model, however, the $SU(5)$ GUT was used to provide the baryon number violating operator [47]. In $SU(5)$ $B-L$ is conserved and as we mentioned

[3] This initial value can be provided by quantum fluctuations during inflation.

earlier if all interactions are in equilibrium, then sphalerons wash out any baryon and lepton number and finally $B = L = 0$. It was already pointed out in the original paper that Bose-Einstein condensates of sfermions may form after reheating. If most of the baryon number is carried by the condensates it is safe from erasure by sphalerons. During expansion of the universe both the critical temperature for condensate formation T_c and temperature of universe T drop, the former proportional to $R^{-\frac{3}{2}}$ the latter to R^{-1} [48], where R is scale factor of the universe. If T_c does not drop below T until sphalerons go out of equilibrium, the baryon number is safe from erasure. After condensate evaporation there is no sphaleron effect anymore and part of the baryon number carried by fermions lead to BAU.

There are also models [49] , [50] that use the same features, flat directions in the ground state and scalars that carry lepton and baryon number, to derive a lepton asymmetry which partially converted to baryon asymmetry through sphalerons. These models contain right-handed neutrinos and use lepton number violating operators to get a net lepton number carried by sleptons and after reheating by leptons. In these models $B - L \neq 0$ initially so there is no need for extended duration of the Bose-Einstein condensates.

The elegance of the Affleck-Dine scenario is its naturalness. There are more scalars (superpartners of fermions)in supersymmetric models so the existence of flat directions is natural. Also there are scalars that carry baryon (lepton) number and we can use them for baryon (lepton) number generation.

Chapter 5

R-Parity Violation and LSP Decay

In the standard model it is impossible to introduce gauge invariant, (super-) renormalizable terms that violate baryon (lepton) number. Furthermore, the $U(1)$ baryon (lepton) number global symmetries are preserved in any order of perturbation theory, and they are good accidental symmetries. With supersymmetry, however, the situation is totally different. The superpotential we have already written for the MSSM is not the most general gauge invariant and renormalizable one. It is possible to add baryon (lepton) number violating terms, simply because there are scalars in the theory that carry baryon (lepton) number. This will result in R-parity violation and LSP decay.

5.1 R-Parity Violation in The Light Field Sector

The only renormalizable, baryon and lepton number violating terms we can add to the MSSM are superpotential terms:

$$F_R = m_1 H_1 L + \lambda_1 L L e^c + \lambda_2 L Q d^c + \lambda_3 u^c d^c d^c \tag{5.1}$$

as well as soft supersymmetry breaking baryon and lepton number violating terms. These terms violate R-parity too. The first term is a source for Majorana neutrino mass, the second and third terms radiatively contribute to the neutrino Majorana mass, the third term results in neutron-antineutron oscillation, and the combination of last two terms lead to proton decay. These all put strong laboratory limits on the parameters m_1 , λ_1 , λ_2 and λ_3 [52].

Even stronger constraints can be derived from cosmology if we insist on preserving any baryon asymmetry produced in the early universe. All of the R-parity violating interactions we have considered are dimension four or less. This means that they are in thermal equilibrium below some temperature T_0 (as we mentioned in chapter 2) and can wash out any asymmetry that was produced at temperatures above T_0. In ref. [51] both laboratory and cosmological limits on these (and higher dimension baryon and lepton number violating) interactions have been derived. Interestingly enough, cosmological limits are generally more restrictive and put limit on the scale of these operators several orders of magnitude above that achieved in laboratory.

5.2 R-Parity Violation in The Presence of Right-Handed Neutrino

There are numerous ways in which one can imagine extending the MSSM. In what is often called the minimal-nonminimal superymmetric standard model (MNMSSM) a single additional gauge singlet chiral superfield, N is added. This extension is realized by simply adding to the superpotential the contribution from superpotential terms involving the N field (see below). The primary motivation for the inclusion of the Higgs singlet is the possibility that it offers for the dynamical generation of the Higgs mixing mass μ. If the N field is a field which acquires a vev determined by mass parameters of the order of the electroweak scale, then with a NH_1H_2 coupling of standard strength (say comparable to a gauge coupling) Higgs mixing of the requisite magnitude is induced. On the other hand, if the mass parameters in the N sector are much larger, say of an intermediate scale, or perhaps of the GUT scale, as might naturally be expected to be in see-saw models, then if the N has a nonzero vev one would naturally expect it to also be of this scale . In such a case one still might imagine inducing a weak scale mixing between the Higgs doublets, at the price of fine tuning the NH_1H_2 coupling to be small to give the hierarchical ratio between the electroweak scale and the N mass scale. Though this small (O(M_W) mixing mass is technically feasible its smallness is part and parcel with the hierarchy problem. The cubic term is required in order to avoid an N-axion like field, in the absence of an explicit μ superpotential term mixing the two Higgs supermultiplets. In a detailed examination of this model [52], it was found that many of the standard Higgs mass relations are altered [1] . If the MSSM Higgs mass relations are found to be experimentally not viable, this model becomes the simplest alternative.

From another point of view, the MNMSSM is of interest as it can easily produce a relatively light LSP [54]. In the MSSM, steadily improving accelerator limits, are pushing up the mass of the LSP. With regards to a dark matter candidate, the best choice in the MSSM appears to be the bino whose mass is typically between 40 GeV and $\sim$ 300 GeV for cosmologically interesting parameters [55]. In the non-minimal model it is quite feasible [56] to have a a light LSP (10 - 50 GeV), which is a state which has a strong admixture the fermionic component of N. Though, cosmologically, a very massive LSP is just as good as a light one (light still referring to O(GeV)), from the point of view of experimental detection, the lighter one is better [57]. We consider an extension of the MSSM which includes singlet Higgs superfield representatins (in three generations) to generate neutrino masses via the see-saw mechanism. This is easily accomplished by the addition to the superpotential of

$$F_\nu = MNN + h_\nu H_1 LN \tag{5.2}$$

[1] It has recently been suggested that N field can have a large mass but a vev in Tev range [53]. In this case standard Higgs mass relations remain unchanged.

Neutrino masses of order $h_\nu{}^2 v_1{}^2/M$ will then be generated for the light (left-handed) neutrinos, where $v_1 = \langle H_1 \rangle$. It should be noted that the interactions induced by the superpotential F_ν do not violate R-parity as the Majorana mass term only violate lepton number in units of two. However, if we take into account

$$F_N = \lambda H_1 H_2 N + k N^3 \tag{5.3}$$

the combination of interactions in F_ν with *either* of the interactions in F_n will result in violation of R-parity. The inclusion of the neutrino mass by see-saw mechanism has many benefits as we mentioned in chapter 1.

5.3 LSP Decay Rate

Now we derive the consequences of the R-parity violation of the full superpotential. Here, R-parity is violated only in the heavy N-field sector. Nevertheless, this R-parity violation shows up in the low energy sector, most notably in the destabilization of the LSP. We derive constraints on the N-field mass parameters as a consequence of the constraints on late-decaying LSP's.

As well as the destabilization of the LSP to which we will turn below, there are other possible low-energy signatures of R-parity violation in the high energy N-field sector. If supersymmetry were exact, then even the combined presence of the F_ν and F_N superpotential terms would not induce (super)renormalizable lepton number violating superpotential terms involving only the light superfields of the theory, due to the nonrenormalization theorems for the superpotential. After supersymmetry breaking the nonrenormalization theorems no longer hold exactly, and lepton number (and hence R-parity)violating effective interactions will be induced in an amount governed by the scale of supersymmetry breaking. This will result in low energy R-parity violating interactions involving standard model superfields of the form of both induced effective superpotential terms such as

$$F_{RX} = m_X H_1 L + \lambda_X L L e^c \tag{5.4}$$

as well as soft supersymmetry breaking lepton number violating terms.

As we will show below, the nature of the R-parity violation emerging at low energies depends decisively on whether it arises from the $H_1 H_2 N$ term or the NNN term in the superpotential. Supersymmetry breaking R-parity violating slepton-Higgs soft mixing terms will arise in a way that depends crucially on the form of the R-parity violation in the N-field sector, and in some cases may not be suppressed for large N-field masses. By appropriate change of basis we may diagonalize the Higgs-lepton superpotential mass mixing and parametrize our lepton number violating effects by λ_X. Soft supersymmetry and R-parity violating mass terms which are induced in the low energy theory may also be rotated into effective trilinear interactions by rediagonalization of mass terms in the slepton-Higgs state space. Lepton number violating renormalizable interactions of this type are constrained by laboratory and cosmological limits as we pointed out before.

As we have mentioned above, the combination of the NH_1L superpotential term with either the NH_1H_2 or NNN superpotential interactions breaks R-parity and hence will destabilize the l'ghtest neutralino mass eigenstate. The nature of the low energy R-parity violation induced, and the rate of the resulting LSP decay will depend on which of these latter terms is responsible.

It is well known that in supersymmetric theories with soft supersymmetry breaking, renormalization group running of soft scalar mass terms [58] may induce, in the low energy sector, soft scalar mass mixings which result from interactions with (super)fields from the high energy sector of the theory [59]. This results, for example, in lepton flavour violating contributions to slepton mass mixings induced by GUT [59], or by heavy singlet see-saw [60] superpotential couplings. So the question that presents itself is whether the introduction of R-parity violation in the heavy N-field sector will be fed down by renormalization group mixing to induce soft R-parity violating slepton-Higgs mass mixings in the low energy theory.

One can see from the diagrams in Figure 1 how such slepton-Higgs soft mixings will be induced by the simultaneous presence of the $h_\nu H_1 LN$ and $\lambda H_1 H_2 N$ superpotential terms. For loop momenta below M_N the diagrams will cancel, but for loop momenta greater than M_N the diagram of Figure 1(b) will be suppressed by the N propagator. The surviving contribution from Figure 1(a) will be proportional to the product of the couplings, the soft supersymmetry-breaking scalar mass squared, and the logarithm of the ratio of the cutoff scale Λ (say of order the Planck scale) to the N-field mass (below which the cancellation between the diagrams is reinstated). This logarithm of the cutoff is the term resummed in the renormalization group mixing, when running the renormalization mass scale from the cutoff scale down to the N-field mass scale. The resulting induced soft mass mixing squared is then

$$\Delta M_{LH}^2 \propto \lambda^\dagger h_\nu m_{\tilde{s}}^2 log(\Lambda/M_N) \tag{5.5}$$

and does not decouple for large M_N. So in this case the R-parity violation in the N-field sector is fed down by the renormalization group to soft scalar Higgs-slepton mixings in the low energy sector, with the resulting implications for lepton number and flavour violation, destabilization of the LSP (see below), and lepto/baryogenesis.

If, on the other hand, the R-parity violation enters the N-field sector via the $kNNN$ term, then renormalization group mixing will not induce soft slepton-Higgs mixing. In this case, the one loop diagrams with the R-parity violating trilinear

F-term and soft supersymmetry breaking mass insertion are shown in Figure 2. As the N-field F-component in figure 2(a) and the N-field scalar in 2(b) carry zero momentum, these diagrams exactly cancel, and there is no induced mixing [2]. Thus the mechanism by which R-parity violation in the N-field sector comes to destabilize the LSP depends crucially on the manner in which that R-parity violation arises.

First let us consider LSP decays induced by the NH_1H_2 superpotential term. With this coupling the renormalization group running generates slepton-Higgs mixing of order the susy breaking scale times the couplings, up to factors of small logs. The LSP can now decay from its $\tilde{H}_2^o$ component. This component has an F-term coupling to a lepton-slepton pair via the $h_e H_2 L e^c$ superpotential coupling. With the slepton-Higgs mixing we have the physical decay to a lepton-Higgs final state. If (in the absence of mixing with the N-field) we would write the LSP as an admixture

$$\tilde{\chi}_o = \alpha\tilde{B}^o + \beta\tilde{W}^o + \gamma\tilde{H}_1^o + \delta\tilde{H}_2^o \tag{5.6}$$

then the decay of the LSP via its H_2 component will then occur at a rate

$$\Gamma \simeq 4\delta^2\lambda^2 h_\nu^2 \Gamma_o \tag{5.7}$$

where

$$\Gamma_o \simeq \frac{m_{\tilde{\chi}_o}}{16\pi}(1 - \frac{m_o^2}{m_{\tilde{\chi}_o}^2}) \tag{5.8}$$

where m_o is the mass of the final state Higgs scalar, and we have assumed that the soft scalar masses, and mass mixings, are of order the Higgs mass.

[2]This cancellation is the same one that allows the decoupling of heavy superfields from tree level interactions of light superfields which are coupled to them, where the heavy field F-term contribution to the light field potential is cancelled by the heavy scalar exchange contribution.

There will also be a contribution to the neutralino decay width from decay of its H_1 component mediated by N-fermion exchange, with a contribution

$$\Gamma \simeq \gamma^2 \frac{\lambda^2 h_\nu^2 v^2 \sin^2\beta}{M_N^2} \Gamma_o \tag{5.9}$$

it is suppressed by the N mass, and will not, in general, be significant.

There is another contribution to the decay of neutralino from decay of its H_2 component to a (light) neutrino plus physical (on mass shell) photon via the diagram in Figure [4] [3] . This mode is the most significant from the observational point of view; it leads to emission of monochromatic photons. The only part of electromagnetic vertex which contributes to the shaded blob in Figure [4] is the induced transition dipole moment. We notice that the first loop in the diagram represents $H_2 L$ mixing and is logarithmically divergent. This is allowed because it is a wavefunction renormalization. The shaded blob, however, must contain N-field propagators to induce lepton number violation. So this mode is eventually suppressed by the N mass too.

Similarly, decays of the LSP may be induced by the NNN superpotential term. As we have discussed above, in this case there are no soft lepton number violating $\tilde{l}h$ mass mixing terms induced by renormalization group mixing. Now the dominant decay modes are those arising from the diagrams of figure 3. We note that figure 3(a) is an induced D-term and contains a loop which is also a D-term. This ensures a non-zero decay rate for the neutralino even when supersymmetry is unbroken, unlike the case for F-terms. Because D-terms do not obey non-renormalization theorems, they can be radiatively induced even in the limit of unbroken supersymmetry; hence in general they appear without suppression factors associated with the scale of supersymmetry breaking. We also note that the induced D-term in figure 3(a) (and its associated component diagrams) is a dimension six term [62]. The component diagrams relevant

[3] Ward-Takahashi identities of electromagnetic gauge invariance ensure that photon emission from external lines do not contribute to the induced loop [61].

to neutralino decay are shown in figure 3(b) to 3(e). The processes of figure 3 dominate over decays induced by tree-level diagrams for large M_N, as the latter are suppressed by eight powers of M_N in rate, whereas the loop induced decays are only suppressed by four powers.

Computing the diagrams of figures 3(b) and 3(c) one finds that they result in a decay rate that is approximately

$$\Gamma \sim \gamma^2 \frac{k^2 {h_\nu}^6}{16\pi(2\pi)^8} \mu^2 \frac{{v_1}^2 m_{\tilde{\chi}_o}}{{M_N}^4} \tag{5.10}$$

whereas the final two diagrams of figure 3 result in a decay rate for the LSP that is approximately

$$\Gamma \sim \gamma^2 \frac{k^2 {h_\nu}^6}{16\pi(2\pi)^8} \frac{{v_1}^2 m^3_{\tilde{\chi}_o}}{{M_N}^4} \tag{5.11}$$

We expect that the Higgsino mass should be at least of the order of the doublet mixing term, and in certain circumstances the doublet mixing term might be substantially smaller; so below we will use the latter of these rate estimates for numerical estimates.

5.4 Comparison with Cosmological Limits

Almost without exception, the LSP decays we are considering are effectively entropy producing decays, ie. they will produce high energy photons. Photon producing decays are known to be highly constrained from both astrophyical and cosmological observations (see eg. ref. [63] for a recent compilation of such limits). These limits generally place constraints in the density-lifetime plane of the decaying particle. We will assume that the LSP, in the absence of its decay, is the dominant form of dark matter and therefore assume that its cosmological density is such that $\Omega_\chi \approx 1$, where $\Omega = \rho/\rho_c$ is the cosmological density parameter. At this density, one finds that the LSP lifetime is constrained so that either $\tau_\chi \leq 10^4$s to avoid affecting the light element abundances produced during big bang nucleosynthesis, or the LSP must be effectively stable with a lifetime $\tau_\chi \geq 10^{24}$s. Astrophysical limits on other R-parity violating interactions were considered in [64].

The decay rates that we derived are clearly dependent on a number of model parameters. In order to get a feeling for the limits imposed by the cosmological constraints we make a few more assumptions. We assume that the LSP is primarily a gaugino (a bino) with mass $m_\chi \approx 150$ GeV. For $|\mu| \sim 1 - 10$ TeV, $\gamma \sim 2 \times 10^{-3} - 2 \times 10^{-2}$ and $\delta \sim 4 \times 10^{-3} - 4 \times 10^{-2}$ and for large $\tan\beta$, $\sin\beta \approx 1$. We can then write (for the decays based on the $H_1 H_2 N$ superpotential term)

$$\tau_\chi \simeq 10^{-25} \left(\frac{150\mathrm{GeV}}{\delta^2 \lambda^2 H_\nu^2 m_\chi} \right) \mathrm{s} \tag{5.12}$$

This unsuppressed (by factors of the N mass) decay is, from the cosmological viewpoint, effectively instantaneous, and not subject to constraint, save that of the absence of LSP dark matter, and those of maintenance (or regeneration) of the baryon asymmetry at temperatures above the electroweak scale.

For LSP decay induced by the kN^3 superpotential term, from the decay width estimate given above we deduce an LSP lifetime of order

$$\tau_\chi \simeq 4 \times 10^{20} h_\nu{}^{-6} k^{-2} \gamma^{-2} \left(\frac{M_N}{10^{12}\mathrm{GeV}} \right)^4 \left(\frac{150\mathrm{GeV}}{m_\chi} \right)^3 \mathrm{s} \tag{5.13}$$

which translates into the limits

$$M_N \leq 5 \times 10^6 h_\nu{}^{3/2} k^{1/2} \ \mathrm{GeV} \tag{5.14}$$

or

$$M_N \geq 5 \times 10^{11} h_\nu{}^{3/2} k^{1/2} \text{ GeV} \tag{5.15}$$

These limits show therefore that even if R-parity violation is inserted in the singlet sector, destabilization of the LSP can indeed occur and R-parity violation of this type is strongly constrained.

5.5 Does R-Parity Violation Erase Lepton Asymmetry?

It has been pointed out [49] that the potential from the full superpotential (except the $\mu H_1 H_2$ term) we considered above plus the $SU(3) \times SU(2) \times U(1)$ D-terms has, among others, the following flat direction [4]

$$\begin{gathered}\tilde{t}_3^c = a \quad ; \quad \tilde{t}^1 = v \quad ; \quad \tilde{b}_3^c = c \\ \tilde{s}_2^c = e^{i\alpha}\sqrt{|\, v\,|^2 + |\, c\,|^2} \quad ; \quad \tilde{d}_1^c = e^{i\phi}\sqrt{|\, a\,|^2 + |\, v\,|^2 + |\, c\,|^2} \\ \tilde{\nu}_e = e^{i\gamma} c \quad ; \quad \tilde{\mu}^- = e^{i\beta}\sqrt{|\, v\,|^2 + |\, c\,|^2}\end{gathered} \tag{5.16}$$

which depends on three arbitrary complex parameters a , v , c and four phases α , β , ϕ , and γ. A vev along this particular flat direction produces a non-zero vev for the scalar operator

$$< \tilde{\mu}^- \tilde{\nu}_e \tilde{b}_3^c (\tilde{t}_3^c)^* > = e^{i(\beta+\gamma)} a^* c^2 \sqrt{|\, v\,|^2 + |\, c\,|^2} \tag{5.17}$$

which violates lepton number by two units. After supersymmetry breaking, this scalar operator will be induced by the singlet (s)neutrino interaction via the diagram of Figure [5]. As we showed earlier in the presence of the NH_1H_2 term there are lepton number violating operators, originating in the slepton-Higgs mixing. If we

[4] With the Higgs mixing term $\mu H_1 H_2$ the same flat direction exists.

insist that these operators don't erase any lepton asymmetry that was produced via the above flat direction before sphalerons go out of equilibrium, we will have [51]

$$\lambda h_\nu m_\delta{}^2 < 10^{-4}\mathrm{GeV}^2 \tag{5.18}$$

but $m_\delta \simeq 1$TeV and even for $h_\nu \sim 10^{-6}$ we get $\lambda < 10^{-4}$, something not typical. The question is: does the NH_1H_2 term erase any lepton asymmetry that model has already produced? It has been pointed out [23] that a baryon asymmetry may be generated from a lepton asymmetry, provided one of the generations of lepton flavours has its lepton number violating interaction in equilibrium, while another does not. In our case it is possible if (after diagnolization) one of the neutrinos has a zero Dirac mass. Therefore, the slepton-Higgs mixing for this generation disappears. We also notice that with the Higgs mixing term $\mu H_1 H_2$ the lepton number violating scalar operator can be induced at one-loop level after supersymmetry breaking via the diagram in Figure [6].

Chapter 6

Conclusion and Outlook

Primarily motivated by the benefits of a massive neutrino, we considered an extension of the MSSM by introducing singlet superfields N in three generations that includes neutrino masses via the see-saw mechanism and has the most general gauge invariant and renormalizable superpotential with respect to singlet superfields. This violates both lepton number and R-parity in the heavy field sector [1] .This full superpotential has both good and bad cosmological implications, originating in supersymmetry. On one hand it leads to a lepton asymmetry via an extension of the Affleck-Dine mechanism. On the other hand R-parity violation results in destabilization of the LSP, the dark matter candidate, and lepton number violating interactions that could potentially erase any lepton (baryon) asymmetry. The strength of these effects crucially depends on the R-parity violating term. If R-parity is violated by the NH_1H_2 term, then LSP decay is too fast. In this case the dominant decay channel is driven by soft supersymmetry breaking terms and its rate depends only logarithmically on the mass of heavy (s)neutrino. Nevertheless, a lepton asymmetry can be saved if one of the generations of lepton flavours is out of equilibrium. When R-parity violation is through the NNN term LSP decay rate (at low energies) is suppressed by powers of the heavy (s)neutrino mass M_N and reasonable constraints on the N-field parameters can be derived.

What is specially interesting is that cosmological arguments provide such

[1] Implications of R-parity violation in the light field sector have been considered in the literature before.

strong constraints, probing possible see-saw sources of R-parity violation to far higher mass scales than could be directly accessed by laboratory experiments.

A final word about the future of experiments both in the field of neutrinos and supersymmetry. What we have considered is a simple see-saw mechanism in the presence of three generations of light and heavy neutrinos. However, if we were to believe all the cosmological and astrophysical hints (from hot dark matter, solar and atmospheric neutrino observations) to massive neutrinos, then either neutrinos are closely degenerate, with a mass of about 2 eV (leading to neutrinoless double beta decay rate observable in the next round of experiments) or a light sterile neutrino exists in nature [65]. In either case the simplest see-saw mechanism scheme would be ruled out. Further data from low-energy neutrinos and from solar neutrino experiments will shed further light on the neutrino sector. The same can be said of the ongoing studies with atmospheric neutrinos. Similarly, a generation of experiments capable of more accurately measuring the cosmological temperature anisotropies at smaller angular scales than COBE, combined with data on structure formation in galactic and extragalactic scales, would presumably shed further light on the need for hot neutrino dark matter.

We also considered the simplest supersymmetric version of the standard model with simplifying assumptions like universal soft supersymmetry breaking terms. The future results of LEP II, and the LHC, will most probably play a decisive role in the search for signals of low-energy supersymmetry and wide regions of parameter space of supersymmetric extensions of the standard model can be tested. This means that the time for a thorough scan of parameter space will come soon and we can no longer make simplified assumptions and just consider simple models. As an example non-universality of soft supersymmetry breaking terms might have significant implications [66]. It is especially interesting to scan parameter space both in comparison with experimental data and cosmological constraints.

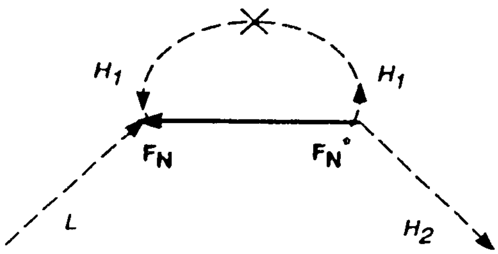

FIG.1(a)

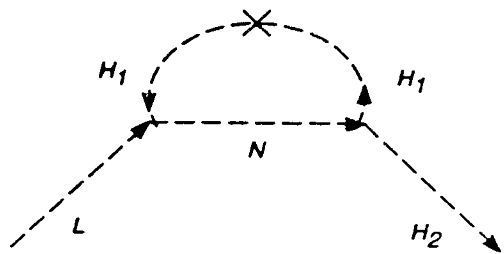

FIG.1(b)

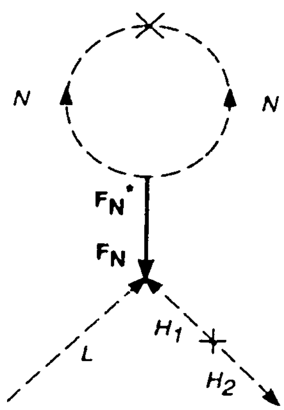

FIG.2(a)

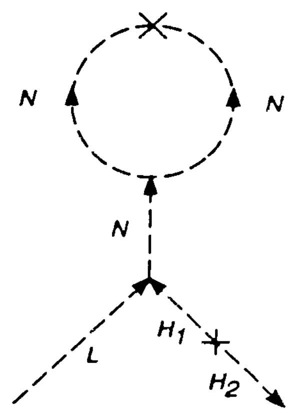

FIG.2(b)

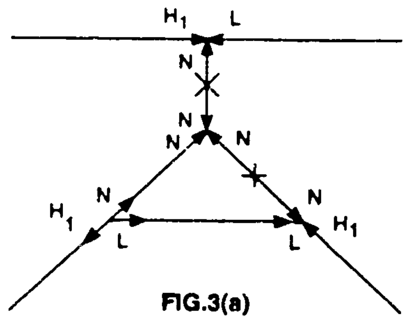

FIG.3(a)

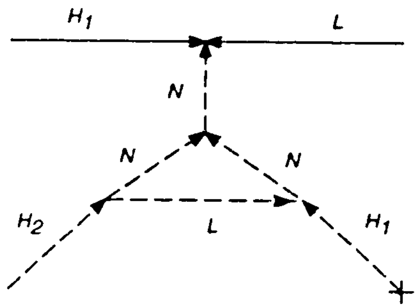

FIG.3(b)

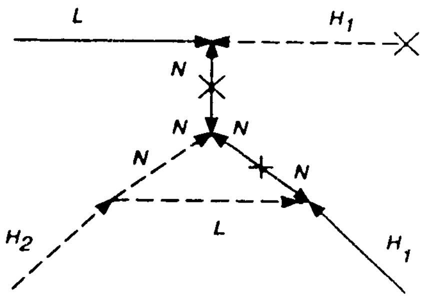

FIG.3(c)

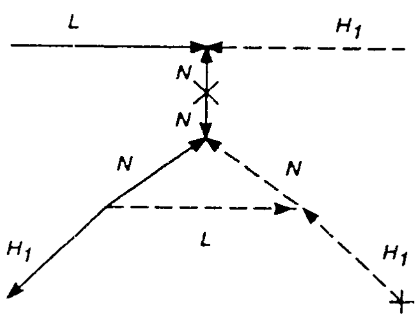

FIG.3(d)

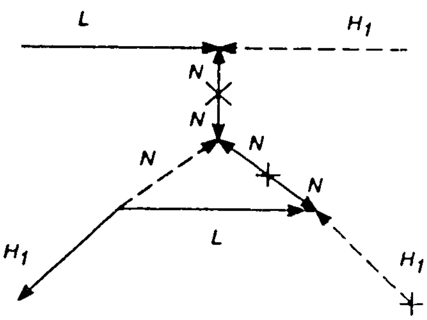

FIG.3(e)

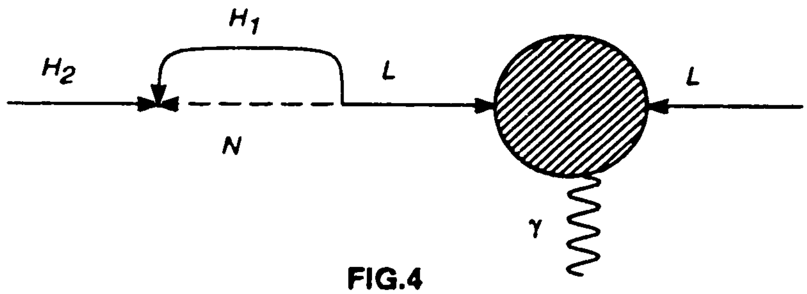

FIG.4

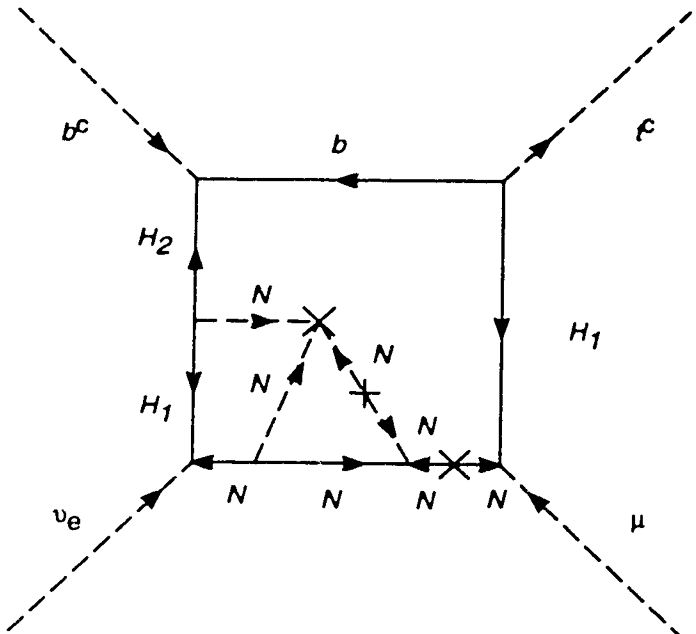

FIG.5

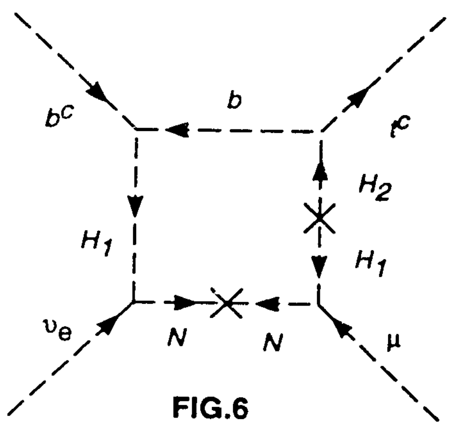

FIG.6

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Appendix A

A superfield $\Phi(x,\theta,\bar{\theta})$ is a function that transforms as follows under the supersymmetry transformation $S(y,\alpha,\bar{\alpha}) = exp[i(\alpha Q + \bar{Q}\alpha - x_\mu P^\mu)]$

$$\Phi(x,\theta,\bar{\theta}) \rightarrow \Phi(x + y - i\alpha\sigma_\mu\bar{\theta} + i\theta\sigma_\mu\bar{\alpha}, \theta + \alpha, \bar{\theta} + \bar{\alpha}) \quad \text{(A.1)}$$

Consider the Taylor's expansion of a superfield in terms of $\theta, \bar{\theta}$. Because $\theta, \bar{\theta}$ are Grassmann variables this series is finished at the $\theta\theta\bar{\theta}\bar{\theta}$term. A left (right) handed chiral superfield is a superfield that satisfies $\bar{D}\Phi = 0 (D\Phi = 0)$. In the L-representation of the supersymmetry algebra

$$\bar{D}_L = -\partial_{\bar{\theta}} \quad \text{(A.2)}$$

so a left-handed chiral superfield in L-representation is a function only of θ and can be written as follows

$$\Phi_L(x,\theta) = \phi(x) + \theta\psi(x) + \theta\theta F(x) \quad \text{(A.3)}$$

with spinor indices suppressed for simplicity. ϕ and F are complex scalars and ψ is a left-handed Weyl spinor. They are called component fields of the superfie.d Φ . Variation of these component fields under a supersymmetry transformation is

$$\delta\phi = \sqrt{2}\alpha\psi \quad ; \quad \delta\psi = \sqrt{2}\alpha F + i\sqrt{2}\sigma^\mu\bar{\alpha}\partial_\mu\phi \quad ; \quad \delta F = -i\sqrt{2}\partial_\mu\psi\sigma^\mu\bar{\alpha} \quad \text{(A.4)}$$

we notice that variation of the highest component (F) of a chiral superfield under supersymmetry transformation is a total derivative. Analogously a right-handed chiral superfield in R-representation (where $D_R = \partial_\theta$) can be written as follows

$$\Phi_R(x,\bar{\theta}) = \phi(x) + \bar{\theta}\psi(x) + \bar{\theta}\bar{\theta}F(x) \qquad (A.5)$$

where ψ is a right-handed Weyl spinor. It is seen that conjugate $\Phi_L{}^\dagger$ of a left-handed chiral superfield Φ_L in L-representation is a right-handed chiral superfield in R-representation. To bring them into the same representation we have to do the following replacement [35]

$$\Phi_R(x,\theta,\bar{\theta}) = \Phi_L(x - 2i\theta\sigma_\mu\bar{\theta},\theta) \qquad (A.6)$$

where Φ_R is now a right-handed chiral superfield in L-representation. The product of any number of left- (right-)handed chiral superfields is a left- (right-)handed chiral superfield. There are also vector superfields, superfields which are real $V = V^\dagger$. Vector superfields are functions of both $\theta,\bar{\theta}$ and in Wess-Zumino gauge are written as follows [33]

$$V(x) = -\theta\sigma^\mu\bar{\theta}V_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x) \qquad (A.7)$$

where V_μ is a vector field, λ is a left-handed Weyl spinor and D is a real scalar field. Variation of these component fields under supersymmetry transformation is

$$\delta\lambda = \alpha\sigma^{\mu\nu}V_{\mu\nu} \quad ; \quad \delta V_\mu = i\alpha\sigma_\mu\bar{\lambda} + i\bar{\alpha}\bar{\sigma}_\mu\lambda \quad ; \quad \delta D = -\alpha\sigma^\mu\partial_\mu\bar{\lambda} + \bar{\alpha}\sigma^\mu\partial_\mu\lambda \qquad (A.8)$$

where $\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. It is seen that variation of the highest component (D) of a vector superfield is also a total derivative. The product of an equal number of left- and right-handed chiral superfields is a vector superfield.

We notice that both chiral and vector superfields contain an equal number of bosonic and fermionic degrees of freedom. In a chiral superfield ψ provides four fermionic degrees of freedom before using Dirac equation and ϕ , F each provide two bosonic degrees of freedom, four in total. In a vector superfield λ provides four fermionic degrees of freedom (again before using Dirac equation) while V provides three and D another bosonic degree of freedom.