3932

NATIONAL LIBRARY

OTTAWA

BIBLIOTHÈQUE NATIONALE

OTTAWA

CANADA

NAME OF AUTHOR...HILDRED I. RAWSON.....

TITLE OF THESIS...A Study of the Relationships and Development of Reading and Cognition

UNIVERSITY...of Alberta (Edmonton)..

DEGREE..Ph.D...............YEAR GRANTED....1969.......

Permission is hereby granted to THE NATIONAL LIBRARY OF CANADA to microfilm this thesis and to lend or sell copies of the film.

The author reserves other publication rights, and neither the thesis nor extensive extracts from it may be printed or otherwise reproduced without the author's written permission.

(Signed)...Hildred Rawson

PERMANENT ADDRESS:

7E - 20 Prince Arthur Av

Toronto 5

Ont.

DATED. May 7.....1969

NL-91

THE UNIVERSITY OF ALBERTA

A STUDY OF
THE RELATIONSHIPS AND DEVELOPMENT OF
READING AND COGNITION

BY

© HILDRED I. RAWSON

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES IN

PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF ELEMENTARY EDUCATION

EDMONTON, ALBERTA

SPRING, 1969

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled A STUDY OF THE RELATIONSHIPS AND DEVELOPMENT OF READING AND COGNITION submitted by HILDRED I. RAWSON in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Marion D. Jenkinson
Supervisor

R. L. Armstrong

S. Hunka

L. von Bertalanffy

Albert J. Kingston
External Examiner

Date Spring, 1969

ABSTRACT

A STUDY OF THE RELATIONSHIPS AND DEVELOPMENT OF READING AND COGNITION

This study was designed to explore relationships between comparable cognitive operations under two sets of conditions: one in which logical operations of conservation, classification, deduction, induction and probability reasoning are required in response to concrete-verbal stimuli; the second in which corresponding operations are required in reading. The purpose of the study was to consider the extent to which cognitive operations available to nine-and-ten-year-old children are likely to be equally available in the two situations; the extent to which a standardized test of reading comprehension indicates the availability of these operations; and the possibility of sex differences in their acquisition in concrete and reading situations.

STEP Reading, Form 4B, and Concrete and Stories tests consisting of corresponding test items were administered to a sample of 100 Grade four children, randomly selected from the population of the regular Grade four classes of the City of Moose Jaw, Saskatchewan.

Statistical procedures for the analysis of the data included t-tests for the significance of sex differences between mean scores; stepwise multiple regression analyses to obtain a clearer indication of the relationship between the experimental tests and STEP Reading;

product-moment correlations between the variables; canonical correlation between the two sets of tests, Concrete and Stories, taken as composites; and factor analyses of the Concrete and Stories tests and STEP Reading.

Results and conclusions

Relations between the Stories tests and sex and STEP Reading and sex were found to be of a low order. On the Concrete tests, however, boys appeared to be significantly in advance of girls on operations of conservation and of inductive and probability reasoning.

Sex differences in levels of reasoning at the concrete-verbal level, not represented by differences in reasoning in reading (Stories tests) or in reading comprehension (STEP) were considered to suggest the possibility of risks in the acquisition of intellectual skills for both boys and girls.

Relations between the Concrete and Stories tests and STEP Reading examined in stepwise multiple regression analyses, suggested that the operations assessed by the Concrete and Stories tests could differ considerably from those assessed by STEP Reading.

Product-moment correlations between Concrete and Stories subtests indicated that the strongest relation in the data was between corresponding tests of conservation ($r = .44$ $p < .001$).

Significant cross correlations occurred throughout the matrix. The strongest of these was between Stories conservation and Stories deduction ($r = .39$ $p < .001$). Approximately one half of the relations in the matrix were not significant (alpha > .05). The common variance ranged from 19 per cent for CCO-Con and SCO-Con to negligible. It appeared that some degree of integration of the logical operations assessed, based mainly on operations of conservation, could be considered to be under way at this level.

There was then the question of the extent of the overall relationship between operations at the concrete level and operations in reading. When the two sets of tests, Concrete and Stories were taken as composites one significant canonical correlation was obtained ($R_c = .55$ $p < .01$). This canonical coefficient of correlation, representing the maximum possible relation between the composites, indicated that approximately 30 per cent of the total variance was common to the two sets of tests. The variance not common could be due to error and to elements unique to each set of tests. Elements unique to each set could represent operations not equally available in concrete and in reading situations. It was considered that comparable operations not equally available in the two situations could be at least as great as those likely to be available in each situation.

In obtaining the highest possible correlation between the composites, the greatest weights were assigned to a single cognitive category, conservation. This category accounted for 20 per cent of the common variance, leaving 10 per cent of the common variance to be accounted for by classification, deduction, induction, and probability together.
It would seem that, of the logical operations assessed, only conservation was likely to be equally available in reading and concrete situations to nine-and-ten-year old children. But it could not be assumed that even concrete conserving responses would be available in reading.

Factor analyses suggested that the elements underlying STEP Reading included basic reading skills and inferring from context. The elements underlying the Concrete and Stories tests represented a wider range of cognitive abilities and a greater elaboration of logical operations.

Implications

The lag observed between the acquisition of operations in reasoning in concrete and in reading situations could be a highly significant finding in view of an important developmental task of this period: the transformation of intelligence from the specific and the concrete to the abstract and formal level of functioning, a transformation which includes the recognition that logical principles of thinking hold across a wide range of content, that they are invariant in form. The bridge would seem to be the printed symbol: children in non-literate societies do not appear to have available the means of achieving this transition.

The study suggests a theory of reading which throws light on the full range of its function in the construction of intelligence, a theory which includes the refinement of perceptual skills, the accomplishment of associative skills, operations at the concrete level and also the extrapolation of cognitive skills to the symbolic level.

Implications were drawn for the preparation of materials and the development of instructional techniques. It may be clearly anticipated that class differences, racial and individual differences, as well as sex differences in cognitive styles will contribute to making these extremely difficult tasks.

ACKNOWLEDGEMENTS

The writer wishes to acknowledge the generous assistance of many individuals during the conduct of this investigation.

Dr. Marion D. Jenkinson has provided advice and stimulating direction throughout all stages of the investigation. Her assistance is gratefully acknowledged.

Dr. S. Hunka, Director, Division of Research Services, and his assistant, Mr. Walter Muir, have given invaluable assistance in the programming of the statistical procedures and during the analysis of the results.

The helpful criticisms of each of the members of the committee were much appreciated.

Dr. Wilfrid W. Toombs, Director and Superintendent, Moose Jaw Public Schools, his staff, and the Principals and teachers of the schools participating in the study provided continuous administrative assistance and encouragement during the period of interviewing the children. The cooperation of the children and of their parents is also gratefully acknowledged.

It was a privilege to have Dr. Albert J. Kingston as my external examiner. His contributions to the study were greatly appreciated.

TABLE OF CONTENTS

Chapter

Chapter

LIST OF TABLES

Table

Table

Table

LIST OF ILLUSTRATIONS

Figure

Figure

Figure

CHAPTER I

INTRODUCTION

One of the concerns in the teaching of reading is that a child's experience and his organization of his experiences should be appropriate for interpreting the subject matter he reads. To ensure as far as possible that these conditions are provided, supplementary concrete experiences of many kinds are introduced by the school. Objects, pictures, experiments, field trips, dramatic productions are both actual and televised experiences in the school environment. These experiences are organized through more or less continuous verbal exchanges among the children and between children and adults. It is generally anticipated that concrete experiences, when they are structured and reported in a verbal communication, will be similarly organized and interpreted when information is presented in printed form which the child is able to decode.

Two problems in reading instruction are clearly recognized in achieving these objectives: that skill in decoding become equal to a child's competency in speaking, as rapidly as possible; and that instruction in reading take account of syntactical and organizational structures specific to printed communications.

There is the question of a possible third problem. Are the operations by which concrete data are structured and outcomes expressed in verbal responses simultaneously available when the data is presented in printed form? Are some of these operations developed in reading before they are available in concrete situations? Are some of the operations which function in organizing concrete data not yet viable in reading situations? The question of concern appears to be the extent to which the printed page itself is a variable in cognitive operations and may contribute to determining the emergence and integration of these operations in their different forms.

Purpose of the Study

The purpose of this study is to explore the relation between children's ability to reason logically concerning concrete experiences and their ability to reason in reading; and to consider the role of reading in the on-going intellectual development of nine-and-ten-year-old children.

A good deal is known about the characteristics of intellectual operations during this period, in particular the sequence and interrelationships of this development. But what is known has been derived from research in which data is presented in one of three ways: perceptually, in concrete situations; verbally, as spoken instructions; or in printed form to be read. Responses also vary in these and in other ways. What we do not seem to know from these studies is the relation between results obtained from one of these procedures and results requiring corresponding operations with an alternative method of presentation of the data. In particular, we do not know in what ways cognitive operations and verbal responses in concrete situations may be differentially related to similar operations when the data is presented in printed form.

The child who is learning to read and learning by reading is shifting constantly from one manner of presentation of problems to alternate forms of presentation. At the same time he is likely to be attempting to acquire new cognitive operations and to improve his skill in emerging operations. In order to assist the child in this complex situation, even to present him with some appropriate questions concerning what he has read, it would seem to be necessary to examine the levels at which he is operating in both concrete and reading situations at a given time and to know to what extent these operations may be alike and unlike for the same logical procedures.

It is hoped that such an investigation will suggest instructional techniques that will ensure a close relation between reading and the cognitive development of the child and contribute to our understanding of the relation of reading to the transformations of intelligence which co-occur with the introduction of literacy (six-to-seven-year-olds), and again with the full acquisition of literacy during early adolescence.

The Problem

The problem in this study is to compare the competency of children in structuring experiences verbally when the data is presented in concrete form with their competency in structuring similar data presented in printed form. Specific cognitive operations are selected for comparison in these situations: they are operations in conserving, in classifying, and in deductive, inductive and probability reasoning.

Performance involving operations with concrete data and operations in reading with comparable data will be compared with scores in reading on a standardized reading test. The question arises of the relative contributions of the selected cognitive operations as measured in concrete and in corresponding reading situations to scores on a standardized test of reading comprehension.

The possibility of sex differences in performance in concrete and in corresponding reading situations on the logical operations assessed will be examined.

One additional question may be of interest: the possibility that relations within the data, Concrete and Stories, will indicate that at the nine-and-ten-year-old level, operations of conserving will be associated with ability in classification; that children who conserve are also likely to be capable of induction, deduction and probability reasoning.

The study will therefore investigate the characteristics of two sets of relationships: the extent to which logical operations of conservation, classification, deduction, probability reasoning and induction, available to nine-and-ten-year-old children in concrete stimulus situations, are also available in reading; and the extent to which these logical operations as assessed by Concrete and Stories tests are related to scores on a standardized test of reading comprehension.

Background of Study

Patterns of thinking and the characteristic ways in which highly divergent cultures structure their social and material environments have been studied in relation to the structure of the language, the technology, world-view, and early child-rearing practices of the cultures.[1,2] Studies of the psychological aspects of acculturation have also considered the formative influences of each of these factors on individual development.[3] More recently, within the effects of acculturation, the effects of confrontation with literacy and with schooling generally have been identified as

---

[1]Jane Bright and William Bright, "Semantic Structures in Northwestern California and the Sapir-Whorf Hypothesis", American Anthropologist, Special Publication, LXVII (October, 1965), 249-58.

[2]Claude Lévi-Strauss, La pensée sauvage (Paris: Plon, 1962).

[3]George Spindler and Louise Spindler, "The Instrumental Activities Inventory: A Technique for the Study of the Psychology of Acculturation", Southwestern Journal of Anthropology, XXI (Spring, 1965), 1-23.

contributing to the acceleration of cognitive development, in particular, operations of conserving, of hierarchical ordering, and of constructing logical equivalences.[4] Bruner described the effects of schooling on the Wolof child:

> The Wolof child who has been to school shows a pattern of intellectual growth that is strikingly similar to patterns familiar in Western society. He early shows the effect of learning to use language outside the context in which reference is supported either by pointing or by the structure of the situation . . . The difference lies, at the very least, in the extent to which and the manner in which children learn to use language as an instrument of thought. School forces him to rely on linguistic encoding as a way of communicating . . . .
>
> The hypothesis I would like to set forth is that there is a greater push toward hierarchical connections in technical cultures than in those that are less technical.[5]

Conservation as well as classification was found to be influenced by schooling. Greenfield reported the following effects of schooling:

> The oldest unschooled [Wolof] bush children (eleven to thirteen-year-olds) show no significant increase in conservation over the eight and nine-year-olds. Only half of the unschooled bush children attain conservation at this late age . . . No further changes in the pattern of conceptual thought was observed in the adults, . . . [The experiment suggests] that, without school, intellectual development defined as any qualitative change, ceases shortly after age nine.[6]

Schooling does not in general provide direct instruction in conservation and classification. It does, however, instruct children in reading. Are some of the dramatic cognitive changes reported in these studies associated with second level language learning? In

---

[4]Jerome S. Bruner, et. al., Studies in Cognitive Growth (New York: John Wiley and Sons, 1966).

[5]Ibid., p. 323.

[6]Ibid., pp. 233-34.

particular, are there interactions, significant for cognitive development, between reading and the concrete experiences of children? The concrete experience of the schooled Wolof children would not appear to have been materially changed on entering the village school.

We do not seem to have considered in detail the possibility of a relation between the emergence and consolidation of intellectual skills and learning to read. Studies in cognitive development appear to have been conducted in almost complete isolation from a consideration of the influence of reading on the child's behavior. We do not know, for example, the reading behavior of a single child whose congnitive development has been studied by Piaget, by Laurendeau and Pinard, by Lovell, Flavell, Bruner and many others. A recent study by Elkind has examined the relation between reading achievement and "non-verbal measures of perceptual de-centration".[7] Many of the studies in reading have been concerned with problems of decoding rather than with the emrgence of cognitive skills through reading.[8]

Research in reading, including the extensive literature on reading readiness and reading disability[9] has seldom considered the levels of development of basic cognitive operations such as conservation in relation to the acquisition of reading skills. It may be equally important that these studies have not examined the effect of

---

[7]David Elkind, John Horn, and Gerrie Schneider, "Modified Word Recognition, Reading Achievement and Perceptual De-centration", The Journal of Genetic Psychology, CVI (1965), 235-51.

[8]Jeanne Chall, Learning to Read: The Great Debate (New York: McGraw-Hill Book Company, 1967).

[9]John Money, ed., The Disabled Reader (Baltimore: John Hopkins Press, 1966).

learning to read on the emergence and consolidation of fundamental cognitive operations such as conservation, classification and deduction.

There are difficulties in examining cognitive operations which are developing and interacting in concrete and reading situations. This study will attempt to present to children who have accomplished the early decoding operations, comparable cognitive tasks in each of these situations. The logical operations will be those which previous studies have identified as emerging or as already becoming integrated in the thinking of nine-and-ten-year-old children when they are operating with concrete data. These logical operations will be required in response to questions based on data presented in both concrete and printed form.

It is hoped that a comparison of the responses in the two situations will suggest some of the significant factors contributing to those changes in patterns of conceptual thought which "push" children who read in a literate society toward realizing more fully their potential for abstraction and logical thinking.

Hypotheses

Several hypotheses arise from the above discussion and will now be presented. Each hypothesis permits a consideration of a possible relation in on-going intellectual development between reasoning in reading and reasoning in related verbal-concrete situations.

1. There will be no significant relation between scores on the following tests and sex (alpha > .05):

    Concrete tests
    Stories tests
    STEP Reading total scores.

2. There will be no significant difference between the mean scores of boys and girls on the following tests (alpha > .05):

    Concrete tests
    Stories tests
    STEP Reading total scores.

3. Subtests of the Concrete tests and subtests of the Stories tests ranged in order as predictors of the criterion STEP Reading will indicate that subtests of each of the predictor tests will be selected in the same order as predictors of STEP, and that each set of predictors will be approximately equally effective in predicting the variance of the criterion (alpha > .05).

4. Relations between scores on the Concrete tests and scores on the corresponding Stories tests assessing the following operations will be significantly different from zero (alpha > .05):

    Conservation
    Classification
    Deduction
    Induction
    Probability.

5. Relations between scores on the Concrete tests assessing each of the following operations will be significantly related to scores on the Stories tests assessing each of the other operations (alpha > .05):

    Conservation
    Classification
    Deduction
    Induction
    Probability.

6. Relations between subscores within the set of Concrete tests and between subscores within the set of Stories tests assessing the following operations will be significantly different from zero (alpha > .05):

   Conservation
   Classification
   Deduction
   Induction
   Probability.

7. Canonical correlation between the set of Concrete tests and the set of Stories tests taken as composites will indicate that the two sets of tests are significantly related (alpha > .01).

8. A weighting system which maximizes the relation between the two sets of tests taken as composites will suggest that the following categories of logical operations contribute approximately equal amounts to the prediction of the common variance of the Concrete and Stories tests:

   Conservation
   Classification
   Deduction
   Induction
   Probability.

9. Factor patterns identified for the following tests will indicate that simple structures underlying these tests will be similar:

   STEP Reading test items
   Concrete and Stories subtests
   Concrete and Stories subtests and STEP Reading total scores taken jointly.

Definition of Terms

Terms which are used generally throughout the study are defined below. Additional terms used in describing the tests of conservation, classification, deduction, probability and inductive reasoning are defined in Chapters IV to VIII in which the construction of these tests is described.

The definitions presented are more numerous than is usual. Logical and psychological terms are defined as closely as possible according to their use by scholars in these fields. Explanations and illustrations of some of the terms are also presented in order to make clear the use of these terms in this study.

Cognition

Cognition is used here as defined by Webster: an act or process of knowing in the broadest sense; specifically, an intellectual process by which knowledge is gained about perception or ideas -- distinguished from affection or conation (conation defined as an instinctually motivated biological striving that may appear in consciousness as volition or desire or in behavior as action tendencies).[10]

Intellectual process

An intellectual process is considered to include a symbolic (representational) component and an operative component. The symbolic

---

[10] Webster's Third New International Dictionary (1961).

component may be an "image-symbol" or a "sign" (or both) as these are defined by Ausubel. The image-symbol consists of "an image that reflects an internalized imitation of a past action or event", the symbol being "a private type of signifier which physically resembles its significate." A sign is "an arbitrary, socially shared type of signifier bearing no physical resemblance to its significate" (the actual object or event). It may be a linguistic representation of "absent realities": motor, sensory, conceptual, propositional.[11]

The operative component in the intellectual process is considered to transform "one reality state into another", on the basis of assumed available structures which relate meanings.[12] The observed outcomes of the transformations may be described in terms of principles governing invariance;[13] of rules of classification for the acquisition of new concepts;[14] of rules of logic relating propositions;[15] in terms of probability,[16] and in other ways.

---

[11]David P. Ausubel, "Neobehaviorism and Piaget's Views on Thought and Symbolic Functioning", Child Development, XXXVI, No. 4 (1965), 1029-32.

[12]Jean Piaget, Play, Dreams and Imitation in Childhood (London: Routledge and Kegan Paul, 1962), p. 67.

[13]Jean Piaget et Bärbel Inhelder, Le développement des quantités physiques chez l'enfant (Neuchatel: Delachaux et Niestlé, 1962).

[14]Earl B. Hunt, et al., Experiments in Induction (New York: Academic Press, 1966), p. 36.

[15]Hans Reichenbach, Elements of Symbolic Logic (New York: The Free Press, 1966), p. 38.

[16]Jean Piaget et Bärbel Inhelder, La genèse de l'idée de hasard chez l'enfant (Paris: Presses Universitaires de France, 1951), pp. 244-61.

Cognitive Operation

Cognitive operation is here defined as an intellectual process by which knowledge is gained about perception or ideas and expressed verbally.

Following Smedslund, a cognitive operation will be considered in this study to involve a percept (based on an initial stimulus input); a representation of the percept (an image); "holding" or reconstruction of the image in the absence of the stimulus; a goal (what the subject is instructed to attain); a decision (a response to a goal), and an inference pattern (an explanation of the decision).[17]

Concept

A concept is an abstract entity. It is defined as the relation, ε, membership in a class. Since a member of a class (an individual) is a concrete instance, it is not a part of the concept. The relation, member of a class, as determined by the criterial property defining such membership, constitutes the concept.

The recognition of criterial properties implies discovering rules for combining previously learned concepts to form a new decision rule. It is therefore assumed in concept learning that the learner has some previously learned concepts. These will be "primitive constructs", for example, the concept of "red" and the still more primitive concept of "color". In this study, following Hunt, primitive

---

[17] Jan Smedslund, Concrete Reasoning: A Study of Intellectual Development, Monograph of the Society for Research in Child Development Serial No. 93, XXIX, (Yellow Springs, O.: Antioch Press, 1964).

concepts are assumed and concept learning is considered to be "represented within the attribute-value notation."[18]

The decision rule for a concept may be represented symbolically:

$$(x) : (x \in F) \equiv_{df} (x)\ f_x,$$

The formula represents the decision rule: when a description, f, may be applied to an object, x, that object may be called an F.

Variable

The term variable will refer to one of the five categories of logical operation dealt with in this study: conservation, classification, deduction, probability, induction.

Conservation

Conservation is here defined as a cognitive operation requiring an inference of invariance and its justification based on the premises that certain conditions of identity for an object and a relation of equivalence between objects hold in the presence of changes in shape and changes in distribution of the material of the objects.

The conditions of identity will refer to the conservation of the substance, weight and volume of the object with change in form.

---

[18] Hunt, et al., Experiments in Induction, p. 14.

Proposition

A proposition is defined as a meaningful set of symbols (a sentence) which can be true or false.[19]

Propositional operation

In propositional operations, molecular sentences are constructed out of atomic sentences, the operations being expressed by the following words (and symbols): "not" ($\sim$); "or" (V) "and" (&); "implies" ($\supset$) and ($\rightarrow$); "equivalent" ($\equiv$).[20]

Synthetic proposition

A synthetic proposition is a proposition which is justified by methods other than logical. It may be justified by direct observation or by more comprehensive inductive procedures.

Tautology

A tautology is a statement that is true whatever be the truth-values of the elementary propositions of which it is composed. A formula which is a tautology remains so when any sentences are substituted for its component atomic sentences in all occurrences.[21]

---

[19] Hans Reichenbach, Elements of Symbolic Logic, p. 67.

[20] Ibid., pp. 22-23.

[21] Ibid., p. 36.

Modus Ponens is a tautological implication of the form:

$$A \;\&\; (A \supset B) \supset B.$$

The Principle of Transitivity is a tautological implication of the form:

$$(A \supset B) \;\&\; (B \supset C) \supset (A \supset C).$$

Modus Tollens is a tautological implication of the form:

$$\sim B \;\&\; (A \supset B) \supset \sim A.$$

The Rule of Identity concerning one proposition is a tautological implication of the form:

$$A \;\&\; A \equiv A.$$ [22]

<u>Deductive Inference</u>

A deductive inference is defined as an intellectual process in which a particular instance of a class of events is assigned the characteristics of a universal class on the basis of its being a member of that class.

A deductive inference consists of premises and necessary conclusion. It proceeds by rules of logic from true propositions to true propositions.[23]

<u>Implicative proposition</u>

An implicative proposition is a sentence in which a conditional "if . . . then" statement is put forward as a hypothesis.

---

[22] <u>Ibid.</u>, p. 38.

[23] <u>Ibid.</u>, p. 67.

<u>Singular proposition</u>

A singular proposition attributes a property to a thing (an object, an individual). It consists of two parts, a name, and what is asserted to be an attribute of what is referred to by the name. If the proposition is true, we say the property is present in the thing and call the thing a positive instance of the property.[24]

A singular proposition may be symbolized: f_x. In language it could represent the statement, "This (object) is square".

<u>Singular terms</u>

Singular terms are expressions which function as subjects in singular propositions.

<u>Individuals</u>

Individuals are instances, things, which may be pointed to, indicated by saying "this", or by being given a proper name.[25] In a formula, an individual may be represented by a name variable: x,y,z,etc.[26]

<u>Property</u>

A property is what is expressed by the predicate of a proposition as being a characteristic of an individual. It refers to

---

[24] *Ibid.*, p. 28.

[25] Susan K. Langer, *An Introduction to Symbolic Logic* (New York: Dover Publications, 1953), p. 129.

[26] A.N. Prior, *Formal Logic* (Oxford: Clarendon Press, 1962), p. 158.

"something physical that things have, a side or aspect or component or character of the things."[27]

General proposition

A general proposition is a statement which mentions a member or members of a class and the class to which these members belong. It consists of two parts: the name of a class; and the name of a class of which it is asserted members of the first class belong (are included in). A general proposition may be symbolized: $F \subset G$. In language a general proposition could be the sentence, "Ducks are birds," or "A duck is a bird."

General terms

General terms are expressions which function as predicates in general propositions.

Class

A class (x) is defined as "the totality of objects having a certain property." It is the totality of objects (x) for which f(x) is true. The notion of a class is, however, an abstraction. The totality is not something which is known empirically except perhaps in those instances in which the class may be indicated by enumerating

---

[27]Rudolf Carnap, Meaning and Necessity: A Study in Semantics and Modal Logic (Chicago: University of Chicago Press, 1956), p. 20.

its members. The designation of a class is a sign of the form:

$$\hat{F} : (x)\ f_x.$$[28]

Domain of discourse

Domain (or universe) of discourse has been defined by De Morgan as a whole of some definite category of things which are under discussion. It is "not the totality of all conceivable objects of any kind whatsoever."[29]

Classification

Classification is defined as a cognitive operation yielding one of the following outcomes:

A criterial property f is abstracted and asserted in a predicate.
A class name is assigned to objects on the basis of common criterial properties.
Class inclusion relations are asserted to hold between classes.

Inductive inference

An inductive inference is defined as an assertion by a thinker who has proceeded from what is known of some members of a class to what he asserts to be true of unknown members of the class. An inductive

---

[28] William Kneale and Martha Kneale, The Development of Logic (Oxford: Clarendon Press, 1962), p. 626.

[29] Ibid., p. 408.

inference is understood in this study as the term is used by von Wright:

> From the fact that something is true of a certain number of members of a class, we conclude that the same thing will be true of unknown members of that class also. If the conclusion applies to an unlimited number of unexamined members of the class we say the induction has led to a generalization.[30]

Probability

Two models of the concept of probability may be identified by reference to the probability symbol $P(^a/_h)$: the frequency model and the range model.[31] On the frequency view the probability of a given h is, "the relative frequency with which 'an event', a, takes place when 'conditions', h, are fulfilled: that is, the probability of a given h is the proportion of h's which are a's."[32] It is the relative frequency of an occurrence of a characteristic or event within a class.

On the basis of the range model, the possibilities available in h are "analysed" into a number of mutually exclusive alternatives, say $h_1 \ldots h_n$. Some of these, say m, are considered "favorable" to a; those which entail $\sim a$ are then "unfavorable." The probability of a given h is then understood as the ratio of favorable alternatives to all the possibilities.[33]

Explanation

To explain an event is "to bring it under a law, that is,

---

[30]George Henrik von Wright, The Logical Problem of Induction (Oxford: Basil Blackwell, 1965), p. 1.

[31]Ibid., p. 92.

[32]Ibid., p. 98.

[33]Ibid., p. 100.

under any uniformity of nature [by profering] a well-established statement of how nature works in this way rather than that. The statement must be capable of disproof by empirical observations."[34]

Concrete

The term "concrete" is used in three contexts in this study. Spelled with a capital in the expression "Concrete tests" it refers to a series of five tests constructed by the investigator for this study. The term "concrete" is also used in the expression "concrete stage of cognitive development" to mean a period of development from approximately seven to eleven years as this has been described by Piaget. In the expression, "a concrete testing situation," concrete refers to the presence of objects which provide perceptual stimuli as contrasted with a "reading test situation," in which the information is presented in printed form.

Limitations of the Study

This study is exploratory: it is in the nature of a survey of a problem. An attempt has been made to examine in selected areas of cognitive behavior the possibility of significant relations between reading and the emergence of logical operations during a particular period of schooling.

---

[34]John Hospers, "What is Explanation?"in Essays in Conceptual Analysis, ed. by Antony Flew (London: MacMillan and Company, 1963), pp. 94-119.

Important aspects of cognitive development have not been examined. Two of these which could be expected to be facilitated both through reading and concrete experiences are operations involving the various aspects of notions of time and space. A more complete consideration of the possible relations between reading and on-going cognitive development would also have included an examination of moral judgments, world-view, perceptual and motor skills, number, and social and emotional behavior. The aspects of cognition selected for study are, however, those which have been considered in the research to have important implications for intellectual development. Nevertheless, the scope and detail with which each of these has been examined is restricted.

The population from which the children for this study were drawn were living in a small western Canadian city, essentially suburban in character. The relevance of findings based on the sample in this study for children in large industrialized centers, in isolated areas, or of other language backgrounds, must remain in doubt.

Significance of Study

The importance of reading for academic progress is generally recognized. It has been pointed out that a new world of ideas and experience is open to the child who reads well and widely. Instruction in reading has accordingly accepted the responsibility of ensuring that children become capable of receiving in full the printed communication;

that they learn to evaluate it and respond to it; and that they succeed in integrating it with other experiences available to them. But more may be needed. We do not seem to know to what extent the basic logical operations necessary for processing the information children receive when they read are available to them. Even if they are available, we do not know if children actually use them when the message has first to be decoded, since factors other than availability of the logical operations (as determined by presenting the problem in a concrete situation), may affect the level of reasoning in reading. The information the child receives in reading will be in the more formal language of the printed page. In reading, too, the supporting, or distracting, contributions from a concrete - verbal presentation will be reduced.

One other matter does not appear to have been fully considered. Reasoning from printed symbols may be particularly effective in the early consolidation of acquisitions in logical thinking. Reading, if this were the case, could be one of the contributing factors in the realization of potential intelligence. From the investigations reported by Bruner it would appear that children who were taught to read began earlier to conserve and to classify. This was determined by retesting the children in concrete situations. Did these children also conserve and classify at the same level in reading? Was the application in reading of these basic cognitive skills related to a subsequent emergence of the ability to reason deductively; to recognize a problem as requiring investigation and to use inductive procedures in solving it; to an ability to think in terms of probability; and, of course, to recognize new applications for the principles of invariance and to classify more accurately?

The optimum use of instruction in reading in the development of intellectual potential would seem to depend on further information concerning how children reason when they read and how they reason in concrete situations. An attempt will be made to examine some of these problems in this study. The findings should contribute to clarifying some of the objectives in teaching children to read.

Overview of the Study

Five categories of operations, considered significant in the cognitive development of nine-and-ten-year-old children have been investigated: conservation, classification, deduction, probability and induction. Concrete tests and Stories tests were constructed by the investigator for each of these categories. These tests together with a standardized test of reading comprehension, have been administered to each subject in the study.

Chapter II presents a review of the research related to the problem to be investigated.

Chapter III discusses the design of the study. It describes the population and the sample on which the study was conducted; the collection of the data; and the pilot study. Data on the reliability and validity of the tests constructed by the investigator are presented. The statistical procedures in the analysis of the data are indicated.

Chapters IV to VIII describe the construction of the tests, Concrete and Stories, for each of the categories examined. Since the tests are somewhat diverse in form they are described separately.

Chapter IV presents the Concrete and Stories tests of conservation of substance, weight and volume.

Chapter V presents the Concrete and Stories tests of classification. These tests include hierarchical and multiplicative class structures and relations.

Chapter VI presents the tests of deductive reasoning. These tests include inferences based on the rules Modus Ponens and Modus Tollens and inferences derived by the Principle of Transitivity.

Chapter VII describes the tests of probability reasoning, Concrete and Stories. The tests assess an understanding of chance distributions, of probability as a proportion of favorable to unfavorable outcomes and as a likelihood of an exact recurrence of a sequence of chance events.

Chapter VIII describes the Concrete and Stories tests of inductive reasoning.

Chapter IX presents the results of the study.

Chapter X presents a brief summary of the study and of the conclusions related to each of the hypotheses. It discusses the implications of the conclusions, suggests limitations in their applicability, and some directions for further research.

CHAPTER II

REVIEW OF RELATED RESEARCH

The impressive intellectual achievements of the pre-school years are becoming well documented. It has been shown that the acquisitions of these years are critical for later cognitive development. Educational intervention during this period has been found to be indispensable if the intellectual potential of many children is to be realized subsequently. Perceptual and motor skills; class names and the basic syntactic structures of the language; representation; notions of perceptual constancy, of space, and of time - these are some of the intellectual accomplishments of early childhood. Their development not only prefigures the more complex logical operations of intelligence; they also permit pre-inferences and pre-implications during a period in which thinking is liable to distortion from centration and effects of field.[1] They represent the operations characteristic of intelligence prior to the development of more mature cognitive skills, including conservation, operations based on the principle of transitivity, class inclusion relations, and notions of chance.

The push for the acquisition of these later cognitive skills and for the control of syntactical structures capable of both representing experience and of classifying and transforming it, appears to be in large

---

[1]Jean Piaget, Les méchanismes perceptifs (Paris: Presses Universitaires de France, 1961), pp. 420-25.

part a function of the culture and most effectively of a literate culture.[2]

The research reported below examines the interactions between the child and his culture by which the child acquires the logical operations of conserving, of classifying, and of deductive, inductive and probability reasoning, and uses these as rules in thinking.

The Acquisition of Intellectual Skills: Evidence from Studies of Conservation

One of the earliest studies pointing to the importance of logical operations to the child in adapting himself to his physical environment was Piaget and Inhelder's, Le développement des quantités physiques (1941).[3] Two of the characteristics of mental development which influenced the progress of this adaptation were also suggested by this study. One was the presence of "décalages," of appreciable time lags between a child's use of a rule of logic in one situation and his application of the rule to other related concrete situations. A second characteristic was a remarkable resistance to the evidence of an experiment during the concrete period, however clearly and repeatedly the evidence was presented.

The existence of a time lag in extending a principle of conservation was suggested by the results of a preliminary experiment reported

---

[2]Jerome S. Bruner, et al., Studies in Cognitive Growth (New York: John Wiley and Sons, 1966), p. 323.

[3]Jean Piaget et Bärbel Inhelder, Le développement des quantités physiques chez l'enfant (2d ed.; Neuchatel: Delachaux et Niestlé, 1962). (Hereinafter referred to as Quantités physiques.)

by Piaget.[4] The percentage of the conserving responses for substance, weight and volume by subjects eight-to-eleven-years are presented in Table 1 (N = 25 for subjects five-to-eleven-years in the experiment).

TABLE 1.- Percentage of successes in conserving substance, weight and volume for children eight-to-eleven-years (Piaget).

Age	8	9	10	11
Substance	72	84	-	-
Weight	52	72	76	96
Volume	28	32	56	80

A similar time lag, but somewhat more extended in relation to age was reported by Lovell and Ogilvie in a study of conservation of substance and weight for subjects eight-to-eleven years.[5,6] The results of this study are presented in Table 2.

TABLE 2.- Percentage of successes in conserving substance and weight for children eight-to-eleven-years (Lovell and Ogilvie).

Age	8-9	9-10	10-11
Substance (N=322)	68	74	86
Weight (N=364)	36	48	74

[4] Ibid., p. xiii.

[5] K. Lovell and E. Ogilvie, "A Study of the Conservation of Substance in the Junior School Child," British Journal of Educational Psychology, XXX (1960), 109-18.

[6] Lovell and Ogilvie, "A Study of Conservation of Weight in the Junior School Child," ibid., XXXI (1961), 138-44. (Hereinafter referred to as Conservation of Weight.)

Lovell and Ogilvie also demonstrated the operation of a time lag in the extension of conservation within a single concept, the conservation of volume.[7] This study was a more extended examination of a problem investigated by Piaget in The Child's Concept of Geometry.[8]

Lovell and Ogilvie's experiments were designed to assess an understanding of three related aspects of volume: interior volume occupied volume, and displacement volume. Their subjects were English children (N = 191) in each of the four years of the junior school (approximately equivalent to the Grades three to six).

On the basis of the evidence as a whole the authors concluded:

> The concept of physical volume . . . develops slowly during the junior school years and it is unlikely that any single test will decide if a child fully understands it.[9]

From the results of tests assessing displacement volume they observed:

> It appears that not until the fourth year of the junior school do 50 per cent of pupils realize that the amount of water displaced by a single cube is independent of the size of the full container, within the limits set by the apparatus used.[10]

The weight of the object was offered as the explanation of the water displaced by 48 per cent of pupils at the second year level (Grade

---

[7]Lovell and Ogilvie, "The Growth of the Concept of Volume in the Junior School Child," Journal of Child Psychology and Psychiatry, II (1961), pp. 118-26. (Hereinafter referred to as Concept of Volume.)

[8]Jean Piaget, Bärbel Inhelder, and Alina Szeminska, The Child's Conception of Geometry (London: Routledge and Kegan Paul, 1960), p. 358.

[9]Lovell and Ogilvie, "Conservation of Weight," p. 125.

[10]Ibid., p. 124.

four), 59 per cent at the third year level, and 37 per cent at the fourth year level (Grade six).

A second characteristic, resistance to the evidence of an experiment, presents a problem in designing appropriate instructional techniques. Piaget has recorded this behavior in a number of protocols. Smedslund also, in a series of studies, confirmed the relative ineffectiveness of repeated demonstration in inducing conservation.[11]

There are various explanations for the child's resistance to the acceptance of experimental evidence for conservation and his vulnerability in the face of conflicting perceptual evidence. For Piaget they seem to sum to:

Tant qu'il n'y a pas de logique ... il n'y a pas de conservation.

For assisting the child in the acquisition of the logic, Piaget has suggested:

> Ce n'est nullement, ... par un simple perfectionnement des méthodes intuitives ou perceptives, car le perception est rigide et irréversible ... il s'agit au contraire de briser les structures perceptives et de construire un système d'opérations pures.[12]

This directive, "smash perceptive structures and build up a system of pure operations", is scarcely itself operational.

---

[11]Jan Smedslund, "The Acquisition of Conservation of Substance and Weight in Children," Scandinavian Journal of Psychology, II (1961), 11-20; 71-84; 85-87; 153-55; 156-60; 203-10. (Hereinafter referred to as Conservation of Substance and Weight.)

[12]Piaget et Inhelder, Quantités physiques, pp. 328-39.

Contemporary learning theories have suggested a number of approaches to this instructional problem. On the basis of learning theory, a concept of conservation would be acquired as a function of reinforcement. It would be inferred in empirical situations as a result of social reinforcements: the significant events would be part of the external world as perceived by the learner. Maturational theory, on the other hand "would assert that a logical structure may not be present from the beginning in children's behavior, but that it develops as a function of nervous maturation and independently of experience."[13] Equilibration theory, a theory of development proposed by Piaget and his associates, puts forward the view that cognitive learning is an internal process dependent on the sensori-motor activity of the learner. Logical rules for making inferences derive from this activity: they are not derived from the properties of the external world nor are they identical with maturation. Progressive development results as the learner coordinates and represents his own actions as figurations and symbolically in a biologically-based relation of assimilation and accommodation to the environment.

A number of instructional procedures for inducing conservation have been suggested by these theories of learning. In general, experiments based on the equilibration theory have included some modification of the environment involving intervening in the assimilation - accommodation equation. Gréco predicted that reducing or extinguishing visual cues would force the child to search for more reliable evidence

[13]Smedslund, "Conservation of Substance and Weight," p. 13.

and this in turn would lead to the acquisition of the concept of conservation. The prediction was not confirmed.[14] Smedslund suggested that increasing the complexity of perceptual cues would induce a state of "cognitive conflict" which could lead to greater effort and success in solving conservation problems. He designed experiments to avoid as far as possible external reinforcement. Complexity was effected by "the simultaneous deformation of an object and additions to or subtractions from another or the same object."[15] Some success was reported in obtaining conserving responses resistant to extinction.

Smedslund reexamined this problem in a subsequent study.[16] He concluded that a shift of emphasis should be made from the relative importance of the ability to resist misleading perceptual cues and also from the role of cognitive conflict in achieving conservation. He suggested rather that "a very close relation is indicated between resistance to extinction of conservation responses and the ability to give good verbal explanations. He added, "this points directly to the importance of some kind of internal structure, supported or at least related to language processes.[17]

---

[14]Pierre Gréco, "Les relations entre la perception et l'intelligence dans le développement de l'enfant: le facteur d'équilibre dans les conservations opératoires et les constances perceptives," Bulletin Psychologie, X (1956-57), 751-65; 833-43.

[15]Smedslund, "Conservation of Substance and Weight," p. 156.

[16]Smedslund, Concrete Reasoning: A Study of Intellectual Development, Monograph of the Society for Research in Child Development, Serial No. 93, XXIX, No. 2 (Yellow Springs, O.: Antioch Press, 1964).

[17]Ibid., p. 30.

There is no suggestion in these studies that presenting a conservation task in a reading situation (a language process) could be examined as a possibly useful technique in establishing "resistance to extinction" of a concept such as conservation.

There is also no indication that a further question has been considered: is it reasonable to affirm that a notion of conservation has become "resistant to extinction" before it has been ascertained that it is equally resistant in a reading situation?

The recognition of invariance and variance as properties of objects in the environment would appear to be a condition for ordering, classifying, and reasoning about these objects. The ability to conserve properties of objects such as quantity, size, area, length, weight, number, volume, in the presence of observed changes in shape and distribution would further seem to be essential in providing stability and clarity in the individual's cognitive structure as he learns by relating new meaningful material to already available concepts and propositions. Concepts of conservation would appear to be among the necessary and basic concepts for the "ideational anchorage" Ausubel refers to as providing the fundamental propositions needed:

> . . . for the acquisition of new meanings; . . . for the psychological organization of knowledge as a hierarchical structure in which the most inclusive concepts occupy a position at the apex of the structure and subsume progressively more highly differentiated subconcepts and factual data; . . .[18]

---

[18]David P. Ausubel, Learning Theory and Classroom Practice, Bulletin No. 1 (Toronto: The Ontario Institute for Studies in Education, 1967), p. 11.

For children during the elementary school years the ability to conserve in situations of increasing complexity has been considered by Piaget to be associated with a process in which the child frees himself from the dominance of perception as the primary criterion in judgment, accepts the existence of alternative points of view and achieves "decentration." This requires:

> ... le passage des rapports égocentriques au groupement des relations objectives ... une nouvelle décentration des rapports perceptifs à l'égard de ce centre illusoire qu'est le moi ...[19]

But the acquisition of perceptual constancy (e.g., constancy of the size of an object at increasing distances) is also, in Piaget's view, a complex development, more or less independent and in advance of the development of the logical structures required for the attainment of notions of conservation. Fundamental differences, he has pointed out, appear to separate perception in its most specific form (effects of field) from the structures which characterize intelligence (operational structures such as those required in conservation).[20] Perception, among other differences, is dependent on the presence of the object; on conditions of space and time providing knowledge of the object by making possible immediate contact with it. Even in the presence of the object, perception does not, however, provide an exhaustive account of the properties of the object, only of the "apparent" ones. On the other hand knowledge of the characteristics of an object even at the level

---

[19]Piaget et Inhelder, Quantités physiques, pp. 49-50.

[20]Jean Piaget, Les méchanismes perceptifs (Paris: Presses Universitaires de France, 1961), pp. 355-365.

of primary perception can be increased by the intervention of "schématisations dues à des activités antérieures."[25] Perception, therefore, remains essentially at the level of "the given," and is not extended by deductive reconstruction even in contact with the object present and given.

Intelligence, however, as Piaget has described its operations, evokes the object in its absence by means of symbols and even in its presence will interpret the object not only by its immediate characteristics, but within a conceptual framework: a perceived square will be interpreted as a particular case of rectangles in general, and as independent of its specific dimensions. Intelligence, even in contact with the object, constantly goes beyond the data of perception to effect an interpretive reconstruction.[22]

In conservation, the interpretive reconstruction provided by intelligence seems to include three operations. One of the earliest appears to be reversibility. Reversibility in this context is the recognition that one may return to the point of departure in the deformation of an object and it will then be "the same" in significant respects.

But reversibility may not yet be "réversibilité vrai"; that is, it may not represent a true argument for the conservation of significant properties of the object in the deformed state. The object may be thought of as "the same" only after it has been returned to

---

[21]*Ibid.*, p. 368.

[22]*Ibid.*, p. 356.

its original form.[23] There are required also the operations of conservation of identity and conservation of a quantitative relation in the deformed state.[24]

Conservation of identity requires the judgment that given a single object A, and a deformation of A producing A′, A = A′. Conservation of a quantitative relation (which may be one of equality or of inequality) requires the recognition that given a relationship, for example, of equality between two objects, A and B, and a deformation of B (B => B′), a relationship of equality continues to hold between A and B′.[25]

Finally, in defence of the judgments of conservation of identity and conservation of a quantitative relation, premises are inserted in an argument which permit a deductive inference.[26]

This deductive inference consists of premises and necessary conclusion. Premise I is given: A = B. Premise II is: If nothing is added and nothing removed in the deformation of an object, it is the same in significant respects in the alternative form. Premise III is to be constructed and inserted in the argument. This premise is: In this action nothing was, in fact, added or removed. The necessary inference follows: In this action of deformation, the

---

[23]Piaget et Inhelder, Quantités physiques, p. 16.

[24]David Elkind, "Piaget's Conservation Problems," Child Development, XXXVIII, No. 1 (1967), 15-27.

[25]Ibid., p. 17.

[26]Piaget, Les méchanismes perceptifs, p. 408.

objects and the relation between the objects remain identical in significant respects. At this point in the development of cognitive operations of conservation it is the deductive argument itself which becomes the invariant. Conservation of substance, weight, volume, etc., become instances of its applicability. When this occurs it appears that children are likely to show astonishment that an adult should inquire about so obvious a matter.

It is at this point also that the child acquires control of the logical operation of equivalence, which is a relation holding between propositions and no longer simply a relation between objects. The propositional argument available to him is the transitivity of equivalence. It is a tautological implication of the form:

$$(p \equiv q) \; \& \; (q \equiv r) \supset (p \equiv r).$$

The significance of conservation in cognitive development appears, therefore, to be the opportunity presented for the emergence and recognition of the function of deductive reasoning in the acquisition of new knowledge and in its validation.

There appears to be little evidence in the literature of the corresponding development of these operations in reading situations. Academic progress would, however, appear to be dependent on the effective use of cognitive skills in response to the more abstract stimuli of the printed page. Moreover, intellectual progress characterized by the acquisition of formal combinatorial operations in early adolescence may be dependent on, or facilitated by, the use of concrete operations in reasoning in response to printed symbols. The achievement of accepting the evidence of an experiment in a concrete situation may also not be complete until such evidence is accepted, if judged adequate, when it is reported on the printed page.

The Stories tests in this study, matched as carefully as possible to the Concrete tests of conservation, are designed to assess the relationship between cognitive operations of conserving in response to printed symbols in reading and operations of conserving when the initial stimuli are concrete.

The Acquisition of Intellectual Skills: Evidence from Studies of Classification

In considering the acquisition of classification it is necessary to follow modern logicians in distinguishing between form and content; and psychologists in distinguishing between information processing variables, the rules by which we combine information, and the content variables, the particular information being structured.

Information processing variables appear to develop along different "dimensions" as the child matures in thinking. In operations of conserving, for example, the dimension available for processing in-put information could be the identity rule; the reversibility rule; or a deductive operation. The same objective stimulus would in each of these instances by "mediated" by a different set of rules. The choice of a rule which provides the greater certainty in a wider range of situations and is at the same time economical of time and effort could be indicative of increasing intellectual maturity.

In classification the dimensions in information processing appear to be wider than in some of the other intellectual operations of the concrete period. In the beginning there is probably only the object: when names are learned they also appear to be thought of as an integral part of the object. The name of the sun is "inside the sun; it's not in the head because it's in the sun".[27] The names of objects can also be "explained" by the attributes of the object:

> . . . an animal is called "cow" because it has horns, "calf" because its horns are still small, "dog" because it is small and has no horns, . . . "car" because it is not an animal.[28]

But with the acquisition of "description" proper, the child comes to use expressions which do not designate an object by a name, but characterize it univocally in a different way, "namely, by means of the statement of a property which belongs only to that object."[29]

A description of an individual object, ix, having the property f may be represented symbolically:

$$f_{ix}.$$

Description soon becomes ambiguous when there are several objects having the same property. A property (for example the color red, or the shape round), may then come to be thought of as something "apart from" an

---

[27]Jean Piaget, The Child's Conception of the World (London: Routledge and Kegan Paul, 1951), pp. 72-74.

[28]Lev Semenovich Vygotsky, Thought and Language, trans. and ed. by Eugenia Hanfmann and Gertrude Vakar (Cambridge, Mass.: M.I.T. Press, 1962), p. 129.

[29]Rudolf Carnap, The Logical Syntax of Language (London: Routledge and Kegan Paul, 1964), p. 42.

object but belonging to or characteristic of objects. This is a new dimension in thinking in which a property is abstracted from the instances in which it may be observed. Oléron defines this operation of abstraction:

> Abstraire consiste dans le fait de dégager d'un ensemble complexe de données certaines d'entre elles auxquelles le sujet va réagir d'une manière élective en négligeant les autres.[30]

Abstraction is a learned behavior: the subject reacts to one aspect of a situation, the one which he has considered apart, and he neglects others; he reacts "as if" these others did not exist. "Abstraire, c'est penser à part ce qui ne peut être donné à part".[31] Later given an array of objects the child will first "think apart" color, then shift and "think apart" shape, size,use, and many other properties for the same array.

On the basis of an abstraction of properties the child moves gradually to "classifying" which is the operation of "putting together what belongs together" by virtue of a property abstracted from and common to all of its members. All members (x) of a class A are seen to be members by virtue of possessing this common character (excluding some other characters). Membership in a class may be symbolized:

$$(x) : (x \in A).$$

Kneale and Kneale point out that there is a difference of the greatest importance between a set that can be specified by enumeration of its members and one that can be specified only by an indication of some

---

[30] Pierre Oléron, Les activités intellectuelles (Paris: Presses Universitaires de France, 1964), p. 42.

[31] Ibid., p. 44.

feature common to all its members.[32]

From the concept of "membership in a class" the child moves first to the notion of class inclusion ($A \subset B$) and finally to recognizing the interrelations of classes: relations of inclusion; exclusion; overlap between classes of the same level; and the additive relation of subclasses to a superordinate class in a hierarchical structure. With rules of class inclusion relations available it becomes a question of applying them logically in a wide range of contexts.

Continuous progress is by no means inevitable: there is again the presence of décalages. In part this may be explained by the fact that the system of class inclusion relations is "completely deductive", so that the number of facts which may be asserted on the basis of class inclusion is remarkable:

> Whatever characteristics we mention, we cannot assert many propositions together without generating, by implication, a whole system of further classes and their relations.[33]

In classificatory behavior, adults, no less than children who are acquiring these operations, are constantly functioning in the several dimensions: description, abstraction, class inclusion, class inclusion relations and the implications which these relations entail. Symbolically the dimensions may be represented as follows:

Description: f_{1x}
Abstraction: $(x):f_{(x)}$
Class membership: $(x):(x \in A)$

---

[32]William Kneale and Martha Kneale, <u>The Development of Logic</u> (2d. ed., rev.; New York: Dover Publications, 1953), p. 183.

[33]Suzanne K. Langer, <u>An Introduction to Symbolic Logic</u> (2d. ed., rev.; New York: Dover Publications, 1953), p. 183.

Class inclusion: $A \subset B$

Class inclusion relations: $A + A' = B$
$A = B - A'$
$B = A \vee A'$

Implication: $(x):(x \in A) \supset (x \in B)$
$(A \subset B) \,\&\, (B \subset C) \supset (A \subset C)$

Studies in the acquisition of classificatory behavior have examined these various dimensions. Inhelder and Piaget in The Early Growth of Logic described the development of description, abstraction, the construction of complementary and null classes, class membership and the understanding and ordering of class inclusion relations.[34] Bruner, in Studies in Cognitive Growth, examined equivalence operations, abstraction, class membership, alternative criteria for class membership, equivalence across cultures and class inclusion and hierarchical ordering.[35] A study by Morf tested the effectiveness of selected instructional techniques in speeding up the acquisition of an operation of class inclusion: the understanding that a class contains more elements than one of its subclasses ($B = A + A'$; $B > A$).[36] He found that subjects at the preoperational level were unlikely to learn this classificatory behavior. One instructional procedure showed some promise. It "consisted of training the children in the ancillary concept of logical multiplication - the recognition that an object can belong to

---

34Bärbel Inhelder and Jean Piaget, The Early Growth of Logic in the Child: Classification and Seriation, trans. by E.A. Lunzer and D. Papert (London: Routledge and Kegan Paul, 1964). (Hereinafter referred to as Growth of Logic.)

35Bruner, et al., Studies in Cognitive Growth.

36Albert Morf,"Apprentissage d'une structure logique concrète (inclusion) effet et limites," Études d'épistémologie génétique, IX (1959), 15-76. (Hereinafter referred to as Une structure logique.)

several different classes at once."[37]

In one of the studies reported in Early Growth of Logic, Inhelder and Piaget investigated children's ability to understand "extension of a class", an operation preliminary to quantifying class inclusion relations. In an experiment involving "yellow primulas, primulas, and other flowers", the nine-to-ten-year old subjects recognized inclusion relations at the 73 per cent level of success.[38] There was a striking difference, however, in the per cent of correct responses to class inclusion questions in an experiment in which animals were substituted for flowers.[39] In this experiment only 27 per cent of the nine-year-olds and 42 per cent of the ten-year-olds answered all inclusion questions correctly.

Piaget suggested that the classification of animals was the more difficult task since the child's own actions of ordering (arranging) were not equally available to him in the two situations. Animals (birds, ducks) have, however, been selected as appropriate material for classifying in the Stories tests in this study (The Ducks Arrive in Spring). It is considered that the subjects in the study have had varied concrete experiences with this material, and, in addition, that this is the kind of material frequently presented to children for reading.

One other study in The Early Growth of Logic has been adapted for use in this investigation: Inhelder and Piaget's study of the multiplication

---

[37]John Flavell, The Developmental Psychology of Jean Piaget (Princeton, N.J.: D. Van Nostrand Company, 1963), p. 375.

[38]Inhelder and Piaget, Growth of Logic pp. 100-10.

[39]Ibid., pp. 110-18.

of classes. For multiplicative classification the authors report a 75 per cent level of correct responses for the nine-to-ten-year-olds tested in a concrete situation.[40] Subjects' ability to produce this classificatory structure in reading is examined in this study in the story, "A City of Long Ago". Multiplicative classes are the familiar adjective plus noun in the noun phrases of a sentence. It is possible that children's assumed familiarity with this structure in speech may contribute to a tendency to overlook the complexities both of its logical properties and of its grammatical derivation.[41]

The importance of the interrelatedness of dimensions of classification in facilitating learning was suggested by the results of Morf's experiment referred to above.[42] Morf compared the outcomes of three instructional procedures.

Training procedure I stressed repeated and varied observations and the quantification of the data by counting. This procedure produced more accurate and more rapid observations, but not class inclusion.

Procedure II introduced free manipulation of the materials. This led to spontaneous and varied activities by the subjects which "prefigured" class inclusion, but did not lead to its discovery.

---

[40] Ibid., p. 178.

[41] Noam Chomsky, Aspects of the Theory of Syntax (Cambridge, Mass.: M.I.T. Press, 1965), p. 9.

[42] Morf, "Une structure logique."

Procedure III was a systematic attempt to teach dimensions of classification related to class inclusion. These included recognizing alternative criteria for classifying; subdividing a class into sub-classes; noting that an element may belong to two or more classes; and constructing the class of intersection by multiplying the defining attributes of two classes.

Procedure III appeared to be effective. Morf concluded:

> After instruction in multiplicative logical relations, all the subjects accepted without difficulty the principle of class inclusion . . . The critical factor, therefore, appeared to be training in an understanding of another logical operation - the logic of multiplicative classification.[43]

It appears from this experiment that the accumulation of facts without the logical operations of classification to summarize and relate the facts does not lead to an effective comprehension of the nature of the material.

The reason for having a concept is to classify,[44] to consider objects or events as somehow "the same" and to give this sameness a name. Economy in thinking is feasible when multiplicity can be named. There are risks in subsuming diversity under a class name but there is no alternative. The "magical number seven plus or minus two" is a limitation of "our capacity for processing information" to be reckoned with.[45]

---

[43] Ibid., p. 75.

[44] Earl B. Hunt, Janet Marin and Philip J. Stone, Experiments in Induction (New York: Academic Press, 1966), p. 19.

[45] George A. Miller, "The Magical Number Seven, Plus or Minus Two. Some Limits on our Capacity for Processing Information," The Psychological Review, LXIII, No. 2 (1956), 81-97.

The ability to relate classes in a hierarchical structure represents a completely new dimension in information processing and retrieval. The implications available when data is structured in this way are impressive. We can say, for example, that something is either this or that; that there is more of these than those; that some but not all are like this; that all the others are like those, but none are like these; put these and these together and this is the name by which they are called; if something is like this then it is also like that. An unstructured "storage" of this information would present insuperable problems in retrieval.

Operations of abstraction and classification not only permit going beyond the information given by structuring the data, they represent the emancipation of the child from the dominance of the given of perception.[46]

Operations of classification in reading appear to require considerable ability in going beyond the given both of the data and of perception. The relations between classes are seldom stated explicitly in what is read; they are implicit within the context. In addition a hierarchical (or matrix) structure may have been used in relating the information within paragraphs and for the organization of the communication as a whole. It is possible therefore that operations of classification in reading will be effective to the degree that they are generalized; that the operation itself is abstracted and independent of content. Such a development may be one of the significant contributions of literacy in actualizing intellectual potential.

---

[46] Jean Piaget, Les méchanismes perceptifs, p. 358.

The Acquisition of Intellectual Skills: Evidence from Studies of Deductive Reasoning

The unit in deductive thinking is the proposition: a sentence which expresses what is true or false. A proposition is anything that is believed, disbelieved, doubted or supposed. It is true or false according to its content and not for reasons of form.

The "logic of propositions" is the study of propositions as unanalysed wholes, and sentences as symbols of such unanalysed propositions:

> If p and q express propositions, then $p \& q$ expresses their conjunction, and $p \vee q$ expresses their disjunction.
>
> If p is the antecedent and q is the consequent in an implicative proposition, then $p \rightarrow q$ expresses their implication.
>
> If p and q express propositions, $p \leftrightarrow q$ expresses their equivalence.[47]

Children at the Grade four level readily construct such sentences:

$\sim p$, the negation-sentence

$p \& q$ the conjunction -

$p \vee q$ the disjunction -

$p \rightarrow q$ the implication -, and

$p \leftrightarrow q$ the equivalence sentence.

The question of interest is the ability of these children to coordinate propositions according to some of the "truths of logic" in the "logic of propositions".

The truths of logic examined in children's thinking in this study were Modus Ponens, Modus Tollens, and the Principle of Transitivity.

---

[47] Georg Henrik von Wright, Logical Studies (London: Routledge and Kegan Paul, 1957), pp. 22-23.

These may be symbolized:

$$p \,\&\, (p \longrightarrow q) \longrightarrow q$$
$$\sim q \,\&\, (p \longrightarrow q) \longrightarrow \sim p.$$
$$(p \longrightarrow q) \,\&\, (q \longrightarrow r) \longrightarrow (p \longrightarrow r).$$

The first, the Law of Modus Ponens, may be read:

If p and q are sentences, p & (p ⟶ q) entails q.

The second, the Law of Modus Tollens, may be read:

If p and q are sentences, ~q and (p ⟶ q) entails ~ p.[48]

The third, the Principle of Transitivity, in the logic of propositions may be stated:

If p implies q and q implies r, then p implies r.

Implication is the fundamental relation between propositions, as inclusion is the fundamental relation between classes. The relational forms of the principle of transitivity hold between classes:

If $A = B$
and $B = C$
then $A = C$

If $A \subset B$
and $B \subset C$
then $A \subset C$

Matalon, in a study described below, examined children's ability to relate propositions by the rules Modus Ponens and Modus Tollens, and to recognize the relation Undetermined and the relations Possible and Not-Possible for propositions asserted together. These relations require some explanation.

For the relations Possible and Not-Possible von Wright has introduced the symbols $M(P/_q)$ and $\sim M(P/_q)$.[49] The symbol $M(P/_q)$ may be read:

---

[48] Ibid., p. 26.

[49] Ibid., p. 89.

p is possible given q. The symbol $\sim M(p/q)$ may be read: p is not-possible given q.

Logical and physical modalities are to be distinguished for each of these relations. A proposition p is "logically" necessary (possible, not-possible) relative to q means that "in all possible worlds" p is necessary (possible, impossible) on evidence of q. The proposition p is then logically not-possible relative to q if it is possible in "no possible world". Von Wright has also explained that the conditions under which something can be "physically" necessary (possible, impossible) are "contingent", that is, they are conditions which (logically) can either be or not be. They are, therefore conditions which hold "in some possible world".[50]

It has been of interest both in this study and in Matalon's study to determine if ten-year-old children recognize the truth of a logical relation of necessity (possibility, impossibility) as a truth in "all possible worlds"; that is, if they recognize the importance of logical form in the relation between propositions. Clearly an inference may be correct and need not imply a logical operation either contingent or "in all possible worlds". A five-and-a-half-year-old was asked to explain how she knew that "all these small pieces weighed the same as the large ball of clay". She replied:

> Oh! vous savez, il y a des choses bizarres dans la vie, il y en a des bien plus bizarres que ça.[51]

---

[50]Ibid., p. 113.

[51]Michel Goustard, et al., "La logique des apprentissages", Vol. X of Études d'épistémologie génétique, ed. by Jean Piaget, (Paris: Presses Universitaires de France, 1959), p. 166.

The inference Undetermined (one cannot know) was also examined in Matalon's study. From the proposition $\sim p$, and the proposition $p \rightarrow q$, the proposition q is undetermined. From q, and $p \rightarrow q$, the proposition p is undetermined. Matalon's research question for each of these inferences was, "What relations must exist between two propositions in order that a subject is willing to go from the truth of one to the truth of the other?"[52]

Matalon predicted that the "natural" reasoning of the younger child would be more dependent on content: that it would be the truth of a fact, a fact sufficiently verified that he could count on it, which would be the point of departure in his thinking.[53] He also predicted that the natural discourse would introduce certain "supplementary" meanings: the fact that two propositions were linked by a speaker would tend to imply that they were related; the fact that a speaker chose one connective rather than another would also contribute supplementary meanings to the statements. He hypothesized that this relation would exist in the case of Modus Ponens and Modus Tollens (see Fig. 1).[54] These forms of deductive inference would then develop early, in advance of the inference Undetermined.

---

[52]E.W. Beth _et al._, "Implication, formalisation et logique naturelle", _Études d'épistémologie génétique_ ed. by Jean Piaget, XVI (Paris: Presses Universitaires de France, 1962), pp. 69-95.

[53]_Ibid._, pp. 6-65.

[54]_Ibid._, p. 70.

Deductive Inferences

(1) $p \rightarrow q$ p ——— q (Modus Ponens)	(2) $p \rightarrow q$ q ——— p (Undetermined)
(3) $p \rightarrow q$ $\sim p$ ——— q (Undetermined)	(4) $p \rightarrow q$ $\sim q$ ——— p (Modus Tollens)
(a) $p \rightarrow q$ ——— (p and q) (Possible)	(c) $p \rightarrow q$ ——— (-p and q) (Possible)
(b) $p \rightarrow q$ ——— (p and -q) (Not-possible)	(d) $p \rightarrow q$ ——— (-p and -q) (Possible)

Fig. 1.--Inferences examined by Matalon in the "Lights" and "Village" tests.

Matalon tested the complete set of implications shown in Figure 1 in two situations: the "Village" tests, considered to be the more intelligible to a young child; and the "Lights" tests in which the relations were more arbitary or probabilistic. The two situations have also been presented to subjects in this study.

Matalon's results for the "Lights" test are presented in Table 3.

TABLE 3.- Number and percentage of correct responses of subjects 9:8 to 10:4 years on test items requiring deductive inferences. (Matalon's "Lights" experiment.)

Inferences	9:8 n=10	10:0 n=10	10:4 n=10	Total	%
(a) Possible	10	9	9	28	90
(b) Not-possible	4	5	7	16	50
(c) Possible	9	9	5	23	80
(d) Possible	9	8	8	25	80
(1) Modus Ponens	9	10	8	27	90
(2) Undetermined	2	2	2	6	20
(3) Undetermined	2	2	1	5	16
(4) Modus Tollens	4	4	7	15	50

In the "Village" test, considered to offer greater content support, the inference Not-Possible was answered correctly by 40 of the 45 six-to-eight-year old subjects. Only the inference Undetermined appeared to present difficulty in the "Village" test. Correct responses for this inference were below the 30 per cent level of success. Matalon concluded that these six-to-eight-year olds were interpreting implication as causality rather than as logical consequence. Two conditions were found to assist young children in manipulating implication: that the propositions express a concrete relation corresponding to the accepted properties of the material; and that the child be able to conceive of the two propositions as belonging to the same domain. All natural thought appears to take content and the factor of a common domain into consideration, he stated:

The importance for later inferential reasoning of learning to operate on the basis of a rule abstracted from content was supported in an experiment reported by Gréco. Gréco examined the outcomes of two procedures for inducing the concept of direct and inverse spatial order. A sequence A B C was presented (colored beads in a tube). The child was required to predict the effect of one and two 180° rotations (a return to the order A B C).

The group which was trained on a mixed set of items containing both one and two rotations was found to be able to generalize to n-rotations and to demonstrate facility in transferring to an analogous situation (a rotating disk). This group had been presented with a more complex situation, forcing them into intense structuring activity in order to solve the problem. What was learned appeared to be a logical system for inferring the relations, permitting a correct conclusion in specific instances. It was this feature, Gréco concluded, which ensured generalization to n-situations and extension to a new situation. Gréco was also impressed with the additional energy and attention required of the subjects in this group.[55]

The studies of children's acquisition of deductive operations reported above would appear to have implications for instructional

---

[55]Pierre Gréco et Jean Piaget, "Apprentissages et connaissance", in Vol. VII of Études d'épistémologie génétique, ed. by Jean Piaget (Paris: Presses Universitaires de France, 1959),68-182.

procedures. Deductive inference, the recognition of a relation of implication or entailment, requires the child to operate with true-false sentences on the basis of rules for determining in what way propositions entail other propositions. During the development of these operations, children appear to require the support of concrete content and an acceptable domain. The abstraction of rules from content appears in Gréco's study to increase their effectiveness.

It would seem from the evidence presented that a planned withdrawal of concrete support would be useful in assisting the child to an appreciation of the distinction between the logical and the contingent as he learns to produce and evaluate arguments.

The most difficult inference for the children in Matalon's study was the recognition that there were conditions in which "one could not know"; that with the information available, an inference remained in doubt. There may be educational as well as inferential obstacles intervening which contribute to this difficulty. For most of the problems presented in school, children are encouraged to assume that "there is an answer". But problems exist for which there is no answer on the basis of the evidence available. The usefulness of accepting uncertainty in reasoning will be seen again in the study of the acquisition of notions of probability.

The principle of transitivity which is fundamental in deductive inference, was not included in Matalon's set. There is the question if instruction based on materials for reading, and tests of comprehension in reading provide sufficient opportunities for developing and examining the progress of children in operations which require both the inferences "one cannot know", and also the basic deductive principle, transitivity. No experimental studies were found in the literature in which inferences of these forms were specifically assessed in reading.

A limited number of tests requiring inferences based on the principle of transitivity and the inference Undetermined have been presented in the Concrete and Stories tests in this investigation. In addition, test items requiring inferences of the forms Modus Ponens and Modus Tollens, and the inferences Possible and Not-Possible are presented.

The Acquisition of Intellectual Skills: Evidence from Studies of Inductive Reasoning

Before a child makes use of inductive operations in a "scientific" experiment to discover the conditions which will account for an as yet unexplained situation he has "acted inductively" in almost all real life situations. He has assumed that the kind of food he ate yesterday, the dog he has known, the fire which hurt will be "the same" in significant respects today. The question of the validity of these inferences he has referred to experience. He takes his experiences to be "fair samples from a larger totality" and inferences of this kind to be "the best mode of reasoning about the unknown".[56] In his use of induction he may be said to be adopting the "policy of a practicalist".[57]

---

[56] Von Wright, The Logical Problem of Induction, p. 160.

[57] Ibid., p. 224.

He may be wrong. He may be mistaken in his reasoning from "all the previously observed" to "this particular instance" (from all previously observed round pink objects were candy to these round pink objects are candy). He may not know a "law of nature" to which to refer an observed event. He may also fail to recognize a situation as "new". But his policy is sound. He has found a necessary way of approaching the world both in ordinary circumstances and in the many critical situations in the life of a child. For his security in real emergencies he needs to develop a great deal of skill in this basic orientation: skill in constructing hypotheses and in testing them against evidence which he carefully assesses as he assembles it.

A critical situation requiring this careful hypothesis testing has been presented in this study in the story of "The Cave". A "laboratory" application of inductive reasoning has been tested in an adaptation of Piaget's experiment introducing a magnet as the new regularity-producing agent in a previously random distribution of events.[58]

To understand the nature of inductive operations, it may be useful to examine recent extensive investigations of the process of inductive reasoning in modern science. The implications of these investigations have been considered by Bocheński in <u>The Methods of Contemporary Thought</u>.[59]

---

[58]Piaget, <u>L'idée de hasard</u>, pp. 76-106.

[59]J.M. Bocheński, <u>The Methods of Contemporary Thought</u> (Dordrecht-Holland: D. Reidel Publishing Company, 1965).

Methods of inference, Bocheński has stated, may be divided into two principal classes: deduction and reduction, of which induction is a subclass. This distinction has provided a framework for his discussion of the characteristics of induction as a method of logical thinking. In all proofs, he states:

> . . . the premises can be so transformed that one is a conditional statement ("if A, then B") and the other is identical either with the antecedent or with the consequent of this statement . . . the two cases can be set out as follows:
>
> (1) if A, then B
A
therefore B.
>
> (2) if A, then B
B
therefore A.[60]

An inference which follows the first pattern is called a deduction; one which follows the second pattern, a reduction. The rule of inference in deduction is Modus Ponens: formal logic supplies both the first premise, "If A, then B", and the basic rule of inference. For reduction, logic is required only for the formation of the first premise, "If A, then B". Logic is not required for the rule of inference. The "logic" in both, however, is the same logic: there is no inductive logic as distinct from deductive logic much less a "logic of research", or a "logic of discovery".[61]

The second pattern, reduction, has always seemed questionable, since in logic it is invalid to infer from the consequent to the antecedent. Actually the problem of validity in reduction is not, in Bocheński's view, as acute as it might appear. In reduction "nothing

---

[60] Ibid., p. 67.

[61] Ibid., p. 94.

is derived from a single statement, say A, which needs to be verified, but from a conjunction of this statement with others (perhaps with some theory or the like), say T.[62] The schema may therefore be:

$$\frac{\begin{array}{l}\text{If A and T, then B}\\ \sim \text{B}\end{array}}{\sim \text{A}}$$

This pattern of reasoning is one which may be demonstrated in the solution of the problem presented in this study in the story "The Cave". The inductive solution following this schema is:
If there is life (A) and oxygen (B) is necessary for life (T):

$$[(A \text{ and } T) \rightarrow B]$$

then the absence of oxygen ($\sim B$) is sufficient to account for the absence of life ($\sim A$). The argument may be symbolized:

$$\sim B \;\&\; [(A \;\&\; T) \rightarrow B] \rightarrow \sim A$$

Actually either $\sim A$ or $\sim T$ may be inferred in this argument. Since T is considered a law and therefore "highly probable", the choice of inference is $\sim A$.

A second characteristic of this inductive form of reasoning is that the propositions A and $\sim A$ are not "given" as in deduction but are constructed by the thinker from summary statements of observations. "There was life in the cave", A, is a summary statement of experiences from previous visits: there were bats, sounds, feelings of security. "There is no life in the cave", $\sim A$, is also a summary statement of observations: no bats, no sounds, the candles go out, the chipmunk does not go into the cave. The problem is to account for $\sim A$. But

---

[62] <u>Ibid.</u>, p. 94.

first it must be recognized as a "new" situation and its characteristics noted in a sequence of protocol statements.

These protocol statements are, in Bocheński's view, "the essential foundation of the whole system" of inductive reasoning.[63] They are a collection of non-ordered statements recording the occurrence of phenomena. In inductive reasoning an attempt is made to "explain" the protocol statements by asserting a general statement from which these observations could be derived. In the test question based on the story of "The Cave", the general statement, or hypothesis, would be "no air would account for no life". The next step is to test if this hypothesis does in fact account for each of the protocol statements.

Verification may proceed in two ways. It may be demonstrated that other hypotheses do not account for the data; and it may be shown that the proposed hypothesis does indeed account for the data. After verification the hypothesis may be inserted as a premise in a hypothetico-deductive argument.

Procedures for determining the truth-value of a hypothesis consist of experiments and observations. In reading, the procedure is the collection of the observations reported and the testing of their veridicality against other information presented, and against their relation to known laws of nature and to experience.

In induction the basic operation is observation and the assembling of observations in protocol statements. But these observations themselves are guided by hypotheses. Bocheński stresses the

---

[63] Ibid., p. 85.

importance of hypotheses in guiding observation and experiment:

> Hypotheses are of great importance for the guidance of observation and hence for the formation of protocol statements. Without them, in most cases, it would not be known what was really being looked for; they give a definite direction to observation. They are therefore the basis of every kind of experiment. Experimentation without a guiding hypothesis is inconceivable.[64]

The question of interest is the extent to which these intellectual skills will be demonstrated by nine-to-ten-year-old children in solving problems requiring inductive reasoning.

Piaget and Inhelder investigated this problem in <u>L'idée de hasard</u> following the interrogation of the subjects on their understanding of a uniform chance distribution. For the study of inductive reasoning a "new" regularity was introduced: the spinner stopped at one point only so that a uniform random distribution was no longer observed.

The material for this second experiment consisted of eight small boxes, of identical appearance but of three different weights. One box of medium weight contained a magnet. A rod of iron not visible to the subject, had been attached to the underside of the spinner.

An inductive approach was considered to be demonstrated by the subject if he recognized that the operation of a new factor in the situation constituted a researchable problem. Inductive reasoning was demonstrated in constructing hypotheses to direct observation and experiment; in producing protocol statements; in testing hypotheses

---

[64]<u>Ibid.</u>, p. 98.

which might explain these protocol statements. Verification of a hypothesis included falsifying a number of alternative hypotheses and showing that the selected hypothesis explained the data. The verified hypothesis could be inserted as a premise in a hypothetico-deductive argument.

The subjects in Piaget's study who had not previously recognized the characteristics of a uniform chance distribution were unable to demonstrate the first step in the inductive process, an inductive approach: they did not show "surprise". Subjects seven-to-nine years demonstrated notable progress in both chance and inductive reasoning. These subjects tended to recognize both the equi-possibilities producing the uniform distribution and the presence of an intervening factor producing regularity. The two intellectual operations, when they occurred together, frequently led to success in testing a hypothesis. The hypotheses of color, speed of the spinner, and weight were proposed and eliminated in that order. The hypothesis, weight, was the most difficult for the younger of these subjects to eliminate. Piaget suggested that the conflict between their expectations and the evidence they produced in their experiments was too strong so that the evidence was ignored.

Subjects nine-to-ten-years old showed a new openness to evidence and greater flexibility in inventing ways to test hypotheses. The hypothesis frequently offered was, "It's something in the box that's stopping the spinner." This may be symbolized:

$$(x) : (A_x \rightarrow B_x).$$

A complete verification of this hypothesis would involve showing that B_x did not occur in the absence of A_x:

$$(x) : (\sim B_x \rightarrow \sim A_x).$$ [65]

Inductive reasoning appears to be an early "practical" orientation to everyday experience. In these first applications inductive operations are limited by the young child's partial control of classification, by his limited knowledge of general laws which govern natural events, and by frequent failure to identify as "new" significant elements in familiar situations. The recognition of regularity appears to be dependent on the prior recognition of chance as the outcome of multiply interacting events. Skill in inductive reasoning appears to be related to the acquisition of disjunctive class relations, of notions of the irreversibility of chance distribution, and of an ability to accept evidence which may conflict strongly with expectations. Later developments in inductive reasoning appear to be associated with flexibility in testing hypotheses and a greater rigor in what is accepted as necessary and sufficient evidence.

The acquisition of skill in inductive reasoning would seem to depend on discipline, training, and experience in a variety of situations both concrete and in reading. The child's safety in many situations in the modern world may depend on his success in proceeding effectively through the various sequences identified as characteristic of inductive operations. His continued progress in mastering important aspects of a changing scientific culture may depend on isolating and

[65]Von Wright, The Logical Problem of Induction, p. 67.

identifying the operations in thinking on which inductive reasoning depend.

The Acquisition of Intellectual Skills: Evidence from Studies of Chance and Probability Reasoning

Piaget and Inhelder have concluded from studies of the development of notions of chance, of proportion, and of probability reasoning that these intellectual skills are not fully developed before the level of formal operations.[66,67,68] They also found that the evolution of operations of chance was independent of and prior to the discovery of the principles of combinations and permutations, basic to judgments of probability. It was, they suggested, the convergence of operations of chance, with those of proportion, combinations and permutations which made possible judgments of probability. This convergence, which occurred at the formal level of intellectual operations was a kind of "choc en retour" of concrete probability reasoning with the developing notions of chance distributions.[69]

The discovery of the existence of chance, as opposed to miracle, caprice, the merely fortuituous, or as an apparent disorder temporarily

---

[66]Jean Piaget and Bärbel Inhelder, The Child's Conception of Space (London: Routledge and Kegan Paul, 1956), pp. 320-52.

[67]Jean Piaget, Les notions de mouvement et de vitesse chez l'enfant (Paris: Presses Universitaires de France, 1946).

[68]Jean Piaget et Bärbel Inhelder, La genèse de l'idée de hasard chez l'enfant (Paris: Presses Universitaires de France, 1951). (Hereinafter referred to as L'idée de hasard.)

[69]Ibid., p. 228.

violating a previous "good order" appears to occur sometime after seven-to-eight years. The discovery occurs, Piaget suggests, as an awareness of a contrast with the recently acquired notions of deductive necessity. It is the discovery in what had appeared before as certain, that "one cannot be sure". Deductive necessity itself was dependent on the prior acquisition of concrete class inclusion $(B = A + A'; A = B - A'; A' = B - A; B > A)$. Both chance and deductive reasoning appear to depend on the acquisition of a further inclusion relation, that of disjunction $(B = A \vee A')$. With disjunctive class inclusion one knows as a certainty that B is either A or A' (e.g., square or round). In chance reasoning, however, the outcome is seen to be insufficiently determined by the operations of addition and subtraction. A new operation, that of chance, is seen to interfere with the operations proper to the system of deduction. There are now two modalities which must be distinguished: the necessary and the possible. They are related to the disjunction of classes, and to the deductively necessary proposition: "If x is a B, it is either an A or an A'." They are also related to the "possible" proposition, "If x is a B, it can be either an A or an A'." These propositions represent two levels of reality, the possible and the necessary. There is now the problem of proportioning the possible, of making it conform in some degree to what is "real". This will require new intellectual skills, including the recognition of the unpredictability of isolated cases; an inventory of the possible and favorable cases; and relating the number of favorable to the whole of possible cases as a proportion. At the concrete level, these operations may only be initial static inclusions. They will not yet, Piaget found, take account of a changing part-whole relationship and they will not recognize irreversibility for large numbers. Reversibility will only be "very unlikely" for small numbers. The discovery of the new reality and its demands on intellectual skill may well be, as Piaget has described it, "momentarily a blow to reason". But "reason reacts sooner or later by explaining chance, and the only way to explain it, is to treat it as if it was, at least in part, composable and reversible, that is, by finding a way to make it determinable in spite of itself".[70]

Experience undoubtedly brings situations which challenge certainty in reasoning to the child's attention. The principle of transitivity is challenged when he observes that A being a friend of B, and B a friend of C, it does not necessarily follow that A is a friend of C. And not only in implicative operations do these challenges present themselves, but in spatial-temporal situations as well. What he considers cause-effect in one set of circumstances does not necessarily intersect cause and effect in another set of events. The "indetermination" which he has discovered will be subject to rational control only later when he has found that considerations of dispersion and probability permit him to assign to an event a fraction of determination.[71]

In L'idée de hasard Piaget and Inhelder have examined the evolution of the notion of chance, and the parallel development of operations of combinations and permutations and of concepts of proportion. In The Child's Conception of Space they have considered the process

---

[70] Ibid., pp. 244-45.

[71] Ibid., p. 253.

of their mutual interaction, the "choc en retour" by which operations basic to probability reasoning are established during the concrete period.[72]

In L'idée de hasard the recognition of physical chance was examined in an experiment involving the distribution of an ordered row of colored marbles resulting from the interferences of their trajectories as they were rolled from end to end of a shallow rectangular box.[73]

Subjects below seven years of age tended to expect the marbles to return to the initial "good order" on the first or after a few teeterings of the box:

> Rol (6:3) Elles resteront (à leur place).
> (The experiment was performed and there was some mixing.)
> E. Et si je continue?
> Toutes les rouges ici et toutes les blanches là.
> Oui, les rouges ici et les autres là.[74]

Somewhat older subjects ten-to-eleven-years predicted the progressive mixing of the marbles but they did not appear to understand clearly that the increasingly random distribution was the result of the multiple chance interferences of the trajectories. Each marble was represented in a drawing as following a regular and uneventful course. Collisions were not represented. These subjects did, however, represent the final result as a mixed distribution. Piaget concluded:

---

[72]Piaget and Inhelder, The Child's Conception of Space, trans. by F.J. Langdon and J.L. Lunzer (London: Routledge and Kegan Paul, 1956), pp. 320-74.

[73]Piaget, L'idée de hasard, pp. 14 and 17.

[74]Ibid., p. 20.

> ... comprendre la mutuelle dépendance de deux mouvements ... c'est sans doute en cela que consiste le caractère propre de la pensée formelle.[75]

Eleven-to-twelve-year-old children predicted continued mixing and they also represented the collisons in drawing the trajectories. They were certain that a return to the initial order would be impossible over a small number of trials but they were uncertain that this would be true for a very large number of trials. At this level the convergence of operations of chance with the discovery of operations of permutations and the "law of large numbers" has not yet occurred. The return of the marbles to an original position is "peu probable" but it could "peut-être" occur. Piaget considered that this remaining difficulty was associated with delay in the development of the recognition of permutations and combinations and of the effect of "large numbers". These operations, which are formal, are achieved only at early adolescence, so that their convergence with notions of chance will be delayed. He examined the basis for the formal nature of operations of combinations and permutations in subsequent experiments in L'idée de hasard.

Children's recognition of the characteristics of a uniform chance distribution was examined in experiments reported in L'idée de hasard. One of these was adapted for measurement at the Grade four level in the present study.[76]

In this experiment a spinner at the center of a board turns freely, and a bar on the spinner points to one of the colored sections of the board when the spinner stops. The questions concern the

---

[75] Ibid., p. 20.

[76] Ibid., p. 71.

distribution of the points of stopping of the spinner.

Subjects, up to about seven years, tended to think one could predict the color at which the spinner would stop. They knew well enough that they themselves were not likely to predict each event correctly but they thought it was legitimate to try to discover how it could be done.

The eight-to-eleven-year-olds recognized that regularity increased with the number of events, but this regularity was thought to be attained more easily for a moderate number of trials than for a great number.

Piaget explained that this restriction in reasoning about chance distributions to small numbers was similar to concrete limitations in thinking in general:

> Procédant par "operations concrètes", le sujet ne parvient à raissonner que sur des données manipulables en fait ou en pensée, c'est-à-dire visibles ou représentables dans le détail. C'est pourquoi il ne parvient pas à dépasser des ensembles de 25 à 30 coups, ... Pour appliquer le même schéma aux "grands nombres" il lui faudrait au contraire la pensée formelle, capable de déduire toutes les combinaisons ...[77]

The third level was marked by the discovery of the "law of large numbers" and the inference of the increasing regularity of the distribution.

The quantification of chance as a proportion was examined by Piaget and Inhelder in experiments requiring the subjects to predict the chances of drawing a card with a cross from a collection of cards with crosses and others without crosses. They were asked to

---

[77] Ibid., p. 93.

estimate "the better chance" for alternative combinations.[78] The operations would appear to involve the disjunctive class inclusion relation and also an interpretation of the probabilities in the relation ($B = A \vee A'$) as a function of the distribution of the number of A's and A′'s. The probability of a single event would be the proportion of the number of favorable cases to the number of possible cases expressed as a fraction:

B		
A	A	
(1)	(0)	impossibility of A′.
(1)	(1)	equal chances of A and A′.
(1)	(2)	the proportions $\frac{1}{3}$, and $\frac{2}{5}$.
(2)	(3)	

Subjects ten and eleven years were likely to conserve the whole, B, and relate the parts to the whole correctly for small numbers. Later developments in quantification of chance and probability reasoning were found to depend on operations of permutation.[79]

In the present study the ability of ten-year old children to recognize the characteristics of chance distributions, and to estimate probability as a proportion has been examined using adaptations of Piagetian studies in this area. In reading, the ability of the subjects in this study to estimate the likelihood of an exact recurrence of a sequence of independent events was examined in Story VI,

---

[78] _Ibid._, pp. 144-72.

[79] Jean Piaget, "Une expérience sur la psychologie du hasard chez l'enfant: le tirage au mort des couples," _Acta Psychologica_, VII (1950), 323-36.

"Home from School". (See Appendix E.)

With the acquisition of new modalities in thinking (modalities which include the possible and the probable as multiply determined), the child's experience of the external world would seem to become less open to explanations of experience in terms of magic, miracle, animism, participation, and even the acceptance of regularity and order may be reconsidered. These revisions could be related in complex ways to both personality development and academic achievement.

There is the question of the extent to which instruction in reading contributes to the emergence of these new and sophisticated intellectual skills.

The Acquisition of Intellectual Skills: Cognitive Operations in Reading

The central purpose of education, "the purpose which runs through and strengthens all other educational purposes", was stated in a recent publication of the National Education Association to be the development of the ability to think.[80]

In developing the ability to think in reading, instruction is generally concerned with questions requiring literal and inferential comprehension, evaluation and appreciation. These operations are required at more advanced levels as greater skill in decoding is

---

[80] Educational Policies Commission, The Central Purpose of American Education (Washington, D.C.: National Education Association, 1961).

is achieved by the child. But even during instruction in decoding when the emphasis is primarily on perceptual skills, operations in reasoning appear to be involved. Operations of conserving may be utilized in understanding that the referent of a letter, for example, of the 'm's' in remember, of 'm' in mark and Mark, is conserved with change of position in the word and with change in the form of the letter. Operations of classifying are also involved in learning to decode: letters are upper cases and lower cases; there are consonant sounds and vowel sounds; consonant sounds are voiced and voiceless, stops and others etc. Inductive and probability reasoning may be involved both in decoding and in acquiring rules for spelling. Classes are again involved in learning word structure: root, stem, affix. Suffixes may be classified as inflectional and derivational. But the development of the ability to think is usually considered to be achieved in reading through instruction in comprehension, evaluation and appreciation. The goal of instruction is said to be "the understanding and interpretation of the meanings embodied in printed symbols".[81]

The dimensions of thinking in reading are suggested in "The Barrett Taxonomy, Cognitive and Affective Dimensions of Reading Comprehension", presented by Clymer in Innovation and Change. Clymer points out that "the taxonomy cannot take into account the background which the reader brings to the comprehension tasks", and that this background "must be a deciding factor in the type or level of comprehension required

---

[81] Miles A. Tinker, Bases for Effective Reading. (Minneapolis: University of Minnesota Press, 1965), p. 39.

by the question".[82] Accepting that limitations in a reader's experience and skill in thinking are not represented, the taxonomy presents five classes of operations required in reading comprehension: literal comprehension, inferential comprehension, reorganization, evaluation and appreciation.

Literal comprehension "focuses on ideas and information which are 'explicitly' stated in the selection." If it is a simple task in literal comprehension, "recognition or recall of a single fact or incident will be required". A more complex task might be "the recognition or recall of a series of facts or the sequencing of incidents in a reading selection." Recognition questions require the student to locate details, the main idea, a sequence, a comparison, a cause and effect relationship explicitly stated by the author. Recall requires producing these responses by memory. Reorganization questions require an analysis or synthesis of the information explicitly stated in the selection. Inferential comprehension is demonstrated by a student:

> ... when he uses ideas and information explicitly stated in the selection, his intuition, and his personal experience as a basis for conjectures and hypotheses. Inferences ... may be either convergent or divergent ... he may or may not be asked to verbalize the rationale underlying his inferences.[83]

Details, main ideas, sequences, etc., are considered to be inferences if they concern what the author might have included or if they involve

---

[82]Theodore Clymer, "What is 'Reading'? Some Current Concepts", in Innovation and Change in Reading Instruction, the Sixty-seventh Yearbook of the National Society for the Study of Education, Part II, edited by Helen M. Robinson (Chicago: University of Chicago Press, 1968), pp. 7-29. (Hereinafter referred to as Innovation and Change.)

[83]Ibid., p. 21.

hypothesizing about what might have happened, what might happen next, the motivations of characters, or the literal meanings of figurative language used by the author.

Evaluation requires a value judgment by the reader of the ideas of the author. Judgment may be by reference to external criteria (other authority) or internal criteria (the reader's experiences, knowledge, values).

Appreciation involves all of these cognitive dimensions and also "the psychological and aesthetic impact of the selection on the reader."[84]

Appropriate questions designed to elicit responses in each of these areas are considered likely to ensure comprehension of the printed communication. Limitations in a specific instance will be due to deficiencies in the background of experience and in the skills in thinking the reader brings to the comprehension task. A question of interest is the nature of these skills in thinking and of the conditions of their development.

Guilford, in his studies over the past twenty years has been concerned to systemize "the abilities having to do with intellectual activities", and he has also considered the teaching of reading in relation to these abilities. Using the methods of factor analysis he had identified some sixty different intellectual abilities. He has found that these can be grouped into "five classes of abilities depending on the basic kind of operation involved". The five classes are:

---

[84] Ibid., p. 22.

> Cognitive abilities associated with the discovery or recognition of information.
>
> A group of abilities having to do with the retention of information.
>
> Two groups of abilities concerned with productive thinking: divergent production in which the object is to produce a variety of logically possible ideas which could follow from given information; and convergent production in which the conclusions arrived at are completely determined by the given information.
>
> A group of abilities having to do with the critical evaluation of information recalled, and of conclusions reached in thinking.[85]

Some information is concrete, some abstract, some concerns the behavior of ourselves and others. Information operated on by a thinker yields products: units, classes, relations, systems, transformations, and implications.

The three categories, abilities, information (contents), and products, interact and their interaction is represented by Guilford in a three-dimensional model for the structure of intellect. In this structure the interaction of a certain kind of operation, a certain kind of content and a certain kind of product is represented by a single cell. These intellectual abilities, Guilford states, may also be regarded as intellectual functions, and reading, when fully developed involves many of these intellectual abilities. A pupil's intellect may be looked upon "as being an organized collection of distinguishable skills, each with certain properties". There is then the possibility of deciding "what kinds of exercises are needed to develop these skills".[86]

---

[85] J.P. Guilford, "Frontiers in Thinking that Teachers Should Know About", The Reading Teacher, XIII, No. 3 (February, 1960), 176-82.

[86] *Ibid.*, p. 179.

There is little doubt, Guilford states, that the teacher of reading has a wealth of opportunities to teach the child to think.

In the development of operations of thinking Guilford suggests that the "comprehension of principles" is essential, principles abstracted by the thinker from the details of particular situations:

> Comprehension of principles is a matter of cognition and takes us at once beyond the stimulus - response model.[87]

Concern for the comprehension of principles of thinking as an outcome of instruction in reading would seem to constitute an important distinction between Guilford's conception of the function of reading instruction and the goal of reading comprehension presented as "the understanding and interpretation of the meanings embodied in printed symbols", their evaluation and appreciation. The focus of the latter appears to be on a reader applying the thinking skills available to him to an instance at hand. Guilford's point of view suggests that instruction in reading may have the potential of serving a further function: the development of cognitive abilities including the ability to recognize principles of thinking as applicable beyond particular situations: the ability to go beyond the products of thinking (units, classes, relations, systems, transformations, implications) to an awareness of the generality involved in their production.

Spache appears to consider the function of instruction in reading comprehension in similar terms. He related his concept of the function of reading to the work of Guilford, and also to the work of Piaget, Newell, Hunt, Gardner, Getzels, Torrance and others whose

---

[87] Ibid.

ideas of cognitive development he specifically refers to in discussing the contributions of allied fields to the teaching of reading. Comprehension in reading requires, and instruction in reading is capable, in Spache's view, of developing the potentialities of the child, including his ability to achieve "strategies of logic". Spache discusses the applicability of the findings of basic research to achieving children's potential through reading instruction:

> Since comprehension is generally recognized as a cognitive or information processing system, direct application of much of this basic research to the teaching of this process is possible. There is the distinct possibility that comprehension develops through a series of what are as yet unrecognized stages. Some of these might involve strategies of logic, such as those we now label word recognition, main ideas, details, inferences, critical reading, and the like. The first step would be to define these behaviors operationally ... and then to study carefully the chronology of their appearance ... A second step suggested by the implications of the research in cognitive development is the exploration of the programing of the child's learning or development of comprehension ... Somewhere in this research program we should include a careful study of cognitive styles in comprehension as well as of the influence of personality traits and the set of the reader.[88]

The importance of questioning and other teaching methods in determining if "strategies of logic" and other principles of thinking will be developed through instruction in reading comprehension has been demonstrated in a number of recent studies.

Huack, using gifted grade-six students as subjects, obtained evidence suggesting that evaluative abilities improved significantly with special instruction in which feedback provided by the teacher was directed toward thinking processes rather than toward responses or

---

[88] George D. Spache, "Contributions of Allied Fields to the Teaching of Reading", in Innovation and Change, ed. by Helen M. Robinson, pp. 237-90.

products of thinking.[89] For this study Huack prepared special materials and unique lesson plans. Guilford also considered that provocative materials and skillful questioning would be needed to exercise intellectual abilities.

Guszak has recently studied the characteristics of teachers' questions and pupils' responses during instruction in reading comprehension. He used four randomly selected classes at each of three grade levels (two, four and six). Question - response exchanges were recorded over a three-day period, then transcribed and analysed.

Guszak was interested in the range of cognitive operations which would be required in responding to the teachers' questions, and the level of reasoning represented in the pupils' responses.

He pointed out that previous evidence suggested that teachers' questions tended to concentrate on literal comprehension to the relative neglect of "higher level comprehension abilities". Guszak's first research question was:

> What kinds of thinking outcomes are teachers seeking to stimulate with their oral questions about reading content?[90]

---

[89]Barbara B. Huack, "A Comparison of Gains in Evaluation Ability Between Gifted and Nongifted Sixth Grade Students", Gifted Child Quarterly, XI (Autumn, 1967), 166-71.

[90]Frank L. Guszak, "Teachers Questions and Levels of Reading Comprehension", in Perspectives in Reading No. 8, The Evaluation of Children's Achievement, edited by Thomas C. Barrett (Newark, Delaware: International Reading Association, 1967), pp. 97-109.

To describe the thinking modes represented by the teachers' questions, a Reading Comprehension Question - Response Inventory was developed. Six categories of questions were identified:

Recognition	:	locating information given in the text.
Recall	:	recalling information given in the text.
Translation	:	demonstrating literal understanding by giving an objective, part-for-part rendering of a communication.
Conjecture	:	anticipating what may happen next in the story. A rationale for the prediction is not required.
Explanation	:	inferring a rationale for a situation from context; inferring a conclusion; stating a main idea.
Evaluation	:	a judgment of worth, acceptability, probability is required. This may be presented in terms of good-bad; yes-no; most likely; this-that. In general an explanation or justification of the value assigned was not solicited.

A total of 1857 question - response sequences were collected in the study. It was found that approximately 70 per cent of teacher questions involved recalling or locating items of information given in the text; .6 per cent required translating information in the text. Inferential type questions (conjecture and explanation) accounted for approximately 14 per cent of the question - response sequences. Evaluation accounted for 15 per cent of these exchanges.

Pupil responses to the questions presented were correct at the 90.4 per cent level. This may not be surprising when it is considered that most of the answers were available on the printed page. Guszak comments on his findings:

> The dramatic finding is the revelation that a single recall solicitation followed by the single congruent recall response is the dominant interaction pattern. From this combination it is apparent that teachers ... are conditioning pupil thinking to the point that they will

respond with a simple fact.[91]

In a recent study Robertson identified a critical factor affecting pupil understanding of printed materials: their knowledge of the meanings of connectives, the small words which link sentences in subordinate and coordinate relations, and link one sentence to another. The connectives of interest in the study included the following: 'if', 'so', 'that', 'when', 'which', 'who', 'and', 'but', 'for', 'yet', 'however', 'although', 'thus', and the "absent" connective for which 'that' could be inserted. For this study a sample of 402 children aged eight to twelve, in grades four to six, was chosen from a stratified random sample drawn proportionate to the larger population strata.

The total student group in grades four to six understood 67 per cent of the sentences having connectives. The understanding level of the Grade four children was 57 per cent; for Grade five children it was 66 per cent. The test sentences were selected from basal reader stories. Robertson concluded:

> Since basal reader stories are materials used at an instructional level, it appears that student comprehension in grades four and five is too low. For independent reading materials such as textbooks in science ... this comprehension level is very low ...[92]

Robertson noted that although children use clauses in speech before they go to school, their understanding of connectives in print is insufficient for a number of years after that.

---

[91] Ibid., p. 108.

[92] Jean E. Robertson, "Pupil understanding of Connectives in Reading", in Forging Ahead in Reading, Vol. XII, Part I., of Proceedings of the Twelfth Annual Convention, International Reading Association (Newark, Delaware: International Reading Association, 1968), p. 588.

Attention to the teaching of the meanings of these connectives would appear also to involve children in understanding relations which hold between propositions, including relations of conjunction, disjunction, implication, and equivalence, as well as possible, not-possible, probable and contingent relations and others. These relations would seem to be among the basic logical operations required for reading comprehension during the elementary school years. Children who master some of these basic skills in thinking in reading may be those who are able to move with confidence toward the acquisition of complex intellectual operations characteristic of the formal level of reasoning.

Summary: Review of Related Research

The research under review describing the cognitive development of children during the middle concrete period presents a vivid account of emerging cognitive skills. Eight, nine and ten year old children are seen to be making progress in recognizing invariance across a variety of changes in form and distribution of the material. They are abstracting the properties of objects as a basis for classification and recognizing relations of inclusion between classes. Nine-and-ten-year-olds are able to quantify some of these relations. They also construct propositions of many forms: conjunctive, disjunctive, implicative and equivalent, and in the research reported they related these propositions by rules of logic. Operations involving inductive reasoning appear to be available to some extent at this level with the beginning of notions of chance, though from Piaget's investigations the latter appear to be limited to operations involving small numbers.

Most of these children also read, and appear to be able to respond with a high degree of accuracy to questions concerning the content of what they have read.

The operations described in most of the research under review involve the thinking of children when they are presented with specific conditions concerning real objects: concrete level thinking. The child is asked to consider what is true at this moment, for these materials, under these circumstances. It seems that he is likely to consider the conclusions he offers as "sometimes" true: there is lag in his ability to apply a solution available in one concrete situation to a problem in a related concrete situation.

Reading would appear to offer the child his first opportunity to venture well beyond the concrete situation, to try to think in the absence of the concrete. The research presented in reading instruction suggests however, that there may be considerable hesitancy in urging the child beyond concrete operations in reading. There is an expressed concern that concrete experience should be kept equal to the task of comprehension. Questions suggested in assessing comprehension appear to stress the recognition of what is explicit in the printed message.

An important developmental task of the concrete level child is, however, to escape the obligations and limitations inherent in his concrete thinking and begin to recognize the universality of the principles of reasoning he is using. An attempt will be made in this study to consider some of the difficulties experienced by children in accomplishing the transition to the more abstract level of operations.

CHAPTER III

DESIGN OF THE STUDY

This chapter will describe the design of the study. It will include a description of the population and the sample, an account of the test instruments administered, and a discussion of the reliability and validity of the tests. Procedures in collecting the data of the pilot study and of the main study, and the statistical treatment of the data are also described.

The Population

The population in this study was 514 children attending Grade four classes in the City of Moose Jaw, Moose Jaw School District No. 1, Saskatchewan, in June, 1966. The classes were located in the thirteen elementary schools in the system. The population from which the sample was drawn did not include children who had been transferred to ungraded classes for the retarded, children in a special school for the educable retarded, children who had been decelerated (remaining in Grade three) and children who had been accelerated (transferred to Grade five from Grade three). Included in the population were children who were older and had been decelerated (the second year in Grade four) and younger children who had been accelerated. Each of the last two groups would represent approximately ten per cent of the population. The sample was therefore drawn from a school population for whom the Grade four curriculum was considered appropriate.

Pupils completing the Grade four year of schooling were selected for this study for two reasons. At this level the children could be expected to have a reasonable competency in the decoding aspect of reading: a study of reading comprehension could be conducted in such a way that errors in word recognition and difficulty with basic sentence structure would be minimized as factors affecting the level of comprehension of the reading materials presented.

A second consideration was of importance for this study: the period was considered critical in the acquisition of cognitive skills. These children had completed their primary education; they were already consolidating the intellectual operations characteristic of the concrete phase of development in preparation for the major advances of early adolescence. It is important that children at this period should be prepared to move forward from a position of strength. Academic progress from this time will depend more and more on the range and quality of thinking skills. It is at this time, too, that thinking skills developed in concrete situations are becoming available in reading. It is essential that they do so to meet the increasing demands of school learning. It is considered possible that factors contributing to underachievement in the later elementary school years could be identified at this earlier period as retardation in specific cognitive skills and in particular in the application of these skills in reading.

The City of Moose Jaw, selected for the study, is primarily a service centre, serving south central and south western Saskatchewan. There are also a number of major industries in the area, as well as a provincial institution for the mentally retarded, a Provincial Technical Institute, and a Department of National Defense air base. A number of ethnic groups are represented in the population including French, English, Ukrainian, and German. The total population is approximately 35,000.[1]

The Sample

The sample for the study consisted of 100 subjects, 50 boys and 50 girls, randomly selected from the Grade four population. The original sample selected included 108 subjects. Eight protocols were discarded as incomplete.

The age range of the subjects was eight-years-seven-months to eleven-years-two-months (one subject at each extreme of the range). The average age was nine-years-eight-months.

The V (Verbal-Meaning) test and the R (Reasoning) test of the SRA Primary Mental Abilities tests Ages 7 - 11 - Form AH[2] had been administered to the Grade four population seven months prior to the present study, in November, 1965. "IQ Estimates" had been obtained using the formula, 2V + R, where V and R are expressed in raw scores,

---

[1]Letter from Wilfrid W. Toombs, Director and Superintendent, Moose Jaw Public Schools.

[2]SRA Primary Mental Abilities Tests, Ages 7-11 - Form AH (Chicago, Ill.: Science Research Associates, 1947).

V is doubled and then added to the R score. The results were available for 95 of the 100 subjects in the sample (five subjects in the sample were not present for this test). The distribution of the scores on this test for the subjects in this study for whom scores were available is presented in Appendix I. The mean "IQ Estimate" of these subjects was 109.7. The median "IQ Estimate" for the Grade four population examined in November, 1965 (N = 475) was 112.

Scores were also available for most of the subjects in the sample on the Iowa Arithmetic Group Test (N = 92), on the Iowa Language Group Test (N = 94), and on the Gates Reading Survey (N = 98). The distribution of scores on these tests are also shown in Appendix I. On the Gates Reading Survey, administered in May, 1965, approximately one year prior to the present study, 82 of the 98 subjects in the sample for whom scores were available scored at or above grade placement; 16 subjects scored four months to nine months below grade placement (below Grade three and nine months in May, 1965). These results suggest that the reading levels of the subjects in the sample may be considered adequate under the conditions of administering the Stories tests (described below) to allow a fair assessment of reading comprehension.

The Experimental Tests

Three types of tests were administered to subjects in this study: a standardized test of reading comprehension; Concrete tests of cognitive operations, CCO; and Stories tests of cognitive operations, SCO.

The Concrete and Stories tests were constructed by the investigator. Both tests were designed to examine cognitive operations at the concrete level of intellectual development as defined by Piaget. The Concrete tests present test items using objects as stimuli (concrete objects); the Stories tests present test items using printed materials as stimuli (reading).

The standardized test of reading comprehension selected was the Cooperative Sequential Tests of Educational Progress, Reading, Form 4B[3] (see Appendix G). The STEP Reading tests have been reviewed by Jackson in The Fifth Mental Measurements Yearbook.[4] He considered that the test items in this series of tests were on the whole well constructed; that they attempted to measure the student's ability to apply his learning in problems of practical consequence. He concluded that technically the STEP series was one of the best available. He added that more complete information on reliability and validity was needed.

Wiseman, who reviewed the reading tests in the STEP series considered that they were useful and efficient. He indicated that factor analysis or other evidence on construct validity and reliability data in addition to calculations based on Kuder-Richardson formula 20, should be made available.[5]

---

[3]Cooperative Sequential Tests of Educational Progress, (Princeton, N.J.: Cooperative Test Division, Educational Testing Service, 1957).

[4]Robert W. B. Jackson, in The Fifth Mental Measurements Yearbook, ed. by O. K. Buros (Highland Press, N.J.: The Gryphon Press), pp. 62-67.

[5]Stephen Wiseman, The Fifth Mental Measurements Yearbook, pp. 752-54.

STEP Reading, Form 4B presents seventy multiple-choice items designed to assess abilities to reproduce ideas, translate ideas, make inferences, analyse motivation and the style of presentation, and to criticize. Separate scores for these categories are not given; a single score represents the number of items answered correctly.

The Concrete and Stories tests, denoted by the symbols CCO and SCO, each consist of a series of test items designed to elicit samples of children's thinking in the five categories of cognitive operations selected for study: conservation, classification, deduction, induction and probability reasoning. The test items CCO and SCO are in general constructed as corresponding pairs. The materials for the Concrete tests were constructed commercially. The stories for the SCO tests are presented in Appendixes A to E.

Collection of Data

All tests were administered during the period May 28 to June 28, 1966. The STEP Reading test was administered to subjects in their own classrooms with the assistance of the Head, Reading and Instructional Services and of the classroom teacher. The Concrete and Stories tests were administered to each subject individually in rooms in his own school assigned by the Principal of the school for this purpose. The investigator was assisted in administering the Stories tests by the Head of the Reading and Instructional Services, and in administering one part in each of two sequences of the Concrete tests (the "Village" tests described in Chapter VI and test items V and VI of the Probability tests described in Chapter VIII) by a test administrator selected by the Director and Superintendent.

Administration of the Tests

The tests were administered in the sequence STEP Reading, the Stories tests, the Concrete tests. Each test series was completed before the next series of tests was administered.

The test items for the Stories tests were administered to all subjects one to two weeks before administering the Concrete tests. It was considered that there was less likelihood of a transfer of responses if the Stories tests were administered prior to the Concrete tests. The variety of incident in the seven stories, together with the time lapse was considered to contribute to reducing the tendency to recognize comparability in the two situations.

The stories were presented in the sequence I to VII to each subject. The subject read a story silently, and when he indicated he had finished he was taken to an adjoining room for the test interview. The interview consisted of the preliminary questions and the test questions. The interviews were tape recorded and the protocols were typed from the tapes according to instructions from the investigator.

The seven stories required approximately forty-five minutes of interview time for each subject.

The test materials for the Concrete tests were arranged at five "stations" in a semi-circle in the testing room. The subject and the investigator moved from one station to the next, following the test sequence conservation, classification, deduction, probability and induction. Two tests within this sequence were administered in an adjoining room (the "Village" test and the test of uniform distribution). Each subject completed the Concrete tests in one interview of approximately forty-five minutes. A number of interviews were conducted on Saturdays in order to complete the tests by the end of June.

The Pilot Study

The three tests, STEP Reading, and the Concrete and Stories tests were administered to seventeen subjects, eleven girls and seven boys, in May 1966, prior to the main study. The subjects for the pilot study were attending Grade four classes in an elementary school in the City of Edmonton, Alberta. They were tested individually by the investigator at the Education Clinic, Faculty of Education, University of Alberta. The test interviews were tape-recorded and typed as described for the main study.

The subjects were selected by the Principal of the school as being representative of the Grade four pupils. Scores for these pupils on tests of intelligence were at various levels within the range 95 to 132.

A number of adjustments were made in the form of the test items as a result of experience in presenting these items in the pilot study. Two test items were omitted from the tests of random distribution on the basis that the time required in administering these items was out of proportion to the information obtained. Two or three test items were added to each of the Stories tests to obtain greater comparability with the Concrete tests. Slight modifications were made in the stories themselves to obtain a lower readability level.

In a first attempt to score the protocols of the pilot study it was clear that more specific criteria for assessing the responses were needed. The criteria adopted are discussed in Chapters IV to VIII. The protocols of the pilot study were re-scored on the basis of the adjusted criteria and a trial run of the data was obtained. The results are presented in Appendix H.

A number of significant relations were obtained between Concrete and Stories variables and between some of these variables and STEP Reading total scores. The results of this trial run of the data suggested that a statistical analysis of data obtained on a random sample of a Grade four population could reveal relationships between logical operations in concrete and in reading situations which might contribute to an understanding of problems of comprehension in reading.

Reliability of the Tests

The reliability of the STEP Reading test was determined by calculations based on the Kuder-Richardson formula 20 using total scores obtained by the 100 subjects in the main study. The mean score was 48.01; the variance was 74.47. The reliability coefficient obtained was $r_{xx} = .85$.

The reliability of the Stories and Concrete tests for the main study was estimated by the test-retest method, and coefficients of correlation were obtained. Subjects were retested after an interval of two weeks. The Concrete tests had not been administered during this period to the subjects retested on the Stories tests. The retest group for the Concrete tests had previously completed both test sequences. The reliability test results are presented in Table 4.

TABLE 4.- Means, standard deviations and test-retest reliability coefficients for total scores CCO and SCO.

Test	$\bar{X}_I$	SD_I	$\bar{X}_{II}$	SD_{II}	n-1	r_{xx}
CCO	44.25	7.03	46.67	5.85	12	.74
SCO	30.00	7.67	30.92	7.93	12	.72

A number of factors may have affected the reliability obtained. For the CCO classification test, instruction had been provided, if needed, for test items I and II in the first test situation in order that the subject could attempt succeeding items. These test items were found to be correctly answered on retest. A similar situation occurred for the CCO tests of probability. There was also some discussion among the children who had been intrigued with the effect of the magnet on the spinner (CCO induction) during the first test and this may have influenced the retest results. It was observed also that children frequently asked the examiner for the correct solutions for Stories test items. These requests were refused as politely as possible but teachers reported that children continued to discuss questions on the Stories when they returned to the classroom.

Readability Levels of the Stories

The readability level of each of the seven stories in Stories tests was estimated using the Dale-Chall Readability Formula.[6, 7] The results for each story are presented in Table 5. The calculations in each case are based on a complete story text. The grade placement of the subjects at the time of testing was Grade four and nine months.

TABLE 5.-Levels of readability of the stories in the Stories tests predicted by the Dale-Chall Readability Formula.

Story	Average Sentence Length	Dale Score	Readability Grade Score
Story I (Race)	12	2	4.55
Story II (Birds)	13	2	4.62
Story III (Ducks)	12	5	5.02
Story IV (City)	12	1	4.39
Story V (Blizzard)	12	2	4.55
Story VI (Home)	12	4	4.86
Story VII (Cave)	10	3	4.61

6 Edgar Dale and Jeanne S. Chall, "A Formula for Predicting Readability: Instructions," Educational Research Bulletin, XXVII, (February 19, 1948), 37-54.

7 George R. Klare, "A Table for Rapid Determination of Dale-Chall Readability Scores," Educational Research Bulletin, (February 13, 1952), 43-47.

It appears that the readability levels of the stories is at or below the grade placement of the children in the sample with the exception of Story III. Preliminary questioning was designed to ensure that each subject was able to report correctly the information in the stories. These questions appeared to contribute mainly to a more accurate recall of detail and of the sequence of incidents. The assistance required with decoding was negligible. The procedure adopted if an item of information was not recalled was to instruct the subject to read aloud that part of the story. The purpose was to insure as far as possible that errors in decoding did not contribute to reducing the level of performance of the subjects on test items requiring the cognitive operations under study.

Reliability of the Scoring

Two markers, the investigator, and a research assistant on the staff of the Ontario Institute for Studies in Education, scored independently the protocols of a random sample of ten subjects. The Kendall Coefficient of Concordance was calculated for each of the ten Concrete and Stories tests. Spearman's coefficient of rank correlation (ρ) is related to Kendall's coefficient of concordance by the formula $\rho = 2W-1$ for the particular case where $m = 2$.[8] The significance of ρ is obtained by using a t given by

$$t = \rho\sqrt{\frac{N-2}{1-\rho^2}}$$ [9]

These calculations are shown in Table 6.

[8] George A. Ferguson, Statistical Analysis in Psychology and Education (2nd ed., New York: McGraw-Hill, 1966), p. 227.

[9] Ibid., p. 220.

TABLE 6.- Kendall coefficients of concordance (W) and Spearman's rank-order correlation coefficients (ρ) for the ten Concrete and Stories tests. (N = 10; m = 2)

Test	W	ρ (2W-1)	$t^{*} = \rho\sqrt{\frac{N-2}{1-\rho^2}}$	p two-tailed test
CCO-Con	.93	.86	4.73	< .01
CCO-Cl	.93	.86	4.73	< .01
CCO-D	.88	.76	3.30	< .02
CCO-P	.91	.82	4.03	< .01
CCO-I	.96	.92	6.63	< .001
SCO-Con	.98	.96	9.68	< .001
SCO-CI	.76	.52	1.72	n.s.
SCO-D	.92	.84	4.35	< .01
SCO-I	.91	.82	4.03	< .01
SCO-P	.89	.78	3.90	< .01

*t = 3.35 p <.01(two-tailed test); df = N - 2

It would appear from the results presented in Table 6 that the criteria for scoring these test items (see Chaps. IV to VIII) could be applied independently with reasonable uniformity with the exception of the criteria for items assessing Stories classification (see Chap. V).

Validity of the Concrete and Stories Tests

The question of the validity of the Concrete and Stories tests requires a consideration of the validity of the test items: the validity of their design for assessing logical operations and their construct validity. The validity of the test situations selected in each of categories for eliciting the cognitive operations under study also requires consideration.

Validity of the design of the test items

Smedslund, in the monograph Concrete Reasoning, discusses a number of problems in the construction of items for testing reasoning from the point of view of ensuring that these items will yield valid results.[10] The rules he proposes are for tests of reasoning in concrete situations, that is, in situations which present concrete stimuli. The rules proposed by Smedslund for ensuring valid results from tests of reasoning using concrete stimuli have been considered in this study in the construction of test items to assess reasoning when the stimuli are printed symbols, in the Stories tests. These rules will now be summarized and discussed in relation to the CCO and SCO test items.

Rule 1, proposed by Smedslund, distinguishes between a perceptual and a cognitive solution for a problem:

> Rule 1. The tasks should not be solvable on the basis of perceptual processes. This can be assured if the initial events are absent at the moment of solution.[11]

[10] Jan Smedslund, Concrete Reasoning: A Study of Intellectual Development, Monograph of the Society for Research in Child Development, Serial No. 93, XXIX, No. 2 (Yellow Springs, O.: The Antioch Press, 1964).

[11] Ibid.

In the administration of the Concrete test items the materials are "covered" during both the presentation of the test questions and the responses of the subject to these questions. When actual covering is not feasible, for example in the "Village" and "Lights" tests, the presentation and manipulation of the test materials in no case suggests the solution to a problem. A response under these conditions has been called a "decision." A decision appears to require what Piaget has called an "anticipatory schema."

Preliminary questions for the Stories tests are designed to ensure that subjects have decoded efficiently. Solutions to the test questions are not, however given in the text of story. The questions may not, therefore, be solved on the basis of decoding and recall. The children frequently pointed to this condition during testing. They would remark, "It didn't tell you that in the story." Although the text of a story was not removed during questioning only two or three subjects considered consulting it. It appeared that they recognized that rereading the text would not provide the answer. It is therefore considered that a decision, as defined, is required in each instance for a story test item.

> Rule 2. The tasks should not be solvable on the basis of readily available hypotheses with a non-logical structure.[12]

This rule requires the distinction in test construction between part-whole, spatial, time, and many psychological relations and logical or inferential patterns of reasoning. Non-logical operations are frequently involved in recognizing cause-effect relationships,

---

[12]*Ibid*.

the motivations of characters in a story, style in a communication, certain ethical concepts such as "fair play," and in general all such relations as "before," "next," "under," "as far as," etc. Non-logical operations are also required in responding to certain "which" questions: "Which reply did a character in a story make?"

A deductive logical structure, on the other hand, consists of premises and necessary conclusion. A test item designed to require such a logical operation for its solution must not be capable of being resolved by an alternative non-logical operation available from experience: for example, that a hot stove will burn if it is touched; that a deep cut will bleed more than a slight cut.

> Rule 3. The possibility of being correct by guessing should be minimized....[13]

To meet this criterion, both a correct decision and a correct explanation of the decision have been required to obtain a score on a test item. In the case of a correct decision in the tests of conservation in which three alternatives are offered (the same, more, less) the correct decision is scored in addition to scores for correct explanations.

> Rule 4. All information available to the subject should be in the form of perceived events. Verbally communicated hypothetical premises should be avoided.[14]

In the Concrete tests the basic premises, for example, the initial relation of equality of the objects, are determined experi-

---

[13] Ibid.

[14] Ibid.

mentally by the subject. In the Stories tests these basic conditions are recalled as read. Hypothetical premises of the form, "if . . . then", from which a solution could be derived, are not suggested in either test sequence.

> Rule 5. It must be ensured that the subject perceives the relevant events.[15]

In the Concrete tests, the subject is required by the preliminary questions to label the material ("That's a red square."); to state a relevant event ("The balls of plasticine are the same now; I weighed them"); and to report the relevant conditions ("If the red light is on, the green light is on"). In the Stories tests the preliminary questions appear to have a similar function. In each situation, the "set" suggested to the subject would seem to be that he consider the information available in arriving at the solutions required and that the relevant information will be the observed and reported facts and conditions.

In the Stories tests oral rereading is required as necessary, to ensure as far as possible that responses to the test questions will represent abilities in the cognitive operations of interest and that these abilities will not be delimited by accidental failures in decoding or recall.

> Rule 6. There should be no differential reinforcement during the test. Every response should get the same mild positive reinforcement.[16]

Throughout the Concrete and Stories tests answers to all test

---

[15] Ibid.

[16] Ibid.

items, right or wrong, are accepted as the considered opinions of the subject. Differential reinforcement is further avoided by requiring an explanation for both correct and incorrect decisions. The subject in either case is asked, "How did you know that?" "Why is that?"

It is considered that this procedure was reasonably successful in maintaining morale since children appeared to defend with equal confidence their correct and incorrect responses. Explanations of non-conserving, non-classifying and other non-logical decisions are valuable diagnostically, as Piaget has demonstrated. These explanations permit a clinical orientation which does not appear to invalidate succeeding test items since instruction or correction does not follow these explanations (the exceptions noted above were considered to affect reliability rather than validity). In several instances, particularly during the Stories tests, correction was requested. The request was "postponed" as politely as possible.

> Rule 7. The same type of materials should be used throughout the items as far as possible, in order to keep constant any effects of the type of materials.[17]

To meet the criterion of percept constancy, the concrete stimulus materials have been kept uniform as far as possible for the test items within each category of cognitive operation. In the conservation tests, for example, materials and procedures are uniform for items testing conservation of substance, weight and volume at levels of complexity I and II. The tests at level of complexity III are based on a single alternative set of materials. For other test sequences

---

[17] *Ibid.*

the material is uniform throughout. In the Stories tests one story situation is, in general, the unit for testing each category of cognitive operation. In some instances two stories are provided. The test items are then repeated for each story.

Percept constancy "between" corresponding Concrete and Story test items has been more difficult to maintain. For the level of sophistication of the children in this sample the comparability of the materials for the corresponding tests would appear to be adequate. The materials in each instance are familiar: clay, sugar, wax, popcorn. In addition, comparability of percept attainment is supported by the procedure of preliminary questioning. The concern in these questions is specifically with the subject's "noticing"[18] and his attention is directed to the information necessary in responding to the goal presented. The questions are not phrased to be "distractors," that is, they do not direct attention to details which "could be noticed" in these situations but which are not relevant to the correct solutions.

It is considered that principles recognized as essential in the construction of tests of reasoning have been satisfactorily followed in designing the Concrete and Stories tests.

Construct validity of the test items

The questions of importance in considering the construct validity of test items for assessing logical operations would appear to be the following:

---

[18] Bertrand Russell, An Inquiry into Meaning and Truth (Great Britain: Penguin Books, 1962), p. 311.

To what may a measure of logical operations be related? How well do the tasks of the tests represent what are considered to be important outcomes in this area?[19]

The blueprint against which to match the tasks of the tests would seem to be the judgment of logicians concerning the structure of the logical sequences under study. Logicians represent these logical sequences in symbolic form, abstracting form from content. For deductive inferences, for example, symbolic formulae represent premises, the rules of logic by which one may validly proceed from premises to a conclusion, and the necessary conclusion. Tests designed to assess deductive reasoning should then be capable of identical representation in symbolic form. Tests designed to assess operations in classification, and in inductive, and probability reasoning should also be capable of such logical representation. The question of the validity of a test task may thus be referred to the correspondence of its symbolic representation to the accepted logical representation for that operation. A test task, for example, which purported to assess an inference derived by the rule Modus Ponens would be considered to have construct validity if the solution was based on the schema $A \;\&\; (A \supset B) \supset B$. Tasks involving class inclusion would be considered to have construct validity if they conformed to recognized principles of class inclusion, such as:

$$(\hat{F} \subset \hat{G}) \supset (x){:}\ \ (x \in F) \;\&\; (x \in G).$$

---

[19]Robert L. Thorndike and Elizabeth Hagen, Measurement and Evaluation in Psychology and Education (2nd ed.; New York: John Wiley & Sons, 1961), p. 172.

An attempt will be made in Chapters IV to VIII to judge the construct validity of the Concrete and Stories test items by this procedure: representing and relating the logical form of the test item to the symbolic representation of the inference it is designed to assess.

It may not be assumed that the thinking which yields logical solutions for the tasks presented by the test items is identical with these symbolic representations. The justifications presented for a logical conclusion will refer to the classes, propositions, and relations represented by the symbols. But the process involved in the subject's thinking (which is not observable), and his verbal presentation justifying his conclusion will follow its own sequence. Justification of an argument may, for example, be presented in the order conclusion, premises, rule of logic, rather than in the accepted logical order. Construct validity does not depend on the patterns of the justifications presented for correct solutions; it is considered to depend on a judgment, based on a symbolic representation, that the task of a test is identical with the logical operation as described by logicians. This correspondence will be examined in the description of the construction of the test items in Chapters IV to VIII.

Validity of the test situations

In addition to the problem of the design and construct validity of the test items, there is the question of the validity of the test situations selected for eliciting responses in each of the categories of cognitive operation of interest. The questions of concern are, "Do the tests of multiplicative classification require the construction of a new class of intersection?" and so on for each of the test situations selected.

It is suggested that for tests of concrete reasoning the validity of the test situations is related to the degree of continuity that has been maintained between the present study and previous studies in these aspects of cognitive development. The situations for testing concrete operations in this study have been adapted from tests originally designed by Piaget and others who have examined his findings. They have been adapted for purposes of measurement at the Grade four level. Modifications suggested by previous research have also, in a number of instances, been adopted. The selection and modifications of the situations have necessarily been influenced by problems of feasibility in a school situation.

Critical appraisals of Piaget's experiments have pointed to the necessity for replications of his studies with statistical controls, cross-cultural controls, and in situations involving reinforcement schedules. Alternative interpretations of Piaget's findings have also been suggested. These appraisals do not appear, however, to have included a serious question of the power of the Piagetian experiments to elicit the particular behavior the experiments were designed to assess.

Some of the Piagetian experiments on which the Concrete test sequences in this study have been based, are available in English language publications, others in French language publications. The latter include the tests of induction,[20] of probability,[21] and the tests of conservation of substance, weight and volume.[22] Replications of Piaget's experiments in conservation have been conducted by Lovell and Ogilvie with British children,[23, 24, 25] and by Elkind with American children.[26] Modifications have been introduced by Smedslund in a series of studies of the acquisition of conservation.[27] It is considered that the Concrete tests of conservation of substance, weight and volume adapted in this study for testing at the Grade four level closely parallel these earlier studies and in some instances a tentative comparison of results may be possible.

The validity of the Stories test situations may be considered to depend on the comparability of the responses required by the test

---

[20] Jean Piaget et Bärbel Inhelder, La genèse de l'idée de hasard chez l'enfant (Paris: Presses Universitaires de France, 1951).

[21] Ibid.

[22] Piaget et Inhelder, Le développement des quantités physiques chez l'enfant (Neuchatel: Delachaux et Niestlé, 1962).

[23] K. Lovell and E. Ogilvie, "A Study of the Conservation of Substance in the Junior School Child," British Journal of Educational Psychology, III (1960), 109-18.

[24] K. Lovell and E. Ogilvie, "A Study of Conservation of Weight in the Junior School Child," British Journal of Educational Psychology, XXXI (1961), 138-44.

[25] K. Lovell and E. Ogilvie "The Growth of the Concept of Volume in the Junior High School Child," Journal of Child Psychology and Psychiatry, II (1961), 118-26.

[26] D. Elkind, "Children's Discovery of the Conservation of Mass, Weight and Volume. Piaget Replication Study II," Journal of Genetic Psychology, XCVIII (1961), 219-27.

[27] Jan Smedslund, "The Acquisition of Conservation of Substance and Weight in Children." Scandinavian Journal of Psychology, II, IV, V 91961); 11-20; 71-84; 153-55; 156-60.

items in these situations with those required in the corresponding Concrete test situations. This comparability will be examined for initial premises, decisions and inference patterns for tests of conservation in Chapter IV. The forms of reasoning required by the Concrete and Stories tests of classification, deduction, induction and probability should, in both test sequences, not only be of precisely the form identified in scholarly works in logic, but also comparable for corresponding test items, comparable in form as distinct from content. The extent of this comparability will be considered in Chapters V to VIII.

Summary: validity of the tests CCO and SCO

The validity of the design of the test items CCO and SCO, considered in relation to suggested principles for the construction of tests of reasoning, was judged to be satisfactory. The construct validity of the test items was considered to depend on the correspondence of the logical structures of the solutions required by the tasks and logical forms of these solutions as identified by logicians.

The validity of the test situations was considered to depend on the continuity maintained with previous research under conditions of feasibility in a school situation and adaptation for purposes of measurement at the Grade four level.

Validity of the test sequences Concrete and Stories for the purposes of this study was also considered to depend on the extent of the comparability achieved between corresponding items of these tests, comparability in logical form as distinct from content.

Each of these problems will be examined in some detail in Chapters IV to VIII, in which the construction of the Concrete and Stories tests will be described.

Statistical Treatment of the Data

The statistical procedures for the analysis of the data were programmed by the Division of Research Services, Faculty of Education, University of Alberta.

Product-moment correlations were obtained between scores on Concrete and Stories subtests, STEP Reading total scores, and sex. This matrix is presented in Table 38, Appendix I.

The significance of sex differences between mean scores on the Concrete variables, the Stories variables, and on STEP Reading total scores was estimated using t-tests for the significance of the differences between mean scores.

Stepwise multiple regression analyses were used to range in order of priority the five subtests of the Concrete tests as predictors of STEP Reading totals; the five subtests of the Stories tests as predictors of the criterion STEP Reading; and the ten Concrete and Stories subtests taken together as predictors of STEP Reading. These analyses also provided an estimate of the per cent of the total variance of STEP Reading predictable by the Concrete tests; the per cent of the total variance of the criterion predictable by the Stories tests; and the per cent predictable by the ten Concrete and Stories tests taken together.

Phi coefficients were calculated for the 70 STEP Reading items.

This matrix is presented in Table 39, Appendix I. Principal-axes factor analysis was applied to the matrix of Phi coefficients. Varimax rotation was applied to the principal-axes factors selected.

Varimax rotation of the principal-axes factors obtained was also applied in the analyses of Concrete and Stories subscores and of these subscores and STEP Reading total scores taken jointly.

The three factor analyses: STEP Reading items; Concrete and Stories subscores; and Concrete and Stories subscores jointly with STEP Reading totals, were undertaken to obtain an indication of the factors which appeared to be common to these tests.

Canonical correlation procedures were applied to determine the maximum possible relation between the Concrete tests and the Stories tests taken as composites; and to obtain an indication of the relative contributions of subscores on these tests to the prediction of the variance common to the two sets of tests, Concrete and Stories.

Summary: Design of the Study

This chapter has presented a brief account of the population and the sample of the study; the tests administered and the procedures in collecting the data; and the pilot study. Data on the reliability of the tests, on the reliability of the scoring of the Concrete and Stories tests, and of the predicted readability levels of the stories were presented.

The validity of the Concrete and Stories test items was discussed in relation to principles for the construction of tests of reasoning, and considered to be satisfactory. Construct validity of the test items was considered to depend on the identity of the logical forms of the tasks of the test items and these forms as defined by logicians. The validity of the Concrete test situations was considered to depend on the continuity maintained with previous research. For the purposes of this study comparability in form between corresponding Concrete and Stories test items will be necessary. Each of these problems will be examined further in the discussion of the construction of the tests which follows.

The statistical procedures for the analysis of the data were indicated. They included product-moment correlations between scores on the Concrete and Stories variables and between these variables and STEP Reading total scores; t-tests for the significance of sex differences between mean scores; stepwise multiple regression analysis to range in order of priority Concrete and Stories subtests as predictors of the criterion STEP Reading total scores; factor analyses; and the calculation of the canonical correlation between the two sets of tests, Concrete and Stories, taken as composites.

The construction of the tests of conservation will now be considered.

CHAPTER IV

TESTS OF CONSERVATION

This chapter will describe the construction of the Concrete and Stories tests of conservation of substance, weight and volume. The tests will be designated CCO-Con-S-W-V and SCO-Con-S-W-V.

A description of the materials for the tests will be followed by descriptions of the operations measured by items CCO and SCO-Con. A discussion of the comparability of decisions and levels of explanation for the corresponding test items CCO-Con and SCO-Con will follow. Procedures in scoring the test items and protocols illustrating the scoring will be presented.

Materials

The materials for the Concrete tests of conservation are shown in Figure 2. They consist of the following:

Plasticine,
Sugar cubes,
A balance scale,
Two clear plastic glasses with water,
Four square display supports, of 1/4 inch cardboard,
9 inches to the side, painted black,
Elastic bands, tongs, a slieer.

The materials for the Stories tests of conservation were the two stories "The Funny Race at the Picnic", and "Jimmy Feeds the Birds". The stories are included in Appendix A.

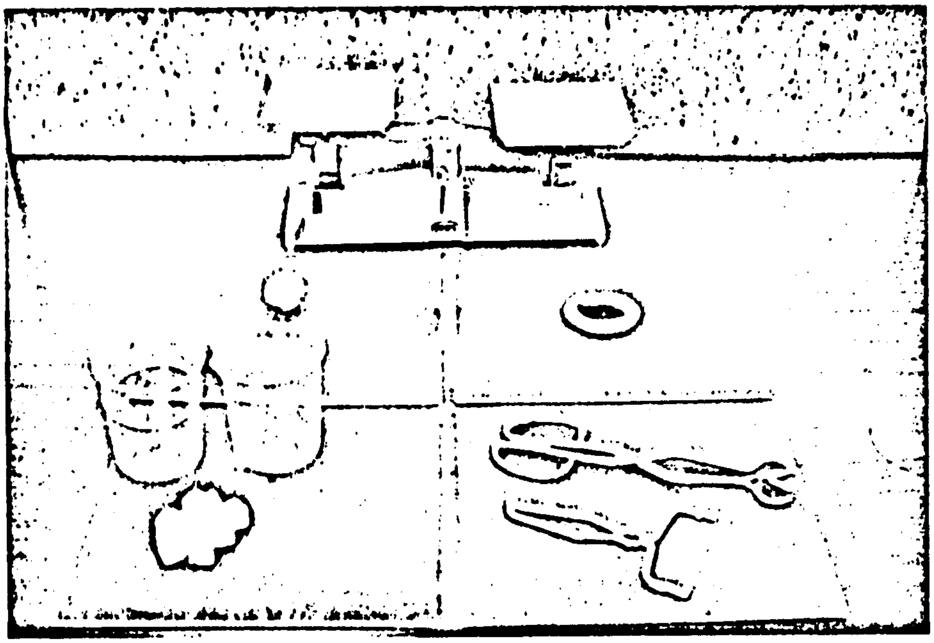


Fig. 2--Test materials: Concrete
tests of conservation.

Operations Measured: Concrete Tests of Conservation

Test items for the Concrete tests of substance, weight and volume were constructed at three levels of complexity. Level of complexity refers to the nature and extent of the deformations in the presence of which substance, weight and volume are to be conserved. At level of complexity I, one of two objects, two balls of plasticine, previously determined by the subject to be equal in weight, is deformed by a simple change in shape with the substance remaining continuous (one object is deformed as a "doughnut"). At level of complexity II, each of the two equal objects is deformed in shape with the substance of one of the objects becoming discontinuous (one of the objects is deformed as a "snake", the other is cut into small cubes which are separated). At level of complexity III, a substance is distributed throughout a space and there are changes in the extent of its visibility (sugar is dissolved in water).

For each of the nine test items justification is requested for both conserving and non-conserving responses. At each level of complexity the form of the test items, the sequencing of the items, and the experimental materials are kept as uniform as possible. Conditions requiring a decision as defined, are considered to be maintained.

In order that test materials may be held as constant as possible, one experimental situation assesses conservation of substance, weight and volume at levels of complexity I and II and one situation assesses these operations at level of complexity III.

Preliminary questions are presented to ensure as far as possible that important characteristics of the materials are perceived by the subject. Objects are named and described as part of the test situation:

> What is this? (clay, plasticine.)
> Make two balls from this clay. Make them about the same size.
> What is this? (balance scale, pans of the scale.)
> Can you find out if the two balls you made are equal? How?
> Make them equal.
>
> What will happen if I put these cubes of sugar in this glass of water?
> What else will happen?
> Mark the level of the sugar water with this elastic band.

In addition to ensuring the perception of the materials, the preliminary questions are intended to suggest a specific orientation or "set", namely, that the gathering of accurate information is an appropriate and relevant activity prior to considering the solution of problems arising from the situation, which require logical thinking.

Three test items at each level of complexity assess the subject's ability to conserve the identity of an object in substance, weight and volume with changes in form, and to conserve a relation of equality in respect to these properties between two objects in the presence of the deformations indicated. At level of complexity III, the conserving decisions require the recognition of these relations with changes involving both space and time: the properties substance, weight and space-occupied by sugar dissolved in water are to be conserved "tomorrow".

The twenty-seven test items measuring Concrete conservation, and the details of their administration are presented in Appendix A.

In order to relate the performance of subjects on items in the Concrete tests of conservation to their performance on items in the Stories test of conservation, corresponding test items and the conditions of administration should be comparable. Operations assessed by the test items of the Stories tests of conservation will now be described.

Operations Measured: Stories Tests of Conservation

Test items SCO-Con-S-W-V were constructed at levels of complexity I, II, III, similar to those in the Concrete tests. The items were administered in the sequence substance, weight, volume, with the exception of item III, conservation of volume level II, which is based on Story II. This item was administered after the three items at level of complexity III.

Test items at levels of complexity I and II, based on Story I, "The Funny Race", were completed in the first interview. Five test items assess conservation of substance, weight and volume in Story 1; four test items assess these operations in Story II, "Jimmy Feeds the Birds" (see Appendix A).

The preliminary questions presented for the stories are designed to ensure as far as possible that the material has been accurately decoded and the information recalled. They would appear to constitute in effect "a second reading" of the story. They are also intended, as are the preliminary questions for the Concrete tests, to suggest the usefulness of the information recalled for the solution of problems requiring thinking. The preliminary questions and responses differ from those which follow in that they do not require relating propositions, deriving conclusions by rules of logic, constructing classes and recognizing relations of class inclusion. Both sets of questions are, however, presented in the context of a continuing conversation concerning the matter at hand: a story to be understood; phenomena to be explained.

Nevertheless it seemed that the children were aware of a transition, not in the sense that "these are the questions that will be counted", rather that the responses required were of another order. The evidence was sometimes a slight pause to consider, a more careful selection or search for words, a modification in the tone of voice which conveyed the sense that a personal judgment was now being offered. Sometimes it was a clear statement of recognition, "It didn't tell you that in the story". In this case, the child was asked if he could figure it out. Responses, right or wrong, were followed by a question of the form, "How did you know that?" This also seemed to contribute to maintaining a uniform conversational tone throughout the interview.

The question of the equivalence of the cognitive operations assessed by corresponding SCO-Con and CCO-Con test items will now be examined. These operations involve "holding" initial relations of equivalence, noticing deformations, deciding questions of conservation of substance, weight, and volume, and showing the reasonableness of the decisions made.

Comparability of the Concrete and Stories Tests

Two kinds of responses are required for each of the test items CCO-Con and SCO-Con: a decision and an explanation of the decision. The conditions for determining comparability for these responses differ, and will be considered separately. The focus of concern in determining comparability for a decision for conservation is the construction of the corresponding test items. The focus in considering comparability for explanations is the logical thinking of the subject. This problem involves identifying levels of rigor represented by the different responses.

Decisions

A decision asserting conservation for each of the test items CCO-Con and SCO-Con is dependent on a relation of initial equality in the data. This relation may be expressed as an equation. Two operations are involved in a decision for conservation: the recognition of the invariance of certain properties of an object with deformation; and the recognition of the invariance of a relation of equality between the two objects with deformation of one or both objects. For the latter operation the initial relation of equality must be "held" in arriving at a decision for conservation. Figure 3 presents the equations for "holding" operations at level of complexity I, and the deformations (symbolized " => ") which are to be "noticed" by the subject. The decisions to conserve a property, and to conserve the initial relation of equality are shown as equations.

Test Items I,II,III Level of Complexity I	CCO-Con	SCO-Con	Operation
Item I	$A_1 = A_2$	A_1	Hold
Substance	$A_2 \Rightarrow A_2'$	$A_1 \Rightarrow A_1' \,\&\, A_1''$	Notice
	$A_2 = A_2'$	$A_1' \,\&\, A_1'' = A_2'$	Conserve property-S
	$A_1 = A_2'$	$A_1 = A_2'$	Conserve relation
Item II	$A_1 = A_2$	$A_1 = A_2 = A_3 \ldots$	Hold
Weight	$A_2 \Rightarrow A_2'$	$A_2 \Rightarrow A_2'$	Notice
	$A_2 = A_2'$	$A_2 = A_2'$	Conserve property-W
	$A_1 = A_2'$	$A_1 = A_2'$	Conserve relation
Item III	A_{1x}	A_{1x}	Hold
Volume	$A_1 = A_2$	$A_1 = A_2$	Hold
	$A_2 \Rightarrow A_2'$	$A_2 \Rightarrow A_2'$	Notice
	$A_{2x} = A_{2x}'$	$A_{2x} = A_{2x}'$	Conserve property-V
	$A_{1x} = A_{2x}'$	$A_{1x} = A_{2x}'$	Conserve relation

Fig. 3.-- Equations representing conserving operations for corresponding test items I, II, III, level of complexity I, CCO-Con and SCO-Con.

A_{2x}: space occupied by A_2

A', A'': one and two deformations of A.

Two of the corresponding operations involving conservation of substance represented symbolically in Figure 3 are not identical. They are the operations for holding (A_1 is not identical with $A_1 = A_2$), and the deformations to be noticed ($A_1 \Rightarrow A_1' \,\&\, A_1''$ is not identical with $A_2 \Rightarrow A_2'$). The actions of deformation performed for these corresponding test items appear, however, to be equal in difficulty as defined: the deformation $A_2 \Rightarrow A_2'$ represents one action on one object; the deformation $A_1 \Rightarrow A' \,\&\, A_1''$ is also one action on one object, a single object is divided into two parts without loss. The significant difference between these corresponding test items appears to be in the holding operation. Test item SCO-Con-S requires the holding of an object which is "destroyed" in the deformation: $A_1 \Rightarrow A_1' \,\&\, A_1''$. This holding operation appears to lack the support in representation of the continued existence of a second object to which it may be observed to be initially equal. This support is available in the corresponding test item CCO-Con-S. An unanticipated complication may, therefore, have been introduced in test item SCO-Con-S. Although the subject could recall the original equality of object A_1 (the ball of wax) to the other balls A_2, A_3, the reference is not made explicit for test item I. It is made explicit for test item II SCO-Con-W ($A_1 = A_2 = A_3 \ldots$).

With the exception of this operation of holding in SCO-Con-S, the equations presented in Figure 3 suggest that at level of complexity I the required operations are comparable for corresponding test items CCO-Con-S-W-V and SCO-Con-S-W-V.

The equations representing holding operations and decisions asserting conservation for test items IV to VI at level of complexity

Test Items IV,V,VI Level of Complexity II	CCO-Con	SCO-Con	Operation
	$A_1 = A_2$	$A_1 = A_2$	Hold
Items IV, V	$A_1 \Rightarrow A_1'$	$A_1 \Rightarrow A_1'$	Notice
Substance and Weight	$A_2 \Rightarrow A_2''$	$A_2 \Rightarrow A_2''$	
	$A_1 = A_1'$	$A_1 = A_1'$	
	$A_2 = A_2''$	$A_2 = A_2''$	Conserve properties-S-W
	$A_1' = A_2''$	$A_1' = A_2''$	Conserve relations
	$A_1 = A_2$	$A_1 = A_2$	Hold
	A_{1x}	A_{1x}'	Hold
Item VI	$A_1 \Rightarrow A_1'$	$A_1 \Rightarrow A_1'$	
Volume	$A_2 \Rightarrow A_2''$	$A_2 \Rightarrow A_2''$	Notice
	$A_{1x} = A_{1x}'$	$A_{1x} = A_{1x}'$	
	$A_{2x} = A_{2x}''$	$A_{2x} = A_{2x}''$	Conserve property - V
	$A_{1x} = A_{2x}''$	$A_{1x} = A_{2x}''$	Conserve relation

Fig. 4.— Equations representing conserving operations for corresponding test items IV, V, VI, level of complexity II, CCO-Con and SCO-Con.

II are presented in Figure 4. These equations suggest that the conserving operations required for corresponding test items are comparable at this level of complexity. For test items VI, volume, the "form" of the objects for holding, A_{1x} and A'_{1x}, are not identical. They appear however to be comparable in the test situation: the space occupied by the ball of plasticine, A_{1x} is given; the space occupied by the chunks of suet, A'_{1x} is also given. Equality of space-occupied is to be determined in each instance in relation to a space-occupied which is given.

There also appears to be a consistent increase in difficulty from level of complexity I to level of complexity II for each of the corresponding test items (compare Figs. 3 and 4). At level of complexity I, one object is deformed by one action $A_2 \Rightarrow A'_2$); at level of complexity II one object is deformed by one action ($A_1 \Rightarrow A'_1$) and a second object is deformed by two actions ($A_2 \Rightarrow A''_2$).

The equations representing holding operations and decisions for conservation for corresponding test items at level of complexity III are presented in Figure 5. The corresponding equations for test items VII and VIII appear to be comparable (see Fig. 5): A..., in Figure 5, represents cubes of sugar; A_1, A_2, A_3, equal pieces of suet. In each case the objects are discrete, equal, and familiar to the subjects.

The equations for the corresponding test items IX suggest that these items are comparable in the operations of "holding", of "noticing" and of conserving the relation of equality; they differ in the property which the subject is instructed to conserve. In each

Test items VII,VIII,IX Level of complexity III	CCO-Con	SCO-Con	Operation
Items VII,VIII	$A \ldots$	$A_1 = A_2 = A_3$	Hold
Substance	$A \Rightarrow A_1''$	$A_1 \Rightarrow A_1''$	Notice
and	$A_1'' \Rightarrow A_1'''$	$A_2 \Rightarrow A_2'''$	
Weight	$A = A_1''$	$A_2 = A_2'''$	
	$A = A_1'''$	$A_1'' = A_2'''$	Conserve properties -S, -W
	$A_1'' = A_1'''$	$A_1'' = A_2'''$	
			Conserve relation
	AM_x ^{a}	$^{a'}AM_x$	Hold
	$AM_x \Rightarrow A_1'M_x$ ^{b}	$^{b'}AM_x \Rightarrow A'M_x$ & $A_2'M_x$	Notice
Item IX	$A_1M_x \Rightarrow A_1''M_x$ ^{c}	$^{c'}A_1'M_x \Rightarrow A_1''M_x$	Notice
Volume	$A_1''M_x \Rightarrow A'''M_x$ ^{d}	$^{d'}A_2'M_x \Rightarrow A_2'''M_x$	Notice
	$A_1'M_x = A_1'''M_x$	$A_1'M_x = A_1''M_x$	Conserve property Mx
	$A_1'M_x = A_1'''M_x$	$A_1'M_x = A_2'''M_x$	Conserve relation

Fig. 5.-- Equations representing conserving operations for corresponding test items VII, VIII, IX, level of complexity III, CCO-Con and SCO-Con.

a: sugar cubes in water
b: level of sugar water
c: level, sugar disintegrating
d: level sugar dissolved (time)

a$'$: quantity of popcorn
b$'$: levels of popcorn (half-cup each)
c$'$: popcorn for the jay (A_1'')
d$'$: popcorn exploded (time) (A_2''')

M_x: space occupied by mass, M

A''' : three deformations of A.

test item M_x is conserved. But in CCO-Con-V the subject is instructed to consider the problem of the conservation of the property M, quantity or mass. There is a further difference. In the deformation $A'''M$, CCO-Con-V, a visible substance becomes invisible (the sugar is dissolved tomorrow). In $A''''M$, SCO-Con-V, an enclosed invisible substance becomes visible (popcorn seeds are exploded).

For each of these test items the operation of conserving a property requires consideration of both mass and space occupied by the mass: M_x for CCO-Con-V requires the decision, "x is conserved"; M_x for SCO-Con-V requires the decision, "M is conserved".

Explanations for these decisions also require a statement of the invariance of each property M and x. In CCO-Con-V the evidence for conservation of M is x is invariant. In SCO-Con-V the evidence for conservation of x is M is invariant. It may be considered that these corresponding test items are comparable in difficulty in respect to the number and the character of the deformations involved in conserving the relation of equality and in explaining the relation which is conserved.

The equations in Figures 4 and 5 suggest that a consistent increase in difficulty obtains from level II to level of complexity III for each of the corresponding test items. At level of complexity II deformation involves two actions on one object ($A_1 \Rightarrow A_1''$); at level of complexity III deformation involves two actions on one object ($A_1 \Rightarrow A_1''$) and three actions on the second object ($A_1 \Rightarrow A_1'''$) and ($A_1M_x \Rightarrow A_1'''M_x$).

Explanations

Three levels of explanation which appear to correspond to levels of development in logical thinking are recognized for decisions in the corresponding test items CCO-Con and SCO-Con. These explanations are responses to questions of the form:

> Why is that?
> How did you know that?
> Is there another way to prove it?

Explanations level zero. -- are statements supporting decisions of non-conservation and statements offering non-logical or perceptually based explanations of decisions for conservation. In the following examples, the first number represents the protocol, the number in brackets the age of the subject:

> 84 (10:0) He gave the sparrows less because they're not so big and they don't eat as much as the jays.

> 15 (9:5) The snake will make the water go lower. The pieces will be higher . . . because the pieces are scattered about in the water and they take up more room than the snake and it makes it heavier and the water rises.

> 11 (9:10) The same amount (stilts and muff) 'cause . . . that was only fair.

> 57 (10:2) It was the same weight because one was carried on his head and you can hardly feel it on the head.

Explanations level 1. -- are arguments in support of conservation containing one or more premises which assert the equality of two objects with deformation. An explanation at this level for the deformation of one object may be symbolized:

$A_1 = A_2$ P Given

$A_2 = A_2'$ P Asserted

$\therefore A_1 = A_2'$ I Inferred, Principle of Transitivity.

For two deformations, an explanation at level I may be symbolized:

$A_1 = A_2$ P Given

$A_1 = A_1'$ P Asserted

$A_2 = A_2'$ P Asserted

$\therefore A_1 = A_2'$ I Inferred, Principle of Transitivity.

58 (10:2) The same amount ($A_1 = A_2'$) because they started out with the same amount ($A_1 = A_2$), and cutting it up into pieces ($A_2 \Rightarrow A_2'$), won't make any difference ($A_2 = A_2'$).

$A_1 - A_2$ P

$A_2 \Rightarrow A_2'$

$A_2 = A_2'$ P Assertion

$\therefore A_1 = A_2'$ I Principle of Transitivity.

At explanation level I, reversibility may be offered in support of a statement of the conservation of a property of an object in the deformed state. This argument in words is expressed in the general form:

> You could roll it back again and it would be the same.

Reversibility is considered by Piaget to be an important intermediate operation in the acquisition of conservation. For the development of deductive reasoning associated with the acquisition of conservation, reversibility appears to have a more restricted function. In an assertion of equality, reversibility may or may not imply equality in the deformed state. In addition, in many instances reversibility is not demonstrable. One cannot, for example, offer observational or experimental evidence of equality on the basis of reversibility in the case of suet fed to birds, of popcorn exploded, of sugar dissolved. Reversibility in such cases is not available for the construction of a synthetic proposition. The child's argument, with reversibility as a premise would be:

> It's the same amount of sugar. If it was back in the cubes it would be the same.

An argument in which reversibility (R) is introduced in support of a premise may be symbolized:

$A_1 = A_2$ P

$A_2 + a \Rightarrow A_2'$

$A_2' - a = A_2$ P (R) Assertion

$\therefore A_1 = A_2'$ I Principle of Transitivity

+ a: deforming action
- a: the reverse of this action (R).

In this argument objective evidence for the premise $A_2' - a = A_2$ is not offered and may not be available. It is not certain that the subject conserves property and relation in the deformed state. A particular event is not shown to be an instance of a class of events having certain characteristics, and sharing significant characteristics of the class. The inference is not, therefore, in the form of a deduction. A similar form of argument would appear to be presented in the following protocol:

> 11 (10:1) They weigh the same ($A_1' = A_2'$). If you put all the cubes (A_2') together there would be the same amount as in the snake (A_1'), ($A_2' - a = A_1'$). You just cut them up ($A_2 = A_2'$); they were the same when they were a ball ($A_1 = A_2$).

$A_1 = A_2$ P

$$A_1 \Rightarrow A_1'$$
$$A_2 \Rightarrow A_2'$$
$$A_2 = A_2' \quad \text{P Assertion}$$
$$A_2'-a = A_1' \quad \text{P Reversibility, Assertion}$$
$$\therefore A_1' = A_2' \quad \text{I Principle of Transitivity}$$

In this protocol the subject asserts, but he does not support with objective evidence, the premise that the object is invariant in the presence of deformation. He infers the invariance of the relation of equality ($A_1' = A_2'$) by the Principle of Transitivity.

Explanation level II. -- includes as premises a tautological implication and the antecedent of this implication in the form of a synthetic proposition. The necessary inference follows by the rule of Modus Ponens. The tautological implication is, "If nothing is added and nothing taken away in an action of deformation the objects remain the same in significant respects." The implication may be symbolized:

$$A' \pm 0 \supset A.$$

The synthetic proposition is a statement supported by observation: "In this deformation it is observed that nothing is in fact added or taken away", (the A/S argument). The synthetic proposition may be symbolized:

$$A' \pm 0 \text{ (is observed to be true).}$$

The argument at level of explanation II may be symbolized for a test item at level of complexity I:

$$A_1 = A_2 \quad \text{P}$$
$$A_2' \pm 0 \supset A_2 \quad \text{P Tautological implication (T)}$$

$A_2' \pm 0$	P	Synthetic premise (Sy)
$\therefore\ A_1 = A_2'$	I	Deductive inference (D) by the rule Modus Ponens.

For test items at level of complexity II this argument may be symbolized:

$A_1 = A_2$	P		
$A_1' \pm 0 \supset A_1$	P	T	
$A_2'' \pm 0 \supset A_2$	P	T	
$(A_1' \pm 0)\ \&\ (A_2'' \pm 0)$	P	Sy	
$\therefore\ A_1 = A_2''$	I	D	Modus Ponens

In the arguments presented below, the question of the conservation of a property of an object in the deformed state is referred to a general proposition:

$$A_1' \pm 0 \supset A_1.$$

The instance of deformation in question is stated to be a member of this general class. This statement is supported by objective evidence. The conclusion follows logically, and is stated as "necessary":

5 (8:6) I know they are the same . . . $(A_1' = A_2'')$. Well . . they are the same balls of clay that we weighed $(A_1 = A_2)$ so they still must be the same $(A_1 = A_1')$ & $(A_2 = A_2'')$ unless this one took some from that one $(A_1 \pm 0 \Rightarrow A_1') \equiv (A_1 = A_1')$ and it didn't, because I watched $(A_1' \pm 0$ and $(A_2'' \pm 0)$. So they're the same (snake and cubes).

$A_1 = A_2$	P	
$(A_1 \pm 0 \Rightarrow A_1') \equiv (A_1 = A_1')$	P	Tautology
$(A_2 \pm 0 \Rightarrow A_2'') \equiv (A_2 = A_2'')$	P	Tautology
$(A_1 \pm 0)\ \&\ (A_2 \pm 0)$	P	Synthetic proposition (observed)
$\therefore\ A_1' = A_2''$	D	Modus Ponens

74 (10:1) They had the same amount. It said in the story that they had half a cup each $AM_x \Rightarrow A'_1M_x \equiv A'_2M_x$. Jimmy's would look more popped (A'''_2M_x). but popping doesn't put any more in ($A'_1M_x \pm 0 \equiv A'''_2M_x$). It was the same seeds. So Jimmy got as much to eat ($A'_1M_x = A'''_2M_x$).

	AM_x		P
	AM_x	$\Rightarrow (A'_1M_x \equiv A'_2M_x)$	
	A'_1M_x	$\pm 0 = A'''_2M_x$	P Tautology
	A'_1M_x	± 0	P Synthetic
	A'_2M_x	± 0	P Synthetic
$\therefore$	A'_1M_x	$= A'''_2M_x$	I Modus Ponens

88 (9:6) It will be the same, because none of the sugar has been taken out to make it lower and no more sugar was put in to make it go higher. ($AM_x \pm 0$). So it has to be the same.

	AM_x		P
	$AM_x \pm 0$	$= A''_1M_x$	P Tautology
	$A''M_x \pm 0$	$= A'''_1M_x$	P Tautology
	$AM_x \pm 0$		P Synthetic
$\therefore$	A''_1M_x	$= A'''_1M_x$	I Modus Ponens

Explanations levels I and II appear to distinguish between subjects who may be at different levels of logical development. Subjects offering an explanation at either level are conserving. At level of explanation I, however, a statement of the invariance of an object in the presence of a given deformation is inserted without proof as a premise in an argument. The conservation of a given relation between one object and a deformed object (or between two deformed objects) is then inferred on the basis of the Principle of Transitivity. At level of explanation II subjects refer an instance of invariance to the general case, "If . . . then". They offer objective evidence that the instance in question is a member of this general class of events and infer that by virtue of its membership in this class the instance shares the properties of the class. They state the inference as a necessity.

Explanation level II is considered to indicate a more advanced level of logical development. It is possible that it also represents a more advanced linguistic development. There are a number of indications in these protocols of the use of logical quantifiers and of new syntactic structures. The logical quantifiers include the expressions "some", "all", "not any", "any", "no". Other expressions which refer to quantity also appear, such as "exactly", "not any more", "never . . . any" and suggest attempts to speak with precision. Sentences occur in which other sentences are embedded using the connectives "unless", "but", "or", "that", "so", "if", "because". Decisions are sometimes prefaced by expressions such as, "I know", "I'm not just sure but I think", "They must be . . .", suggesting awareness of the self as a decision maker, and also perhaps indicating a certain degree of confidence in intellectual operations as a source of new knowledge.

Scoring

Each of the nine test items in the test sequences CCO-Con and SCO-Con requires a decision and an explanation of the decision. The range of scores for each test item is 0 to 3. The range of scores CCO-Con and SCO-Con is in each case 0 - 27. A summary of the scoring is presented in Figure 6.

Scoring Tests of Conservation

Levels of Complexity	Test Items CCO-Con SCO-Con	Decisions	Explanation Level I	Explanation Level II	Range of Scores CCO-Con	Range of Scores SCO-Con
Level I	Substance	0-1	0-1	0-1		
	Weight	0-1	0-1	0-1		
	Volume	0-1	0-1	0-1		
	Range	0-3	0-6		0-9	0-9
Level II	Substance	0-1	0-1	0-1		
	Weight	0-1	0-1	0-1		
	Volume	0-1	0-1	0-1		
	Range	0-3	0-6		0-9	0-9
Level III	Substance	0-1	0-1	0-1		
	Weight	0-1	0-1	0-1		
	Volume	0-1	0-1	0-1		
	Range	0-3	0-6		0-9	0-9
Levels I, II, III	Range	0-9	0-18		0-27	0-27

Fig. 6. -- Scores assigned for decisions and for explanations levels I, II, CCO-Con-S-W-V and SCO-Con-S-W-V.

A decision asserting conservation is assigned a score 1. A non-conserving decision is assigned a score 0.

Explanations are identified at three levels: zero, I, II. Explanations offered in support of non-conservation are considered to be level zero; the response is scored 0, 0, 0. Non-logical explanations in support of conservation are considered to be level zero; the response is scored 1, 0, 0.

An explanation at level I is assigned a score 1. An explanation at this level is identified as one which supports conservation in an argument consisting of a premise asserting the conservation of a property in the presence of deformation followed by an inference, based on the Principle of Transitivity of the conservation of an initial relation of equality. Total score assigned the response is 1, 1, 0, if an explanation at level II is not subsequently presented.

An explanation at level II for a decision of conservation is assigned total scores 1, 1, 1. An argument at this level presents a tautological premise and a premise which is a synthetic proposition (stated to be true on a basis of observation). An inference derived by the rule Modus Ponens follows from these premises. It appeared that subjects who responded at this level also demonstrated increased facility in using quantifying expressions, connectives, and more complex grammatical structures.

Scored protocols of tests of conservation are presented in Appendix A.

Summary: Tests of Conservation

The Concrete and Stories tests of conservation of substance, weight and volume were designed to assess subject's ability to recognize the invariance of these properties with deformations of varying degrees of complexity. For each decision, conserving or non-conserving, an explanation was elicited. The decisions and also the levels of explanations offered were considered to identify behaviors characteristic of sequences in logical development.

The question of interest in presenting these tests to children at the Grade four level was the relationship between their responses in conserving in a concrete stimulus situation and in a reading situation (Stories tests). It was considered that a solution to this question might be available if test items and test situations could be constructed which would correspond in significant respects in these two stimulus situations.

In order to determine the comparability of the test situations, two aspects of correspondence were considered: the operations of holding and noticing which would be associated with the deformations presented; and the patterns of logical inference which could be involved in explaining a decision of conservation. Success in achieving comparability in the first of the two aspects appeared to depend on the skill of the test constructor; success in the second, on the level of logical sophistication in thinking of the subject. A symbolic representation of each of these aspects of correspondence was found helpful in assessing comparability.

The symbolic representations presented suggested that acceptable levels of test comparability for operations of holding and noticing were achieved. These representations also clarified the nature of certain unanticipated differences in these operations as required by the test situations.

Levels of explanations, presented symbolically, appeared to suggest that there were distinctions between the logical operations required which could be indicative of sequences in the acquisition of intellectual skills. At level of explanation 1 properties were asserted to be conserved; proof was not offered. A logical inference, based on the Principle of Transitivity was available to subjects who presented these premises. At level of explanation II properties were asserted to be conserved and proof was offered. The proof required two new premises: a tautology and a synthetic premise, the antecedent of the tautological implication. A deductive inference, based on Modus Ponens, was available to subjects who presented these premises.

It was considered that explanation level II represented a more advanced logical operation. It also appeared to require for its expression additional complex linguistic structures. The question of interest will be the relation between these operations as observed in response to concrete stimuli and responses requiring similar operations when the stimuli consist of printed symbols.

CHAPTER V

TESTS OF CLASSIFICATION

This chapter will describe the construction of the Concrete and Stories tests of classification. The tests will be designated CCO-Cl and SCO-Cl.

Terms required in describing the tests of classification will be defined and illustrated. Materials used in the tests will be presented. The operations of classification measured will be identified and the comparability of the decisions for the corresponding tests items will be considered. Procedures in scoring the tests will be presented.

Definition of Terms

The following terms relating to classification were defined in Chapter I: concept; class; classification; property; singular and general propositions; and singular and general terms. Additional terms required in identifying the class structures and relations to be assessed are defined below.

Extension of a class

The extension of a class is the range of applicability of the decision rule which specifies membership in the class.

Intension of a class

The intension of a class is defined by the decision rule which assigns the criterial properties determining membership in the class. Intension of a class may be represented symbolically:

$$(\hat{x}) : f(x).$$

The symbol can be read: every x which is a member of the class $(\hat{x})$ has the intensional property f.

Intension and extension are characterized by the following distinction: the greater the intension, that is, the defining attributes f of a class, the smaller is its extension, and alternatively. For example, the class "red squares" has a greater intension than the class "squares". The extension of the class "red squares" is smaller than the extension of the class "squares". There are more "squares" than "red squares".

Class inclusion

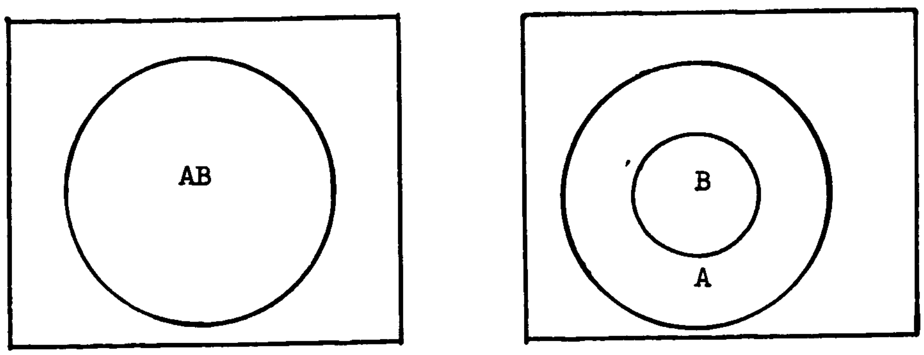
Class inclusion is a relation which holds between the extensions of classes. The relation of inclusion is to be distinguished from the relation, member of a class, which holds between a class and an individual. The relation of class inclusion may be symbolized:

$$A \subset B \equiv (x): (x \in A) \supset (x \in B).$$

The formula indicates that the extension of class A is included in the extension of the class B if and only if all members of class A are also members of class B.

Complete class inclusion

Complete class inclusion is defined as the relation in which all members of one class are also members of a second class. The relation of complete inclusion may be coextensive or not-coextensive. The two possible cases are that every A is a B and every B is an A so that A's and B's are coextensive; and the case in which this is not so.[1] The two cases are represented in Figure 7.



Every A is a B.
Every B is an A.
A and B are coextensive.
$(A \subseteq B)$

Every B is an A.
B and A are not-coextensive.
$(B \subset A)$

Fig. 7.--Class inclusion: complete inclusion, coextensive and not-coextensive.

Partial class inclusion

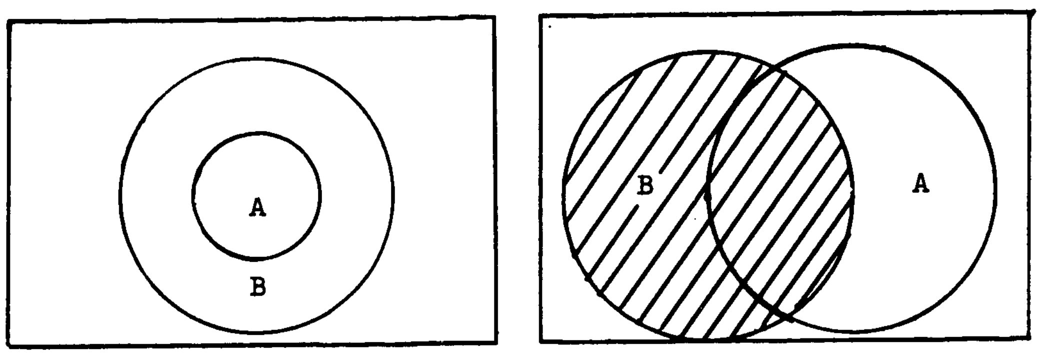
In the partial class inclusion relation there are also two possibilities: there is the case in which although not every B is an A, every A is a B; and the case in which this is not so.[2] These

---

[1]Arthur N. Prior, Formal Logic (Oxford: Clarendon Press, 1962), p. 108.

[2]Ibid.

relations are represented in Figure 8.



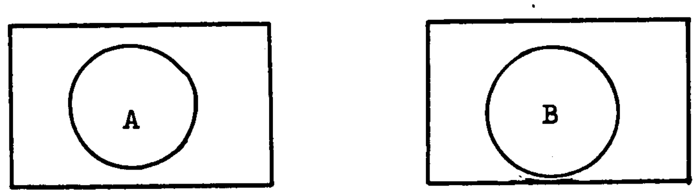
Every A is a B.
Not every B is an A.

At least one B is an A.
At least one B is not an A.

Fig. 8.--Class inclusion: partial inclusion.

Complete class exclusion

In complete class exclusion no member of A is a member of B. This relation is represented in Figure 9. The formula $(x):(x \in A) \supset \overline{(x \in B)}$, can be read, "For every x, x is a member of A implies that x is not a member of B."

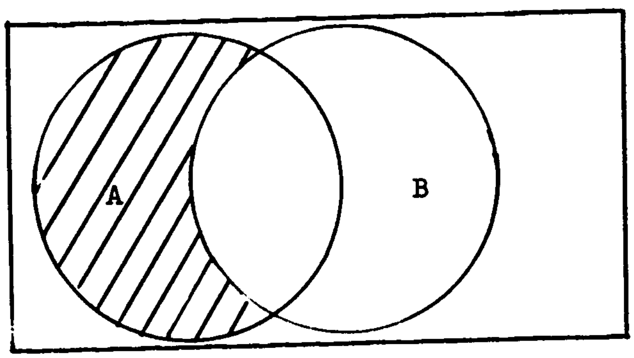


No A is a B.
No B is an A.
$(x):(x \in A) \supset \overline{(x \in B)}$.

Fig. 9.--Class inclusion: complete class exclusion.

Partial class exclusion

In the partial class exclusion relation some members of one class are not members of a second class. The relation is represented in Figure 10. The shaded area represents the A's which are not B's. The formula $(\exists x):(x \in A) \ \& \ (\overline{x \in B})$ can be read, "There is some x (at least one) such that x is a member of A and it is not the case that x is a member of B.



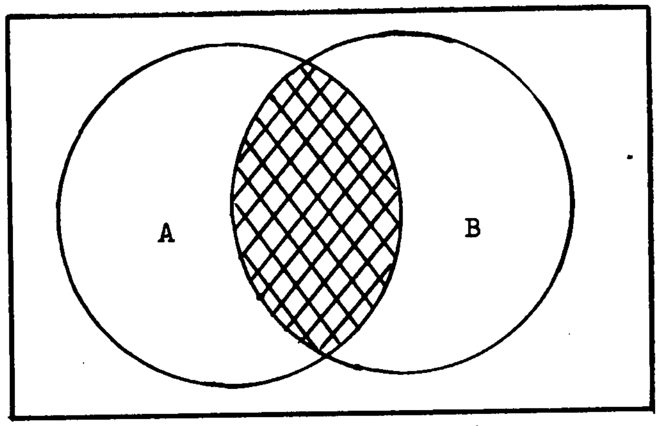
Some A's are not B's.

$(\exists x):(x \in A) \ \& \ (\overline{x \in B})$

Fig. 10.--Class inclusion: partial class exclusion.

Joint class inclusion

In the joint class inclusion relation, the extensions of two classes overlap so that some A's are B's and some B's are A's. This relation is represented in Figure 11. The formula $(\exists x):(x \in A) \ \& \ (x \in B)$ can be read, "There is some (at least one) x such that x is a member of A and x is a member of B.



Some A's are B's.
Some B's are A's.

$(\exists x):(x \in A) \& (x \in B)$

Fig. 11.--Class inclusion: joint class inclusion.

Empty class

An empty class, symbolized ⊗, is defined as the class which has no members. It is characterized by the property that for every x, (x), x is not a member of A.[3] This relation may be symbolized:

$$(x):(x \notin A)$$

Although nothing belongs to the empty class, the empty class can itself be a member of another class. For example, in the symmetrical classification of a collection as circles and squares in which the subclasses are red and white squares, and red and white circles, the class white circles could have no members. It would be the empty class. A hierarchical classification in which an empty class is included as a member is shown in Figure 12.

---

[3]Patrick Suppes, Introduction to Logic (Princeton, N.J.: D.Van Nostrand Company, 1957), p. 184.

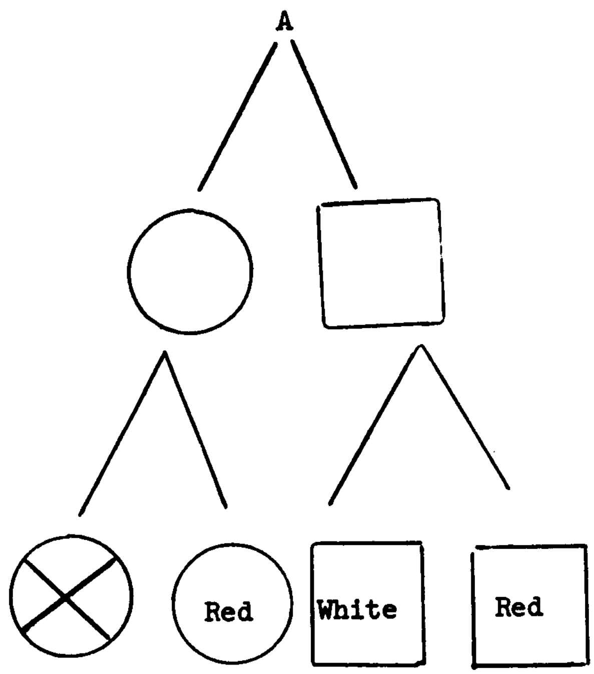


Fig. 12.--Class inclusion: the empty class.

Complementary class

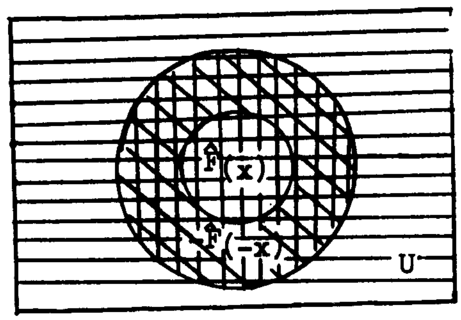
The complementary class is that class each of whose members is the negation of a member of its related class. It is therefore the "complement" of that class. For example, for every x, if x is a member of the class F, there is one further element, -x, which is a member of the class -F. The class -F (not -F) is the complement of the class F. The relation of complementarity may be symbolized:

$$\hat{F}(x) \supset - \hat{F}(-x)$$

The complementary class is usually referred to by the word "other", followed by the name of the superordinate class to which both F and -F belong. They are expressions of the form:

Birds and other animals (animals that are not birds),
Ducks and other birds (not-ducks),
Pintail and other ducks (not-pintail).

The complementary class is represented in Figure 13. Here the class F, and its complement -F, within a universe of discourse, U (animals), could represent the sentence, "Ducks and other birds are feathered animals".



$\hat{F}(x) \supset -\hat{F}(-x)$

U = Universe of discourse

Fig. 13.- Class inclusion: the complement of a class.

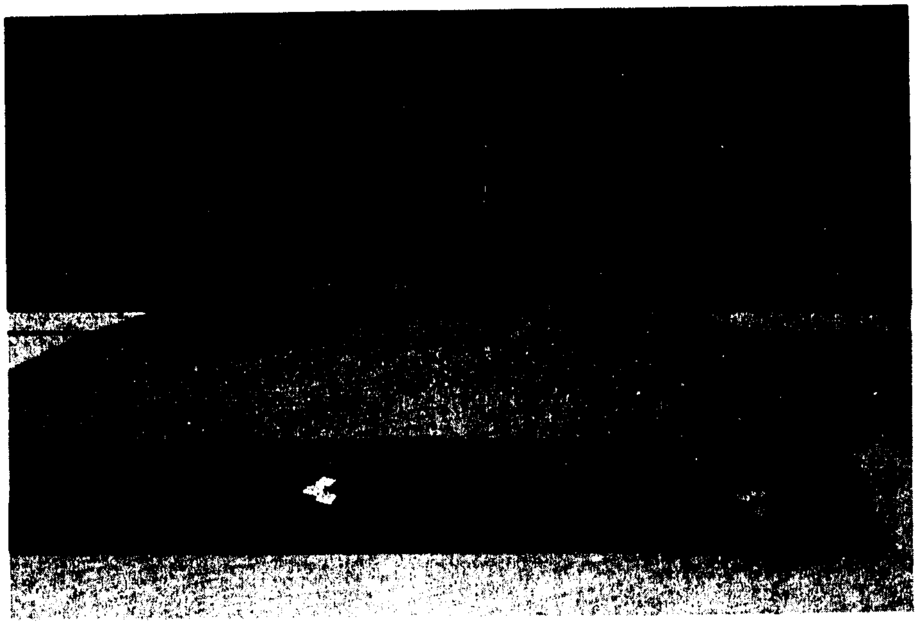
The materials for the Concrete and Stories tests of classification will now be indicated and summaries of the test items presented.

Materials

The test materials for the Concrete tests of classification are shown in Figure 14. The design of the counters follows closely the material constructed by Smedslund for experiments reported in Concrete Reasoning.[4] The materials include:

---

[4]Jan Smedslund, Concrete Reasoning: A Study of Intellectual Development. Monograph of the Society of the Society for Research in Child Development, Serial No. 93, XXIX, No. 2 (Yellow Springs, O.: Antioch Press, 1964). (Hereinafter referred to as Concrete Reasoning.)



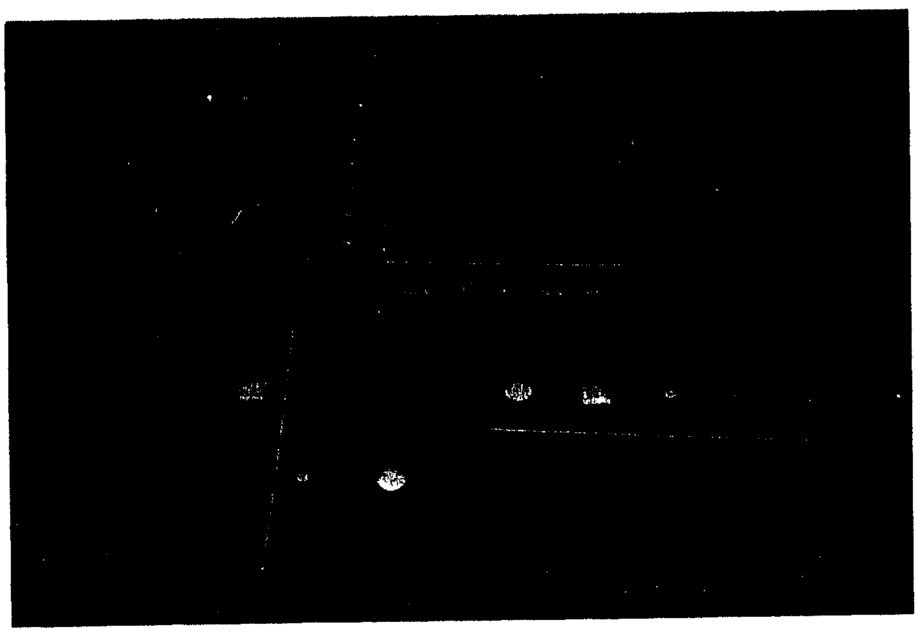


Fig. 14. -- Test materials: Concrete
tests of classification.

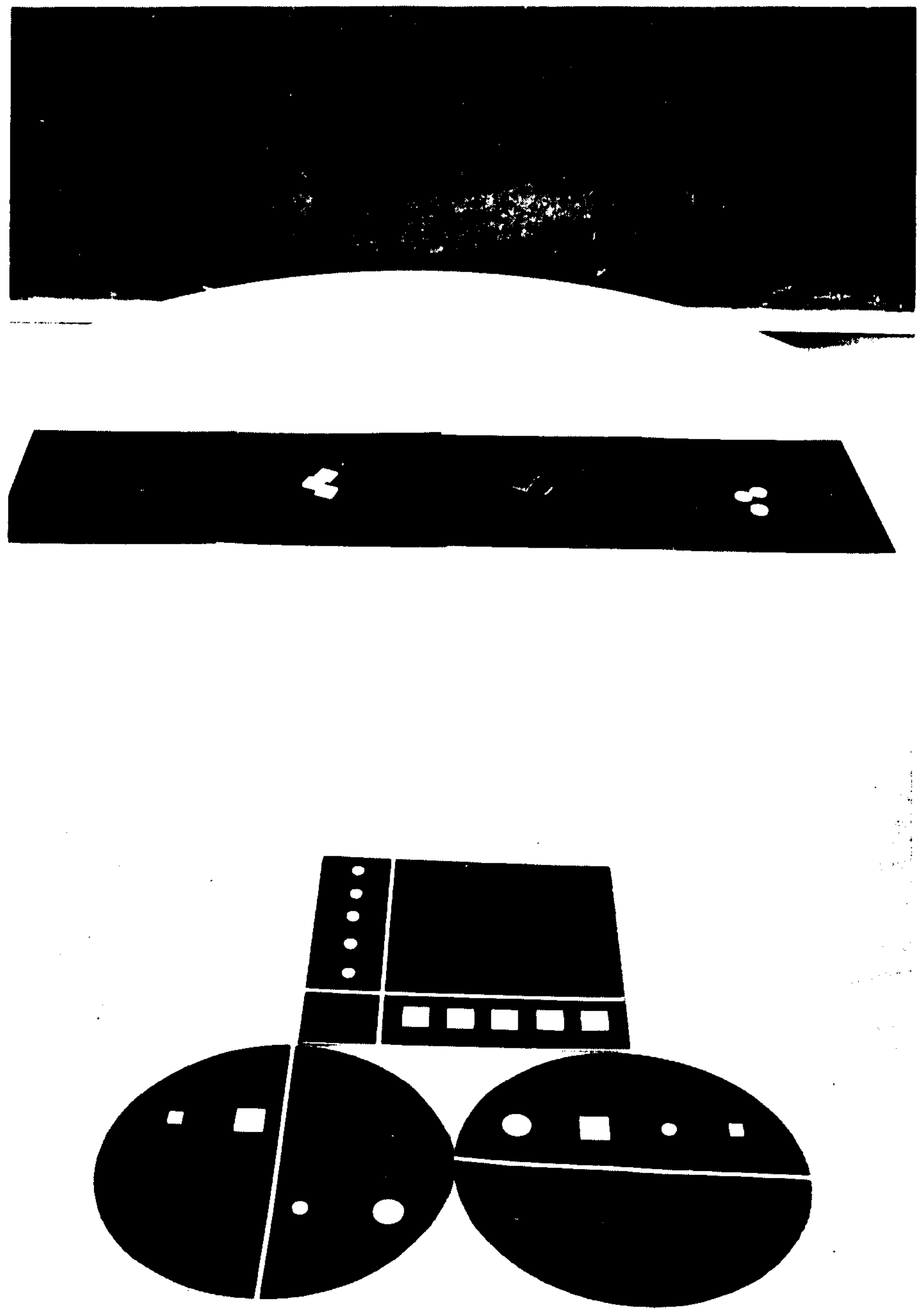


Fig. 14. -- Test materials: Concrete tests of classification.

14 square cardboard display supports, black, side 9",
10 red round counters, diameter 4/5",
3 white round counters, diameter 4/5",
3 red square counters, side 4/5",
3 white square counters, side 4/5",
2 white cardboard rods 9" long, 1/10" wide,
Patterns I, II, III,
A small collection of counters on a display support as specified for test item XVI (see Appendix B).

The stories for the Stories tests of classification are included in Appendix B. They are "The Ducks Arrive in Spring" and "A City of Long Ago".

Test Items Concrete and Stories

The nineteen test items of the Concrete tests of classification are summarized in Figure 15. They are shown in Appendix B. These tests are adapted for measurement at the Grade four level from experiments designed by Inhelder and Piaget for the study of class inclusion and multiplicative classification.[5] Modifications of these tests suggested by Smedslund's study have also been adopted.[6]

The test items of the Concrete tests require the hierarchical ordering of classes; the recognition of class inclusion relations; the construction of predicates describing the extensions of dichotomous classes; the construction and ordering of multiplicative classes and of alternative classes of intersection.

---

[5] Bärbel Inhelder and Jean Piaget, The Early Growth of Logic in the Child, trans. by E.A. Lunzer and D. Papert (London: Routledge and Kegan Paul, 1964), pp. 59-99 and 151-95. (Hereinafter referred to as The Early Growth of Logic.)

[6] Smedslund, Concrete Reasoning.

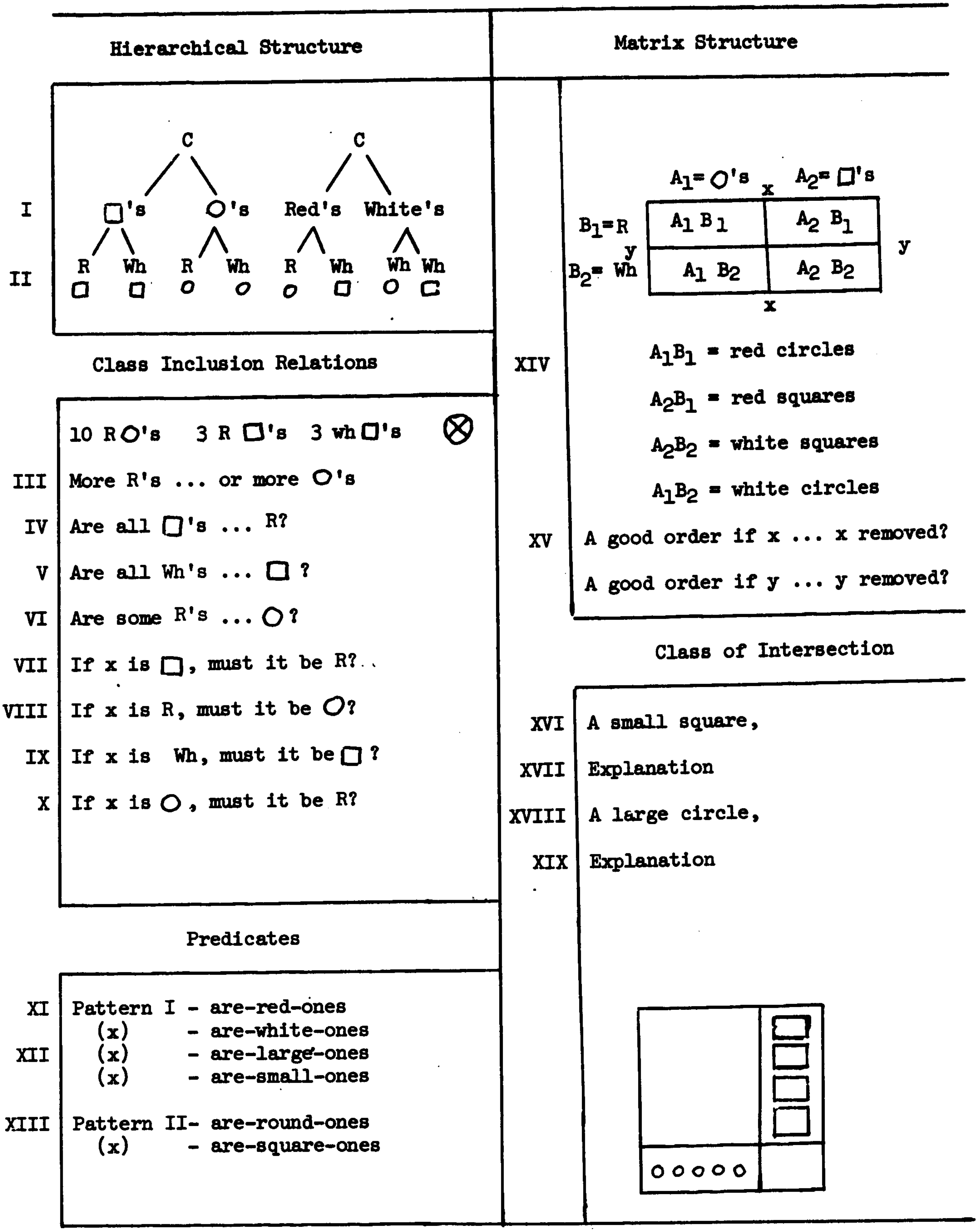


Fig. 15.--Summary of test items, Concrete tests of Classification.

The stimulus material was kept as uniform as possible throughout. Decisions, as defined, were assured as far as possible by covering the stimulus materials before presenting the problems.

Test items for the Stories tests of classification are summarized in Figure 16. The test items and details of their administration are presented in Appendix B. The ten test items assess hierarchical class inclusion relations; the construction of predicates which define the extensions of classes; the construction of multiplicative classes in a matrix structure; and the construction and description of the class of intersection. Tests items measuring these operations presented by Inhelder and Piaget in their experiments involved the complementary class and the class of intersection.[7]

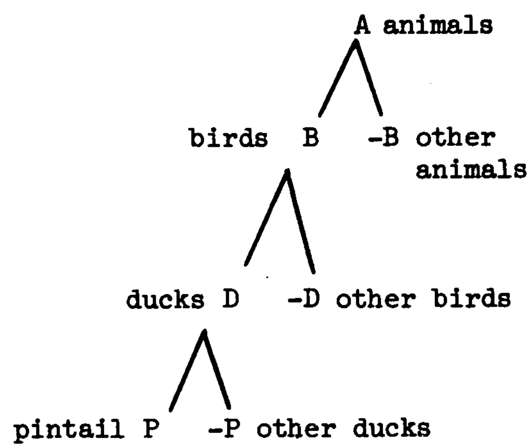
The preliminary questions presented for Story III and Story IV are designed to ensure comprehension of the material. They are capable of being answered directly from the story and constitute in effect a recapitulation of the sentences of the story. The questions and responses for Story III, for example, begin with the first sentences of the story:

> What kind of birds come back first ...?
> [The first birds to arrive are ducks ...]
>
> What kind of ducks arrive first?
> [... the first ducks to arrive are pintail.]
>
> How do the pintail get their food?
> [These ducks can live off the land ...]

---

[7]Inhelder and Piaget, The Early Growth of Logic, pp. 119-50 and 176-87.

SCO-Cl Test Items

Test Item	Hierarchical Structure
	

Class Inclusion Relations

Test Item	
I	More ducks or more pintail? Are pintail ducks?
II	More ducks or more birds? Are ducks birds?
III	More birds or more animals? Are birds animals?
IV	If all the birds flew away would there be some ducks left?

Predicates

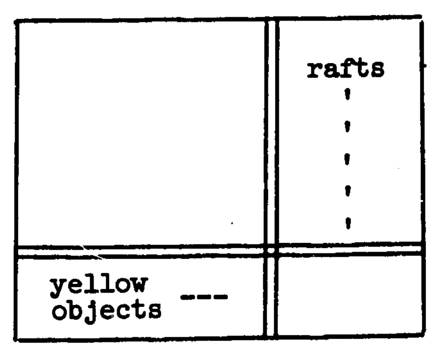
Test Item	
V	$\hat{x}$ - defined by time of arrival
VI	$\hat{x}$ - defined by place of arrival (Habitat)
VII	$\hat{x}$ - defined by feeding habits

Matrix Structure

Habitat

Test Item	Habit		ponds	lakes
VIII		1	surface feeders	⊗
IX		2	⊗	diving ducks

Class of Intersection



X a raft painted yellow (a yellow raft)

Fig. 16.--Summary of test items, Stories tests of classificatior

The questions continue to follow the order of the sentences in the story (see Appendix B). The detail recalled in answering these questions will be the "raw material" for the operations of classification required by the test items.

Classificatory operations in a reading situation appear, however, to require special restructuring of the information, given the style and complexity of continuous prose. The child reads:

> The first birds to arrive are ducks, and the first ducks are the pintail.

That is, he reads:

> ... birds ... are ducks, and ... ducks are ... pintail.

Inclusion relations in this sentence involve reversing this sequence (a shift from "some" to "all"):

> ... ducks are birds; pintail are ducks.

The preliminary questions which begin, "what kind of ...?" may offer some assistance. They permit a response which is a class name, a first step in classifying. The test questions which follow ask for inclusion relations and their quantification: "pintail are ducks", "all ducks are birds;" "there are more birds than ducks." Succeeding preliminary questions permit responses which are in the form of predicates, essential information for later operations in constructing alternative predicates and in multiplicative classification. In the testing situation there is no marked transition between the first form of questioning and the second: in reading comprehension involving classification it would also seem that one first assembles the information, then proceeds to process this information according to the class structures and relations available. The testing situations, Concrete and Stories, have been designed to follow, as far as possible, this procedure in thinking.

The operations of classification measured by the tests CCO-Cl and SCO-Cl will now be described.

Operations Measured: Classification

Operations of classification are considered to be components in an intellectual process by which one reality state is transformed into another in the context of assumed available class and class inclusion structures. The operations may be indicated symbolically by "logical operators"; the outcomes of the intellectual processes indicated by the operators may be observed as verbal responses. These responses may be represented symbolically as well as in sentences of the language.

Symbolic representation is useful in identifying the classificatory form of a response independently of the content on which the operation is performed. The following nine operations in classification are assessed in this study; a symbol representing the operation, "the operator", is shown in brackets:

Abstraction	$(\hat{x})$
Predication	(ϕ, ψ, θ)
Quantification	(x), $(\exists x)$, $(\sim\exists x)$
Addition	(&)
Multiplication	(X)

Complement of a class	$\hat{F}(x) \supset -\hat{F}(-x)$
Empty class	$\otimes$
Hierarchical class structure and relations	$\hat{K} = \hat{F} + \hat{G}$, $\hat{F} = \hat{F} + \hat{F}'$ etc.

Matrix class structure and relations

	F	G
A	AF	AG
A′	A′F	A′G

These operations require some clarification in relation to their assessment in the Concrete and Stories tests of classification.

<u>The operation of abstraction</u> ($\hat{x}$).--is a procedure whereby, given the condition " ... " upon x, we form the class $\hat{x}$ ... whose members are just those objects x which satisfy the condition.[8]

In this operation a class name is formed from a predicate. The operation is represented in the formula:

$$(\hat{x}):f_{(x)}$$

In language this could be the sentence, "Birds are feathered animals." Operations of abstraction are required in test items I, II, XIV and XV (see Fig. 15), and items I to IV, VIII, IX (see Fig. 16).

<u>Predicate operations (ϕ, ψ, θ)</u>.-- are procedures by which propositions are formed out of names. From individual names x, y, z ... or class names $\hat{x}$, $\hat{y}$, $\hat{z}$ ... the propositions formed may be represented as:

$$\phi x,\ \psi y,\ \theta y \ldots,$$

$$\theta\hat{x},\ \psi\hat{y},\ \phi\hat{z} \ldots.$$

---

[8]Willard Van Orman Quine, <u>From a Logical Point of View</u> (Cambridge, Mass.: Harvard University Press, 1964), p. 87.

In language these propositions could be sentences such as:

These counters are-all-large-ones. ϕx.

Pintail are-surface-feeders. $\psi\vartheta$

Predicate operations are assessed in test items XI to XIII (see Fig. 15), and items V to VII (see Fig. 16).

<u>The operation of quantification (x), (∃ x), (~∃ x)</u>.--requires an understanding of expressions such as "all", "every" (x); "some" (∃ x); "more" (>); and "no", "not any" (~∃ x) which describe the relations between the extensions of classes.

There are five and only five ways in which the extensions of two classes may be related; and between any two extensions, provided only that neither is null, one and only one of these relations holds. If the extensions of the two classes are represented by circles these relations may be represented by a letter (H, X, I, C and its reverse, ⊃). The French mathematician Gergonne in 1816 surveyed these possible relations of extension, developing a new theory described by Kneale as a "sub-structure for syllogistic". These relations are shown in Figure 17.[9]

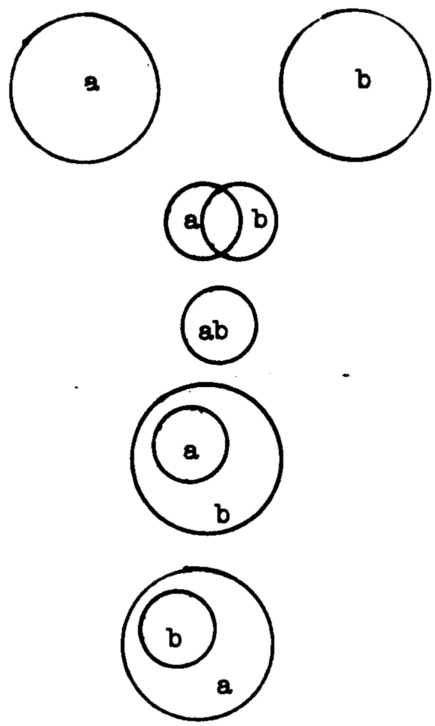
Each of the five relations between the extensions of classes may be expressed in a statement using one of the quantifying expressions "every", "all", "some", "no". These statements are shown in Figure 18.[10]

The quantification of the relation between the extensions of two classes is assessed by items III to X (see Fig. 15), and I to IV (see Fig. 16).

---

[9]William Kneale and Martha Kneale, <u>The Development of Logic</u> (Oxford: Clarendon Press, 1962), pp. 350-51.

[10]<u>Ibid</u>.



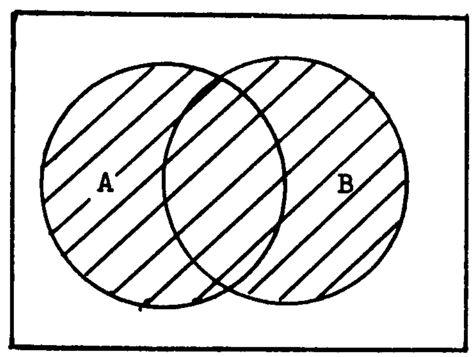
a	H	b	est hors de
a	X	b	s'entre-croise avec
a	I	b	est identique à
a	C	b	est contenu dans
a	$\supset$	b	contiens

Fig. 17.--Class inclusion relations: the possible relations between extensions of two classes.

a	H	b	No a is a b
a	X	b	Some a is b. Some a is not b. Some b is not a.
a	I	b	Every a is b. Every b is a.
a	C	b	Every a is b. Some b is not a.
a	$\supset$	b	Every b is a. Some a is not b.

Fig. 18.--Class inclusion relations: the quantification of the relation between the extensions of two classes.

The operation addition of classes (&).--is a procedure for constructing a new class by including in it the members of other classes. The outcome of this operation is represented in Figure 19. The class K is the sum of the members of classes A and B; it contains as its members those and only those elements which belong to at least one of the classes A and B.[11] An element of K may belong to both A and B and it may also belong to A or to B only. (Compare Figure 11, in which the new class contains only those elements which are joint members of A and B).



K = (A & B)

Fig. 19.- The construction of a new class: addition of classes.

The operation of addition of classes is basic to the hierarchical ordering of classes. In a hierarchical structure the superordinate class is the sum of its subclasses. Operations of addition underlie the hierarchical structures shown in Figures 15 and 16, and are involved in test items based on these structures.

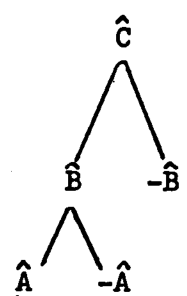
---

[11]Susanne K. Langer, An Introduction to Symbolic Logic (New York: Dover Publications, 1953), p. 140.

The operation multiplication of classes (X).--is a procedure for constructing a new class which is the "logical product" or the common part of two classes (s'entre-croise avec). The multiplicative class was represented in Figure 11, as the intersection of the two classes A and B.

The construction of the class of intersection of two classes is required in test items XVI to XIX (see Fig. 15), and in test item X (see Fig. 16).

The operation complement of a class (—).--is a procedure for constructing a new class which is the negate of a given class (see Fig. 13). A class B (see Fig. 20), will be the sum of subclasses A and − A, the negate of the subclass A. The formula, B = A + (-A), could represent the sentence, "Pintail and other ducks (-A) are birds".



$$\hat{B} = \hat{A} + (-\hat{A})$$

Fig. 20.--The construction of a new class: complement of a class.

The construction of the complement of a class is required in test items I to IV (see Fig. 16).

The operation empty class (⊗).--is a procedure for constructing a new class which is defined by the properties that it has no members and is itself a member of another class (see Fig. 12). Inclusion relations involving the empty class are assessed in test items III to X (see Fig. 15), and VIII, IX (see Fig. 16).

<u>The operation hierarchical class structure</u>.-- is the procedure for ordering classes in a superordinate-subordinate relation in which certain additive and disjunctive relations between the classes hold true. A class K and the subclasses F and G are hierarchically structured if the following relations are true:

$$\hat{K} = \hat{K} + \hat{G}$$
$$\hat{K} - \hat{F} = \hat{G}$$
$$\hat{K} - \hat{G} = \hat{F}$$
$$(x):(x \,\varepsilon\, K) \supset (x \,\varepsilon\, F) \vee (x \,\varepsilon\, G).$$

The operation of ordering classes hierarchically may continue as long as common categorial attributes are available. The attributes shape and color are used symmetrically in the hierarchical structure shown in Figure 21. Other criterial attributes such as size might have been available.

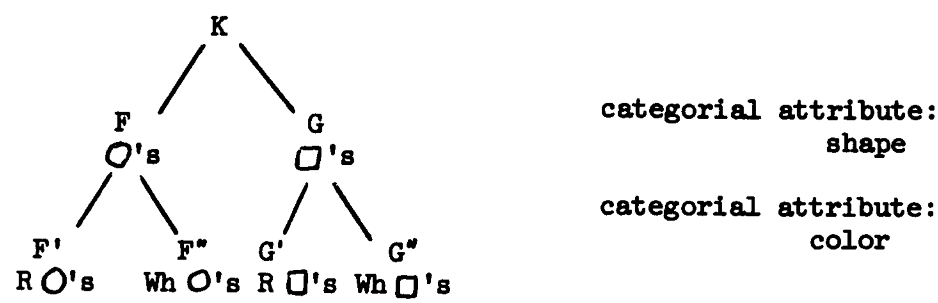


Fig. 21.--A hierarchical class structure.

Inclusion relations in hierarchical class structures are assessed in test items III to X (see Fig. 15), and I to IV (see Fig. 16). Implicative relations between propositions which are comparable to class inclusion relations are assessed as deductive operations (see Chap. VI).

The operation matrix class structure.--is the procedure for ordering multiplicative classes. In the matrix structure the categorial attributes are contiguous.

Multiplicative classes in a matrix structure are assessed in test items XIV and XV (see Fig. 15), and items VIII and IX (see Fig. 16).

The comparability of corresponding Concrete and Stories test items measuring operations in classification will now be considered.

Comparability of Concrete and Stories Tests

Test items CCO-Cl and SCO-Cl were designed to assess operations within the following dimensions of classification:

Abstraction and membership in a class,
Class inclusion relations,
The construction of predicates,
Multiplicative classes and relations.

The comparability of the decisions in classification for corresponding test items in these five dimensions will be considered. Symbolic representation will be used to identify and compare the logical forms of the responses required.

Abstraction and class membership

CCO-Cl test items I and II require the abstraction of the properties shape and color as a basis for ordering a random collection of objects into two, and four, mutually exclusive (no mixing of properties across classes) and exhaustive classes (no elements remaining unclassified).

The corresponding operations SCO-Cl require the recognition of the properties which determine membership in the classes pintail (x): p(x); ducks (x):$d_{(x)}$; and birds (x):$b_{(x)}$. The defining properties p, d, b, must permit the construction of mutually exclusive and exhaustive classes. Some of the criterial properties were given in the story; others were to be derived from experience (as was also knowledge of geometric shapes).

The two test situations appear to be directed to a common dimension in classification: the abstraction of properties and the assigning of membership in a class on the basis of these defining properties.

Inclusion relations

Test items CCO-Cl and SCO-Cl assessing class inclusion relations are based on the class structures shown in Figure 22.

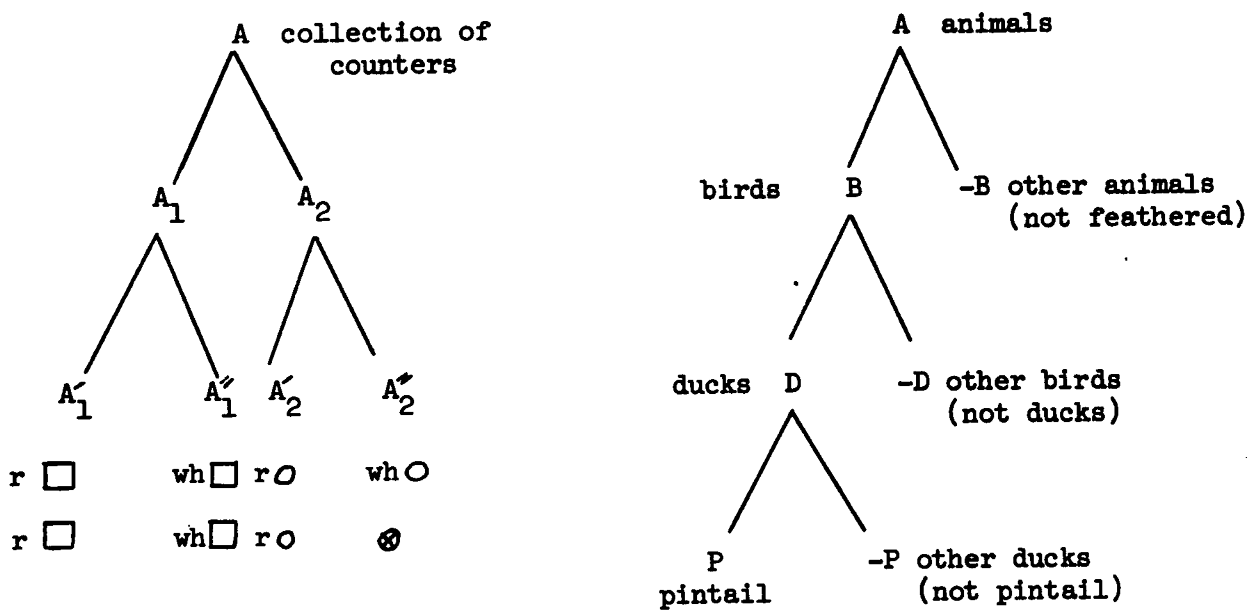


Fig. 22.--Hierarchical class structures: class inclusion relations CCO-Cl and SCO-Cl.

The hierarchical structures shown in Figure 22 suggest that differences in the formation of classes within the hierarchies may affect the assessment of inclusion relations. The Concrete class structure presents the empty class; the Stories structure presents the complementary class. Inclusion relations involving these structures are not necessarily equal in difficulty.

Two further differences affecting decisions concerning inclusion relations appear to be more closely associated with reading situations as such and may on this basis be justified in this testing situation. The first, discussed above, involves the manner of presentation of the information relating to inclusion. The subject reads:

> The first birds to arrive are ducks, and
> the first ducks are the pintail.

He must restructure this information to derive inclusion relations.

The second difference frequently associated with a reading situation concerns the universe of discourse. In a concrete situation the universe of discourse is usually given. In this concrete test situation the examiner explains, "These are counters." In a reading situation the domain under discussion is usually implicit: in the story, "The Ducks", for example, the reader must infer the universe of discourse which is the world of animals, their needs and habits. He must derive inclusion relations within this context. That this proved to be a difficult operation for these subjects was suggested by the frequent response, "It didn't say anything about animals in the story." It is suggested that the problems of restructuring the data, and determining the universe of discourse, as required in this Stories situation is inherent in reading comprehension and that these may be considered to be reasonable differences between concrete and stories situations.

The comparability of Concrete test items III and IV, and corresponding Stories items I and II, may be examined in Figure 23.

Item	CCO-Cl	Item	SCO-Cl
III	$\overset{R}{\square}$ $\overset{Wh}{\square}$ $\overset{R}{\bigcirc}$ $\otimes$ More R's ... or more $\bigcirc$'s $\hat{x}R = \hat{x}\overset{R}{\square} + x\overset{R}{\bigcirc}$ $\hat{x}R > \hat{x}\overset{R}{\bigcirc}$ $\hat{x}\overset{R}{\bigcirc} \subset \overset{R}{\hat{x}}$	I	More ducks or more pintail? Are pintail ducks? $\hat{x}D = \hat{x}P + \hat{x}\,(-P)$ $\hat{x}D > \hat{x}P$ $\hat{x}P \subset \hat{x}D$
IV	$\overset{R}{\bigcirc}$ $\overset{R}{\square}$ $\overset{Wh}{\square}$ $\otimes$ Are all $\square$'s ... R? $\hat{x}\square = \hat{x}\overset{Wh}{\square} + \hat{x}\overset{R}{\square}$ $\hat{x}\square > \hat{x}\overset{R}{\square}$ $\hat{x}\overset{R}{\square} \subset \hat{x}\square$	II	Are there more ducks or more birds? Are ducks birds? $\hat{x}B = \hat{x}D + \hat{x}(-D)$ $\hat{x}B > \hat{x}D$ $\hat{x}D \subset \hat{x}B$

Fig. 23.--Class inclusion: comparability of corresponding test items CCO-Cl-III-IV and SCO-CI-I-II.

The generalized logical form of the inclusion relations shown in Figure 23 is:

$$B = A + A'$$
$$B > A$$
$$A \subset B$$

A point of difference between the CCO-Cl test items and the SCO-Cl test items is associated with the occurrence of the complementary class in the SCO-Cl items. The generalized form of the SCO-Cl-I-II items indicates this difference:

$$B = A + (-A)$$
$$B > A$$
$$A \subset B$$

The comparability of test items CCO-Cl-V and IX and the corresponding test items SCO-Cl-III-IV may be examined in Figure 24.

	CCO-Cl		SCO-Cl
Item V	R ○ R □ Wh □ ⊗	Item III	Are there more birds or more animals?
	Are all the Wh's ... □ ?		Are birds animals?
	$\hat{x}\ Wh = \hat{x}\ \overset{Wh}{\square} + \otimes$		$\hat{x}\ A = \hat{x}\ B + \hat{x}(-B)$
	$\hat{x}\ Wh = \hat{x}\ \overset{Wh}{\square}$		$\hat{x}\ A > \hat{x}\ B$
	$\hat{x}\ Wh \subseteq x\ \overset{Wh}{\square}$		$\hat{x}\ B \subset \hat{x}\ A$
IX	R □ R ○ Wh □ ⊗		
	If x is a Wh, must it be □ ?	IV	If all the birds flew away would there be some ducks left?
	Are all Wh ... □ ?		
	$\hat{x}\ Wh = \hat{x}\ ^{Wh} + \hat{x}\ \otimes$		$\hat{x}B = \hat{x}D + \hat{x}(-D)$
	$\hat{x}\ Wh \subsetneq x\ \overset{Wh}{\square} + x\ \otimes$		$\hat{x}B \subseteq \hat{x}D + \hat{x}(-D)$

Fig. 24.--Class inclusion: comparability of corresponding tests CCO-CI-V and IX and SCO-Cl-III-IV.

The symbolic representation of the CCO-Cl test items in Figure 24 indicates the role of the empty class in these inclusion relations. The outcome for CCO-Cl-V and IX is complete class inclusion: every A is a B and every B is an A; A and B are coextensive (see Fig. 7). In the corresponding test item SCO-Cl-III the relation is also complete class inclusion but the class B is not-coextensive with the class A: every B is A; B and A are not coextensive (see Fig. 7). In SCO-Cl-IV the class B is coextensive with the classes D and -D.

The logical forms of these corresponding test items may be generalized:

$B = A + \otimes$	$B = A + (-A)$
$B = A$	$B > A$
$B \subseteq A$	$B \subset A$

The difference between these corresponding test items is the distinction in class inclusion between the coextensive and not-coextensive inclusion relation. All inclusion relations between classes are relations between the extensions of the classes: in hierarchical class structures between the extensions of subclasses and of a superordinate class. In these respects the logical operations involved in these corresponding test items are identical. The differences introduced are in the structures of the classes to be related: the empty class in the CCO-Cl tests and the complementary class in the SCO-Cl tests. The differences in coextensiveness follow.

Four additional Concrete test items measuring class inclusion relations were presented. Items VI, VII, VIII, involve the quantifier "some". Item X assesses the coextensive inclusion relation. These items may be represented symbolically:

Item VI: Are some of the R's ... $\bigcirc$'s?

$$\hat{x}\, R = \hat{x}\, \overset{R}{\bigcirc} + \hat{x}\, \overset{R}{\square}$$

$$(\exists x):(x \varepsilon R) \supset (x \varepsilon \bigcirc)$$

Item VII: If x is a $\square$, must it be R?

$$\hat{x}\square = \hat{x}\, \overset{R}{\square} + \hat{x}\, \overset{Wh}{\square}$$

$$(\exists x):(x \varepsilon \square) \supset (x \varepsilon\ Wh)$$

Item VIII: If x is an R must it be $\bigcirc$?

$$\hat{x}R = \hat{x}\, \overset{R}{\bigcirc} + \hat{x}\, \overset{R}{\square}$$

$$(\exists x):(x \varepsilon\ R) \supset (x\ \varepsilon \square)$$

The inclusion relations measured by the Concrete and Stories tests appear to be similar in the generalized form: for each test item it is a question of the additive relation between subclasses and superordinate class and the quantification of the relation between a superordinate class and one of its subclasses. The differences observed have been associated with the characteristics of the subclass: the empty class of the CCO-Cl test items and the complementary class of the SCO-Cl test items. These differences appeared to be clarified by a symbolic representation of the inclusion relations involved.

The construction of predicates

Concrete test items XI-XIII and Stories items V-VIII measure the construction of predicates which describe the extensions of dichotomous classes. The Concrete test items are based on Patterns I and II (see Fig. 14). The categories available are color, size and shape. For the Stories tests dichotomous classes are to be constructed from the group, "ducks that arrive in spring". The available categories, mentioned in the story, are time of arrival, place of arrival, and feeding habits. Predicates in each instance are expansions of these categories: red and white; large and small; round and square; and early and late; ponds and lakes; surface-feeders and diving ducks.

The classes, categories and predicates required are summarized in Figure 25. It is not suggested that the specific items are comparable, rather that the kind of decision required is the same for the Concrete and Stories. Different categories are available in each situation. It is considered that the expansions of these categories are appropriate for concrete and reading situations and that they were approximately equally available in the presentation of the materials.

CCO-Cl			SCO-Cl		
Item	Class	Category	Item	Class	Category
XI	$\hat{x} \ldots R_x$ $\ldots Wh_x$	Color - red - white	V	$\hat{x} \ldots T_{1x}$ $\ldots T_{2x}$	Time - early - late
XII	$\hat{x} \ldots L_x$ $\ldots S_x$	Size - large - small	VI	$\hat{x} \ldots P_{1x}$ $\ldots P_{2x}$	Place - ponds - lakes
XIII	$\hat{x} \ldots \;\; x$ $\ldots \;\; x$	Shape - round - square	VII	$\hat{x} \ldots H_{1x}$ $\ldots H_{2x}$	Habit - surface-feeders - diving-ducks

Fig. 25.--The construction of predicates, CCO-Cl-XI to XIII and SCO-Cl-V to VII.

<u>Multiplicative classes</u>

Concrete test items XIV and XV and Stories items VIII and IX (see Figs. 15 and 16), require the construction of multiplicative classes and their positioning in a matrix structure.

The logical sequence in the construction and ordering of multiplicative classes appears to be the abstraction of criterial properties, the multiplication of these properties to form new classes, and the positioning of the classes in a matrix relationship. This sequence is not necessarily an "operational" order in solving these problems. The test items intersect the "operations" at the point of inquiring for the matrix relationships of the constructed classes. The sequence is represented in Figure 26.

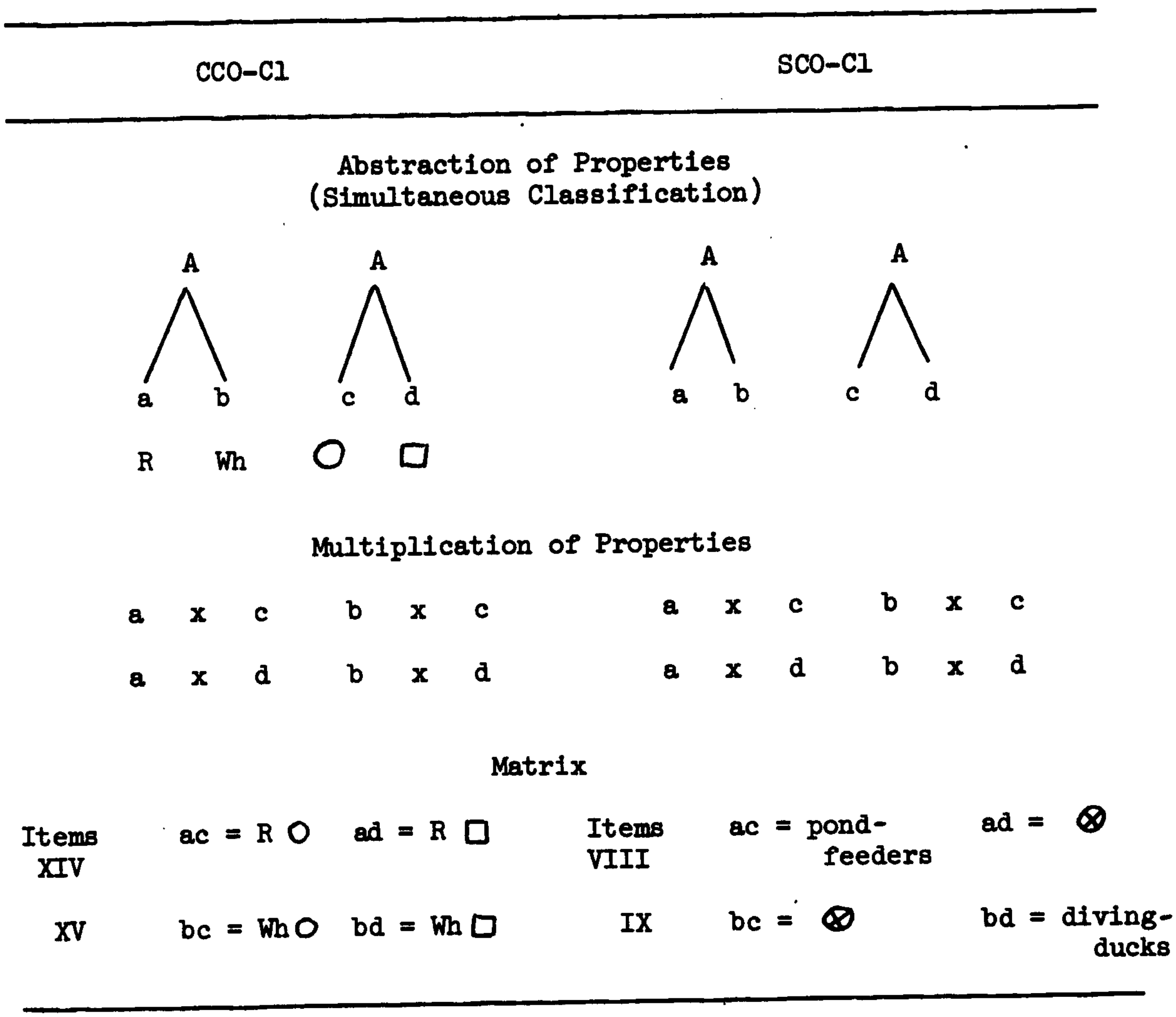


Fig. 26.--Multiplicative classes and their relations.

Concrete test items XVI to XIX and Stories item X also require the construction of the multiplicative class: in this case, the class of intersection of two class (see Figs. 15 and 16). The abstraction of the criterial properties and their multiplication to form the new class of intersection is represented in Figure 27.

CCO-Cl	SCO-Cl

Abstraction of Properties

large □ small ○ yellow raft

Multiplication of Criterial Properties

Items		Item	
XVI	a x d = large ○	X	a x b = yellow raft
XVII	b x c = small □		
XVIII			
	Explanation		Explanation
XIX			

Fig. 27.--Multiplicative classes: the intersection of two classes.

An examination of Figures 27 and 28 suggest that a satisfactory level of comparability may be postulated between Concrete and Stories test items assessing operations of multiplicative classification.

Scoring

The scoring of test items CCO-Cl and SCO-Cl follows the pattern of scoring the tests of conservation and test items in subsequent test sequences. A correct response for each test item is scored 1; an incorrect response is scored 0. Explanations are elicited for correct and incorrect responses. The scoring for the Concrete and Stories tests of classification is shown in Figure 28.

Test Item CCO-Cl SCO-Cl	Range of Scores CCO-Cl	 SCO-Cl
Construction	0 - 2	
Inclusion	0 - 8	0 - 4
Predicates	0 - 3	0 - 3
Matrix	0 - 2	0 - 2
Intersection	0 - 4	0 - 1
Range of Total Scores	0 - 19	0 - 10

Fig. 28.--Scoring for tests of classification.

Responses from the protocols assigned a score 0, and those assigned a score 1 are presented in Appendix B.

Some of the difficulties experienced by subjects in the study in classifying correctly with the material presented may be seen in the following examples selected from the protocols:

Construction

37 (9:6) Divide it into parts, half in one, half in the other. [Instructions repeated.] Well ... maybe seven in one and seven in the other.

Inclusion

75 (9:6) More round ones. Because when I put them on the table ... I picked up more round ones.

49 (9:7) More pintail 'cause it said in the story they come in flocks of hundreds. They come first.

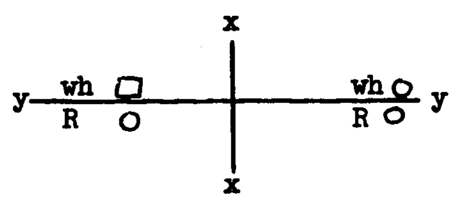
17 (10:1) [Are ducks birds?]
No, they're better. They swim and fly.

Predicates

4 (10:7) This is a red circle and this is a white one ... etc.
Each item is described.

Matrix

16 (10:0) The first attempt presents a diagonal:



[Good order? (x ... x)]
I don't know ... uh ...
[Can you fix it?]
Yes. [Attempt two is correct.]

Intersection

83 (9:7) No, you can't. You could only do it with two things. You can't put circles and squares together.
[This subject also failed to recognize the categories shape and size for Patterns I and II.)

SCO-Cl

79 (9:6) They'd have to set out two things ... maybe a raft and put some treasures on it.
[Instructions repeated.]
They didn't set out anything.
[They'd like to show they belong to both streets.]
No, I don't know. I can't tell them what to do.

Summary: Tests of Classification

Test items CCO-C1 and SCO-C1 were constructed to measure fundamental dimensions in classification: abstracting criterial properties which define membership in a class; inferring class inclusion relations; constructing predicates which describe the extensions of classes; and constructing and ordering multiplicative classes. A symbolic representation of the logical sequences involved in these decisions suggested that basic relations of inclusion, and fundamental procedures in abstraction, predication, and class construction were common to the two test sequences, CCO and SCO. The differences which occurred were in the forms of the classes to which these fundamental procedures were to be applied. In particular, the empty class did not occur in the corresponding test items for CCO and SCO; and the complementary class was not presented in Concrete test situations.

The dimensions of classification selected for examination would appear to represent a reasonable sampling of these behaviors at the Grade four level.

Procedures in scoring were indicated. Examples selected from the protocols were presented to illustrate some of the difficulties experienced by the subjects in responding to test items of classification.

Concrete and Stories tests of deduction will now be presented.

CHAPTER VI

TESTS OF DEDUCTION

This chapter will describe the construction of the Concrete and Stories tests of deduction. The tests will be designated CCO-D and SCO-D.

Logical terms used in describing the tests of deductive reasoning will be defined.

The materials used in the Concrete and Stories tests of deduction will be presented, followed by a description of the test items and an account of the inferences assessed by these test items. The comparability of corresponding test items Concrete and Stories will be considered and procedures for scoring the tests will be presented.

Definition of Terms

A number of additional terms used in describing the tests of deduction will be defined to ensure as far as possible that expressions introduced in the discussion of deductive reasoning will be used as these are defined by logicians. The following terms used generally in this study in discussing deduction were defined in Chapter I: deductive inference; proposition; propositional operation, synthetic proposition and tautology. The additional terms are defined below.

Logical truth

A sentence which expresses truth because of its form, and independently of its content, expresses formal or logical truth.[1]

Logical analysis

Logical analysis is the formal treatment or "rational reconstruction of linguistic expressions of which sentences are the most important.[2] The purpose of this formal treatment is to determine the extent to which the results of thinking as expressed in sentences are valid or invalid; whether statements are compatible; in general whether statements are related in such a way that one follows logically from the other according to the truths of logic.

The statements, "This sentence follows logically from that one", or, "If this sentence is true then (on logical grounds), that one is also true", mean that the sentence which logically follows is derivable from the sentence(s) which preceded it by rules of inference (Modus Ponens, Modus Tollens, the Principle of Transitivity, etc).

Object statements

Object statements concern our knowledge of matters of fact. They are statements which deal with "object questions", that is,

---

[1]Georg Henrik von Wright, Logical Studies (London: Routledge and Kegan Paul, 1957), p.2.

[2]Hans Reichenbach, Experience and Prediction (Chicago: University of Chicago Press, 1938, pp. 5-7.

they "have to do with the objects of a domain under consideration, such as inquiries regarding their properties and relations".[3]

Object statements vary with the extent of our knowledge. In addition, statements concerning knowledge are usually implicit in an elliptic form of speech in which we do not mention matters of fact to which the actual statements refer. The logical statement itself may be represented by the formula:

$$p \,\&\, (p \longrightarrow q) \longrightarrow q.$$

This could be read as in the "Lights" test:

> The red light is on, implies the green light is on.
> The red light is on (p).
> Therefore the green light is on (q).

Object statements are implicit in this inference. They include suppositions about the laws of electricity, assumptions about the wiring, the source of power, the connection with the source of power, etc. For each inference asserted by a subject in the "Village" and "Lights" tests and in the Stories tests, many object statements are assumed as implicit knowledge. Whereas the logical expression of an inference may be symbolized $p \,\&\, (p \longrightarrow q) \longrightarrow q$, an extended formula in which r represents the object statements implicit in this inference, could be symbolized:

$$r \,\&\, [p \,\&\, (p \longrightarrow q)] \longrightarrow q.$$

<u>Modality</u>

Modality is a "property" of propositions. This property may be expressed in monadic predicates: p is-necessary; p is-possible; p is-not-possible. These modalities may be examined as dyadic

---

[3]Rudolf Carnap, <u>The Logical Syntax of Language</u> (London: Routledge and Kegan Paul, 1964), p. 277.

predicates or relations between propositions, that is, as relative modalities.[4] For relative modalities von Wright has introducted the symbol $M(^p/_q)$. This symbol may be read: p is possible given q. The symbol $\sim M(^p/_q)$ may be read: p is not-possible given q. The symbol $\tilde{M}(^p/_{\sim q})$ may be read: $\sim$ p is possible given $\sim$ q. The symbol $\sim M(^{\sim p}/_q)$ may be read: not-p is not-possible given q, or by verbal convention, p is necessary given q.[5] The symbol $N(^p/_q)$ expresses the same relation of necessity between p and q.

In language, modality is expressed by the modals "may", "could", "might", "must" etc., associated with the verb. Modality may also be expressed by adverbs such as "necessarily", "possibly" and by predicate adjectives such as "possible" and "impossible".

Compatibility

The compatibility of one proposition with another is expressed by the symbol $M(^p/_q)$. Compatibility is a mutual modal relation and can not be established by considering the modal predicate of one of the propositions alone.[6] With possible propositions only other possible propositions can be compatible.[7] If each of the two propositions is possible and the two are also mutually possible, they are compatible. Compatibility is therefore determined by the relation of the propositions to premises, and their relation to object statements.

---

[4]von Wright, Logical Studies, p. 89.

[5]Ibid.

[6]Ibid., p. 92.

[7]Ibid., p. 101.

An assertion of compatibility or incompatibility is a judgment. Modality and probability may be related if it is possible to assign a unique and non-negative number expressing the "degree" to which a given proposition p is possible relative to q.[8] The probability will be a number not less than 0 and not greater than 1. The value of $\sim M(^p/_q)$ would be 0. The value of $N(^p/_q)$ would be 1.

A simple matrix of assigned probabilities of p relative to q is represented in Figure 29. 'p' and 'q' represent propositions.

Probability of $^p/_q$	$N(^p/_q)$	$M(^p/_q)$	$\sim M(^p/_q)$
1	1	0	0
½	0	1	0
0	0	0	1

Fig. 29. --Modality and the probability of a proposition p relative to q.

Materials

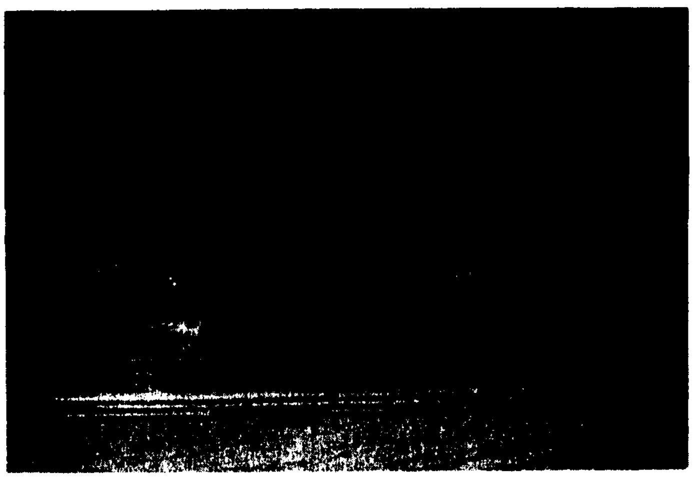
Three test situations are presented for the Concrete tests of deduction: the "Village" and "Lights" test situations, designed by Matalon,[9] and a test situation designed by the investigator which involves the Principle of Transitivity. The materials for these tests are shown in Figure 30.

---

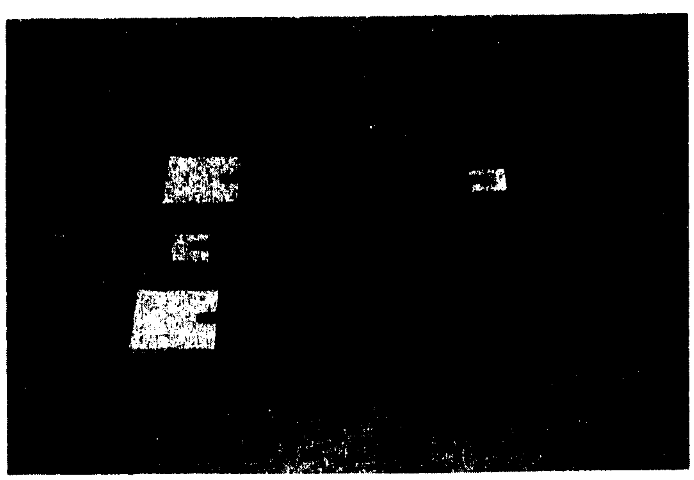
[8] Ibid., p. 115.

[9] Benjamin Matalon, "Étude génétique de l'implication", in Implication, formalisation et logique naturelle, Vol.XVI of Études d'épistémologie génétique, ed. by Jean Piaget (Paris: Presses Universitaires de France, 1962), pp. 69-95. (Hereinafter referred to as Étude de l'implication.)

The "Village" Test



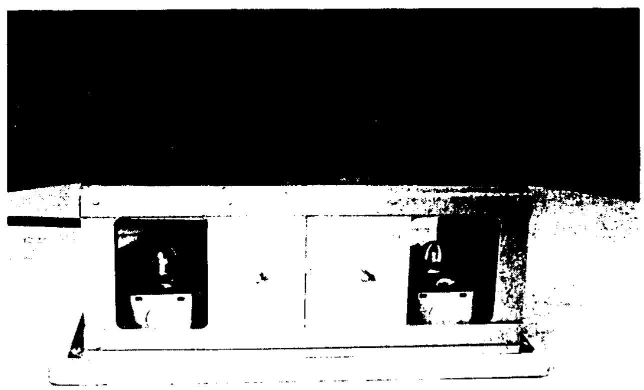
The "Lights" Test



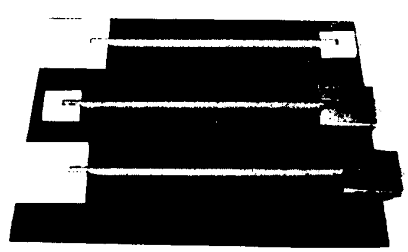
Test of Transitivity

Fig. 30--Test materials: Concrete tests of deduction.

The "Village" Test



The "Lights" Test



Test of Transitivity

Fig. 30--Test materials: Concrete tests of deduction.

The "Village" test materials. -- consist of varnished wooden blocks, representing houses, a Post Office, a School, a church, an apartment block, and trees. A small carved wooden figure represented a man. The materials were assembled on an 20" x 14" x ½" block representing the village, the village square, and a road leading to the square painted black. The figure of the man was placed on this road leading into the village. One of the houses stood beside this road.

The "Lights" test materials. -- consist of a metal box fitted with two electric light bulbs, green and red, positioned behind two sliding windows. The lights were wired to be operated by a switch according to the rule, "When the red light is on, the green light is on".

The Principle of Transitivity test materials. -- consist of square counters with small square openings cut out of one side of the square to permit insertion of a small green rod. The wooden rods used were 6" rods from a set of Tinker Toys. The counters required were as follows:

4 white square counters 1½" to side,
3 red square counters 3/4" to side,
4 white square counters ½" to side,
2 red square counters 3/4" to side.

The "Village" and "Lights" tests were selected to permit a comparison of the results obtained for deductive reasoning in the present study with results reported in Matalon's study. A test of reasoning based on the Principle of Transitivity was considered to extend the Concrete deductive operations which could be related to reasoning in reading as measured by the Stories tests. The test items are shown in Appendix C; they are summarized in Figure 31.

The Stories tests of deduction are based on two story situations: Story III, "The Ducks Arrive in Spring" and Story V, "The Prairie Blizzard". These stories are presented in Appendix C. Test questions based on the stories are designed to follow as closely as possible the inferences assessed by the Concrete tests. The Stories items are also shown in Appendix C. They are summarized in Figure 32.

Test Items Concrete and Stories

Five test items CCO-D require deductive inferences based on the rules Modus Ponens, Modus Tollens and the Principle of Transitivity. Four test items required the recognition that an inference was undetermined by the premises. Eight items required judgments of the compatibility or incompatibility of two propositions. The rule of inference and the formula for each of the test items is shown in Figure 31.

Test items for the "Village" and "Lights" tests were constructed parallel in form and followed in general the pattern of the test items presented by Matalon for these test situations.[9] Each test item was constructed to require a decision as defined, and an explanation of the decision. The stimulus material was held constant throughout each test situation.

---

[9]Matalon, "Études de l'implication".

Summary of Inferences CCO-D

Test Item	Rule of Inference	Formula
I	Modus Ponens	$PO \& (PO \longrightarrow H) \longrightarrow H$
II	Undetermined	$(\exists \sim PO):(\sim PO \longrightarrow \sim H) \sim (\sim PO):(\sim PO \longrightarrow \sim H)$
III	Undetermined	$(\exists H):(H \longrightarrow PO) \sim (H):(H \longrightarrow PO)$
IV	Modus Tollens	$\sim H \& (PO \longrightarrow H) \longrightarrow \sim PO$
V	Compatibility "Village" tests	$M(^{H}/_{PO}$
VI		$\sim M(\sim ^{H}/_{PO})$
VII		$M(^{H}/_{\sim PO})$
VIII		$M(\sim ^{H}/_{\sim PO})$
IX	Compatibility "Lights" tests	$M(^{R}/G)$
X		$\sim M(^{R}/_{G})$
XI		$M(\sim ^{R}/_{G})$
XII		$M(\sim ^{R}/_{\sim G})$
XIII	Modus Pontens	$R \& (R \longrightarrow G) \longrightarrow G$
XIV	Undetermined	$(\exists \sim R):(\sim R \longrightarrow \sim G) \sim (\sim R):(\sim R \longrightarrow \sim G)$
XV	Undetermined	$(\exists G):(G \longrightarrow R) \sim (G):(G \longrightarrow R)$
XVI	Modus Tollens	$\sim G \& (R \longrightarrow G) \longrightarrow \sim R$
XVII	Transitivity	$(C_1 \longrightarrow C_2) \& (C_2 \longrightarrow C_3) \longrightarrow (C_1 \longrightarrow C_3)$

Fig. 31. -- Summary of inferences,test items CCO-D.

Stories test items were constructed to parallel as closely as possible the deductive inferences measured by the Concrete tests. The inferences assessed by the Stories Tests are represented in symbolic form in Figure 32. The inferences measured by the Concrete and Stories tests will now be described.

Deductive Inferences Measured

The inferences measured by the Concrete and Stories tests are of two general forms: the necessary and undetermined; and the possible and not-possible.

Necessary inferences follow tautologically from the premises by the rule corresponding to the tautology. The rules by which necessary inferences are derived in the Concrete and Stories tests are Modus Ponens, Modus Tollens, and the Principle of Transitivity. For example, from the premise PO (He has been to the Post Office) and the premise PO $\longrightarrow$ H, the inference H (He has passed the house) follows by the rule Modus Ponens:

$$PO \,\&\, (PO \longrightarrow H) \longrightarrow H.$$

Similarly in test item CCO-D-XIII, the inference G (The green light is on) follows from R (The red light is on) and R $\longrightarrow$ G:

$$R \,\&\, (R \longrightarrow G) \longrightarrow G.$$

In test items CCO-D-IV and CCO-D-XVI (see Fig. 31) the inferences $\sim$ PO and $\sim$ R follow by the rule Modus Tollens:

$$\sim H \,\&\, (PO \longrightarrow H) \longrightarrow \sim PO.$$

$$\sim G \,\&\, (R \longrightarrow G) \longrightarrow \sim R.$$

Summary of Inferences SCO-D

Test Item	Form of Inference	Formula
"The Ducks"		
I	Modus Ponens	$\sim I \,\&\, (\sim I \longrightarrow D) \longrightarrow D$
II	Undetermined	$[(\exists F:(F \longrightarrow D)]\sim[(F):(F \longrightarrow D)]$
III	Undetermined	$[(\exists D):(D \longrightarrow \sim F)]\sim[(D):(\longrightarrow \sim F)]$
IV	Modus Tollens	$\sim D \,\&\, (IM \longrightarrow D) \longrightarrow \sim IM$
V	Transitivity	$(M \longrightarrow T) \,\&\, (T \longrightarrow Sh) \longrightarrow (M \longrightarrow Sh)$
"The Blizzard"		
VI	Compatibility	$M(^{C}/_{FH})$
VII		$\sim M(^{\sim C}/_{FH})$
VIII		$\sim M(^{C}/_{\sim FH})$
IX		$M(^{\sim C}/_{\sim FH})$
X		$M(^{Rd}/_{\sim C})$
XI	Modus Ponens	$C \,\&\, (C \longrightarrow FH) \longrightarrow FH$
XII	Modus Tollens	$\sim C \,\&\, (FH \longrightarrow C) \longrightarrow \sim FH$
XIII	Modus Ponens	$FH \,\&\, (FH \longrightarrow C) \longrightarrow C$
XIV	Modus Tollens	$\sim FH \,\&\, (C \longrightarrow FH) \longrightarrow \sim C$
XV	Undetermined	$[(\exists Rd):(Rd \longrightarrow C)]\sim[(Rd):(Rd \longrightarrow C)]$
XVI	Undetermined	$[(\exists H):(H \longrightarrow C)]\sim[(H):(H \longrightarrow C)]$
XVII	Transitivity	$(FH \longrightarrow R) \,\&\, (R \longrightarrow PH) \longrightarrow (FH \longrightarrow PH)$

Fig. 32. -- Summary of inferences, test items SCO-D.

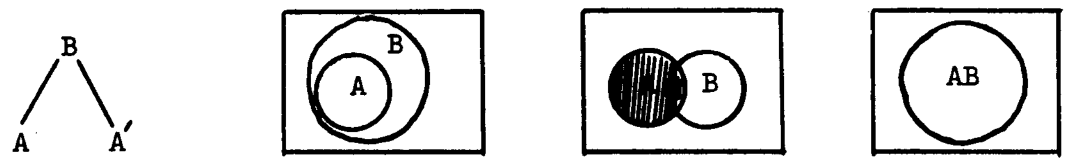
In test item CCO-D-XVII the rule to be applied is the Principle of Transitivity. In this test item perceptual aspects (the question of what "looks right" by color or shape may intervene to occupy attention and supercede the intellectual operation of searching for and applying a rule.

The inference Undetermined, which means "one may not infer with certainty", is required for test items CCO-D-II-III and XIV and XV (see Fig. 31). The logical basis for uncertainty derives from the nature of the premises: one may not infer with certainty from what is known to be true of "some" occurrences of an event to "all" occurrences of this event. From the fact that for some occurrences it is true that if the green light is on ($\exists G$) the red light is also on ($\exists G$): ($G \longrightarrow R$), one may not validly infer ($\sim$) that for "all" cases of the green light being on, the red light will also be on:

$$(\exists G):(G \longrightarrow R) \sim (G):(G \longrightarrow R).$$

An analogous situation was presented in the discussion of class inclusion relations in Chapter V. It was noted that from the case that "some A's are B's" one may not conclude either that all A's are B's or that all B's are A's. Class inclusion relations involving the quantifiers "some" and "all" are illustrated for complete coextensive and not-coextensive relations, and for partial class inclusion relations (see Fig. 33).

From the relations of inclusion shown in Figure 33 it is



$(A)\ (A \subset B)$	$(A)\ (A \subset B)$	$(\exists A)\ (A \epsilon B)$	$(A)\ (A \epsilon B)$
$B = A + A'$	$(\exists B)(B \epsilon A)$	$(\exists B)(B \epsilon A)$	$(B)(B \subset A)$
			$A \subseteq B$

Fig. 33. -- Class inclusion relations: additive, complete and partial inclusion and complete coextensive inclusion relations.

clear that a predicate which describes the extension of one class, for example, A, may describe either a part or a whole of the extension of the class in which it is included. Knowing only the predicate of the class A and the fact of its inclusion in the class B, the extent of this inclusion is undetermined.

Deductive inference concerns, not the relations between classes as defined by their predicates, but rather the relations between propositions as unanalysed wholes. Propositions, however, may also be asserted to be true of "some" occurrences or for "all" occurrences; what is asserted to be true of "some" occurrences may or may not hold true for "all" occurrences. In going from an assertion which is true of "some" to an assertion purported to be true for "all", the inference required is "one cannot know with certainty"; "one may not validly infer."

In test item CCO-D-II, for example, from the premises $(\exists \sim PO)$ and $(\sim PO \longrightarrow \sim H)$, one may not validly infer $(\sim)$ that for all occasions, $(\sim PO)$, the same condition will hold true. The symbolization of the inference Undetermined:

$$[(\exists \sim PO):(\sim PO \longrightarrow \sim H)] \nvdash [(\sim PO):(\sim PO \longrightarrow \sim H)]$$

may be read:

> From the fact that on some occasions not going to the Post Office implies not passing the house, one may not validly infer that on all occasions of not going to the Post Office the man did not pass the house.

In addition to the modality "necessity" and the inference Undetermined test items assessing the logical modalities Possible and Not-Possible were presented. These inferences are summarized for test items CCO-D-V to VII in Figure 31. An inference Possible or Not-Possible depends on a judgment of the compatibility of the propositions with a basic premise and with object statements. The basic premise for these test items is:

> One must pass the house to go to the village.

An object statement would be:

> The Post Office is only one of the places to go in the village; he might not go to the Post Office, but it is possible that he could have gone to another place in the village.

For test items CCO-D-IX to XVII, the "Lights" tests, the basic premise to which compatibility is to be referred is:

If the red light is on (R), the green light is on (G).

Object statements would be expected to refer to conditions of the wiring, connections etc. The subject in these test items is required to assume the appropriate object conditions and to reason on the basis of "if ... then" premises.

In administering these tests of deductive reasoning one gradually realized that it was an impressive fact that each of these children (there was one exception only) "knew" immediately that a state of affairs which he could not "see" (e.g., the condition of a light behind a closed window) could be known to him with certainty (or known by him to be uncertain) by means of an intellectual operation. He did not question that an unknown and unseen state of affairs could be deduced from conditions he did know. The one child who stated that he could not know if he did not see was persuaded to try on the basis of the rule. He too succeeded on some of the inferences, and on none of them did he appear to be offering a random guess. Moreover, when children presented with these problems stated that they did not know, the response seemed to mean, "I cannot figure that one out". It was frequently expressed in just these words. This appeared to imply an acceptance of responsibility for eventually "coming to know". Certainly there was an element of stress in these responses; but this did not at all appear to be the stress of fear or shame or guilt. It seemed rather to express a kind of "cognitive responsibility" now in process of being assumed by the child. The contrast was quite marked between the responses:

> I don't know that one.
> I can't figure that one out.

and the tone of freedom from intellectual responsibility which seemed to be expressed by the child who began by saying:

> I can't know if I don't see!

but who later accepted the responsibility for trying "to see".

It has been suggested that inferential reasoning is supported by object statements available to the child. Problems in applying object statements to inferences is discussed more fully below for the

Stories test items assessing deduction since they appeared to have a more crucial role in the reading situation.

The inferences for test items SCO-D-I-XI-XIII follow from the rule Modus Ponens (see Fig. 32). For test item SCO-D-I the inference was symbolized:

$$\sim I \;\&\; (\sim I \longrightarrow D) \longrightarrow D \qquad \sim I = \text{no ice}$$

The object statement, "Ducks live on open water" would appear to be essential in support of the inference "There are ducks" (D). An even more fundamental object statement was assumed: "No ice" means the ice will have melted so that there will now be stretches of open water.

The inferences for test items SCO-D-XII-XIV follow from the rule Modus Tollens. These were symbolized:

$$\sim C \;\&\; (FH \longrightarrow C) \longrightarrow \sim FH$$

$$\sim FH \;\&\; (C \longrightarrow FH) \longrightarrow \sim C$$

For these inferences the logical conclusions $\sim$FH (They have not reached the farm house) and $\sim$C (They have not found the cut), appeared to involve an appreciable sense of the dangers inherent in the story situation both to the people on the frozen river and to the mother expecting the baby. This awareness may have intruded as a distractor resulting in difficulty in "selecting out" the logical elements in the situation in order to consider the problem from this point of view. In the testing situation object statements which were expressed by some of the children seemed to reflect their concern. The statements expressing this concern were, however, irrelevant from a logical point of view. The statements these children were considering included the following:

They could go through the ice and drown.
They could freeze to death.
One of the horses could break a leg.
They could lose their way and go in circles.
If they didn't find the cut maybe the father
could help the mother get the baby.

Object statements in reading also appeared to require to a greater degree than a laboratory or concrete-type situation, an ability to visualise spatial-temporal relations. This seemed to be demonstrated in test item SCO-D-VII (see Fig. 32), in which the inference followed from the Principle of Transitivity. For this test the premises were presented as implications:

If they reach the farm house, they will reach the road.
If they reach the road they will reach the patient's house.

In this test item a number of children telescoped into one place the farm house and the patient's house, and eliminated "the road". The spatial-temporal considerations in the premises no longer applied, so that the logical problem no longer existed. These conditions again seem to be inherent in reading situations and should on this basis be included in examining reasoning in reading.

It was a remarkable fact in these Stories tests that every nine-to-ten-year-old child in the sample accepted without hesitation the contrary-to-fact premises presented and proceeded to offer inferences derived from these premises.

It is by no means a universal achievement to be willing to speculate on the possible and not only on the real. In a recent study of the acculturation of the Blood Indian people of Alberta the Spindlers found that only to a very limited extent was conditional and conjectural thinking possible to the young adult of this vigorous and prosperous culture. They reported that the Blood people:

> ... were as unable to retrospect conditionally as to project into the present and future conditionally. Things are as they are, or were as they were .... Most Blood simply do not engage in conditional thinking .... It is a cognitive system that is decidedly divergent from that which is an integral part of Western culture.[10]

The acculturative problem of the Blood, as it is presented in this study, would appear to be precisely the problem of a successful transition from a concrete level of inference to the domain of the logic of propositions which includes accepting contrary-to-fact premises. The Spindlers conclude:

> The Blood live in and think about the world as it seems to be to them. Western man lives in the world to a larger extent as he would like it to be, or thinks it should be.[11]

It would seem that a good many nine-to-ten-year-old children in our culture are willing to accept and consider the implications of "contrary-to-what-is" premises: "They have reached the farm house"; "The rivers and lakes are frozen over" etc. It would seem that resources for this development must be available to these children in the culture.

This aspect of inferential behaviour (reasoning from contrary-to-fact premises) was not considered in designing the tests of deduction in this study. Significantly, perhaps, it did not occur to the examiner that it was a matter in question for the age of the children in the sample. It seemed also that the possibility of not proceeding on the basis of a contrary-to-fact premise did not occur to the children.

---

[10]George Spindler and Louise Spindler, "The Instrumental Activities Inventory; a Technique for the Study of the Psychology of Acculturation", <u>Southwestern Journal of Anthropology</u>, XXI (Spring, 1965), 1-23.

[11]<u>Ibid</u>.

If it is correct to assume that inferential behavior on the basis of contrary-to-fact premises was readily available to the children in this study, then the developmental sequences by which it was achieved would already have been accomplished. Knowing some of the effective stimuli for this learned behavior would be of considerable interest. An examination of a number of anthropological studies of the classificatory systems of the languages of different cultures suggests one of these possible sources. Languages differ widely in their manner of ordering the environment.[12, 13, 14] In not all languages are exclusive and exhaustive terminological systems used. Murdock pointed out that "the assumption that cultures . . . have in common a uniform system of classification . . . a single basic plan", has led to much confusion in describing cognitive orientations.[15] The western classificatory pattern, "A is included in B; A is a B", was found to be inadequate for describing Indian folk taxonomies. It was suggested that these taxonomies were better represented by a "sphere of influence" model.[16]

---

[12] William C. Sturtevant, "Studies in Ethnoscience" American Anthropologist, Special Publication, LXIV (June, 1964), 99-131.

[13] George Peter Murdock, "The Common Denominator of Cultures". in The Science of Man in the World Crisis, ed. by Ralph Linton (New York: Columbia University Press, 1945), pp. 123-42. (Hereinafter referred to as The Common Denominator).

[14] Jane O. Bright and William Bright, "Semantic Structures in North Western California and the Sapir-Whorf Hypothesis", American Anthropologist, Special Publication, LXVII (October, 1965), 249-58. (Hereinafter referred to as Semantic Structures).

[15] Murdock, "The Common Denominator".

[16] Bright and Bright, "Semantic Structures".

Exclusive and exhaustive classification and the logic of class inclusion would be likely to be learned most effectively if these were already incorporated in the language of the culture. The development of inferential skills, including the ability to reason on the basis of contrary-to-fact, "as if" premises, may depend on a level of mastery of class structures and class inclusion relations since these appear to be related in logical form to the logic of propositions.

Observing the children in this study in the test situations involving deductive reasoning, it seemed clear that behaviors underlying the ability to derive the inferences assessed had been learned to a quite remarkable level of competency. These children understood that intellectual operations could be substituted for perceptual evidence in arriving with certainty at knowledge of a state-of-affairs. Their selection of object statements were in most instances appropriate and adequate. They accepted and reasoned without hesitation from contrary-to-fact premises. There is the further question of the availability of rules of logic by which valid inferences are derived, and the extent to which these may be equally available to nine-and-ten-year-old children if comparable conditions for reasoning can be provided in concrete and in reading situations.

The comparability of corresponding test items CCO-D and SCO-D will now be considered for items assessing Necessary inference, Undetermined inference, and judgments of the compatibility of propositions.

Comparability of Concrete and Stories Tests

In determining the comparability of test items designed to assess Necessary deductive inference, a matter of first importance is that the inferences for corresponding test items should follow from the same logical rules. The formulae presented in Figure 34 suggest that this requirement has been met: corresponding test items are based on the rules Modus Ponens, Modus Tollens and the Principle of Transitivity.

Comparability of Deductive Inferences
CCO-D and SCO-D

Modus Ponens

Test Item	CCO-D	Test Item	SCO-D
I	$PO \;\&\; (PO \rightarrow H) \rightarrow H$	I	$\sim I \;\&\; (\sim I \rightarrow D) \rightarrow D$
XIII	$R \;\&\; (R \rightarrow G) \rightarrow G$	XI	$C \;\&\; (C \rightarrow FH) \rightarrow FH$
		XIII	$FH \;\&\; (FH \rightarrow C) \rightarrow C$

Modus Tollens

Test Item	CCO-D	Test Item	SCO-D
IV	$\sim H \;\&\; (PO \rightarrow H) \rightarrow \sim PO$	IV	$\sim D \;\&\; (IM \rightarrow D) \rightarrow \sim IM$
XVI	$\sim G \;\&\; (R \rightarrow G) \rightarrow \sim R$	XII	$\sim C \;\&\; (FH \rightarrow C) \rightarrow \sim FH$
		XIV	$\sim FH \;\&\; (C \rightarrow FH) \rightarrow \sim C$

Transitivity

Test Item	CCO-D	Test Item	SCO-D
XVI	$C_1 \rightarrow C_2$	XVII	$FH \rightarrow Rd$
	$C_2 \rightarrow C_3$		$Rd \rightarrow PH$
	$C_1 \rightarrow C_3$		$FH \rightarrow PH$

Fig. 34. -- Comparability of deductive inferences assessed by corresponding test items CCO-D and SCO-D.

The inferences Undetermined for corresponding test items CCO-D and SCO-D are presented in symbolic form in Figure 35. It would appear that the logical forms of these inferences are comparable. Premises CCO-II and XIV are, however, negative in form: He has "not" gone to the Post Office"; "The red light is out". Negative premises of this form do not occur in Stories tests. This condition could represent a difference in the operations involved in these corresponding situations assessing the inference Undetermined.

Comparability of Inferences Undetermined CCO-D and SCO-D

II H Undetermined ("Village" CCO)

$[(\exists \sim PO):(\sim PO \rightarrow \sim H)] \nvdash [(PO):(\sim PO \rightarrow \sim H)]$

D Undetermined (The Ducks, SCO)

$[(\exists F):(F \rightarrow D)] \nvdash [(F):(F \rightarrow D)]$

III PO Undetermined ("Village" CCO)

$[(\exists H):(H \rightarrow PO)] \nvdash [(H):(H \rightarrow PO)]$

F Undetermined (Ducks SCO)

$[(\exists D):(D \rightarrow \sim F)] \nvdash [(D):(D \rightarrow \sim F)]$

XIV G Undetermined (Lights CCO)

$[(\exists \sim R):(\sim R \rightarrow \sim G)] \nvdash [(\sim R):(\sim R \rightarrow \sim G)]$

XV Rd Undetermined (The Blizzard SCO)

$[(\exists Rd):(C \rightarrow Rd)] \nvdash [(Rd):(C \rightarrow Rd)]$

XV R Undetermined.

$[(\exists G):(G \rightarrow R)] \nvdash [(G):(G \rightarrow R)]$

XVI C Undetermined.

$[(\exists H):(H \rightarrow C)] \nvdash [(H):(H \rightarrow C)]$

Fig. 35. -- Comparability of inferences Undetermined, Concrete and Stories tests of deduction.

Decisions for corresponding test items CCO-D and SCO-D requiring judgments of the compatibility of propositions are presented in symbolic form in Figure 36. The logical forms of these decisions appear to be comparable.

Judgments of the Compatibility of Propositions
CCO-D and SCO-D

M:Possible

CCO-D Test Item		SCO-D Test Item	
V	$M(^{H}/_{PO})$	VI	$M(^{C}/FH)$
VII	$M(^{H}/_{\sim PO})$	X	$M(^{Rd}/_{\sim C})$
VIII	$M(^{\sim H}/_{\sim PO})$	VIII	$M(^{\sim C}/\sim FH)$
IX	$M(^{R}/_{G})$		
XI	$M(^{\sim R}/_{G})$		
XII	$M(^{\sim R}/_{\sim G})$	IX	$M(^{\sim C}/_{\sim R})$

$\sim$ M:Not Possible

VI	$\sim M(^{\sim H}/_{PO})$	VII	$\sim M(^{\sim C}/_{FH})$
X	$\sim M(^{R}/_{\sim G})$	VIII	$\sim M(^{C}/\sim FH)$

Fig. 36. -- Comparability of judgments of the compatibility of propositions CCO-D and SCO-D.

The symbolic representations of the inferences required by corresponding test items CCO-D and SCO-D suggest that there is an identity of logical form for the inferences requiring judgments of the compatibility of propositions.

Premises in the negative in the Concrete test items CCO-D-II and XIV may have introduced differences in the cognitive operations involved.

Problems affecting corresponding test items could also include the presence or absence of contrary-to-fact premises, and specific problems in assembling object statements. To some degree these differences could be considered inherent in, and probably appropriate to reading situations as compared with laboratory-like situations designed to assess cognitive operations.

Scoring

Scoring procedures for the tests of deduction were similar to that for previous tests: a correct decision and a correct logical explanation was assigned a score 1; if either was incorrect the score assigned was 0. The range of scores for the Concrete tests of deduction was 0 to 17. The range of scores for the Stories tests was also 0 to 17.

Protocols presented in Appendix C illustrate responses assigned the scores 1 and 0. They suggest some of the difficulties of the subjects in recognizing the inference Undetermined. The judgments Possible and Not-Possible also seemed difficult, particularly in reading. The problem is illustrated in the protocol below:

SCO-D

VII	23(9:6)	0	Yes, they might go another way but most likely they went through the ice. The father would have to help the mother get the baby. For: $\sim M(^{FH}/_{\sim C})$.

Summary: Tests of Deduction

The inferences assessed by the Concrete and Stories tests of deduction were distinguished by the characteristics of the rules of inference on the basis of which the conclusions could be derived.

Valid inferences followed tautologically by the rules Modus Ponens, Modus Tollens and the Principle of Transitivity.

The conclusion, "One may not validly infer",was indicated in deriving a proposition quantified by "all", from a proposition quantified by "some". This relation between propositions was considered analagous to the relation of class inclusion between two classes in which the extension of one class was quantified by "some" and the extension of a second class by "all".

The inferences Possible and Not-Possible referred to the mutual compatibility of two propositions with respect to a premise and to object statements.

Contrary-to-fact premises appeared to be accepted by the children in the study as a basis for inference. The contribution of early language learning which made available exclusive and exhaustive classifications and therefore implicative class inclusion relations was considered likely to be significant in this achievement.

Subjects in the study also appeared to accept that intellectual operations could be substituted for perceptual evidence when they were presented with premises appropriate for deductive inference.

The appropriate use of object statements was considered to present certain difficulties in reading, including the situations presented in the Stories tests. A subject's attention could be directed by the story to compelling object statements with the result that he would tend to substitute these statements for the logical inferences required.

Consideration of the comparability of corresponding test items was based on a symbolic representation of the inferences required. On this basis, corresponding test items appeared to represent the logical structures indicated and to require inferences identical in form.

The introduction of a negative premise in test items CCO-D-II and CCO-D-XIV (see Fig. 35), could have contributed to a difference in the difficulty of arriving at the inference Undetermined for these items.

Scoring of the tests of deduction followed the procedures for assigning scores for other test sequences: correct inference and a correct logical explanation was assigned a score 1. If either was incorrect, the score assigned was 0. The range of scores CCO-D and SCO-D was 0 to 17.

The construction of the tests of inductive reasoning will now be presented.

CHAPTER VII

TESTS OF INDUCTIVE REASONING

This chapter will describe the construction of the Concrete and Stories tests of inductive reasoning. The tests will be designated CCO-I and SCO-I.

The terms required in discussing the test items will be defined and the materials for the tests will be presented. This will be followed by a description of the test items and a discussion of the comparability of corresponding test items. Procedures in assigning scores will be indicated.

Definition of Terms

Two terms "inductive inference", and "explanation", were defined in Chapter 1. Additional terms required in discussing the tests of inductive reasoning are defined below. The definitions follow closely Bocheñski's use of these terms.[1]

Reduction

Reduction is defined as a process of inference in which we infer the antecedent from a conditional statement and its consequent.[2]

---

[1] J.M. Bocheñski, The Methods of Contemporary Thought (Dordrecht-Holland: D. Reidel Publishing Company, 1965).

[2] *Ibid.*, p. 92.

Reductive reasoning presents a difficult problem in justification since in logic it is invalid to infer from the consequent to the antecedent. A reductive inference is represented by the schema:

If A, then B
B
―――――――――
Therefore A.

Reductive and deductive (Modus Ponens) schemas may be compared (see Fig. 37).

Reductive Schema	Deductive Schema (Modus Ponens)
$A \longrightarrow B$	$A \longrightarrow B$
B	A
$\therefore A.$	$\therefore B.$

Fig. 37. -- Reductive and deductive schemas.

Induction

Induction is described by Bocheński as a subclass of reductive reasoning in which the antecedent is a generalization (all A) of the consequent (some B).[3] The reduction proceeds regressively: from the consequent "some B is true" we infer the antecedent "all A is true." The process is therefore ampliative: it proceeds from some individuals (some B), which do not comprise all the members of the class in question, to the general (all A).

In induction the truth of the reductive inference is supported by two procedures. The first is that of associating the antecedent with a theory or law, T:

$$\frac{\begin{array}{l} A \,\&\, T \longrightarrow B \\ B \end{array}}{\therefore A \,\&\, T}$$

[3]*Ibid.*, p. 92.

The second procedure involves the experimental verification of the hypothetical premise. These procedures, association with a law, and experimental verification, could not be applied in the case of the inference Undetermined as described in deduction and which also proceeded from "some" to "all":

$$[(\exists A):(A \longrightarrow B)] \sim [(A):(A \longrightarrow B)]$$

Verification

Verification is a procedure for determining the truth value of a hypothesis. It may proceed either by confirmation or falsification. For confirmation the method is to accumulate protocol statements based on observation and experiment which show that occurrences of the phenomenon are explained by the hypothesis. For falsification the method is to eliminate possible alternative hypotheses by showing that these do not account for the protocol statements.

Conditions

Conditions are formal, that is, logical explanations of observed events. Conditions are to be distinguished from causal relations which may hold between events.

Explanatory statements in inductive reasoning specify at least one of the following conditions as governing the observed phenomena: a sufficient condition; a necessary condition; or a necessary and sufficient condition.

A sufficient condition is a valid statement of the form, "If A, then B": if A is given, then B is also given. The condition may be represented symbolically:

$$(A):(A_x \longrightarrow B_x).$$

A necessary condition is a valid statement of the form, "If B, then A" : A is a necessary condition for B in the case that if A were not given, B could not occur. This condition may be represented symbolically:

$$(A):(\sim B_x \longrightarrow \sim A_x).$$

A necessary and sufficient condition is one in which both the above statements are valid. It is expressed as, "A, if and only if B." This condition may be represented by combining the two statements:

$$(A):(A_x \longrightarrow B_x) \; \& \; (\sim B_x \longrightarrow \sim A_x).$$

Phenomena

Phenomena are understood to mean simply events observable by the senses.[4]

State of affairs

A state of affairs is represented by means of propositions; properties and relations are represented by means of concepts.

Regularity

A regularity is defined as a non-random state of affairs which follows the intervention of a controlling or determining agent (an intervening constant) in a previously random distribution of events. The regularity introduced by the constant is presumed to be

[4] Ibid., p. 95.

"explainable" by the operation of a law.

Laws

Laws are general statements, accepted as true, about how nature works. In inductive inquiry laws "are involved throughout the whole process," that is, "both initially and throughout the sequence of reasoning." They may serve as points of departure for inquiry just as well as they mark its termination. . ." Laws are not only generalizations at which we arrive "after we have established the facts; they play a part in determining what the facts are: they sometimes function precisely as premises. . .or as test hypotheses."[5]

Protocol statements

Protocol statements are a non-ordered collection of observations of phenomena. They are usually guided by "working hypotheses." Protocol statements have been described as "data in search of a law."[6]

Materials

The materials for the Concrete tests of inductive reasoning are shown in Figure 38. Eight rectangular blocks 5/8" x 5/8" x 2", painted black, of three different weights (medium, light and heavy) are positioned by the subject on the eight colored sections of the board. A spinner at the center of the board is capable of turning freely before the positioning of the blocks. The subject has

---

[5]Abraham Kaplan, The Conduct of Inquiry (San Francisco: Chandler Publishing Company, 1964), pp. 89-90.

[6]*Ibid.*

observed this condition during the tests of uniform chance distribution administered prior to the tests of induction (see Chap. VIII). One of the blocks of medium weight contains a magnet capable of attracting iron attached to the underside of the spinner.

The material for the Stories tests is Story VIII, "The Cave" (see Appendix D).

Fig. 38. -- Test materials: Concrete tests of induction.

observed this condition during the tests of uniform chance distribution administered prior to the tests of induction (see Chap. VIII). One of the blocks of medium weight contains a magnet capable of attracting iron attached to the underside of the spinner.

The material for the Stories tests is Story VIII, "The Cave" (see Appendix D).

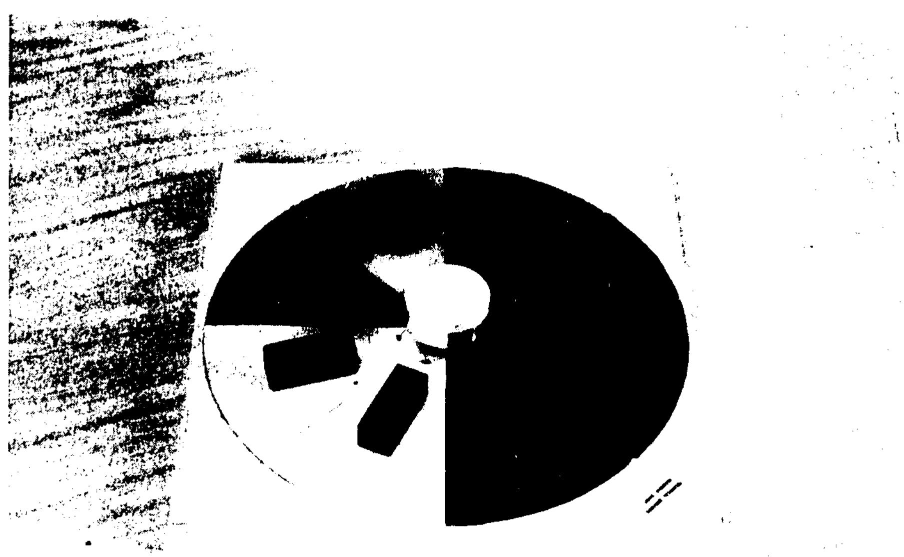


Fig. 38. -- Test materials: Concrete tests of induction.

Test Items Concrete and Stories

The Concrete tests were adapted from tests described by Piaget in La genèse de l'idée de hasard (see Figure 39). The Stories tests are summarized in Figure 40.

Concrete Tests of Induction

I	Recognition that an observable regularity is a "new" situation requiring an explanation.
II	Hypothesis type 1: a dichotomy (classificatory). The hypotheses tested are: $A \vee \sim A$; $B \vee \sim B$
III	Hypothesis type 11: propositional. The hypotheses tested are: $A_x \longrightarrow B_x$; $\sim B_x \longrightarrow \sim A_x$
IV	Assertion of an inductive inference and its vertification by falsification: $(A):(\sim B_x \longrightarrow \sim A_x)$. The hypothesis of weight is falsified.
V	Assertion of an inductive inference and its verification by confirmation. For all observed instances "this block only" is required. $(A):(A_x \longrightarrow B_x)$.
VI	Assertion of an inductive inference supported by a law (a magnet could attract the spinner, $A \& T \longrightarrow B$): $B \& [A \& T \longrightarrow B] \longrightarrow A \& T$.

Fig. 39. -- Summary of Concrete tests of induction.

Stories Tests of Induction

I Recognition of a "new" regularity: "Were things really different at the cave or were the boys just imagining it?"

II Hypothesis type I or II. A hypothesis must be intended to explain two or more of the protocol statements; its limitations in explaining the observations must be recognized:

> It could be an animal in the cave that frightened the bats away and is frightening the chipmunk. But that wouldn't make the candles go out.

III Assertion of an inductive inference to be verified by confirmation and/or by falsification:

> Something must have happened in the cave to frighten the bats and make the candles go out and now the chipmunk won't go in either.
>
> Something must have happened in the cave because if nothing had happened everything would be just the same, the bats would be there, the boys could explore with the candles, etc.

IV Assertion of an inductive inference supported by a law: air is necessary for life and air is necessary for combustion (A & T ⟶ B). The absence of air (~ B), could account for the absence of life and the candles going out. (~ A). The inference is of the form:

$$\frac{\begin{array}{l} A \,\&\, T \longrightarrow B \\ \sim B \end{array}}{\therefore \sim A}$$

Since T, the law, is accepted as true, the inductive inference will be ~ A (no air)

Fig. 40. -- Summary of Stories tests of induction.

Comprehension questions which precede the Stories tests (see Appendix D), follow closely the sequence of events reported in the story. These events constitute the set of observations or protocol statements needed for the subsequent operations of inductive reasoning. Protocol statements are similarly assembled by the preliminary questions for Concrete tests.

Comparability of Concrete and Stories Tests

The comparability of corresponding test items CCO-I and SCO-I may be considered by reference to Figure 41.

Inductive reasoning has been considered to follow a logical and possibly a development sequence observable in the behaviors: recognition; the construction of classificatory and propositional hypotheses to account for protocol statements; the assertion of an inductive inference; an attempt to verify the inductive inference by adducing necessary and/or sufficient conditions or by subsuming the protocol statements under a law. This sequence is followed in testing.

Test items CCO-I-I and SCO-I-I appear to be comparable. Each requires the recognition of the effect of an intervening constant. Recognition of the evidence for the constant was assisted in each case by preliminary questions designed to assist in assembling relevant protocol statements.

Comparability of Corresponding Test Items, Induction

CCO-I

Test item I

Recognition of an intervening constant.

Test item II

Hypothesis construction, classificatory:

$$A \vee \sim A$$

Test item III

Hypothesis construction, propositional:

$$A_x \longrightarrow B_x$$

or

$$\sim B_x \longrightarrow \sim A_x$$

Test item IV

Inductive inference verified by falsification:

$$(A):(\sim B_x \longrightarrow \sim A_x)$$

Test item V

Inductive inference verified by confirmation:

$$(A):(A_x \longrightarrow B_x)$$

Test item VI

Inductive inference verified by subsuming the evidence under a law:

$$\begin{array}{l} A \;\&\; T \longrightarrow B \\ \underline{B \qquad\qquad} \\ A \;\&\; T \end{array}$$

SCO-I

Test item I

Recognition of an intervening constant.

Test item II

Hypothesis construction, classificatory or propositional:

$$A \vee \sim A$$

$$A_x \longrightarrow B_x$$

$$\sim B_x \longrightarrow \sim A_x$$

Test item III

Inductive inference verified by falsification and/or by confirmation:

$$(A):(A_x \longrightarrow B_x)$$

or

$$(A):(\sim B_x \longrightarrow \sim A_x)$$

Test item IV

Inductive inference verified by subsuming the evidence under a law:

$$\begin{array}{l} A \;\&\; T \longrightarrow B \\ \underline{\sim B \qquad\qquad} \\ \therefore \sim A \end{array}$$

Fig. 41. -- Comparability of corresponding tests of induction.

Test items CCO-I-II, III and SCO-I-II also appear to be comparable. In these test items, hypotheses stated as dichotomies, would seem to be classificatory in structure: what is to be tested by a hypothesis of this form is a concept rather than a state of affairs. To test a state of affairs hypotheses are presented as a relation between propositions. In the classificatory form of hypothesis, the intervening constant is said to belong either to a class A (events or objects) having the property, a, or to the complement of the class A, that is to the class not-A, having the property, -a. The subject hypothesizes, "It is a magnet or not-a-magnet":

> These two are magnets. I'll try them against the spinner and see. Yes, they make it move, they're magnets.

In a propositional hypothesis, he considers a state-of-affairs in terms of the compatibility of propositions:

$$M\ ({}^{p1}/_{p2})$$

$$M\ ({}^{p1}/_{\sim p2})$$

It is possible both A and B are magnets.

It is possible A is a magnet and B is not a magnet.

For purposes of scoring, however, the distinction between classificatory and propositional hypotheses which seemed to be recognizable for the Concrete test items, was much less clear for the Stories tests and was abandoned for both test items. A criterion for both CCO-I and SCO-I hypothesis construction, was therefore established as follows:

> The hypothesis (classificatory or propositional) must be intended to account for more than one protocol statement at a time, and recognition of its limitations must be indicated.

Test items CCO-I - IV and V and SCO-I - II appear to be comparable. Verification by falsification and by confirmation were closely associated in the responses to test item SCO-I - III. It was necessary to abandon this distinction in scoring these tests:

> Something's happened in the cave or the boys could go in and explore. Whatever it was, it made the candles go out and the bats fly away. They wouldn't fly away if nothing had happened.

Test items CCO-I - VI and SCO-I - IV are considered to be comparable: each requires that the inductive inference be supported by subsuming the evidence for it under a law. It is not necessary for the subject to specify magnetism, oxygen, cave gas: it is sufficient that he suggest "something in the block," "something the matter with the air."

The inductive behaviors elicited by the Concrete and Stories tests appear to be comparable in form and to represent recognized sequences in inductive reasoning.

Scoring

For each test item CCO-I and SCO-I a score 1 was assigned for a correct response; a score 0 for an incorrect response. The range for CCO-I was 0 to 6; the range for SCO-I was 0 to 4.

Protocols illustrating responses scored 0 and responses scored 1 are included in Appendix D.

Summary: Tests of Induction

Induction was defined as a subclass of reduction: a process of inference in which the antecedent is inferred from a conditional statement and its consequent. In induction the antecedent is a generalization (all A) of the consequent (some B). The verification of an inductive inference was described as involving the association of the antecedent with a law of nature; and showing that sufficient and/or necessary conditions accounted for occurrences of the accumulated protocol statements.

Test items CCO-I and SCO-I were examined for the comparability of the operations in inductive reasoning required. The test items were considered to be comparable for the following sequence of inductive operations:

(a) Recognition of a constant intervening in a multiply determined state of affairs.

(b) The construction of hypotheses to account for protocol statements describing the action of the constant. The hypotheses could be classificatory or propositional. A hypothesis must be constructed to account for two or more observations, and the limitations of a hypothesis which fails to account for all of the protocol statements must be recognized.

(c) The assertion and verification of an inductive inference.

In scoring, the following types of responses were considered to be unrepresentative of inductive reasoning and were scored 0:

(a) Hypotheses accounting for one observation only.

(b) Hypotheses which could not be tested experimentally.

(c) Inferences supported by authority without experimental evidence.

(d) Explanations which accounted for only one observation at a time.

The verification of inductive inferences by falsification and confirmation appeared to proceed by a process of judging propositions for compatibility. Inductive inferences verified by falsification (necessary conditions) and/or by confirmation (sufficient conditions) were scored 1. Inductive inferences verified by bringing the protocol statements under a law of nature were scored 1.

The construction of the tests of probability reasoning will now be presented.

CHAPTER VIII

TESTS OF PROBABILITY REASONING

This chapter will describe the construction of the Concrete and Stories tests of probability reasoning. The tests will be designated CCO-P and SCO-P.

Terms required in describing the test items will be defined. Materials for administering the tests, and a summary of the test items CCO-P and SCO-P will be presented. A discussion of the notions of chance and probability measured by the Concrete and Stories tests will follow.

Definition of Terms

Terms used in describing the tests of probability reasoning are defined below. A definition of the term "probability" was given in Chapter I.

Conditional probability

Conditional probability is concerned with questions of the type, "What is the probability that event B will occur given that event A has already occurred," If event B (and C...) can be considered not-conditioned by A the probability approaches zero.

Subjective probability

Subjective probability is defined as probability reasoning in which past experiences, given as uncertainties are applied to the testing of hypotheses concerning the likelihood of future experiences.

Equiprobability

Equiprobability is a property of a set of events in which each event in the set has the same chance of occurrence, being subject to the same set of chance interacting forces as each other event in the set.

Irreversibility

Irreversibility is a property of a set of objects or events when these are subject to the action of multiple interacting forces.

Materials

Materials for the Concrete tests of probability are shown in Figure 42. For the tests of random distribution the apparatus consists of a teeter box, which is a metal tray 12" x 9" mounted on a transverse axis. To reduce noise it was lined with felt. A small partition divides a row of wooden beads into two parts: eight light-colored and eight dark-colored beads on each side of the partition.

For the tests of uniform distribution the apparatus consists of a spinner balanced at the center of a 12" square block marked with a circle of eight equal wedges, each painted a different color. The spinner turns freely, coming to a stop at one of the pairs of wedges. A line marking the diameter of the spinner indicates the

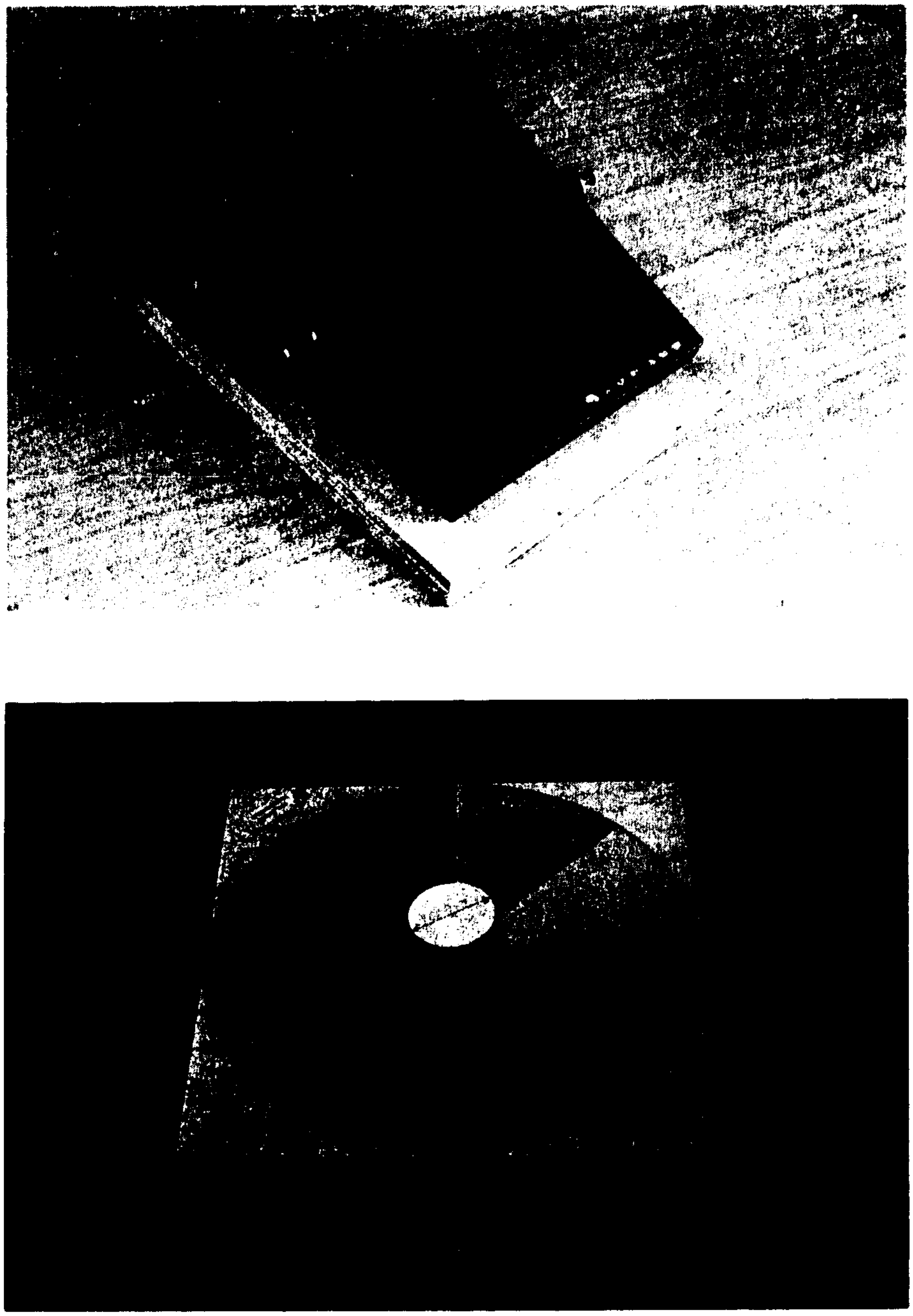


Fig. 42. -- Test materials: Concrete tests of probability.

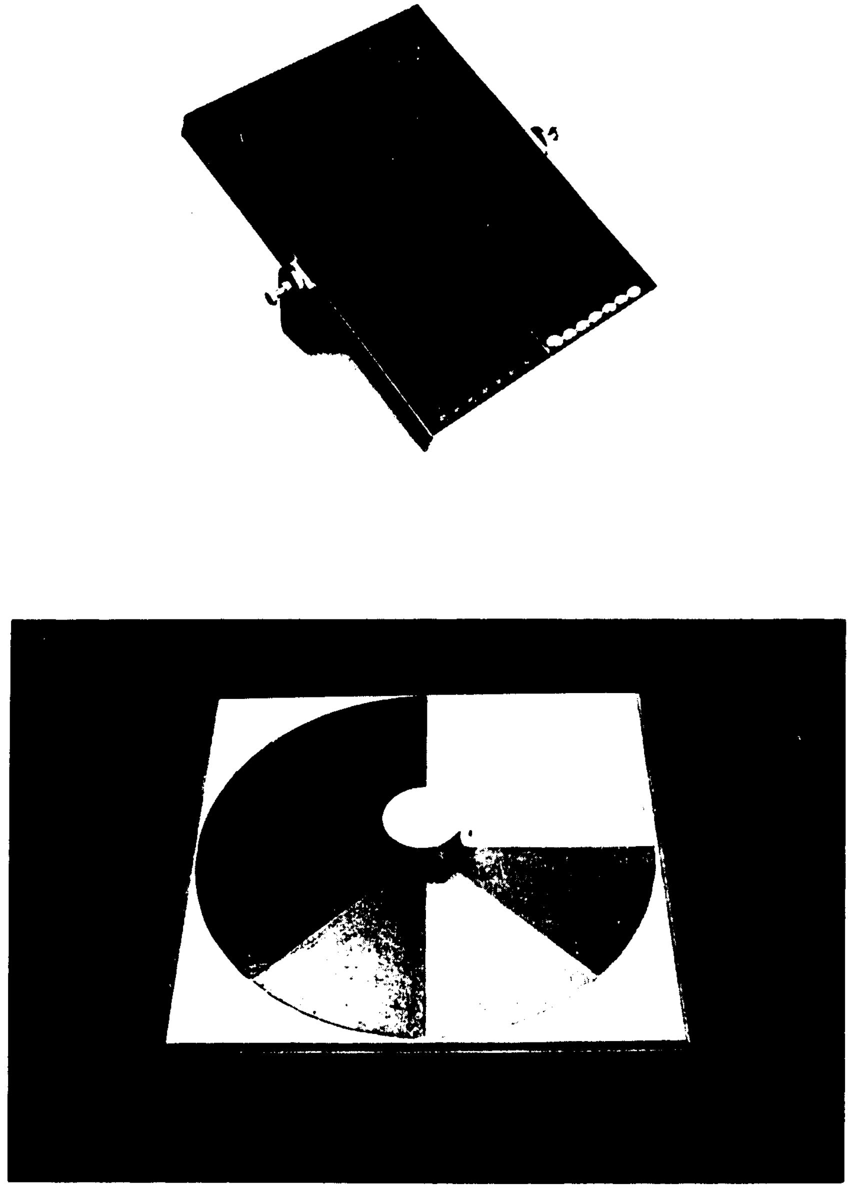


Fig. 42. -- Test materials: Concrete tests of probability.

point at which it comes to rest.

A small white cardboard box and additional beads are used for the test items assessing the quantification of probability.

The Stories tests of probability reasoning are based on Story VI, "Home from School" (see Appendix E).

Test Items Concrete and Stories

Test items CCO-P were adapted for this study from experiments reported by Piaget and Inhelder in La genèse de l'idée de hasard chez l'enfant.[1] The authors in this study investigated the development of notions of irreversibility associated with the effect of multiply interacting forces; the development of notions of uniform chance distribution; and the quantification of probability. The concern of the studies in L'idée de hasard was to discover developmental sequences in probability learning and to identify relations between earlier acquisitions and later achievements. The Concrete tests in the present study have been selected from this research and adapted for purposes of measuring operations of probability reasoning which appear to be available at the nine-to-ten-year-old level.

Test items in the Stories tests of probability reasoning assess subjects' understanding of conditional probability as defined: the probability that event B will occur given that event A has already occurred. In these test situations a complex sequence of independent

---

[1]Jean Piaget et Bärbel Inhelder, La genèse de l'idée de hasard chez l'enfant (Paris: Presses Universitaires de France, 1951). (Hereinafter referred to as L'idée de hasard.)

chance events are to be referred to past experiences in testing the likelihood of a future occurrence of just this sequence of events. Explanations of these estimates of conditional probability are required at two levels, specific and general:

> Why do you think it is (not) likely "these things" will happen just like this ....
>
> Can you explain why "things like this" are (not) likely to happen just like this ....

The judgments of probability measured by test items CCO-P and SCO-P are summarized in Figure 43.

Judgments of Probability Measured

Test Item	Concrete	Test Item	Stories
I	Irreversibility for small numbers		Given: $\{E_j\}$, a sequence of independent events at T_1.
II	Irreversibility for large numbers		To determine:
III	$P(^a/_h)$: 1/3	I	Conditional probabilities of the sequence $E_i \ldots E_j$
	$P(^a/_h)$: 2/6	IV	recurring at T_2 and T_3.
	$P(^a/_h)$: 1/4		Hypothesis testing on the basis of subjective probabilities.
V	Equiprobability	II	Level I of explanation for T_2.
		III	Level II of explanation for T_2.
VI	Uniform distribution for large numbers given conditions of equiprobability of events.	V	Level I of explanation for T_3.
		VI	Level II of explanation for T_3.

Fig. 43. -- Judgments of probability, Concrete and Stories.

Judgments of Probability Measured

An examination of the judgments of probability measured by test items CCO-P and SCO-P, shown in Figure 43, suggests that complementary, rather than comparable, aspects of the development of notions of probability have been assessed by the Concrete and Stories tests. The notions of chance assessed in the Concrete tests involve the concepts of irreversibility, equiprobability and of the quantification of probability. In the Stories tests it is a question of a level of acquisition of a notion of conditional probability.

There is an expected commonality in these tests in that items in each of the sequences assess notions of probability. In the Concrete tests the progressive random and uniform chance distributions presented are the outcomes of multiply interacting forces; in the Stories tests, as frequently in social life and in reading, these multiply interacting forces yield a conditional probability which requires to be judged as a subjective probability on the basis of past experience.

In conditional probability there are two cases: events in a time sequence each of which is conditioned to some degree by a preceding event; and events in a time sequence which are essentially independent, each, however, the outcome of a set of multiple interacting forces. There is the question of the developmental dependencies of these notions of conditional probability: of the processes in experience which assist the child in acquiring an awareness of conditional probability and of its generality in life and in reading. Notions of conditional probability eventually replace earlier non-probability elements in pre-causal and non-logical notions in on-going cognitive development: notions of miracle, and animistic, artificialistic, and finalistic orientations[2] are replaced by probabilistic, inductive and deductive orientations in thinking.

It is not clear that an awareness of conditional probability is likely to develop from a sense of the uniqueness of each day's experience. It is hypothesized that such awareness will depend on the "choc en retour" of probabilistic notions effectively illuminating the familiar and more or less unexamined aspects of the child's day to day experiences. The relationship anticipated involves the priority in development of notions of irreversibility and equiprobability in concrete situations and the application of these notions to interpreting conditional probability in life and in reading. In the Stories tests the conditional probability amounts to zero. It is suggested that an understanding of this conditional probability is dependent not simply on a subjective awareness of past experiences, but of an awareness of past experiences, already capable of being assessed under conditions in which the notions of the chance operation of multiply interacting forces have been available to the child and applied to interpreting these experiences.

The understanding of conditional probability in the reading situation would be expected, on the basis of this hypothesis, to be

---

[2]Monique Laurendeau and Adrien Pinard, Causal Thinking in the Child (Montreal, Que.: Institute for Psychological Research, 1962).

emergent only for the nine-to-ten-year-old children in this study. The questions of interest, then, include the relation of prior notions of concrete probability as assessed by the Concrete tests to the notions of conditional probability as assessed by the Stories tests as well as the relation of the level of acquisition of these notions in the reading and concrete situations to other aspects of cognitive development.

The small number of the test items presented in the tests of probability reasoning will permit a tentative estimate only of trends which may indicate possibilities for further investigations.

Scoring

Test items CCO-P and SCO-P were assigned a score 1 if the response was correct, and a score 0 if incorrect. The range of scores for each test sequence was 0 to 6. Responses scored 1 and 0 are shown in Appendix E.

Summary: Tests of Probability Reasoning

Six Concrete test items (see Appendix E), based on test situations designed by Piaget and Inhelder for the study, L'idée de hasard measured subjects' understanding of irreversibility and equiprobability of chance distributions, and the quantification of probability.

In the Stories tests multiply interacting forces yielded a sequence of independent events to be judged subjectively as a conditional probability. The question of concern is the extent to which the development of probability reasoning as measured by the Concrete tests will be related to probabilistic reasoning in reading as measured by the Stories tests. There is the further question of the relation of probability reasoning in these situations to other aspects of cognitive development assessed in the study. The number of test items CCO-P and SCO-P is small, and the areas of probability reasoning examined are limited, so that tentative estimates only of trends in these relationships will be available. These trends may, however, suggest possibilities for further investigation.

CHAPTER IX

RESULTS

Introduction

This Chapter will present the statistical results related to three main objectives of the investigation. One objective was to examine the relations between outcomes assessed by the Concrete tests and outcomes assessed by the Stories tests on operations of conservation, classification, deduction, probability reasoning and induction at the Grade four level. Relations between the variables, relations within each variable set, and relations between the two composites will be of interest.

A second objective was to investigate the possibility of sex differences between scores on the Concrete and Stories variables and on the standardized test of reading comprehension, STEP Reading.

The third objective was to relate and compare operations as assessed by the Concrete and Stories tests and operations in reading comprehension as assessed by STEP Reading. Factor analysis will be used in an attempt to uncover underlying factorial structures of the phenomena under investigation.

The study is exploratory. In interpreting the results of this type of study great caution is necessary, since errors associated with sampling, with the unreliability of the tests, errors in assigning scores, etc., may significantly affect the results obtained. Cross-validation studies and studies testing specific hypotheses suggested by the statistical results will be necessary to validate the significance of the discovered relationships.

The order of presentation of the results and the statistical procedures applied are indicated below.

To allow an initial examination of the data the average per cent of correct responses on subtests of the Concrete and Stories tests will be presented. These should provide a preliminary indication of the characteristics of the data and permit some tentative comparisons with the results of previous studies.

Product-moment correlations and t-tests of the significance of differences in mean scores will be used in assessing relations between scores on the test variables and sex.

Stepwise multiple regression analysis[1] will be used to indicate how well Concrete and Stories variables taken separately and together are able to predict STEP Reading total scores, to range in order the Concrete and Stories variables as predictors of STEP Reading and to seek the minimum number of predictors for the criterion STEP Reading.

Product-moment correlations between scores on the Concrete and Stories variables, and between scores within each set of variables will be used to assess the relations among these variables for this sample of Grade four children.

Canonical correlation will be used to investigate the relation-

---

[1]M. A. Efroymson, "Multiple Regression Analysis," in Mathematical Methods for Digital Computers, eds. A. Ralston and H. S. Wilf (New York: John Wiley and Sons, Inc., 1960), pp. 191-212.

ship between the two sets of variables, Concrete and Stories. This procedure provides a weighting system which, when applied to the test scores, maximizes the relation between two sets of variables.[2] The canonical correlation will indicate the amount of the variance common to the two sets of variables taken as composites.

Factor analysis, as indicated, will be used to uncover underlying dimensions of the phenomena studied.

Mean Per Cent of Correct Responses
Concrete and Stories Tests

The mean per cent of correct responses for the Concrete and Stories tests of conservation, classification, deductive, inductive and probability reasoning are shown in Table 7.

TABLE 7.-Mean per cent of correct responses, Concrete and Stories Tests. (N = 100).

Variable	%	Variable	%
CCO-Con(n[a]=27)	54	SCO-Con(n=27)	35
CCO-Cl(n=19)	64	SCO-Cl(n=10)	27
CCO-D(n=17)	53	SCO-D(n=17)	51
CCO-I(n=6)	47	SCO-I(n=4)	34
CCO-P(n=6)	56	SCO-P(n=6)	49
Average CCO(n=75)	56	Average SCO(n=64)	39

[a]n=number of test items

[2]William W. Cooley and Paul R. Lohnes, Multivariate Procedures for the Behavioral Sciences (New York: John Wiley and Sons, Inc., 1962).

Inspection of Table 7 suggests that the mean per cent of correct responses on the Concrete variables were generally higher than for the corresponding Stories variables; the mean per cent of correct responses for CCO-D and SCO-D items appear to be at approximately the 50 per cent level.

It would seem that for these nine-and-ten-year-old subjects, some of the cognitive operations available in response to Concrete test items were not necessarily available in a reading situation as measured by the Stories tests. It may therefore be of interest to examine the mean per cent of correct responses on subtests of the Concrete and Stories tests.

<u>Per cent of correct responses: conservation, CCO and SCO</u>

In assessing conservation, the conserving of substance, weight and volume were tested separately, each at three levels of complexity. A score of 1 was assigned for asserting conservation at each level of complexity and for explanations at levels 1 and 11. The mean per cent of responses asserting and defending conservation are shown in Table 8.

TABLE 8.-Mean per cent of responses asserting and defending conservation of substance, weight and volume CCO-Con and SCO-Con. (N = 100; n = 9).

Variable	CCO-Con %	SCO-Con %
Substance	64	43
Weight	57	27
Volume	40	36

The mean per cent of correct responses CCO-Con-S-W-V may be seen in Table 8 to be of decreasing magnitude. This is consistent with the results reported by Piaget and by Lovell (see Tables 1 and 2, Chap. 11, above).

The per cent of correct responses SCO-Con-S-W-V may be seen in Table 8 to be somewhat lower than the per cent of correct responses on corresponding CCO-Con-S-W-V tests. It would seem possible that the pattern of acquisition of conservation and perhaps the rate of acquisition may not be identical in reading and in concrete situations.

The mean per cent of correct responses in asserting conservation at levels of complexity 1, II, III, CCO-Con-S-W-V and SCO-Con-S-W-V are shown in Table 9. Scores for defending conserving decisions are not included in these mean percentages.

TABLE 9.--Mean per cent of responses asserting conservation, levels of complexity I, II, III, CCO-Con-S-W-V and SCO-Con-S-W-V. (N = 100; n = 3 at each level of complexity).

Variable	CCO-Con % Level of Complexity I	II	III	Average	SCO-Con % Level of Complexity I	II	III	Average
Substance	94	88	60	80	72	53	66	64
Weight	88	78	54	73	40	34	44	40
Volume	65	68	33	55	43	49	51	48

The average percentages for responses asserting conservation for nine-to-ten-year-old subjects, shown in Table 9 appear to be comparable to those reported by Lovell for ten-to-eleven-year-old subjects (see Table 2, Chap. 11). The average percentages for conserving substance and weight, CCO-Con, are comparable to the results reported by Piaget for nine-year-olds (see Table 1). The average percentage for conserving volume, CCO-Con (55 per cent), is comparable to Piaget's findings for ten-year-olds (see Table 1).

The mean per cent of correct responses for asserting conservation of substance, weight and volume at each of three levels of complexity are also shown in Table 9. For CCO-Con-S-W-V the per cent of correct responses tend to decrease as the level of complexity increases; and to decrease also for the categories, from substance, to weight, to volume. The per cent of correct responses for SCO-Con are lower than for CCO-Con at levels of complexity I and II for substance, weight and volume. They are slightly higher for substance and volume at level of complexity III. These results may be due to chance or to differences in the difficulty of the test items, Concrete and Stories. There would appear in general to be a tendency for a more uniform level of correct responses to occur on the Stories tests than on the Concrete tests of conservation. Conserving responses on the Concrete tests appear to be rather more liable to extinction as the level of complexity increases than on the Stories tests.

Per Cent of correct responses:
classification CCO and SCO

The mean per cent of correct responses for items assessing hierarchical and multiplicative classification, CCO and SCO are presented in Table 10.

TABLE 10.-Mean per cent of correct responses, subtests of classification, CCO and SCO.
(N = 100).

Variable		CCO-Cl %	Variable		SCO-Cl %
Class inclusion	(n^a= 8)	73	Class inclusion	(n = 4)	24
Multiplicative			Multiplicative		
Classification	(n = 6)	48	Classification	(n = 3)	27
Predicates	(n = 3)	60	Predicates	(n = 3)	30

^{a}n = number of test items (see Figs. 15 and 16, Chap. V).

It may be seen in Table 10 that the average per cent of correct responses for the Concrete classification variables are higher than for the SCO-Cl variables.

It also appears that the per cent of correct responses for Concrete class inclusion relations are likely to be higher than for Concrete multiplicative classification as measured by these tests. On the Stories tests, the differences in the mean per cent of correct responses on these classification variables appears to be slight.

Per Cent of correct responses: deduction, CCO and SCO

The per cent of correct responses for subtests of deductive reasoning CCO and SCO are shown in Table 11.

TABLE 11.-Mean per cent of correct responses, subtests of deductive reasoning, CCO and SCO. (N = 100).

Variable	CCO-D %	Variable	SCO-D %
Modus Ponens (n^a= 2)	97	Modus Ponens (n = 3)	86
Modus Tollens(n = 2)	71	Modus Tollens(n = 3)	67
Undetermined (n = 4)	21	Undetermined (n = 4)	13
Possible (n = 6)	65	Possible (n = 3)	48
Not-Possible (n = 2)	55	Not-Possible (n = 2)	70
Transitivity (n = 1)	7	Transitivity (n = 2)	25
"Village" tests (n = 8)	67		
"Lights" test (n = 8)	48		

[a]n = number of test items (see Figs. 31 and 32, Chap. VI).

It may be seen in Table 11 that a slight tendency may be present for the average per cent of correct responses for CCO-D-Modus Ponens, Modus Tollens, Undetermined and Possible inference to be higher than for the corresponding SCO-D items. The trend is reversed for Not-Possible inference and the Principle of Transitivity; for these items the mean per cent of correct responses were higher as measured by the Stories tests.

The per cent of correct responses reported in Table 11 for the Concrete variables are somewhat higher than the percentages reported by Matalon for nine-to-ten-year-olds on corresponding test items (see Table 3, Chap. II).

Matalon's results were based on the "Lights" tests. The results reported in Table 11 for the CCO-D variables include responses to items from both the "Village" and the "Lights" tests. Matalon considered the "Village" items to be the more "intelligible." The per cent of correct responses for the "Village" items were higher in the present study (67 per cent) than for "Lights" test items (48 per cent). These results would seem to be in agreement with Matalon's judgment of the relative difficulty of the two concrete test situations. Combining responses for the two test situations could account for the higher per cent of correct responses in the present study.

Matalon reported that for each of these tests the items requiring the inference Undetermined proved to be the most difficult. For the Concrete tests 21 per cent of responses requiring the inference Undetermined were correct (Matalon reported 20 per cent). On the Stories tests 13 per cent of responses for the inference Undetermined were correct.

<u>Per Cent of correct responses: induction CCO and SCO</u>

The mean per cent of correct responses on subtests of inductive reasoning CCO and SCO are shown in Table 12.

The average per cent of correct responses on tests of inductive reasoning suggest a tendency for a higher percentage of correct responses to occur on the Concrete test items.

In particular, subjects in this study appeared to recognize a "new" situation as requiring an explanation more readily in the Concrete than in the Stories situation. Only if a situation is

TABLE 12.-Mean per cent of correct responses, subtests of inductive reasoning, CCO and SCO. (N = 100).

Variable	CCO-I n^a	CCO-I %	SCO-I n	SCO-I %
Recognition	1	87	1	69
Hypotheses Construction	2	36	1	20
Inductive Inference	3	20	2	13

[a]n = number of test items (see Figs. 39 and 40 Chap. VII).

recognized as requiring inductive investigation will hypotheses be proposed and tested. Apparently in both test situations some subjects recognized the situation as "new" but did not search for an explanation.

Summary: mean per cent correct responses CCO and SCO

The mean per cent correct responses on total scores and subscores of the Concrete and Stories tests were presented in Tables 7 to 12. There appeared to be a tendency in the tests of conservation, classification, and induction (see Table 7), for the per cent of correct responses to be higher on the average for the Concrete tests (54 per cent; 64 per cent; 47 per cent), than on the Stories tests (39 per cent; 27 per cent; 34 per cent). For total scores deduction and probability reasoning the average per cent correct responses for the Concrete and Stories tests approached the 50 per cent level.

On the Concrete tests of conservation, the "time lags" reported by Piaget and Lovell between acquisitions of conservation of substance, weight and volume (see Tables 1 and 2) occurred also in this data. These time lags were not as clearly evident in the Stories data. On the Concrete tests, also, conserving responses appeared to be rather more liable to extinction as the complexity increased, than on the Stories tests (see Table 9).

Higher average percentages of correct responses occurred in the Concrete data on the subtests of classification (73 per cent, 48 per cent, 60 per cent), than in the Stories data (24 per cent, 27 per cent, 30 per cent). (See Table 10.) Higher average percentages of correct responses also occurred on subscores of Concrete tests of induction (87 per cent, 36 per cent, 20 per cent) than on the corresponding Stories subtests (69 per cent, 20 per cent, 13 per cent). (See Table 12.) For these operations, as for operations in conservation, it would seem that procedures available in concrete stimulus situations might not be equally available in reading: in response to graphic symbols some of these correct responses appear to be liable to extinction.

The relations between the Concrete and Stories variables and sex and between STEP Reading total scores and sex will now be examined.

Correlations between Concrete and Stories Variables, STEP Reading Total Scores and Sex

Product-moment correlations, using 1 and 0 for sex, between the Concrete variables, the Stories variables, STEP Reading totals and sex are presented in Table 13.

TABLE 13.-Product-moment correlations between the Concrete variables, the Stories variables, STEP Reading total scores and sex (N = 100; n = 50; n = 50).

Variable	Sex r	p
CCO-Con	.18	<.05*
CCO-Cl	.09	n.s.
CCO-D	.12	n.s.
CCO-P	.19	<.05*
CCO-I	.32	<.001***
Total CCO	.29	<.01**
SCO-Con	.07	n.s.
SCO-Cl	.11	n.s.
SCO-D	.14	n.s.
SCO-P	.04	n.s.
SCO-I	.08	n.s.
Total SCO	.13	n.s.
STEP Reading	-.09	n.s.

It may be seen in Table 13 that the correlations with sex appear to be significant for three of the Concrete variables: CCO-Con, CCO-P, and CCO-I, and for total scores CCO. The correlations between the other variables and sex do not appear to be significantly different from zero.

Significance of Differences between Mean Scores of Boys and Girls on Concrete and Stories Variables and STEP Reading

The results of t-tests for the significance of the differences between the mean scores of boys and girls on Concrete and Stories variables and STEP Reading are presented in Table 14.

It may be seen in Table 14 that the mean scores of boys appear to be significantly higher on the three Concrete variables noted in Table 13 to be significantly correlated with sex: CCO-Con, CCO-P, CCO-I, and on mean total scores, Concrete.

The mean scores of boys on the Stories variables and STEP Reading totals do not appear to be significantly different from the mean scores of girls (range: $t = 1.28$ to $t = .237$, n.s.).

Correlations between the variables CCO-Con-S-W-V and sex are shown in Table 15.

It would appear that the significantly higher mean score of boys on CCO-Con ($t = 1.95$ $p<.05$), is related to a superior performance in conserving substance and weight (see Table 15).

TABLE 14.-t-tests for the significance of differences between the mean scores of boys and girls on Concrete and Stories variables and STEP Reading. ($n = 50$; $n = 50$; $df = 98$).

Variable	$\bar{X}_1$	$\bar{X}_2$	SD_1	SD_2	t	p
CCO-Con	15.76	13.26	6.44	6.22	1.954	< .05
CCO-Cl	12.42	11.921	2.63	3.14	.854	n.s.
CCO-D	8.78	8.28	1.85	2.15	1.231	n.s.
CCO-P	3.56	3.06	1.15	1.38	1.950	< .05
CCO-I	3.42	2.16	1.93	1.86	3.292	< .001
Total CCO	43.94	38.68	8.90	8.72	2.96	< .01
SCO-Con	9.82	8.90	5.75	6.88	.718	n.s.
SCO-Cl	2.90	2.44	2.07	2.02	1.113	n.s.
SCO-D	8.84	8.10	2.66	2.67	.237	n.s.
SCO-P	3.02	2.84	2.22	2.07	.413	n.s.
SCO-I	1.44	1.26	1.36	1.00	.748	n.s.
Total SCO	26.02	23.54	9.13	10.05	1.279	n.s.
STEP Reading	45.74	47.68	11.01	11.83	.840	n.s.

$\bar{X}_1$ = mean score, boys SD_1 = standard deviation, boys

$\bar{X}_2$ = mean score, girls SD_2 = standard deviation, girls

TABLE 15.-Product-moment correlations between the variables CCO-Con-S-W-V and sex. (n = 50; n = 50).

Variable	Sex r	p
CCO-Con-S	.18	<.05*
CCO-Con-W	.25	<.01**
CCO-Con-V	.08	n.s.

It could be of interest that on the Concrete tests of reasoning boys, nine-and-ten-years, appear to be in advance of girls in conserving substance and weight, in recognizing the role of chance and estimating probability, and in reasoning inductively. The mean scores of boys on tests of these operations on the Stories tests were not significantly different from the mean scores of girls. On the standardized test of reading comprehension, STEP Reading, no significant difference between the mean scores of boys and girls was found (see Table 14). The apparent advantage of the boys in reasoning in these areas in concrete stimulus situations does not seem to be represented in higher mean scores in reading comprehension as measured by either the Stories tests or by STEP Reading. For the girls the problem may be occurring in reverse: the reasoning skills of nine-and-ten-year-old girls in Concrete situations may be rather less adequate than in a symbolic language situation such as reading. Sex differences in the development of reasoning and reading skills at the nine-and-ten-year-old level, and the possible implications of these differences for the later intellectual development of both boys and girls, may be overlooked when no sex differences on mean scores on reading comprehension tests are reported in the research.

Summary: relations of CCO, SCO, STEP Reading and sex.

Product-moment correlations between scores on the ten Concrete and Stories variables and sex (see Table 13), suggested that scores on three Concrete tests CCO-Con, CCO-P, and CCO-I were significantly related to sex ($r = .18$ $p<.05$; $r = .19$ $p<.05$; $r = .32$ $p< .001$). Correlations between Concrete classification, deduction and sex, and between the Stories variables and sex and between STEP Reading total scores and sex were not significantly different from zero. Similar results were obtained from t-tests for the significance of differences of mean scores of boys and girls on these variables. The mean scores of boys were significantly higher than the mean scores of girls on Concrete conservation (substance and weight), and Concrete inductive and probability reasoning. No significant differences were found between mean scores on the Stories tests or STEP Reading.

It appeared possible on the basis of these results that potentially important early sex differences could be occurring in the development of reasoning in reading and reasoning in concrete stimulus situations at the nine-and-ten-year-old level. A standardized test of reading comprehension did not appear to reveal these differences. They were suggested by the results of tests of reasoning in concrete and reading situations in which the logical operations were held as nearly comparable as possible for each situation.

Relationships between the Concrete and Stories tests and STEP Reading will now be investigated. Stepwise multiple regression analysis will be applied to obtain an indication of the ability of the Concrete and Stories tests, separately and together to predict the variance of STEP Reading total scores, and to range in order the Concrete and Stories tests as predictors of STEP Reading.

Stepwise Multiple Regression Analyses

Stepwise multiple regression analyses was applied to obtain a clearer indication of the relationship of the Concrete and Stories tests to STEP Reading.

In stepwise multiple regression analyses one variable at a time is added, the variable added at each step being the one which makes the greatest improvement in the prediction of the criterion. The variable most highly correlated with the criterion is selected first for this purpose. As each new variable is added the overall significance of the regression equation at that stage is determined.

The correlation between the Concrete and Stories subtests and STEP Reading are presented in Table 16.

TABLE 16.-Product-moment correlations between Concrete and Stories variables and STEP Reading. (N = 100).

Concrete Variable	STEP Reading r	p	Stories Variable	STEP Reading r	p
CCO-Con	.16	< .1	SCO-Con	.27	< .01
CCO-Cl	.20	< .05	SCO-Cl	.28	< .01
CCO-D	.22	< .05	SCO-D	.25	< .01
CCO-P	.18	< .1	SCO-P	-.12	n.s.
CCO-I	-.10	n.s.	SCO-I	.19	< .05

It may be seen in Table 16 that for the Concrete tests, variable CCO-D is the most highly correlated with STEP Reading ($r = .22$ $p < .05$), and for the Stories tests, variable SCO-C1 is the most highly correlated with STEP ($r = .28$ $p < .01$). These variables will be selected first in the stepwise multiple regression analyses using Concrete and Stories subtests separately as predictor variables and STEP Reading as the criterion variable. When the ten subtests are used as predictors, the variable SCO-C1 is selected first since this variable is the most highly correlated of the ten CCO and SCO variables with the criterion STEP Reading ($r = .28$ $p < .01$). These variables are selected first since they will predict the greatest amount of the variance of the criterion. Interrelationships between predictor variables will affect the subsequent order of selection.

The results of the stepwise multiple regression analysis using the five subtests of the Concrete tests as predictors of STEP Reading total scores are presented in Table 17. A value of 'F' at the 95 per cent level of confidence was accepted in each analysis.

It may be seen in Table 17 that the five subtests of the Concrete tests predict approximately 13 per cent of the variance of STEP Reading. The variable CCO-D initially selected as the most highly correlated of the Concrete variables with STEP (see Table 16), predicts 5.02 per cent of the variance of STEP Reading predictable by the Concrete tests. Three variables CCO-D, CCO-I and CCO-C1, selected at steps 1, 2, 3, predict 10.6 per cent of the 13.44 per cent of the variance of STEP predictable by the Concrete tests.

At step 2 in the analysis the variable selected as contributing

TABLE 17.–Results of the stepwise multiple regression analysis using the five subtests of the Concrete tests as predictors and STEP Reading total scores as criterion. (N=100).

Order of Selection	Source of variance added	F-ratio	p of F	df	β	Per cent variance predicted
1	CCO-D	5.18	.02	98	2.28	5.02
2	CCO-I	4.11	.02	97	2.67	
					-1.72	2.80
3	CCO-Cl	3.79	.01	96	2.31	
					-1.91	
					1.73	2.78
4	CCO-P	3.50	.01	95	2.11	
					-2.02	
					1.63	
					1.56	2.24
5	CCO-Con	.292	.02	94	1.89	
					-1.96	
					1.42	
					1.61	
					.81	.60
	Total					13.44

Regression equation:

$$\hat{Y} = 1.89\ \text{CCO-D} - 1.96\ \text{CCO-I} + 1.42\ \text{CCO-Cl} + 1.61\ \text{CCO-P} + .81\ \text{CCO-Con} + 26.64$$

most to the prediction of the variance of the criterion is Concrete induction, CCO-I. The negative beta coefficient of the variable CCO-I ($\beta = -1.96$) suggests that CCO-I may be a "suppressor" variable to some extent at least. This could be the case if the variable CCO-I has a zero or near-zero correlation with the criterion and a relatively high correlation with a valid predictor of the criterion.[1]

[1]Philip H. Du Bois, An Introduction to Psychological Statistics (New York: Harper and Row, 1965), p. 184.

Table 18 presents the 5 x 5 correlation matrix showing the interrelationships between the Concrete variables, and the relation of each variable to the criterion STEP Reading totals.

TABLE 18[a].-Intercorrelations between the Concrete variables and the relations between these variables and STEP Reading total scores. (N = 100).

Variable	2	3	4	5	STEP Reading
CCO-Con 1	24**	24**	02	-006	16
CCO-Cl 2		23*	16	11	20*
CCO-D 3			26**	17	22*
CCO-I 4				11	-10
CCO-P 5					18

[a]Decimal points and diagonal elements are omitted.
* $p < .05$
** $p < .01$

It may be seen in Table 18 that the variable CCO-I, selected at step 2, is highly correlated with the variable CCO-D ($r = .26$ $p < .01$), a valid predictor of STEP. The variable CCO-I has a "near-zero" relation to STEP ($r = -.10$ n.s.). It would appear that CCO-I could be a suppressor variable on the basis of its relation to CCO-D and STEP Reading.

The relatively small amount of the variance of STEP Reading predictable by the five Concrete tests (13.44 per cent), could be accounted for in part by error: the reliability quotient of Concrete tests was only $r = .74$ (see Table 4). It could in part be an indication that the Concrete tests and STEP Reading measure aspects of cognitive behavior which differ considerably in the operations involved. This question will be considered on the basis of a factor analysis of STEP Reading test items and of subscores of the Concrete and Stories tests and STEP Reading total scores taken jointly.

The results of stepwise multiple regression analysis using the five subtests of the Stories tests as predictors of STEP Reading total scores is presented in Table 19.

TABLE 19.-Results of stepwise multiple regression analysis using the five subtests of the Stories tests as predictor variables and STEP Reading total scores as criterion. (N = 100).

Order of Selection	Source of variance added	F-ratio	p of F	df	β	Per cent variance predicted
1	SCO-Cl	8.37	.004	98	2.89	7.87
2	SCO-Con	6.81	.002	97	2.30 2.22	4.44
3	SCO-P	6.26	.0009	96	2.32 2.75 -2.15	4.04
4	SCO-D	5.28	.0009	95	1.91 2.19 -2.35 1.46	1.84
5	SCO-I	4.59	.001	94	1.73 2.07 -2.19 1.52 1.30	1.45
	Total					19.64

Regression equation:

$$\tilde{Y} = 1.73\ \text{SCO-Cl} + 2.07\ \text{SCO-Con} - 2.19\ \text{SCO-P} + 1.52\ \text{SCO-D} + 1.30\ \text{SCO-I} + 36.50.$$

It may be seen in Table 20 that the five Stories tests predict approximately 20 per cent of the variance of the criterion STEP Reading. Approximately 16 per cent of the variance predictable is accounted for by the three tests SCO-Cl, SCO-Con, and SCO-P.

The negative beta for the variable SCO-P ($\beta = -2.19$, step 5), raises the question that SCO-P may be to some extent a suppressor variable in this situation. The interrelations between the Stories variables which affect the ordering are shown in Table 20.

TABLE 20[a].-Intercorrelations between the Stories variables and the relations between these variables and STEP Reading total scores. (N = 100).

Variable		2	3	4	5	STEP Reading
SCO-Con	1	25**	39***	08	27**	27**
SCO-Cl	2		31**	14	06	28**
SCO-D	3			01	22*	25**
SCO-1	4				-10	19
SCO-P	5					-12

[a]Decimal points and diagonal elements are omitted.
* $p < .05$
** $p < .01$

It may be seen in Table 20 that the variable SCO-P has a "near-zero" relation to the criterion STEP Reading ($r = -.12$ n.s.), and a highly significant relation to the variable SCO-Con, a valid predictor of STEP, selected at step 2. It appears that SCO-P could be functioning in this situation as a suppressor variable.

The relatively small amount of the variance of STEP Reading predictable by the five Stories tests (approximately 20 per cent) suggests the possibility that the Stories tests are assessing cognitive operations which differ considerably from the operations assessed by STEP Reading. Error due to unreliability of the Stories tests (r = .72, see Table 4) and to other sources could also be contributory.

The results of stepwise multiple regression analysis using the ten Stories and Concrete subtests as predictors and STEP Reading as criterion are presented in Table 21.

It may be seen in Table 21 that the ten subtests of the Concrete and Stories tests taken together are able to predict 27.65 per cent of the variance of STEP Reading. Approximately 16 per cent of the variance predictable is accounted for by three of the Stories tests; 19 per cent is accounted for by four of the ten subtests. Some of the variance predictable by the Stories tests could be explained by reading skills common to the two testing situations. Some of the variance predictable could be explained by cognitive operations common to the Concrete and Stories tests and STEP Reading. The variance of STEP Reading not predictable by the Concrete and Stories tests would then be explained by cognitive operations not common to the Concrete and Stories tests and the criterion by other features not common to the two test situations, and by error.

TABLE 21.-Results of stepwise multiple regression analysis using the ten subtests of the Concrete and Stories tests as predictor variables and STEP Reading totals as criterion. (N = 100).

Order of Selection	Source of variance added	F-ratio	p of F	df	β	Per cent variance predicted
1	SCO-Cl	8.37	.005	98	2.89	7.87
2	SCO-Con	6.81	.002	97	2.22	4.44
3	SCO-P	6.26	.001	96	-2.15	4.04
4	CCO-Cl	5.71	.001	95	1.89	3.02
5	CCO-P	5.06	.001	94	1.48	1.84
6	CCO-I	4.58	.001	93	-1.38	1.59
7	CCO-D	4.33	.001	92	1.55	1.96
8	SCO-D	4.03	.001	91	1.31	1.40
9	SCO-I	3.79	.001	90	1.28	1.33
10	CCO-Con	3.40	.001	89	- .45	.16
	Total					27.65

Regression equation:

$$\tilde{Y} = 1.53\ \text{SCO-Cl} + 1.64\ \text{SCO-Con} - 2.35\ \text{SCO-P} + 1.56\ \text{CCO-Cl} + 1.42\ \text{CCO-P} - 1.92\ \text{CCO-1} + 1.38\ \text{CCO-D} + 1.36\ \text{SCO-D} + 1.26\ \text{SCO-I} - .45\ \text{CCO-Con} + 42.54.$$

Summary: stepwise multiple regression analysis

Three stepwise multiple regression analyses were undertaken to obtain a clearer indication of the relationship of the Stories tests, and the ten Concrete and Stories tests taken together, to the criterion STEP Reading.

Three of the five variables of the Concrete tests predicted approximately 11 per cent of the 13 per cent of the variance of STEP Reading predictable. Three of the five variables of the Stories tests predicted approximately 16 per cent of the 20 per cent of the variance of the criterion predictable. The ten subtests taken together predicted approximately 28 per cent of the variance of the criterion Four of the ten variables, SCO-Cl, SCO-Con, SCO-P and CCO-Cl predicted approximately 19 per cent of the variance predictable.

Some of the variance of STEP Reading predicted by the ten Concrete and Stories variables taken together would be expected to be accounted for by reading skills common to the Stories tests and STEP Reading; some of the variance predicted would be expected to be accounted for by cognitive operations common to the predictors and the criterion. The variance of STEP Reading not predicted by the Concrete and Stories tests could be considered to be related to cognitive operations not common to the predictors and the criterion and to other differences, including error, between the Concrete and Stories tests and the criterion, STEP Reading.

Relations between the Concrete and Stories variables and between the two sets of variables will now be examined. Product-moment correlations between Concrete and Stories variables and canonical correlation between the two composites will be presented.

Correlations between the Concrete and Stories Tests

An examination of the relations between scores on the Concrete tests and scores on the Stories tests assessing conservation, classification, deduction, induction and probability reasoning was undertaken to determine the extent to which cognitive operations available to subjects in response to concrete stimuli for each of these categories would be concurrently available in a reading situation as measured by the Stories tests. Test items, designed to be as nearly comparable as possible for each situation, were presented for each category (see Chaps. IV to VIII).

Relations within each set of variables will also be examined to permit an estimate of the extent to which correct responses for a particular set of operations, for example Concrete conservation, are likely to be associated with correct responses for other Concrete operations within the set.

Correlations between variables, Concrete and Stories Tests.

The matrix of product-moment correlations between variables of the Concrete and Stories tests is presented in Table 22.

It may be seen in Table 22 that the strongest relation within the data is between the variables CCO-Con and SCO-Con ($r = .44$ $p < .001$). The variance of SCO-Con predictable by CCO-Con is approximately 20 per cent of the total variance.

Eleven of the twenty-five correlations in the matrix of correlations between the Concrete and Stories variables are significant ($p < .05$); five of these appear to be highly significant ($p < .01$). Significant correlations are distributed throughout the matrix but there appears to be a tendency for significant correlations between Concrete and Stories variables to be based on Stories conservation and Stories deduction.

The pattern of correlations between the Concrete and Stories variables shown in Table 22 suggests that some of the logical operations assessed will be likely to be available to these Grade four subjects in both Concrete and Stories test situations. The range of the variance predictable by corresponding test scores is from 19.4 per cent (r = .44) to negligible (r = .01).

TABLE 22[a].-Product-moment correlations between variables of the Concrete and Stories tests. (N = 100).

Variable	2	3	4	5	6	7	8	9	10
CCO-Con 1	25**	24**	-01	19*	44***	25**	21*	06	05
CCO-Cl 2		29**	11	16	20*	13	23*	26**	07
CCO-D 3			17	26**	25**	13	30**	09	02
CCO-P 4				11	23*	07	13	17	19*
CCO-I 5					15	-01	25**	21*	-02
SCO-Con 6						25**	39***	27**	08
SCO-Cl 7							31**	06	14
SCO-D 8								22*	01
SCO-P 9									-10
SCO-I 10									

[a]Decimal points and diagonal elements are omitted.

* $p < .05$
** $p < .01$
*** $p < .001$

The product-moment correlation between total scores CCO and total scores SCO (see Appendix I) is r = .53 (p < .01). The maximum possible relation between the two sets of variables taken as composites will be determined by canonical correlation procedures.

Correlations between subscores within the Concrete data and subscores within the Stories data are also shown in Table 22.

Within the Concrete data significant correlations occur between CCO-Con and CCO-Cl (r = .25 p < .01); between CCO-Con and CCO-D (r = .24 p < .01); between CCO-Cl and CCO-D (r = .29 p < .01); and between CCO-D and CCO-I (r = .26 p < .01). The range of the variance predictable within the Concrete data is from 8.4 per cent (r = .29) to negligible (r = -.01).

SCO-Con, SCO-Cl and SCO-D are significantly related within the Stories data (r = .25 p < .01; r = .39 p < .001; r = .31 p < .01). The range of the variance predictable within the Stories data is from 15 per cent to negligible (r = -.01).

It would appear that within both the Concrete and Stories situations the children who conserve are likely to succeed in classification and deduction. Within the Stories data children who conserve are, in addition, likely to succeed in probability reasoning (r = .27 p < .01). Within the Concrete data inductive and deductive reasoning are likely to be associated (r = .26 p < .01). A limited process of integration of the logical operations assessed appears to be under way within each situation, Concrete and Stories. Within each situation some of the logical operations assessed appear to be excluded from what may be an emerging trend toward integration. The operations which appear to be excluded from this trend are Concrete probability and Stories induction. The relation between CCO-P and each of the three variables CCO-Con, CCO-Cl and CCO-D is not significantly different from zero ($r = -.01$; $r = .11$; $r = .17$, n.s.). The relations between SCO-I and each of the variables SCO-Con, SCO-Cl, SCO-D and SCO-P is not significantly different from zero ($r = .08$; $r = .14$; $r = .01$; $r = -.10$, n.s.).

The pattern of relations observed between the Concrete and Stories tests suggests that some of the variables will contribute greater amounts than others to the prediction of the variance common to the two sets of tests. This will be examined by considering the weights assigned to the variables in determining the maximum possible correlation between the two sets of variables taken as composites.

Canonical Correlation: the maximum possible relation between the two sets of variables

In calculating the maximum possible correlation between the two sets of variables, Concrete and Stories, taken as composites, one significant canonical correlation was obtained. The weighting system for this canonical correlation is presented in Table 23.

The maximum canonical correlation between the two sets of variables Concrete and Stories may be seen in Table 23 to be $R_c = .549$ ($p < .01$). This canonical correlation may be seen to be based primarily on Concrete and Stories conservation. The weights assigned these variables are .808 and .912. Concrete probability and Stories deduction also contribute to the prediction of the maximum correlation but to a lesser degree (.412 and .316). The contribution of Stories probability

TABLE 23[a].-Canonical correlation between the two composites, Concrete and Stories, based on scores on the variables. (N = 100).

	Canonical Correlation	.549
	Chi square	49.85
	Degrees of freedom	25.
	Probability	< .01

	Test data	Normalized weights
Concrete Data	1. Conservation	808
	2. Classification	210
	3. Deduction	248
	4. Probability	421
	5. Induction	254
Stories Data	6. Conservation	912
	7. Classification	218
	8. Deduction	316
	9. Probability	143
	10. Induction	-016

[a]Decimal points are omitted.

and of Stories induction to maximizing the relation between the two composites appears to be negligible (.143 and -.016).

The maximum correlation between the composites (R_c = .549 $p <$ .01) indicates that approximately 30 per cent of the total variance is common to the two sets of tests, Concrete and Stories. Conservation (CCO-Con and SCO-Con: $r = .44$ $p < .001$) would then account for 20 per cent of the common variance, leaving 10 per cent of the common variance to be accounted for by classification, deduction, probability and induction together.

This result lends support to the conclusion that the relation between the Concrete and Stories tests while significant is not strong; and that the greater part of the variance common to the tests taken as composites can be accounted for by a single cognitive category: conservation. It would appear that the extent to which these Grade four children recognize comparability in the Concrete and reading situations (Stories), and are able to apply identical logical operations of classification, deduction, probability reasoning and induction in solving these problems, is minimal. Only Concrete and Stories situations requiring the recognition of invariance are likely to be solved in both situations.

The greater variance not common to the two sets of tests, that is, 70 per cent of the total variance, would be accounted for in part by error, and in part by elements unique to each test situation, including operations not equally available to subjects in the Concrete and Stories testing situations. Error would be due to unreliability of the tests, to differences in comparability of the corresponding test items, to the possibility of errors in scoring, and to other sources. Problems in assessing reliability were discussed in Chapter III. Precise comparability was not completely achieved. The extent to which corresponding test items were considered comparable was discussed in Chapters IV to VIII. The possibility of errors in decoding was controlled as far as possible in two ways. The readability levels of the Stories were held to a level considered to be appropriate for the subjects of the study (see Table 5). Assistance was also given if a problem occurred in decoding, in recall, or in interpreting the literal meaning of a statement. Error could have been introduced by the unequal lengths of the stories. It would have been preferable if more than one story had been available for each category assessed.

In addition to variance accounted for by error the results lend support to the conclusion that the variance unique to each test situation may be accounted for by differences in the difficulty of solving identical logical problems from data presented in a concrete form and in reading. This variance which is unique would seem to be appreciably greater than the amount of the common variance (10 per cent) accounted for by classification, deduction, probability reasoning and induction together.

Summary: Correlations

The correlation matrix of order 10 x 10 Concrete and Stories variables was presented in Table 22.

The strongest relation in the data was between the variables CCO-Con and SCO-Con ($r = .44$ $p < .001$). Eleven of the twenty-five correlations in the matrix of correlations between the CCO and SCO variables were significant ($p < .05$). Seven of the eleven significant correlations were based on the Stories variables SCO-Con and SCO-D.

The distribution of significant correlations within the Concrete data suggested that some integration of the logical operations assessed by the Concrete tests was under way at the Grade four level.

The distribution of these correlations within the Stories data suggested that some integration of these operations was also under way in reading with the possible exception of Stories inductive reasoning. The correlations between SCO-I and other variables were not significantly different from zero (range: $r = .14$ to $r = -.10$, n.s.).

An attempt was made using canonical correlation procedures to determine the maximum possible relation between the two sets of variables, Concrete and Stories. One significant correlation was obtained: $R_c = .55$ ($p < .01$). The variance common to the two sets of tests taken as composites was 30 per cent of the total variance.

The weighting system of this canonical correlation was presented in Table 23. The greatest weights (.808 and .912) were assigned to the variables CCO-Con and SCO-Con. The product-moment correlation between these variables was seen in Table 22 to be $r = .44$ ($p < .001$). The amount of the common variance of the two composites accounted for by conservation would then be 20 per cent. The remaining 10 per cent of the common variance would be accounted for by classification, deduction, probability and induction together.

The maximum correlation between the two sets of variables ($R_c = .55$ $p < .01$), was considered to lend support to the conclusion that the logical operations assessed were tending to become available in both the Concrete and Stories situations at the Grade four level; they were unlikely to be equally available in the two situations.

The weighting system presented was considered to lend support to the conclusion that operations of conservation were more likely to be available in both concrete and reading situations than the other logical operations assessed. It appeared that the recognition of invariance in both stimulus situations could be basic to the emergence of other logical operations at the Grade four level. The progress of the nine-to-ten-year-old subjects in applying in reading the logical operations of classification, deduction, probability reasoning and induction, even to the extent that these were available in Concrete situations, was minimal.

An attempt will be made to uncover factors common to the Concrete and Stories tests and STEP Reading. Factors underlying these tests could suggest the characteristics common to the operations measured by the tests.

Factor Analyses

Three analyses were undertaken in an attempt to uncover factors common to the Concrete and Stories tests and to STEP Reading. The first analysis was based on the items of the STEP Reading test. The second was based on Concrete and Stories subscores. The third was based on Concrete and Stories subscores taken jointly with STEP Reading total scores.

Varimax rotations of the principal-axes factors were applied to each of the three variable sets. In each analysis, unities were entered into the diagonal of the correlation matrix.

The three analyses yielded three major groupings of factors each with various subfactors. The groupings, referred to as Clusters I, II, III, are summarized in Figure 44.

Factor Groupings

Factor			Per Cent Common Variance
		Cluster 1: Reading Comprehension Skills	
1	C	Basic reading comprehension skills	30
2	IC	Inferential reasoning from context	20
3	P	Recognizing the purposes of the writer	17
4	D	Reading for detail	17
5	A	Critical appraisal	16
		Cluster II: Cognitive Operations	
6	G	General reasoning	24
7	IN	Recognition of invariance	25
8	L	Logical reasoning, Stories	20
9	CD	Concrete deduction	16
10	ID	Inductive - deductive inference	14
		Cluster III: Cognitive Functioning in Reading	
11	GR	General reasoning, reading	25
12	INR	Recognition of invariance, reading	25
13	ICR	Inferential reasoning, reading	16
14	LR	Logical reasoning, reading	19
15	CDR	Concrete deductive reasoning, reading	16

Fig. 44.-Factors obtained from analyses based on STEP Reading items; on Concrete and Stories subscores; and based on Concrete and Stories subscores and STEP Reading total scores taken jointly.

Factor analysis based on
STEP Reading items

Items on STEP Reading provided a matrix of 70 x 70 Phi coefficients. This matrix is shown in Appendix I. The unrotated factor matrix of the principal-axes factors is presented in Table 25.

Varimax rotation of the principal-axes factors is presented in Table 26.

It may be seen in Table 26 that 29.14 per cent of the total variance of STEP Reading is accounted for by the five factors obtained. The five factors constitute Cluster I (see Fig. 44).

Factor 1, accounting for approximately 30 per cent of the common variance, loads highly on items 21 to 31 and 33 to 35, Part 1, and items 30, 31 and 35, Part II. These items were examined for possible common elements representing the tasks in reading comprehension assessed. The items were found to require the subject to select the "main idea" from four alternatives; to select statements which assign to a character the motivations ascribed to him in the text; and to choose the correct meaning for idiomatic expressions. (See Co-operative Sequential Tests of Educational Progress, Reading, Form 4B, Appendix G.) The STEP Reading items with high loadings on Factor 1 were considered to assess basic reading comprehension skills, the skills recognized as necessary for a first level understanding of the material read. Factor 1 can be called "Basic reading comprehension skills." It will be referred to as Factor C.

Factor 2, accounting for 20 per cent of the common variance, may be seen in Table 26 to load highly on STEP items 2, 6, 18 and 19, Part I; and items 3, 8, 10, 11, 17, and 20, Part II. These items were also examined for the common elements in reading comprehen-

TABLE 24[a].-Unrotated principal-axes factors based on STEP Reading items. (N = 100)

ITEMS	FACTORS 1	2	4	5	6	h^2
Part I						
1	-167	064	262	118	-083	122
2	398	058	313	-019	018	260
3	040	-022	281	162	130	124
4	126	422	-311	-243	207	392
5	195	233	-302	-244	023	244
6	204	270	322	-061	007	222
7	018	276	-205	-400	316	378
8	186	333	189	-204	225	274
9	-067	119	-303	-023	-107	122
10	339	-055	-031	020	254	184
11	371	221	036	-220	285	318
12	005	311	129	154	185	171
13	180	285	152	193	190	210
14	151	177	-103	-000	362	196
15	197	007	126	114	050	070
16	022	153	280	-452	347	427
17	148	034	388	-021	227	225
18	260	285	143	-368	-017	305
19	355	142	293	-054	-164	262
20	256	035	160	099	-226	153
21	475	-164	-078	008	116	272
22	608	176	039	132	024	420
23	630	-009	-130	-041	042	417
24	601	-110	-237	001	003	430
25	493	055	-275	-230	225	425
26	443	-413	-073	-123	026	389
27	301	-213	-209	080	-058	190
28	515	-326	-186	-052	-154	432
29	240	-435	158	-051	-292	360
30	392	-438	-207	-002	-022	389
31	480	-437	-061	-073	-210	474
32	303	-028	-079	-300	-198	288
33	362	-309	-008	-283	-083	314
34	335	-201	-247	-244	-296	361
35	492	-374	014	-336	-226	545

TABLE 24[a].-(continued)

Part II

1	030	026	256	213	345	232
2	128	321	165	393	085	309
3	267	450	109	018	473	510
4	469	253	076	190	231	379
5	174	500	267	230	332	514
6	104	164	062	238	110	110
7	126	356	183	134	020	194
8	368	022	555	043	029	447
9	003	309	203	171	107	177
10	310	372	307	035	494	575
11	269	284	327	107	237	328
12	175	013	080	319	052	141
13	169	133	155	330	049	181
14	378	282	101	346	170	381
15	362	280	052	066	203	257
16	294	019	204	281	095	217
17	373	007	504	154	006	416
18	183	046	197	073	121	094
19	314	097	211	201	123	208
20	368	108	207	075	175	225
21	340	396	042	156	024	299
22	186	034	062	400	213	245
23	249	280	210	092	167	221
24	234	151	184	027	063	116
25	184	318	180	195	089	213
26	506	125	107	115	285	378
27	306	095	124	049	396	277
28	431	180	200	082	085	272
29	608	088	065	254	022	447
30	531	259	161	175	078	412
31	583	149	217	019	109	422
32	280	001	029	028	057	083
33	159	081	143	446	174	282
34	131	061	048	164	038	052
35	586	248	065	187	186	479
SSQ	7.908	4.020	3.037	2.794	2.640	20.399
Per cent of common variance	38.768	19.706	14.888	13.695	12.932	100.000
Per cent of total variance	11.286	5.744	4.339	3.992	3.772	29.133

[a]Decimal points are omitted.

TABLE 25[a].-Varimax rotation of the principal-axes factors based on STEP Reading items. (N = 100).

ITEMS	FACTORS 1	2	3	4	5	h^2
Part I						
1	082	251	-132	-052	180	122
2	178	431	109	-049	169	260
3	-078	152	-019	-152	267	124
4	-046	-044	536	251	-195	392
5	134	-018	337	239	-234	244
6	-081	434	148	019	065	222
7	-069	-064	540	-019	-278	378
8	-102	313	405	-035	-012	274
9	-013	-180	031	251	-161	122
10	258	033	268	-055	203	184
11	139	229	495	-017	017	318
12	-130	036	-010	391	006	171
13	018	083	025	443	080	210
14	-005	-053	415	016	144	196
15	086	149	038	-000	198	070
16	-132	258	418	-367	-184	427
17	-145	306	149	-258	202	225
18	050	397	298	036	-233	305
19	134	485	019	077	046	262
20	144	294	-130	140	100	153
21	455	063	177	-003	172	272
22	546	151	100	087	286	420
23	517	179	275	168	119	417
24	581	054	201	189	113	430
25	421	013	486	095	-045	425
26	600	019	029	-161	035	389
27	402	-093	-023	103	090	190
28	651	036	-035	073	011	432
29	426	171	-351	-159	-024	360
30	602	-122	-045	-056	075	389
31	662	095	-159	-043	001	474
32	336	183	062	084	-267	228
33	499	130	012	-154	-155	314
34	510	030	-060	139	-277	361
35	653	234	-066	-123	-211	545

TABLE 25[a].-(continued)

Part II

1	-081	155	285	-346	023	232
2	-080	-007	035	477	271	309
3	-015	497	-060	492	-134	510
4	238	271	059	469	160	379
5	004	191	186	549	-376	514
6	-079	071	096	118	275	110
7	-135	347	197	110	-068	194
8	105	588	-026	-155	255	447
9	-122	-046	056	395	033	177
10	-000	633	-159	384	-032	575
11	-002	552	014	140	-052	328
12	065	057	-036	076	356	141
13	299	-008	103	-052	-280	181
14	084	075	284	339	416	381
15	094	148	384	198	200	257
16	147	251	-119	095	330	217
17	162	570	064	-229	094	416
18	102	192	106	-168	085	094
19	98	-057	216	244	244	208
20	192	422	025	101	009	225
21	042	343	356	224	-049	299
22	057	-021	051	026	488	245
23	101	082	108	439	009	221
24	157	043	151	256	-040	116
25	-003	046	119	429	114	213
26	288	085	414	144	309	378
27	152	-046	440	015	241	277
28	281	097	358	236	018	272
29	392	205	172	309	356	447
30	556	-027	087	071	299	412
31	444	338	147	-112	278	422
32	190	126	118	032	127	083
33	038	014	-041	-015	527	282
34	175	045	053	-021	-125	052
35	510	137	134	-072	420	479
SSQ	6.098	4.094	3.545	3.506	3.156	20.399
Per cent of common variance	29.943	20.068	17.348	17.157	15.472	100.000
Per cent of total variance	8.711	5.848	5.064	5.009	4.508	29.141

[a]Decimal points are omitted.

sion assessed. They appear to involve inferring meaning from context. They are of the form:

Why did Ben have a chance to ....

Johnny's home is ....

The job of a player in the line is ....

Which animal was friendly ....

The inferences required are in each case "given" in the text; what is assessed appears to be an ability to read and understand the appropriateness of the given inference. Factor 2 can be called "Inferential reasoning from context." It will be referred to as Factor IC (see Fig. 44).

Factor 3, accounting for approximately 17 per cent of the common variance, may be seen in Table 26 to load highly on items 4, 7, 8, 11, 14, 16 and 25, Part I; and 26 and 27, Part II. This Factor appears to involve an understanding of the intentions of the writer (in some cases of a character in the story.) The items present questions of the form:

The writer wants us to know ....

Why does the writer tell us ....

How does the poet try to keep you interested ....

Factor 3 can be called "Recognizing the purposes of the writer." It will be referred to as Factor P.

Factor 4, also accounting for approximately 17 per cent of the common variance, may be seen in Table 26 to load highly on item 13, Part I; and items 2, 3, 4, 5, 23 and 25, Part II. It loads moderately on item 12, Part I and items 1, 9, and 10, Part II. The items appear to require a careful reading for detail. They are usually presented in a form requiring the completion of a sentence: supplying a subject (a class name); a predicate (a detail descriptive of the class); or choosing an alternative predicate. The detail to be recalled is important in recognizing the main idea, or the purpose of the writer, or in making an inference from context:

> The reader ... (should think about the work of ....)
>
> (Miss Parker) ... was the one who ....
>
> (This one)... is a friendly letter.
>
> The writer thinks ... (this is a good book)
>
> (The buck) ... is the friendly animal.
>
> ... air is dead.
>
> ... no movement of the air.

Factor 4 can be called "Reading for detail." It will be referred to as Factor D.

Factor 5, accounting for approximately 15 per cent of the common variance, loads highly on items 14, 22, 33 and 35, Part II. These items appear to involve a critical appraisal of a writer's point of view and some appreciation of the limitations of his presentation. The test items are of the form:

> Why does the writer tell you ...?
>
> The writer does NOT tell you ... .
>
> The writer feels that ... .
>
> The writer does NOT mention ... .

Factor 5 can be called "Critical appraisal." It will be referred to as Factor A.

The five factors of Cluster 1 based on STEP Reading items appear to assess reading comprehension skills generally recognized as important at the Grade four level. Cluster 1 may be called "Reading Comprehension Skills." (See Fig. 44). These reading comprehension skills described by the Factors of Cluster 1 appear to be applied in the STEP Reading test in interpreting character and motivation and in appreciating human situations and human idiosyncrasies. These are important intellectual skills. It is a question if they represent the full range of intellectual operations available in reading to nine-and-ten-year-old children.

Two additional factor analyses will now be presented: one based on Concrete and Stories subscores; and one based on these subscores and STEP Reading total scores taken jointly.

Factor analysis based on
Concrete and Stories subscores

In Table 26, a matrix of order 27 x 27 presents correlations between Concrete and Stories subscores and between these scores and STEP Reading totals. Variables 1 to 26 are subscores of the Concrete and Stories tests; variable 27, STEP Reading total.

The unrotated principal-axes factors of the 26 Concrete and Stories subscores are presented in Table 27.

Varimax rotation of the principal-axes factors is presented in Table 28.

The five factors obtained account for 44.45 per cent of the total variance. They will be referred to as Cluster II (see Fig. 44).

TABLE 26[a].-Product-moment correlations of Concrete and Stories subscores and STEP Reading totals. (N = 100).

	Variables	1	2	3	4	5	6	7	8	9	10
1	CCO-Con-S										
2	CCO-Con-W	720									
3	CCO-Con-V	544	642								
4	CCO-Cl-Inclusion	049	218	132							
5	CCO-Cl-Multiplication	229	196	151	168						
6	CCO-D-Modus Ponens	034	052	002	-010	054					
7	CCO-D-Tollens	210	141	002	115	126	087				
8	CCO-D-Undetermined	056	203	123	137	-031	060	195			
9	CCO-D-Possible	237	189	152	342	194	134	468	276		
10	CCO-D-Not-possible	017	037	097	045	-121	185	047	-046	-005	
11	CCO-D-Transitivity	031	-017	024	-190	042	-117	064	005	-042	095
12	CCO-I	-038	058	026	218	026	-001	108	182	203	060
		245									
13	CCO-P	014	043	-063	094	076	055	175	016	156	129
		-096	114								
14	SCO-Con-S	418	370	287	253	121	004	061	138	393	013
		-015	290	196							
15	SCO-Con-W	223	301	283	180	026	-146	-080	064	165	-006
		-080	-042	146	574						
16	SCO-Con-V	288	312	345	179	001	003	095	169	254	103
		-024	102	227	497	494					
17	SCO-Cl-Inclusion	209	238	190	051	071	049	106	127	048	-037
		-079	-022	029	286	060	170				
18	SCO-Cl-Multiplication	067	133	100	140	214	136	069	053	205	074
		-073	062	193	267	189	156	193			
19	SCO-D-Modus Ponens	145	204	196	086	020	-139	120	051	005	-058
		109	181	-089	212	086	158	263	161		

TABLE 26[a].-(continued)

Variables	1	2	3	4	5	6	7	8	9	10
20 SCO-D-Modus Tollens	169 -007	088 134	074 013	040 296	-011 082	060 233	102 311	004 095	179 341	-112
21 SCO-D-Undetermined	064 -182	-057 050	031 143	114 207	080 072	033 089	088 176	-012 081	212 120	122 046
22 SCO-D-Possible	046 -060 330	-034 243	038 132	116 362	232 276	072 174	118 014	081 193	308 229	085 256
23 SCO-D-Not-Possible	241 007 037	264 120 311	203 171	149 394	244 175	-122 219	076 072	199 134	236 356	094 230
24 SCO-D-Transitivity	094 -004 128	104 132 210	147 083 172	100 227	102 040	134 191	154 209	171 207	269 251	073 346
25 SCO-I	095 -179 -020	134 -019 -008	-073 -038 258	-018 135 -017	124 054	-086 020	-070 107	241 189	048 057	-207 083
26 SCO-P	012 009 -027	077 212 070	069 165 263	217 255 100	172 244 -095	-114 178	061 028	075 156	168 213	-117 172
27 STEP Reading	144 -020 117	158 -102 119	116 183 247	264 229 111	035 203 187	-082 244 -121	234 301	149 -009	278 145	-141 228

[a]Decimal points and diagonal elements are omitted.

TABLE 27[a].-Unrotated principal-axes factors based on subscores of the Concrete and Stories tests. (N = 100).

	Variables	FACTORS 1	2	3	4	5	h^2
1	CCO-Con-S	570	-557	216	176	-008	712
2	CCO-Con-W	609	-606	195	119	023	790
3	CCO-Con-V	524	-563	182	075	172	660
4	CCO-Cl-Inclusion	395	176	139	-191	024	244
5	CCO-Cl-Multiplication	315	030	052	062	-182	140
6	CCO-D-Modus Ponens	044	169	532	253	-201	418
7	CCO-D-Modus Tollens	302	252	312	427	042	436
8	CCO-D-Undetermined	313	050	020	205	-106	154
9	CCO-D-Possible	565	321	296	106	-044	523
10	CCO-D-Not-Possible	055	110	474	-006	333	351
11	CCO-D-Transitivity	-048	-050	-157	350	639	560
12	CCO-I	292	371	-141	158	432	455
13	CCO-P	258	326	252	-313	034	336
14	SCO-Con-S	771	004	-074	-251	052	665
15	SCO-Con-W	518	-179	-064	-612	096	688
16	SCO-Con-V	604	-108	068	-273	162	482
17	SCO-Cl-Inclusion	377	-119	-108	277	-330	353
18	SCO-Cl-Multiplication	397	182	037	-084	-269	271
19	SCO-D-Modus Ponens	422	016	-511	302	101	540
20	SCO-D-Modus Tollens	423	150	-371	289	-094	431
21	SCO-D-Undetermined	251	337	182	-101	-208	263
22	SCO-D-Possible	455	470	-053	-106	-009	441
23	SCO-D-Not Possible	556	041	-287	-018	012	393
24	SCO-D-Transitivity	425	262	-021	349	-061	375
25	SCO-I	163	-123	-310	-028	-604	503
26	SCO-P	359	181	-316	-207	259	371
	Sums of Squares	4.691	2.048	1.681	1.594	1.543	11.556
	Percent of common variance	40.593	17.723	14.542	13.790	13.352	100.000
	Percent of total variance	18.042	7.877	6.464	6.129	5.934	44.447

[a]Decimal points are omitted.

TABLE 28[a].-Varimax rotation of the principal-axes factors based on subscores of the Concrete and Stories tests. (N = 100.)

	Variables	FACTORS 1	2	3	4	5	h^2
1	CCO-Con-S	021	-819	-053	179	-077	712
2	CCO-Con-W	053	-874	-065	114	-072	790
3	CCO-Con-V	046	-804	-067	032	075	660
4	CCO-Cl-Inclusion	462	-114	-064	108	033	244
5	CCO-Cl-Multiplication	173	-154	-094	214	-179	140
6	CCO-D-Modus Ponens	049	-007	293	573	029	418
7	CCO-D-Modus Tollens	079	-084	-133	612	173	436
8	CCO-D-Undetermined	082	-148	-190	279	-106	154
9	CCO-D-Possible	470	-148	-148	507	045	523
10	CCO-D-Not-Possible	175	-092	210	209	473	351
11	CCO-D-Transitivity	-318	-075	-372	-045	559	560
12	CCO-I	201	087	-492	136	383	455
13	CCO-P	548	063	092	101	111	336
14	SCO-Con-S	603	-417	-346	-013	-087	665
15	SCO-Con-W	605	-394	-063	-398	-065	688
16	SCO-Con-V	495	-449	-161	-070	063	482
17	SCO-Cl-Inclusion	-023	-266	-244	274	-384	353
18	SCO-Cl-Multiplication	382	-061	-101	207	-262	271
19	SCO-D-Modus Ponens	-042	-150	-710	032	-103	540
20	SCO-D-Modus Tollens	059	-055	-582	196	-218	431
21	SCO-D-Undetermined	403	105	030	273	-118	263
22	SCO-D-Possible	535	119	-317	200	-017	441
23	SCO-D-Not Possible	301	-227	-480	000	-146	393
24	SCO-D-Transitivity	141	-056	-382	452	-050	375
25	SCO-I	009	-046	-109	-022	-699	503
26	SCO-P	357	-015	-435	-208	103	371
	Sums of Squares	2.772	2.909	2.360	1.8991	1.616	11.556
	Percent of common variance	23.988	25.170	20.426	16.430	13.986	100.000
	Percent of total variance	10.662	11.187	9.078	7.303	6.216	44.446

[a]Decimal points are omitted.

Factor 6, in Cluster II, accounting for 24 per cent of the common variance obtained high loadings on eight subscores, one or more from each logical category assessed except induction. They were subscores 14, 15, and 16, SCO-Con-S-W-V; subscores 21 and 22, SCO-D-Undetermined and Possible inference; subscore 13, CCO-P; and subscores 4 and 9, CCO-Cl-Inclusion and CCO-D-Possible inference. Factor 6 may be called "General reasoning." It will be referred to as Factor G.

Factor 7, accounting for 25 per cent of the common variance, loads highly on conservation: very high on Concrete conservation of substance, weight and volume and high on Stories conservation (subscores 1, 2, 3, 14, 15, 16). All other subscores obtained low or negligible loadings on this factor. Factor 7 can be called "Recognition of invariance." It will be referred to as Factor IN. The high loadings obtained on Factor IN suggests that the recognition of invariance is of fundamental importance in the development of reasoning.

High loadings on subscores 14, 15, and 16, SCO-Con-S-W-V were obtained on both Factor G and Factor IN. This suggests that the recognition of invariance in reading will consist of elements associated with general reasoning in reading. The elements associated with general reasoning in reading, Factor G, may be related to reasoning from graphic symbols; the elements associated with Factor IN could represent logical operations in conserving in reading.

Factor 8, in Cluster II, accounting for approximately 20 per cent of the common variance obtained high loadings on subscores SCO-D-Modus Ponens, Modus Tollens and Not-possible inference. This is the only factor on which these three Stories subscores in deductive reasoning (items 19, 20, 23) obtained high loadings. High loadings on these three Stories deduction items are associated with high loadings on CCO-D-Transitivity, SCO-P and CCO-I (subscores 11, 26 and 12). Factor 8 can be called "Logical reasoning, Stories." It will be referred to as Factor L.

Factor 9, accounting for approximately 16 per cent of the common variance, obtained high loadings on the Concrete deduction subscores Modus Ponens, Modus Tollens, Possible inference and SCO-Con-W (items 6, 7, 9, and 16), and on the Stories deduction subscore Transitivity (item 24). Factor 9 can be called "Concrete deduction." It will be referred to as Factor CD.

Factor 10, accounting for approximately 14 per cent of the common variance, obtained the highest loading on SCO-I (item 25) and a moderate loading on CCO-I (item 12). It obtained high loadings on two CCO-deductive inference subscores: Not-Possible inference and Transitivity (items 10, 11). There was a moderate loading on SCO-Cl-inclusion (item 17). This item obtained low loadings on other factors. Factor 10 can be called "Inductive-deductive inference." It will be referred to as Factor ID.

It is of interest that deductive inferences involving judgments of the logical compatibility of propositions (Possible and Not-Possible) are differentially associated with Factor CD and Factor ID. The inference Possible is associated with Concrete deductive reasoning: Modus Ponens and Modus Tollens. The inference Not-Possible is associated with induction both Concrete and in reading: SCO-I and CCO-I.

Varimax rotation of the principal-axes factors based on Concrete and Stories subscores has produced a reasonable simple structure solution. Cluster II, consisting of Factors 6 to 10 may be called "Cognitive Operations." (See Fig. 44.)

A factor analysis of the Concrete and Stories subscores jointly with STEP Reading total scores will now be presented.

Factor analysis based on Concrete and Stories subscores and STEP Reading total scores

The correlation matrix for the Concrete and Stories subscores and STEP Reading total scores (27 variables) was presented in Table 26.

The variables significantly related to STEP Reading total scores may be seen in Table 26 to be principally items from the Stories data. They are variable 17, SCO-Cl-Inclusion ($r = .30$ $p < .01$); variable 23, SCO-D-Not-Possible inference ($r = .23$ $p < .01$); variable 20, SCO-Modus Tollens ($r = .25$ $p < .01$); and variables 14, 15, 16, SCO-Con-S-W-V ($r = .23$; $r = .20$; $r = .25$ $p < .05$). One item only of the Concrete data is significantly related to STEP Reading: variable 4, CCO-Cl-Inclusion ($r = .26$ $p < .01$).

Common elements associated with reading comprehension skills would be expected to contribute to these significant correlations between Concrete and Stories subscores and STEP Reading. An attempt will be

TABLE 29[a].-Unrotated principal-axes factors based on Concrete and Stories subscores and STEP Reading totals. (N = 100)

Variables		FACTORS 1	2	3	4	5	h^2
1	CCO-Con-S	563	555	215	072	172	706
2	CCO-Con-W	601	603	212	019	125	786
3	CCO-Con-V	514	559	252	130	119	672
4	CCO-Cl-Inclusion	406	176	124	039	192	249
5	CCO-Cl-Multiplication	306	032	049	064	037	103
6	CCO-D-Modus Ponens	033	171	472	305	206	388
7	CCO-D-Modus Tollens	316	250	226	279	401	452
8	CCO-D-Undetermined	318	049	047	192	165	170
9	CC)-D-Possible	572	321	233	219	077	538
10	CCO-D-Not-Possible	037	115	573	173	071	378
11	CCO-D-Transitivity	052	047	007	474	485	465
12	CCO-I	269	376	020	433	263	471
13	CCO-P	266	326	241	040	308	332
14	SC)-Con-S	764	006	017	164	225	662
15	SCO-Con-W	518	177	006	235	572	682
16	SCO-Con-V	605	106	115	139	235	465
17	SCO-Cl-Inclusion	395	122	228	318	193	361
18	SCO-Cl-Multiplication	381	185	038	054	120	198
19	SCO-D-Modus Ponens	420	017	447	236	333	543
20	SCO-D-Modus Tollens	431	149	394	013	267	434
21	SCO-D-Undetermined	254	336	115	209	148	256
22	SCO-D-Possible	449	471	019	091	095	441
23	SCO-D-Not Possible	559	041	253	134	006	396
24	SCO-D-Transitivity	420	263	031	062	335	362
25	SCO-I	179	127	470	387	161	445
26	SCO-P	332	186	149	481	120	412
27	STEP Reading	421	019	235	448	061	437
Sums of Squares		4.834	2.048	1.688	1.644	1.592	11.806
Per cent of common variance		40.945	17.350	14.298	13.921	13.486	100.000
Per cent of total variance		17.903	7.586	6.252	6.087	5.897	43.724

[a]Decimal points are omitted.

TABLE 30[a].-Varimax rotation of principal-axes factors based on Concrete and Stories subscores and STEP Reading totals. (N = 100).

	Variables	FACTORS 1	2	3	4	5	h^2
1	CCO-Con-S	024	-816	-099	026	169	706
2	CCO-Con-W	050	-874	-091	040	102	786
3	CCO-Con-V	027	-813	-058	078	028	672
4	CCO-Cl-Inclusion	472	-118	-028	053	092	249
5	CCO-Cl-Multiplication	191	-166	-071	096	156	103
6	CCO-D-Modus Ponens	088	-019	193	-247	531	388
7	CCO-D-Modus Tollens	126	-075	-002	113	646	452
8	CCO-D-Undetermined	112	-134	-204	135	281	170
9	CCO-D-Possible	509	-148	-048	128	488	538
10	CCO-D-Not-Possible	155	-119	537	-097	207	378
11	CCO-D-Transitivity	-355	-091	359	447	037	465
12	CCO-I	177	062	308	569	134	471
13	CCO-P	554	054	100	-075	078	332
14	SCO-Con-S	593	-428	-123	328	-064	662
15	SCO-Con-W	568	-402	-072	050	-436	682
16	SCO-Con-V	478	-454	-024	150	-084	465
17	SCO-Cl-Inclusion	025	-249	-455	149	261	361
18	SCO-Cl-Multiplication	400	-078	-083	109	113	198
19	SCO-D-Modus Ponens	-040	-154	-237	678	031	543
20	SCO-D-Modus Tollens	087	-051	-342	522	186	434
21	SCO-D-Undetermined	433	103	-071	-042	227	256
22	SCO-D-Possible	547	100	-002	335	143	441
23	SCO-D-Not Possible	304	-232	-231	443	-026	396
24	SCO-D-Transitivity	174	-064	-083	377	423	362
25	SCO-I	052	-020	-660	-026	-069	445
26	SCO-P	317	-038	126	482	-250	412
27	STEP Reading	243	-150	-560	003	205	437
	Sums of Squares	2.892	2.956	1.869	2.251	1.838	11.805
	Percent of common variance	24.494	25.039	15.835	19.065	15.567	100.000
	Percent of total variance	10.710	10.948	6.924	8.336	6.806	43.724

[a]Decimal points are omitted.

made to uncover common factors underlying these relationships.

The unrotated principal-axes factors based on subscores of the Concrete and Stories tests and STEP Reading total scores are presented in Table 29.

Varimax rotation of the principal-axes factors is presented in Table 30.

Varimax rotation of the principal-axes factors based on Concrete and Stories subscores and STEP Reading total scores produced five factors accounting for 43.7 per cent of the total variance. These are Factors 11 to 15, Cluster III (see Fig. 44).

Factor 11, accounting for 24.5 per cent of the common variance, obtained high loadings on the same items in Cluster III, which included STEP Reading totals, as Factor G, in Cluster II. Factor 11 can be called "General reasoning, reading." It will be referred to as Factor GR.

Factor 12, accounting for 25 per cent of the common variance, obtained high loadings on subscores CCO-Con-S-W-V and SCO-Con-S-W-V. All other subscores obtained low or negligible loadings on Factor 12. Factor 12 can be called "Recognition of invariance, reading." It will be referred to as INR.

Factor 13, accounting for 15.8 per cent of the common variance obtained a high loading on STEP Reading, item 27. Associated with the high loading on STEP Reading, were high loadings on items 25, 17, and 10. These three items, SCO-I, SCO-Cl-Inclusion, CCO-D-Not-Possible inference, obtained high and moderately high loadings on Factor ID Cluster II. Factor 13 may be called "Inferential reasoning in reading." It will be referred to as Factor ICR.

A re-examination of the factors obtained in Cluster I, Reading Comprehension Skills, may suggest some of the elements in inferential reasoning common to STEP Reading and subscores 25, 17 and 10 which obtain high loading on Factor ICR together with variable 27, STEP Reading totals.

It would appear from the analysis of STEP Reading items (see Table 25), that Factor IC could be relevant to the association of variable 25, SCO-Induction, and STEP Reading totals in Factor ICR. A consideration of the nature of the STEP Reading items obtaining high loadings on IC and their possible relation to test items SCO-I would appear to be indicated.

The STEP Reading items with high loadings on Factor IC appear to require the following specific reading skills: interpreting meaning in embedded sentences (for example item 10, Part II); inferring attitude (items 11 and 17, Part II); inferring intention (item 20, Part II). These inferences, which may be identified as specific reading comprehension skills, appear to be available in the text, that is, available from context. They require "receiving" the printed communication "as a whole." This involves understanding relations between separate parts of the communication, whether these parts are included (embedded) in a single sentence, or involve relations between paragraphs.

The inferences assessed by the SCO-I test items also require receiving the printed communication as a whole. But they appear to require processing the communication by means of recognized sequences in inductive reasoning rather than by means of context clues. The inductive reasoning involved appears to be based to a greater extent on hypothesis construction and testing. In SCO-I a problem requiring these inductive procedures is to be recognized by the reader and the inference he proposes is to be verified using statements in the text as "protocol" statements considered to be capable of being subsumed under a law.

There would seem to be no question that the SCO-I test items and the STEP Reading test items are each assessing important inferential skills. That these skills are related would seem to be supported by the association of SCO-I subscores and STEP Reading totals (items 25 and 27), each obtaining high loadings on Factor ICR.

That they may also diverge has been considered in the above discussion. It would appear likely that inferential reasoning assessed as the reading skill of "inferring from context" would contribute to performance in "inductive reasoning" as assessed by the Stories tests; and conversely, that the availability of logical operations of "inductive reasoning" would be likely to contribute to "inferential reasoning from context" as assessed in STEP Reading.

The basis for the association in Factor ICR, of variable 10, CCO-D-Not-Possible inference and STEP Reading total (high loadings on variables 10 and 27), may also be considered. Test items CCO-D-Not-Possible inference require the recognition of a relation of incompatibility between propositions $\sim M(^{p}1/p_2)$. STEP Reading items with high loadings on Factor P, of Reading Comprehension Skills, also require the selection of the proposition which is "not-compatible" with propositions in the text: for example, items 8 and 25, Part I; item 16, Part II. These items are of the form:

> Why does the writer tell us Johnny was NOT willing ....
>
> The writer does NOT mention ....
>
> Jonathan Bing did NOT forget ....

The basis for the association in Factor ICR of variable SCO-Cl-Inclusion and STEP Reading total (high loadings on variables 17 and 27) may be considered. STEP Reading items with high loadings on Factor D in Reading Comprehension Skills, Cluster I, appear to be in a form requiring recall of a subject (a class name) appropriate for a given predicate and the selection of a predicate descriptive of a given subject on the basis of information in the text. These reading skills would appear to be basic to the rather more difficult operations in classification assessed in the Stories tests including recognizing inclusion relations implied by "some" and "all" and constructing alternative predicates descriptive of a subject. Grade four subjects appear to be capable in varying degrees of the operations in classification as assessed in the Stories tests and as assessed as reading comprehension skills in STEP Reading.

It may be seen in Table 30 that variables with high loadings on Factors GR and INR obtain negligible loadings on Factor ICR (Cluster III).

Factor 14, in Cluster III, Cognitive Functioning in Reading, accounting for 19 per cent of the common variance, obtained high loadings on subscores of deductive, probability and inductive reasoning. Variables 19, 20, 23, 26 from the Stories data required inferences based on

Modus Ponens and Modus Tollens, Not-Possible inference, and probability reasoning. Variables 11 and 12 from the Concrete data required logical operations involving induction. Factor 14 may be called "Logical reasoning in reading". It will be referred to as Factor LR.

It may be seen in Table 30 that variables obtaining high loadings on Factor LR, obtain low or negligible loading on Factor ICR.

Factor 15, accounting for 15.6 per cent of the common variance loads highly on the Concrete variables 6, 7, 9: CCO-D-Modus Ponens, Modus Tollens, and Possible inference. It loads moderately on SCO variables 15 and 16, involving deductive reasoning: SCO-Con-W and SCO-D-Transitivity. Factor 5 may be called "Concrete deductive reasoning, reading". It will be referred to as Factor CDR.

It may be seen in Table 30 that variables obtaining high loadings on Factor CDR obtain low or negligible loadings on Factor ICR.

It may also be seen in Table 30 that the four variables with high loadings on Factor ICR: SCO-I; SCO-Cl-inclusion; CCO-D-Not-Possible inference; and STEP Reading totals, obtain negligible loadings on Factors GR, INR, LR and CDR. It would appear that in the simple structure solution obtained in this third analysis, the dimensions of Factor ICR are unique to this factor in Cluster III. Elements with high loadings on Factor ICR appeared to have characteristics in common with Factors IC, P and D in Cluster I, Reading Comprehension Skills.

The five Factors of Cluster I may be considered to be representative of reading comprehension skills generally included in a test of reading comprehension at the Grade four level.

The five Factors of Cluster II may be considered to describe operations in reasoning available in varying degrees at the nine-and-ten-year-old level in concrete and reading situations.

The five Factors of Cluster III may be considered to describe cognitive operations in reading. Factor ICR in Cluster III would appear to describe more specifically aspects of reading comprehension likely to be assessed by a standardized test of reading comprehension. These aspects would seem to have certain elements in common with the tests SCO-I, CCO-D-Not-Possible inference and SCO-CI-Inclusion.

Summary: factor analysis

Three factor analyses were undertaken in an attempt to uncover underlying common factors of the Concrete and Stories tests and STEP Reading. The analyses were based on STEP Reading items; on Concrete and Stories subscores; and on Concrete and Stories subscores jointly with STEP Reading total scores. Varimax rotations of the principal-axes factors were applied to each of the variable sets.

Varimax rotation of the principal-axes factors based on STEP Reading items produced five factors accounting for 29.14 per cent of the total variance (see Table 26). Varimax rotation of the principal-axes factors based on Concrete and Stories subscores produced five factors accounting for 44.5 per cent of the total variance. A similar procedure based on Concrete and Stories subscores and STEP Reading total scores taken jointly yielded five factors accounting for 43.7 per cent of the total variance.

A summary of these analyses is shown in Figure 45.

Summary: Factor Analyses

Cluster I

High Loading Items, STEP	Reading Comprehension Skills Factors		Predicted % Common	Variance % Total
Part I: 21 to 31, 33, 34, 35; II: 30, 31, 35	1 C	Basic reading comprehension skills	8.7	29.9
I: 2, 6, 18, 19; II: 3, 8, 10, 11, 17, 20	2 C	Inferential reasoning from context	5.9	20
I: 4, 7, 8, 11, 14, 16, 25; II: 26, 27	3 P	Recognizing the purposes of the writer	5.1	17
I: 12, 13	4 D	Reading for detail	5.0	17
II: 1, 2, 3, 4, 5, 9, 10, 23, 25	5 A	Critical appraisal	4.5	15

Cluster II

High Loading Variables CCO and SCO	Cognitive Operations Factors			
4, 9, 13, 14, 15, 16, 21, 22	6 G	General reasoning	10.7	24
1, 2, 3, 14, 15, 16	7 IN	Recognition of invariance	11.2	25
11, 12, 19, 20, 23, 26	8 L	Logical reasoning, Stories	9.1	20
6, 7, 9, 15, 24	9 CD	Concrete deduction	7.3	16
10, 11, 12, 17, 25	10 ID	Inductive - deductive inference	6.2	14

Cluster III

High Loading Variables CCO, SCO, STEP Totals	Cognitive Functioning in Reading Factors			
4, 9, 13, 14, 15, 16, 21, 22	11 GR	General reasoning, reading	10.7	25
1, 2, 3, 14, 15, 16	12 INR	Recognition of invariance, reading	10.9	25
10, 17, 25, 27	13 ICR	Inferential reasoning, reading	6.9	16
11, 12, 19, 20, 23, 26	14 LR	Logical reasoning, reading	8.3	19
6, 7, 9, 15, 24	15 CDR	Concrete deductive reasoning, reading	6.8	16

Fig. 45.--Summary: factor analyses, Clusters I, II, III.

Summary: Results

An examination of the mean per cent of correct responses on variables of the Concrete and Stories tests suggested a trend in the data for the higher mean percentage to occur on the Concrete variables. In general it appeared that correct responses were unlikely to be equally available in the two test situations.

Product-moment correlations between the Concrete and Stories variables and sex, and STEP Reading and sex, were presented in Table 13. Three of the Concrete variables were significantly related to sex: CCO-Con and CCO-P (r = .19, p .05) and CCO-I (r = .32, p .001). On Concrete total scores boys also appeared to be significantly in advance of girls (r = .29 p .01). The relations between the Stories variables and sex and between STEP Reading totals and sex were not significantly different from zero (range: r = .07 to r = .14, n.s.).

The results of t-tests for the significance of differences between the mean scores of boys and girls on Concrete and Stories variables and STEP Reading were presented in Table 14. The mean scores of boys were significantly higher on three Concrete variables: CCO-Con, CCO-P and CCO-I (t = 1.95, p. .01), and on total scores, Concrete (t = 2.96 p .01). There was no significant difference between the mean scores of boys and girls on the Stories variables or on STEP Reading (range: 5 = 1.279 to 5 = .237 n.s.).

The possibility was considered that differences of mean scores on tests of Concrete logical operations, associated with no significant differences between mean scores on tests of reading, might imply divergences in intellectual development involving some risk for both girls and boys.

Stepwise multiple regression analysis was applied in three analyses to obtain a clearer indication of the relationship between STEP Reading and the Concrete and Stories tests (see Tables 17, 19, 21). The five subtests of the Concrete tests predicted approximately 13 per cent of the variance of STEP Reading total scores. The five Stories variables predicted approximately 20 per cent of the variance of the criterion. The ten Concrete and Stories variables together predicted approximately 28 per cent of the variance of STEP Reading.

When the Concrete tests were used as predictors of STEP, the variables CCO-D, CCO-I and CCO-Cl predicted approximately 11 per cent of the 13 per cent of the variance of the criterion predictable. When the Stories tests were used as predictors, the variables SCO-Cl, SCO-Con and SCO-P, in that order, predicted the greater part of the variance predictable by the Stories tests. When the ten Concrete and Stories tests were used together as predictors SCO-Cl, SCO-Con, SCO-P, and the variable CCO-Cl predicted over two-thirds of the variance of the criterion predictable by these tests. It appeared that in this situation the variable SCO-P could be functioning as a "suppressor" variable.

The greater amount of the variance of STEP Reading predictable by the Stories variables was considered likely to be accounted for in part by reading skills common to the two tests. Some of the variance predictable by the ten Concrete and Stories variables could be explained by cognitive operations common to the predictors and the criterion; the greater variance of the criterion which was not predictable by the Concrete and Stories variables would then be explained by cognitive operations not common to the tests, to other conditions not common to predictors and the criterion, and to error.

Product-moment correlations between variables of the Concrete and Stories tests were shown in Table 22. Cross-correlations between the variables and relations within each set of variables were discussed.

The strongest relation within the data was between the variables CCO-Con and SCO-Con ($r = .44$, $p \quad .001$). Significant relations occurred between Concrete and Stories variables throughout the matrix, suggesting that some of the logical operations in addition to conservation were available in both reading and concrete situations to subjects at the Grade four level.

Significant relations also occurred within the Concrete data and within the Stories data. The variance predictable within the Concrete data ranged from 8 per cent ($r = .29$) to negligible ($r = .01$). It appeared that children who conserved tended to succeed in classification and deduction; children who conserved on the Stories tests tended also to respond correctly on the Stories tests of probability. This suggested that a process of integration of logical operations, based on conservation, could be considered to be emerging at the Grade four level both in concrete and in reading situations.

Canonical correlation between the two sets of variables was calculated to determine the maximum possible relation between the sets, Concrete and Stories, taken as composites. One significant canonical correlation was obtained ($R_c = .55$ $p < .01$). This result suggested that approximately 30 per cent of the total variance was common to the two sets of tests, Concrete and Stories. The greatest contribution to the prediction of the common variance was shown in the weighting system to be from the variables CCO-Con and SCO-Con. (.808 and .912). The product-moment correlation between the variables CCO-Con and SCO-Con was seen in Table 22 to be $r = .44$ ($p < .001$). CCO-Con and SCO-Con then accounted for approximately 20 per cent of the variance common to the two sets of variables, Concrete and Stories. The remaining 10 per cent of the common variance would be accounted for by classification, deduction, induction and probability together. Of the logical operations assessed, conservation was the most likely to be equally available in the Concrete and reading situations to these nine-to-ten-year-old children.

The variance not common to the two sets of tests, 70 per cent of the total variance, could be accounted for by error, and by factors unique to each set of tests. Error could be due to unreliability of the tests; to the absence of precise comparability between corresponding test items (discussed in Chapters IV to VIII), and to other sources. It was considered that the remaining variance, consisting of elements unique to each test and not accounted for by error, represented operations not equally available in the Concrete and Stories test situations to these Grade four children.

Three factor analyses were undertaken in an attempt to uncover factors common to the following tests: the items of STEP Reading; Concrete and Stories subtests; and Concrete and Stories subtests taken jointly with STEP Reading total scores.

The analysis of STEP Reading items yielded five factors accounting for approximately 29 per cent of the total variance. The factors were considered to identify reading comprehension skills generally regarded as important at the Grade four level. In the analysis of the Concrete and Stories subscores, five factors accounting for approximately 44 per cent of the total variance, were obtained. These factors were considered to identify cognitive operations available to some degree at the Grade four level. The analysis of the Concrete and Stories sub-scores and STEP Reading total scores taken jointly yielded five factors accounting for 44 per cent of the total variance. These factors were considered to represent cognitive functioning in reading. The factors obtained in the three analyses were summarized in Figures 44 and 45.

The variable STEP Reading totals obtained a high loading on Factor ICR, Cluster III, and negligible loadings on other factors in Cluster III representing logical functioning. Stories variables obtaining high loadings on Factor ICR in association with STEP were considered to represent abilities in class inclusion relations, inferring from context, and recognizing relations of incompatibility between propositions. Their association with STEP appeared to be based on complementary rather than identical elements. The relation suggested was between a reading skill of inferring from context and a cognitive skill of inferring a relation holding between propositions and between classes, in which the solution not available in the text.

Chapter XI will present a brief summary of the study and of the results and conclusions. Limitations of the study and some implications of the results will be indicated.

CHAPTER X

SUMMARY CONCLUSIONS AND IMPLICATIONS

This study was an attempt to investigate the characteristics of two sets of relationships: the extent to which logical operations in conservation, classification, deduction, induction and probability reasoning, available to nine-and-ten-year-old children in concrete - verbal situations, were also available in reading; and the extent to which these logical operations are assessed by the Concrete and Stories tests were related to scores on STEP Reading, a standardized test of reading comprehension.

The significance of sex differences between mean scores on the Concrete and Stories tests and on STEP Reading was also of interest.

This chapter will present a brief summary of the study and of the results and conclusions. Some of the limitations in the application of the results will be indicated. The theoretical and educational implications of the results will be considered, and suggestions for further research presented.

Summary of the Study

Experimental tests, Concrete and Stories were constructed for each of the five categories of cognitive operations assessed. The categories were selected on the basis of agreement in the research reviewed that these operations were among those which were important in early intellectual development. The Concrete test situations assessing these operations were adapted from studies by Piaget and Inhelder and their associates. The Stories and the tests based on the stories were constructed by the investigator. They were designed to assess operations as nearly as possible comparable to those assessed by the Concrete tests.

The logical operations required by the test items, the validity of the items and problems related to the comparability of corresponding test items, Concrete and Stories, were discussed. The reliability of the tests and of the scoring, and the readability levels of the stories were also considered.

The tests, Concrete and Stories and a standardized test of reading comprehension, STEP Reading Form 4B, were administered to a sample of 100 children, 50 boys and 50 girls. Subjects were selected on a random basis from the children attending regular Grade four classes in the City of Moose Jaw, Saskatchewan.

The statistical procedures applied in the analysis of the data from the Concrete and Stories tests and STEP Reading included product-moment correlations with sex; t-tests for the significance of the differences between the mean scores of boys and girls; product-moment correlations between the variables; and canonical correlation between the two sets of variables, Concrete and Stories, taken as composites. Stepwise multiple regression analysis was used to obtain a clearer indication of the relation between the Concrete and Stories tests and STEP Reading. Factor analysis was undertaken in an attempt to uncover underlying simple structures of the experimental tests and of STEP Reading.

The results of the analyses of the data from the three sets of tests, Concrete, Stories and STEP Reading and the conclusions based on the results will now be summarized.

Results and Conclusions of the Investigation

The results of the investigation will be summarized in relation to the hypotheses tested.

Hypothesis I

> There will be no significant relation between scores on the following tests and sex (alpha >.05):
>
> Concrete tests
> Stories tests
> STEP Reading total scores.

The correlations between each of the five Stories variables and sex were of low order (range: r = .14 to r = .04, n.s.). The relations between total scores on STEP Reading and sex were also of low order (r = -.09 n.s.).

Three of the Concrete variables were significantly related to sex: CCO-Con (r = .18 p <.05); CCO-P (r = .18 p <.05) and CCO-1 (r = .32 - <.001). Total scores Concrete were significantly related to sex (r = .29 p <.01).

An examination of the relations between subscores of the Concrete tests of conservation and sex indicated that boys in the study were significantly in advance of girls in conserving substance and weight, and defending these decisions, but were unlikely to be in advance of girls in conserving volume. Conservation of substance was significantly related to sex (r = .19 p <.05); conservation of weight was also significantly related to sex (r = .25 p <.01). The relation between CCO-Con-V and sex was of low order (r = .08, n.s.).

Hypothesis I was accepted for STEP Reading and for the Stories tests. It was rejected for total scores Concrete and for Concrete subtests, with the exception of classification and deduction ($r = .09$ n.s., $r = .12$ n.s.).

It was concluded that boys at the Grade four level in this study were likely to be significantly in advance of girls in reasoning in concrete situations involving conservation, judgments of probability, and inductive inference. They were unlikely to be in advance of girls in reasoning in reading as assessed by the Stories tests, or in reading comprehension as measured by a standardized test of reading comprehension.

Hypothesis 2

> There will be no significant difference between the mean scores of boys and girls at the Grade four level on the following tests (alpha >.05):
>
> Concrete tests
> Stories tests
> STEP Reading total scores.

The mean scores of boys were significantly higher than the mean scores of girls on CCO-Con ($t = 1.95$ $p < .05$); on CCO-P ($t = 1.95$ $p < .05$); on CCO-1 ($t = 3.29$ $p < .001$); and on total scores, CCO ($t = 2.96$ $p < .01$). There was no significant difference between the mean scores of boys and girls on the Stories variables (range: $t = 1.37$ to $t = .41$, n.s.); on total scores SCO ($t = 1.28$, n.s.) or on STEP Reading total scores ($t = .84$, n.s.).

Hypothesis 2 was accepted for the Stories variables and for STEP Reading. It was rejected for the Concrete variables with the exception of the variables classification ($t = .85$ n.s.), and deduction ($t = 1.23$ n.s.).

It was concluded that boys were likely to be in advance of girls in concrete reasoning in operations involving conservation, induction and probability reasoning. Boys were unlikely to obtain higher scores on tests assessing these operations when comparable data was presented in printed form for reading, or on a standardized test of reading comprehension.

Sex differences in reasoning at the Concrete level, not represented in reading comprehension as measured by STEP Reading, and reasoning in reading as measured by the Stories tests, were considered to involve some risk in the acquisition of intellectual skills for both boys and girls during the concrete period of development (see Chap. X). The finding of no sex differences in reading comprehension could obscure the possibility that boys were progressing in advance of girls in some of the fundamental logical operations at the concrete level but with some delay in the use of these principles in reading. It could also obscure the possibility that girls could be operating on the level of language without an adequate understanding of basic logical principles. These discrepancies could be significant for the subsequent intellectual development and for the school achievement of both boys and girls.

Hypothesis 3

> Subtests of the Concrete tests and subtests of the Stories tests ranged in order as predictors of the criterion STEP Reading will indicate that subtests of each of the predictor tests will be selected in the same order as predictors of STEP and that each set of predictors will be approximately equally effective in predicting the variance of the criterion.

Three stepwise multiple regression analyses were undertaken to obtain a clearer indication of the relationship of STEP Reading to the Concrete tests, the Stories tests, and the ten Concrete and Stories variables taken together.

The five variables of the Concrete tests predicted approximately 13 per cent of the variance of STEP Reading. Three of the five variables, CCO-D, CCO-1 and CCO-Cl, ranged in that order, predicted 10.6 per cent of the variance of the criterion predictable by the Concrete tests. CCO-D predicted 5 per cent of the total variance predictable by the Concrete tests. It appeared possible that in this situation CCO-1 was functioning as a "suppressor" variable.

The five variables of the Stories tests predicted approximately 20 per cent of the variance of the criterion, STEP Reading. Three variables, SCO-Cl, SCO-Con and SCO-P, ranged in that order, predicted 16 per cent of the variance predictable by the Stories tests. SCO-Cl predicted approximately 8 per cent of the variance predictable by the Stories tests.

The ten variables of the Concrete and Stories tests taken together predicted approximately 28 per cent of the variance of STEP Reading total scores. Approximately 19 per cent of the variance predictable was accounted for by four tests: SCO-Cl, SCO-Con, SCO-P and CCO-Cl.

Hypothesis 3 was rejected. The order of selection of the predictors of the criterion differed for the Concrete tests and for the Stories tests. In addition, when the ten subtests were ranged in order as predictors, Stories subtests were selected at steps 1, 2 and 3. The amount of the variance of STEP Reading predictable by the Concrete and

Stories tests also differed. The Concrete tests predicted 13 per cent of the variance of the criterion. The Stories tests predicted a greater amount of the variance, 20 per cent. The amounts of the variance predicted were not large: when the two tests were taken together as predictors, it was less than 30 per cent of the total variance.

It was concluded that the cognitive operations assessed by the Concrete tests and by the Stories tests could differ considerably from those assessed by STEP Reading. Some of the variance of STEP Reading predictable by the Stories tests would be expected to be related to reading comprehension skills common to the two tests. Some of the variance predictable by the Stories tests would be expected to be associated with cognitive operations common to the predictors and the criterion.

The greater variance of STEP Reading not predictable by the Stories and Concrete tests taken together would suggest that predicators and criterion were assessing essentially different cognitive operations and that other differences between the tests could also explain some of the variance of STEP Reading not predictable by the ten Concrete and Stories tests.

Hypothesis 4

> Relations between scores on the Concrete tests and scores on the corresponding Stories tests assessing the following operations will be significantly different from zero (alpha > .05):
>
> Conservation
> Classification
> Deduction
> Induction
> Probability

Two of the five product-moment correlations between corresponding Concrete and Stories variables were significant: SCO-Con and CCO-Con ($r = .44$ $p < .001$); and SCO-D and CCO-D ($r = .30$ $p < .01$). Relations between SCO-Cl and CCO-Cl; between SCO-P and CCO-P; and between SCO-1 and CCO-1 were negligible (range $r = .17$ to $r = -.02$ n.s.).

Hypothesis 4 was accepted for the relations between corresponding Concrete and Stories tests of conservation and of deduction. It was rejected for the relations between corresponding tests of classification, induction and probability.

It was concluded that problems requiring identical logical operations could nevertheless be different in difficulty for Grade four children when the data were presented in a concrete stimulus situation and in printed form to be read. Some of these nine-and-ten-year-old children were likely to conserve in both situations; a smaller number would recognize comparability and apply operations of deduction in the two situations; very few would succeed equally well in both situations in recognizing comparability and in applying operations of classification, of probability, and of inductive reasoning.

Hypothesis 5

> Relations between scores on the Concrete tests assessing each of the following operations will be significantly related to scores on the Stories tests assessing each of the other operations (alpha > .05):
>
> Conservation
> Classification
> Deduction
> Induction
> Probability

Significant relations involving variables other than corresponding Concrete and Stories variables were distributed throughout the cross-correlation matrix (range: $r = .26$ to $r = .19$ $p < .05$). Four of these relations were highly significant ($p < .01$): the relations between CCO-D and SCO-Con; CCO-Con and SCO-Cl; CCO-Cl and SCO-P; and CCO-1 and SCO-D. Six others were significant ($p < .05$). Ten of the twenty correlations were not significant (range: $r = .15$ to $r = -.01$, n.s.).

Hypothesis 5 was accepted for the cross correlations which were found to be significant ($p < .05$). It was rejected for the equal number of relations found to be below this level of probability.

It was concluded that to some degree different logical operations across concrete and reading situations were likely to be related and interdependent in the cognitive development of nine-to-ten-year-old children. The extent of this interdependence could not, however, be determined on the basis of relations observed between operations considered two at a time.

Hypothesis 6

> Relations between scores within the set of Concrete tests and between scores within the set of Stories tests will be significant (alpha > .05).

Five of the ten correlations within the matrix of correlations for each set of tests, Concrete and Stories, were significant (range: $r = .39$ $p < .001$ to $r = .19$ $p < .05$). The range of variance predictable within the Concrete data was from approximately 8 per cent to negligible; within the Stories data the range was from 15 per cent to negligible. The strongest relation in the data was between the variables SCO-Con and SCO-D ($r = .39$ $p < .001$).

Hypothesis 6 was accepted for the following significant relations occurring within the Concrete data: CCO-Con with CCO-Cl, CCO-D and CCO-1; CCO-Cl with CCO-D; and CCO-D and CCO-1. It was also accepted for the following significant relations occurring within the Stories data: SCO-Con with SCO-Cl, SCO-D and SCO-P; and SCO-D with SCO-Cl and SCO-P. It was rejected for the remaining five relations within each set which were not significant ($p < .05$).

It was concluded that a process of integration of the logical operations assessed, probably based on operations of conservation, could be considered under way within each situation, Concrete and Stories. In each situation children who conserved were likely to succeed in classification and deduction. In reading these children were also likely to succeed in probability reasoning. In each situation however, some operations appeared to be excluded from a possible trend toward integration: probability reasoning in concrete situations and induction in reading.

Hypothesis 7

> Canonical correlation between the set of Concrete tests and the set of Stories tests taken as composites will indicate that the two sets of tests are significantly related (alpha > .01).

One significant canonical correlation ($p < .01$) was obtained between the two sets of variables, Concrete and Stories, taken as composites. The canonical correlation obtained was $R_c = .55$.

Hypothesis 7 was accepted. The canonical coefficient of correlation ($R_c = .55$), representing the maximum possible relation between the composites indicated that approximately 30 per cent of the total variance was common to the two sets of tests; 70 per cent of the total variance would then be accounted for, in part by error, and in part by the presence of elements unique to each test situation.

Error would be due to unreliability of the tests, differences in comparability of corresponding test items, and to other sources. The test-retest reliability coefficient obtained for the set of Concrete tests was $r = .74$ ($p < .01$); for the set of Stories tests the reliability coefficient was $r = .72$ ($p < .01$). Difficulties in assessing reliability were discussed Chapter III. Exact comparability of corresponding test items, Concrete and Stories, was not completely achieved. Differences which could affect comparability were discussed in Chapters IV to VIII.

The variance not common to the two sets of tests taken as composites, which was not due to error, could be considered to represent differences in the availability of comparable logical operations in concrete and in reading situations at the Grade four level. The amount of the variance not common to the two composites could be expected to be fairly large, at least as great as the variance common to the two sets of tests.

It was concluded that a developmental process was under way at the Grade four level, which was resulting in operations in reasoning, available in either concrete or in reading situations, becoming available in both situations. This process, however, was in an early phase of its evolution. It could not be generally assumed that an operation available in a concrete situation, or in reading, would be likely to be available in the alternative situation.

Hypothesis 8

> A weighting system which maximizes the relation between the two sets of tests taken as composites will suggest that the following categories of logical operations contribute approximately equal amounts to the prediction of the common variance of the Concrete and Stories tests:
>
> Conservation
> Classification
> Deduction
> Induction
> Probability

In obtaining the maximum possible correlation between the composites, the highest weights were assigned to a single cognitive category, Concrete conservation (.808) and Stories conservation (.912). The product-moment correlation between these variables was $r = .44$ ($p < .001$). This would indicate that approximately 20 per cent of the variance common to the two composites was accounted for by conservation alone. The variance common to the two sets was 30 per cent of the total variance. This would leave 10 per cent of the common variance to be accounted for by classification, deduction, induction and probability together.

Hypothesis 8 was rejected. The contributions of the various categories of logical operations to maximizing the relation between the sets was markedly unequal.

It was concluded that the extent to which these Grade four children were able to recognize comparability in concrete and reading situations and apply identical operations of classification, deduction, probability reasoning and induction, was minimal. Only situations involving conservation could be expected to be recognized as comparable for concrete data, and data presented in printed form for reading.

The developmental process noted above as under way at the Grade four level, by which operations in reasoning, available in either concrete or in reading situations, were becoming available in both situations, could be considered to be restricted mainly to conserving in the two situations. The amount of the variance common to concrete and reading situations, which would be accounted for by operations of conservation, could be as much as twice the amount of the common variance accounted for by logical operations of classification, deduction, induction and probability reasoning together.

Hypothesis 9

> Factor patterns identified for the following tests will indicate that simple structures underlying these tests will be similar:
>
> STEP Reading test items
> Concrete and Stories subtests
> Concrete and Stories subtests and STEP Reading total scores taken jointly.

Three analyses were undertaken in an attempt to uncover factors common to the Concrete and Stories tests and STEP Reading. The three analyses yielded three major groupings or clusters of factors. Cluster I, Reading Comprehension skills, was based on an analysis of

STEP Reading items. The five factors obtained, accounting for approximately 29 per cent of the total variance, were considered to represent Basic reading comprehension skills; inferential reasoning from context; Recognizing the purposes of the writer; and Critical appraisal. The factors obtained would appear to describe the skills generally recognized as the dimensions of reading comprehension (see Chap. II). These dimensions were given as Literal and Inferential Comprehension; Reorganization; Evaluation; and Appreciation. Literal Comprehension referred to the basic comprehension skills of recognizing and recalling information (details, main idea etc.) explicitly stated by the author. Inferential reasoning from context required inferring details, main ideas, the motivations of characters which the author might have included, but did not specifically mention. Evaluation and Appreciation appeared to include operations represented in each of the Factors, Critical appraisal and Recognizing the purposes of the author. For many selections in reading, however, it would seem that the dimensions Evaluation and Appreciation could go beyond the operations in STEP Reading represented by the two factors, Recognizing purposes and Critical appraisal (see Chap. II).

Cluster II, Cognitive Operations, was based on an anlysis of Concrete and Stories subscores. The five factors obtained, accounting for approximately 45 per cent of the total variance, were considered to describe operations in reasoning available in varying degrees to nine-and-ten-year-old children.

Cluster III, Cognitive functioning in Reading, was based on an analysis of Concrete and Stories subscores and STEP Reading total scores taken jointly. The factors obtained, accounting for approximately 45 per cent of the total variance, were considered to describe operations of reasoning in reading.

In Cluster III, STEP Reading obtained a high loading on Factor ICR, Inferential reasoning in reading. Three items SCO-1, SCO-Cl - Inclusion, and CCO-D - Not-Possible inference obtained high loadings on Factor ICR in association with STEP Reading. The same three items obtained high loadings on Factor ID, Cluster II, Inductive - Deductive inference.

The question of the association of the three items and STEP Reading with high loadings on Factor ICR was important from the point of view of hypothesis 9, which stated that factor patterns obtained from the three analyses would be similar, and, by implication, that in each analysis test items could be considered to assess comparable cognitive operations.

High loadings on the three items SCO-I, SCO-Cl - Inclusion, and CCO-D - Not-Possible inference in association with STEP on Factor ICR, Cluster III, appeared to be reasonable on the basis of an examination of items obtaining high loadings on Factors IC, P, and D of Cluster I. STEP items obtaining high loadings on Factor P required the selection of a statement "not - compatible" with a statement in the text (The writer does NOT mention....); STEP items obtaining high loadings on Factor D required the recall of a class name and the recognition of an appropriate predicate of a class name given in the text. These operations involved skills which could be expected to contribute to logical operations of inductive and deductive reasoning and classification in reading, but to a limited extent only. The association of items SCO-I, SCO-Cl - Inclusion, and CCO-D - Not-Possible with STEP involved inferential reasoning from context; the association of these items with Factor IC, Cluster II, involved more extensive logical operations in which inferences were derived by rules of logic. No other items obtained high loadings on Factor ICR in association with STEP.

Hypothesis 6 was rejected. Factor patterns obtained in the analysis of STEP Reading items (Cluster I), were considered to differ from the factor patterns obtained from the analysis of the Concrete and Stories items (Cluster II), and from the analysis of Concrete and Stories items and STEP Reading totals taken jointly (Cluster III).

It was concluded that STEP Reading and the Concrete and Stories tests were assessing essentially different cognitive skills. The skills assessed in each case were important. Both reading comprehension skills assessed by STEP Reading and logical operations assessed by the Concrete and Stories tests could be accepted as feasible and appropriate objectives in the education of elementary school children. The simple structure solutions obtained for Cluster I, "Reading comprehension skills," and for Cluster III, "Cognitive functioning in reading." suggested that current procedures in assessing cognitive operations in reading focused on comprehension skills requiring inferring from context to the relative exclusion of reasoning skills requiring basic logical operations.

Implications of the Study

The problem of constructing a theory of reading which would include both the perceptual aspects and the cognitive operations in reading has been a concern in the reading field for some time. The results of this study suggest the possibility of developing a theory of reading which can account for its unique function in the transformation of intelligence during the elementary school years.

In early childhood, children learn to think on the basis of "sound-symbols": sound-symbols learned in the context of concrete experiences. When these children begin to decode printed symbols, they "decode into sound"; in reading for meaning, they recall the concrete experiences which they have learned to associate with these sounds. They are decoding into a medium which they already know quite well how to manage: sound and sense experience. They are achieving "functional" literacy: the ability to receive the message on the printed page. Printed symbols intervene in a primarily verbal-concrete operation.

Reasoning from printed symbols appears to be different in difficulty from reasoning on the basis of verbal - concrete data. The full scope of this problem has not been explored in this study, but it would seem that inherent in the reading situation is the possibility of the discovery that basic operations in thinking hold across a wide range of content; that operations involving class inclusion, implication, probability, conservation are isomorphoric across content. The bridge would appear to be the printed symbol: not the letters only, but quantifying expressions, connectives, propositions, as abstract symbols.

On the basis of this early transformation of intelligence, children become capable of reasoning from symbols of many kinds: mathematical and logical symbols; the formulae of physics and chemistry; and for modern children, perhaps the symbols of the unambiguous language of the computer.

But the elementary school child begins with symbols which represent speech sounds. If, in reading, he is required only to translate printed symbols into sound which convey the message of the author, the full development of his intellectual potential may not be realized.

There may also be an optimum time during which the transformation of intelligence may be initiated. It appears to be already under way during the elementary school years as children begin to develop structures in thinking capable of achieving certainty, and other structures appropriate for investigation and discovery and for adapting thinking to emerging notions of probability. The nine-to-ten-year-old children in this study accepted that intellectual operations could be substituted for perceptual evidence in arriving at knowledge of a state-of-affairs. They did not question that one could reason from contrary-to-fact premises. It appears that operations at these levels may not occur until much later in non-literate societies, as Bruner and others have observed. It would seem to be a general function of reading to prepare the intellectual transformations which tend to ensure that the potential for abstract reasoning at early adolescence becomes a reality. A theory of reading should, it would seem, account for and describe this function.

In the absence of an adequate theory of reading it becomes necessary to raise the question, what does the young child stand to gain by learning to read? Children in this study could be considered to be close to or above an expected level of competency in reading: they decoded the printed materials presented to them; they identified the main idea, recalled details; related in sequence ideas presented in the text; inferred from context; distinguished between what was said and what was not in fact said; and considered the style, the purpose, and the accuracy of the information presented. These and similar skills they applied in the standardized test of reading comprehension in understanding character, motivation, attitudes, social situations. In the Stories tests they also applied these skills in an attempt to understand and cope with the natural environment and to appreciate difficulties of people of other times and places in coping with their environments. In addition, they appeared to enjoy and appreciate what they read. These are important goals of instruction; they follow from current thinking concerning the function of reading in education.

But most of these skills, with the exception of decoding, children could acquire, it would seem, by living and talking with mature adults and other children, and in participating in social processes during the first ten years of life. They are, in the main, achievements in socialization. Is reading intended, during the elementary school years, to supplement, perhaps substitute for other ways of socializing children? Although achievement in reading has not been evaluated directly in terms of social competence, it is reasonable to expect that reading should contribute to developing social awareness.

The critical question, however, remains: has reading a function other than contributing to the socialization of elementary school children, and as part of this process, ensuring that they are able to receive, in full, communications of many kinds in printed form? Is reading a "tool" subject or does it have content in its own right?

This study appears to suggest that the unique function of reading is the development of abstract intelligence and that this process may be most effectively studied during the period in which an intelligence bound to verbal-concrete reasoning is beginning to be transformed into an intelligence capable of operating on the basis of form. The forms acquired appear to include relations of inclusion, exclusion, complementarity, disjunction, multiplication, which hold between classes; form as implication and compatibility, and the condition, undetermined, which hold between propositions; form as principles governing invariance and probability; form as procedures in induction. The forms are invariant; it is the content of the classes, the propositions, the materials to be conserved, which change. At a critical period in intellectual development reading provides the prime opportunity for allowing the child to discover that operations in reasoning are independent of concrete content, and independent of ideational content. Society does not provide a second chance if this one has not been utilized. It does not appear to have as yet invented an alternative to printed symbols in reading for initiating the transition to abstract logical intelligence.

A theory of reading should also account for the difficulty experienced by many children in learning to read. Reading disability can be readily shown by adequate diagnostic procedures to be multiply determined. The skill of a reading clinician is presently demonstrated in considering the significance of these contributing factors and in implementing effective instructional procedures for the condition as evaluated. Many children who present reading disabilities are able children in other respects than in reading. For others, physical, perceptual, motor-coordination, associative memory, social, emotional and instructional difficulties and deficiencies may be found; some of these also occur in children who learn to read. The implication of this study would appear to be that for the children who fail to read, a printed communication, from the letters to the propositions, does not "mean" something which can be manipulated in thinking by rules known to apply in the concrete-verbal world. The severely disabled reader of average intelligence could be considered to have a near-total disability in operating with data other than that which is presented at a verbal-concrete level. Certainly the symptoms identified in diagnosis contribute to the problem. But treatment directed toward eliminating or compensating for contributing factors (considered to be perceptual, associative, emotional, etc.) could leave a main problem relatively unaltered. Reading disability treated in this way could teach children to "read" but leave them unable to learn by reading throughout their school careers.

A unifying concept in this complex problem could be the notion of a continuum in difficulty in reasoning in the absence of concrete-verbal stimuli. Near-total disability would occur at the lower end of the continuum; the non-academic, underachieving, able child would be functioning below the median in this respect.

It is important to test for what one is attempting to achieve in instruction. A theory of reading which can account for its unique contribution to intellectual development, and for the problems of children who experience great difficulty in effecting the transformation of early concrete intelligence, should also indicate directions in assessing sequences in achieving the transition to logical intelligence.

The results of this study appear to suggest that an intricate sequence of earlier integrations of operations precede the emergence of new forms of reasoning in reading. It will be desirable to determine some of these interdependencies and to find ways of describing the position of a particular child on this continuum. It seems unlikely that tests which assess achievement in reading comprehension will be considered adequate to describe childrens' progress in logical operations in reading.

A theory of reading which includes the transformation of concrete-verbal intelligence will need to be translated into specific objectives in an appropriate sequence with a curriculum designed to achieve these objectives. Materials and instructional techniques will need to be prepared and their effectiveness assessed in relation to the objectives decided on. This will require the coordinated efforts of theoreticians, investigators, departments of education, principals, teachers, and technicians. Coordination of this kind requires administrative direction of a high order.

The intellectual capacity of a generation of children in our own society, and of the children of societies attempting to cope with acculturation should not, it would seem, be entrusted to a multiplicity of smaller working units of lesser dimensions and more restricted goals.

Limitations of the Study

The applications of the results of this study are limited in respect to the population to which the present findings may be generalized and to a number of conditions associated with the exploratory nature of the study.

The population from which the sample was drawn consisted of Grade four children living in the relatively unpressured, cheerful atmosphere of a small urban center in western Canada, under the wide skies and close to the open spaces of the prairie. It is not known if children maturing in other environments learn to reason in reading at similar rates or in similar ways. For the children in this study, operations in conserving were the most readily available for both concrete data and in reading. For children in other environments this may not be the case.

The investigation as conducted also imposed a number of limitations on the application of the results. The forms of logical reasoning examined were not exhaustive, and for those which were studied, the range of the operations examined was necessarily restricted. Comparability between corresponding Concrete and Stories test items was not completely achieved in all cases. The reliability of the tests was not high (CCO: $r = .74$; SCO: $r = .72$ $p < .01$). A proper assessment of reliability was difficult for two reasons: the unexpected interest of the children in the tests and in being "chosen" to participate in the experiment seemed to result in learning from discussions between test and retest; and the condition that repeated demonstrations and instruction be given if a subject failed on those test items on which the possibility of correct responses on succeeding items depended usually resulted in a correct initial response on retest.

Readability levels for the stories appeared to be satisfactory for subjects in the study, but for the purpose for which the stories were used it would have been preferable if their length had been equal and if more than one story situation could have been provided for each of the categories of operations assessed.

The study was in the nature of an exploratory survey of a possible problem in the teaching of reading. Completeness in examining particular aspects of this problem was restricted in the interests of obtaining an estimate of its dimensions and an indication of trends and relationships within the problem.

Decisions in implementing the results of an exploratory study of this kind must be based on careful detailed research by many investigators.

Suggestions for Further Research

Three general directions for further research may be presented for consideration.

The first involves replication studies of reasoning in reading based on children who are maturing under many different social conditions. It would be of value to assess the validity of the results reported in the present study over language and other cultural differences and for children in other physical and economic situations.

The second direction for research which may be suggested concerns developmental sequences in the acquisition of reasoning in reading. This would involve cross-sectional studies. These studies could provide information on interrelated and interacting factors affecting the development of reasoning in concrete-verbal and in reading situations.

A third direction involves the design and testing of instructional techniques in reading comprehension. In particular, it would seem the need is for the development of new forms of questioning by both children and the instructor in which the responses required are logical operations. Questions testing literal and inferential comprehension, evaluation and appreciation tend to require as responses statements available in the text or hypothesized statements. These are equivalent only to premises in a logical argument. It may be necessary in formulating questions requiring logical thinking to experiment with orienting teaching procedures not so directly to obtaining correct answers, but rather to understanding the processes in reasoning demonstrated by the child as he presents and defends both his correct and incorrect responses.

Concluding Statement

This has been an exploratory study. It has considered the patterns of children's responses as they reason in the presence of what they see, and touch, and hear, and manipulate, and these patterns as they occur in modified form when the concrete experiences have been translated into printed symbols which are first to be decoded.

Teachers of reading know that children must clearly understand what has been presented to them in printed form. They have thoroughly examined what children must learn to do in order to receive in full a printed communication. Teachers have also known the importance to children of the experiences and attitudes they bring to the printed page. What appears to emerge from this study is that both of these conditions may be fully realized and well prepared for and there still remains a problem: children in learning to read are unlikely to process the information from the printed page by the logical operations which are becoming available in concrete life situations.

This is a problem, but it appears also to be an opportunity of prime importance. Reasoning from printed symbols children begin to construct an intelligence capable of operating on the basis of form independent of content. Yet this too should be added: it will be necessary at the same time to ensure that children remain capable of returning to the concrete with a vivid appreciation of the colour, the subtlety and the delight of the unique. An intelligence at these levels of sophistication will permit the child to participate in modern complex society which others of similar intelligence have created.

B I B L I O G R A P H Y

BIBLIOGRAPHY

Ausubel, David P. "Neobehaviorism and Piaget's Views on Thought and Symbolic Functioning." Child Development, XXXVI, No. 4(1965), 1029 - 32.

________. Learning Theory and Classroom Practice. Bulletin No. 1, The Ontario Institute for Studies in Education, Toronto: The Ontario Institute for Studies in Education, 1967.

Beth, E.W.; et al. Implication, formalisation et logique naturelle. Vol. XVI of Études d'épistémologie génétique. Edited by Jean Piaget. Paris: Presses Universitaires de France, 1962.

Bocheński, J.M. The Methods of Contemporary Thought. Dordrecht-Holland: D. Reidel Publishing Company, 1965.

Bright, Jane; and Bright, William. "Semantic Structures in North-western California and the Sapir-Whorf Hypothesis." American Anthropologist, Special Publication, LXVII (October, 1965), 249 - 58.

Bruner, Jerome S.; et al. Studies in Cognitive Growth. New York: John Wiley and Sons, 1966.

Buros, O.K., ed. The Fifth Mental Measurements Yearbook. Highland Press, New Jersey: The Gryphon Press.

Carnap, Rudolf. Meaning and Necessity: A Study in Semantic and Modal Logic. Chicago: University of Chicago Press, 1956.

________. The Logical Syntax of Language. London: Routledge and Kegan Paul, 1964.

Cattell, Raymond B., ed. Handbook of Multivariate Experimental Psychology. Chicago: Rand McNally and Company, 1966.

Chall, Jeanne. Learning to Read: The Great Debate. New York: McGraw-Hill Book Company, 1967.

Chomsky, Noam. Aspects of the Theory of Syntax. Cambridge, Massachusetts: The M.I.T. Press, 1965.

Clymer, Theordore. "What is 'Reading'?: Some Current Concepts", in Innovation and Change in Reading Instruction. The Sixty-seventh Yearbook of the National Society for the Study of Education, Part II. Edited by Helen M. Robinson. Chicago: University of Chicago Press, 1968, pp. 7 - 29.

Cooley, William W.; and Lohnes, Paul R. Multivariate Procedures for the Behavioral Sciences. New York: John Wiley and Sons, 1962.

Cooperative Sequential Tests of Educational Progress. Princeton, New Jersey: Cooperative Test Division, Educational Testing Service, 1957.

Dale, Edgar; and Chall, Jeanne. "Formula for Predicting Readability: Instructions." Educational Research Bulletin, XXVII (February, 1948), 37 - 54.

De Hirsch, Katrina; et al. Predicting Reading Failure. New York: Harper and Row, 1966.

Du Bois, Philip H. An Introduction to Psychological Statistics. New York: Harper and Row, 1965.

Educational Policies Commission. The Central Purpose of American Education. Washington, D.C. National Education Association, 1961.

Efroymson, M.A. "Multiple Regression Analysis", in Mathematical Methods for Digital Computers. Edited by A. Ralston and H.S. Wilf. New York: John Wiley and Sons, 1960.

Elkind, David. "Children's Discovery of the Conservation of Mass, Weight and Volume. Piaget Replication Study II." Journal of Genetic Psychology, XCVIII (1961), 219 - 27.

________; and Schneider, Gerrie. "Modified Word Recognition, Reading Achievement and Perceptual De-centration." The Journal of Genetic Psychology, CVI (1965), 235 - 51.

________. "Piaget's Conservation Problems". Child Development, XXXVIII, No. 1 (1967), 15 - 27.

Fisher, R.A.; and Yates, F. Statistical Tables for Biological, Agricultural and Medical Research. London: Oliver and Boyd, 1938.

Flavell, John. The Developmental Psychology of Jean Piaget. Princeton, New Jersey: D. Van Nostrand Company, 1963.

Goustard, Michel; et al. La logique des apprentissages. Vol. X of Études d'épistémologie génétique. Edited by Jean Piaget. Paris: Presses Universitaires de France, 1959.

Gréco, Pierre. "Les relations entre la perception et l'intelligence dans le développement de l'enfant: le facteur d'équilibre dans les conservation opératories et les constances perceptives." Bulletin Psychologie, X (1956-57).

Gréco, Pierre: et Piaget, Jean. Apprentissages et connaisance. Vol. VII of Études d'épistémologie génétique. Edited by Jean Piaget. Paris: Presses Universitaires de France, 1959.

Guilford, J.P. "Frontiers in Thinking That Teachers Should Know About." The Reading Teacher, XIII, No. 3 (February, 1960), 176 - 82.

Guszak, Frank L. "Teachers Questions and Levels of Reading Comprehension", in Perspectives in Reading No. 8. The Evaluation of Children's Reading Achievement. Edited by Thomas C. Barrett. Newark, Delaware: International Reading Association, 1967, pp. 97 - 109.

Horst, Paul. Psychological Measurement and Prediction. Belmont, California: Wadsworth Publishing Company, 1966.

Hospers, John. "What is Explanation?" Essays in Conceptual Analysis. Edited by Antony Flew. London: Macmillan and Company, 1963.

Huack, Barbara B. "A Comparison of Gains in Evaluation Ability Between Gifted and Nongifted Sixth Grade Students." Gifted Children's Quarterly, XI (Autumn, 1967), 166 - 71.

Hunt, Earl B.; et al. Experiments in Induction. New York: Academic Press, 1966.

Inhelder, Bärbel; and Piaget, Jean. The Early Growth of Logic in the Child: Classification and Seriation. Translated by E.A. Lunzer and D. Papert. London: Routledge and Kegan Paul, 1964.

Kaplan, Abraham. The Conduct of Inquiry. San Francisco: Chandler Publishing Company, 1964.

Klare, George F. "A Table for Rapid Determination of Dale-Chall Readability Scores." Educational Research Bulletin (February, 1952), 43 - 47.

Kneale, William; and Kneale, Martha. The Development of Logic. Oxford: Clarendon Press, 1962.

Langer, Suzanne K. An Introduction to Symbolic Logic. New York: Dover Publications, 1953.

Laurendeau, Monique; and Pinard, Adrien. Causal Thinking in the Child: A Genetic and Experimental Approach. Montreal, Canada: Institute of Psychological Research, 1962.

Lévi-Strauss, Claude. La pensée sauvage. Paris: Plon, 1962.

Lovell, K.; and Ogilvie, E. "A Study of the Conservation of Substance in the Junior School Child." British Journal of Educational Psychology, XXX (1960), 109 - 18.

_______. "A Study of Conservation of Weight in the Junior School Child. "British Journal of Educational Psychology, XXXI (1961), 138 - 44.

_______. "The Growth of the Concept of Volume in the Junior School Child". Journal of Child Psychology and Psychiatry, II (1961), 118 - 26.

Matalon, Benjamin. "Étude génétique de l'implication", in Implication, formalisation et logique naturelle. Vol. XVI of Études d'épistémologie génétique. Edited by Jean Piaget. Paris: Presses Universitaires de France, 1962.

Miller, George A. "The Magical Number Seven, Plus or Minus Two. Some Limits on our Capacity for Processing Information." The Psychological Review, LXIII, No. 2 (1956), 81 - 97.

Money, John, ed. The Disabled Reader. Baltimore: The John Hopkins Press, 1966.

Morf, Albert. "Apprentissage d'une structure logique concrète (inclusion) effets et limites." Études d'épistémolgie génétique, IX (1959), 15 - 76.

Murdock, George Peter. "The Common Denominator of Cultures," in The Science of Man in the World Crisis. Edited by Ralph Linton, New York: Columbia University Press, 1945.

Oléron, Pierre. Les activités intellectuels. Paris: Presses Universitaires de France, 1964.

Piaget, Jean. Les notions de mouvement et de vitesse chez l'enfant. Paris: Presses Universitaires de France, 1946.

_______. "Une expérience sur la psychologie du hasard chez l'enfant: le tirage au mort des couples." Acta psychologica, VII (1950), 323 - 36.

_______; et Inhelder, Bärbel. La genèse de l'idée de hasard chez l'enfant. Paris: Presses Universitaires de France, 1951.

_______; The Child's Conception of the World. London: Routledge and Kegan Paul, 1951.

_______; and Inhelder, Bärbel. The Child's Conception of Space. Translated by F.J. Langdon and J.L. Lunzer. London: Routledge and Kegan Paul, 1956.

________; Inhelder, Bärbel; and Szeminska, Alina. The Child's Conception of Geometry. London: Routledge and Kegan Paul, 1960.

________. Les méchanismes perceptifs. Paris: Presses Universitaires de France, 1961.

________. Play, Dreams and Imitation in Childhood. Translated by C. Gattegno and F.M. Hodgson. London: Routledge and Kegan Paul, 1962.

________; et Inhelder, Bärbel. Le développement de quantités physiques chez l'enfant. Neuchatel: Delachaux et Niestlé, 1962.

Prior, A.N. Formal Logic. Oxford: The Clarendon Press, 1962.

Quine, Willard Van Orman. From a Logical Point of View. Cambridge, Massachusetts: Harvard University Press, 1964.

Ralston, Anthony; and Wilf, Herbert S.; eds. Mathematical Methods for Digital Computers. New York: John Wiley and Sons, 1960.

Reichenback, Hans. Experience and Prediction. Chicago: University of Chicago Press, 1938.

________. Elements of Symbolic Logic. New York: The Free Press, 1966.

Robertson, Jean E. "Pupil Understanding of Connectives in Reading," in Forging Ahead in Reading. Vol. XII, Part I of Proceedings of the Twelfth Annual Convention, International Reading Association. Newark, Delaware: International Reading Association, 1968, pp. 581 - 88.

Russell, Bertrand. An inquiry into Meaning and Truth. Great Britain: Penguin Books, 1962.

Smedslund, Jan. "The Acquisition of Conservation of Substance and Weight in Children. I. Introduction". Scandinavian Journal of Psychology, II (1961), 11 - 20.

________. "The Acquisition of Conservation of Substance and Weight in Children. II. External Reinforcement of Conservation of Weight and of the Operations of Addition and Subtraction." Scandinavian Journal of Psychology, II (1961), 71 - 84.

________. "The Acquisition of Conservation of Substance and Weight in Children. IV. An Attempt at the Extinction of the Visual Components of the Weight Concept." Scandinavian Journal of Psychology, II (1961), 153 - 55.

________. "The Acquisition of Conservation of Substance and Weight in Children. V. Practice in Conflict Situations without External Reinforcement." Scandinavian Journal of Psychology, II (1961), 156 - 60.

_______. Concrete Reasoning: A study of Intellectual Development, Monograph of the Society for Research in Child Development, Serial No. 93, XXIX, No. 2. Yellow Springs, Ohio: The Antioch Press, 1964.

Spache, George D. "Contributions of Allied Fields to the Teaching of Reading", in Innovation and Change in Reading Instruction. The Sixty-seventh Yearbook of the National Society for the Study of Education, Part II. Edited by Helen M. Robinson. Chicago: University of Chicago Press, 1968, pp. 237 - 90.

Spindler, George; and Spindler, Louise. "The Instrumental Activities Inventory: A Technique for the Study of the Psychology of Acculturation." Southwestern Journal of Anthropology, XXI (Spring, 1965), 1 - 23.

Science Research Associates Primary Mental Abilities Tests. Chicago, Illinois: Science Research Associates, 1947.

Sturtevant, William C. "Studies in Ethnoscience." American Anthropologist. Special Publication, LXIV (June, 1964), 99 - 131.

Suppes, Patrick. Introduction to Logic. Princeton, New Jersey: D. Van Nostrand Company, 1957.

Tinker, Miles A. Bases for Effective Reading. Minneapolis: University of Minnesota Press, 1965.

Toombs, Wilfred W. Letter.

Vygotsky, Lev Semenovich. Thought and Language. Translated and edited by Eugenia Hanfmann and Gertrude Vakar. Cambridge, Massachusetts: The M.I.T. Press, 1962.

Webster's Third New International Dictionary. 1961.

Wright, Georg Henrik von. The Logical Problem of Induction. Oxford: Basil Blackwell, 1965.

_______. Logical Studies. London: Routledge and Kegan Paul, 1957.

APPENDIXES

APPENDIX A

TESTS OF CONSERVATION

The Funny Race at the Picnic

The grade four class invented a "funny" race for their picnic. They chose six boys to be in the race. Each boy was to carry a ball of wax and walk as fast as he could to the finish line. Every boy was given a ball with just the same amount of wax in it. That was only fair.

It was certainly a funny race. The boys lined up and each stood there with his round ball of wax, every ball the same size. But it was a very hot day. The wax got softer and softer. It started to run down between their fingers. It began to go into funny shapes.

Now the rule was, "No wax is to fall on the ground." So the boys had to act fast. One boy rolled his wax around and around his two fists, like a muff. Another boy made his was flat like a pancake and stuck it on top of his head. Another rolled his wax into a couple of doughnuts and stuck his fists through them. One boy squeezed his wax so hard it shot up through his fingers like a jack-in-the-box. The boy on the end of the line had a funny one. It had long legs and looked like a little old man on stilts.

It was lucky there was a picnic table handy. The boys put their funny wax figures on the table so everyone could see them.

Then the arguments started. Some children said the boy with two doughnuts ended up with more wax than anyone else. Others said that the boy who put the pancake on his head had the lightest load to carry. Some said the boy with the muff had brought back the most wax of all, because it was so thick. Another said the jack-in-the-box was funny, they liked it best, as it was the biggest and the heaviest. One said the little man on stilts was the best because it was lightest of all.

What do you think?

Jimmy Feeds the Birds

Jimmy lived in a cabin on the side of a mountain. The snow was very deep in winter and the days were short. He could see the birds and other wild animals hunting for food in the woods as he walked home from school. The birds came into the clearing around the cabin to look for food and he could watch them as he ate his own meals.

Jimmy decided to make feeding stations for the birds and set out food for them. The chickadees and sparrows and Canada jays would like suet. They needed the fat to keep them warm and suet was good solid fat.

Jimmy took a large chunk of suet and carefully cut it into three equal pieces. He took one solid piece outside and nailed it high up on the trunk of a tree for the chickadees. He chopped one piece up into very tiny bits and put them on top of an old stump for the sparrows. The third piece he cut into middle-sized chunks. He swept a place clear of snow and put all the chunks on the ground for the jays.

Jimmy stood in the door of the cabin and looked at what he had set out for the birds.

"Now that's a funny thing," he said to himself.

"I start out with three whole pieces the very same and just look at them now! One's still a big piece. Another is a few chunks. And the other is just a lot of little bits."

"Will the sparrows have as much to eat as the chickadees?" he wondered.

Suddenly Jimmy remembered his pet crow. Now the crow would like some of Jimmy's popcorn. There was only a cupful of seeds left. So Jimmy measured half a cup of seeds for his pet crow, and kept half a cup for himself. He put the seeds for the crow in a bowl and set the bowl under the porch so that the snow would not cover the seeds.

That night Jimmy's mother popped the corn that he had saved for himself. You know what happened. Jimmy had three bowls of popcorn!

And, of course, Jimmy wondered again if his crow had as much food to eat as he had!

Stories Tests of Conservation

The Stories tests of conservation, based on Stories I and II, consist of nine tests, three tests at each of three levels of complexity. Justification is required for conserving and non-conserving responses to each test item:

> Why is that? How did you know that? How do you explain that? How else could you be sure of it? How is it that they are still the same (not the same) when you change the shape?

Preliminary questions are presented before the test questions for each story.

<u>Preliminary questions, "The Race"</u>

> This was a funny race, wasn't it!
> What did the boys make with the soft balls of wax?
> What else did they make? Why was the wax soft?
> How did they divide the wax among the boys? Why was that?
> What was the rule about the wax for this race?
> Why did they make that rule?
> Where did the boys put their funny figures at the end of the race?

<u>Preliminary questions, "The Birds"</u>

> Jimmy decides to feed suet to the birds around the cabin.
> What kind of birds is he feeding?
> Jimmy starts off with one big piece of suet. What does he do with this big piece?
> Now he has three pieces of suet. What are they like?
> So Jimmy has three equal pieces of suet. Why does he make them equal?
> Jimmy gives one of these pieces to the chickadees. How does he fix it for the chickadees?
> He gives one of these pieces to the jays. How does he fix the piece for the jays?
> How does he fix the piece for the sparrows?
> Jimmy fed his pet crow some popcorn seeds. How much popcorn seed did he feed the crow? How much did he keep for himself?

Test items: level of complexity I

Three test items at level of complexity I are based on the deformation of one object with the substance remaining continuous. The deformation is described as part of the story situation.

Test item I, conservation of substance

> One boy changed the ball of wax he started out with into two doughnuts. Was there more wax in the two doughnuts than in the ball, or was there the same amount or was there less wax?

Test item II, conservation of weight

> One of the boys rolled his ball of wax round and round his two fists like a muff. Would this muff be the same weight as the ball, or was it heavier or was it lighter?

Test item III, conservation of volume

> Suppose we put the two doughnuts the boy made from his ball of wax into half a bowl of water and mark the level the water comes up to (gesture). Then suppose we take the doughnuts out of the water (gesture) and put the muff the boy made from his ball into the water (gesture). Will the level of the water be higher for the doughnuts than for the muff or will the water levels be the same for both, or will the level be lower for the doughnuts?

Test items: level of complexity II

Three test items at level of complexity II are based on one deformation of each of two subjects.

Test item IV, conservation of substance

> You remember the one that looked like a little old man on stilts. Would this little man have the same amount of wax in it as the muff, or would it have less wax or would it have more wax?

Test item V, conservation of weight

> Was the pancake lighter than the jack-in-the-box, or was it the same weight or was the pancake heavier than the jack-in-the-box?

Test item VI, conservation of volume

StoryII

> Jimmy has two cardboard boxes the very same size and shape. The solid piece of suet he fixed for the chickadees fills one of these boxes exactly. If he packs the little pieces for the sparrows into the other box, will the box be full, or will it be partly filled, or will he need an extra box to hold these pieces?

Test items: level of complexity III

Test items at level of complexity III, based on the Story II, involve two deformations of each of two objects.

Test item VII, conservation of substance

> Did Jimmy give the jays as much suet to eat as the sparrows, or did the jays get more, or less suet to eat than the sparrows?

Test item VIII, conservation of weight

> Jimmy cut one of the pieces of suet into big chunks and put these big chunks out for the jays. Will these chunks be heavier than the suet he cut into small pieces for the sparrows, or will they be the same weight, or will they be lighter?

Test item IX, conservation of mass

> You remember Jimmy gave his pet crow the popcorn seeds and he had the popcorn! Did the crow have as much "food" to eat as Jimmy, or did it have more food, or did it have less food?

Concrete Tests of Conservation

The Concrete tests of conservation consist of nine tests, three tests at each of three levels of complexity. The subject determines the initial equality of the test materials using a balance scale (see Fig. 2, Chap. IV). For conserving and non-conserving responses justification is required as for Stories tests of conservation.

Test items: level of complexity I

Three test items at level of complexity I are based on one deformation of one of the two equal balls of platicine. The deformation is performed by the experimenter. Ball and "doughnut" are each placed at the center of a display support, and covered by two display supports.

Test item I, conservation of substance

Is there more plasticine in the "doughnut" than in the ball, or is there less, or is there the same amount of plasticine in the "doughnut" as there is in the ball (pointing)?

The order of the terms "more", "less", "the same", is randomized.

Test item II, conservation of weight

Suppose we put the "doughnut" in one pan and the ball in the other pan of the balance scale. Would the ball weigh the same, or more or less than the "doughnut"?

Test item III, conservation of volume

A glass containing water is placed at the center of a third display support. Ball and "doughnut", on display supports are uncovered. The experimenter presents a preliminary question:

If I drop this ball of plasticine into this glass of water what will happen? What else will happen?

There is also a preliminary experiment. The ball is placed by the experimenter in the glass of water. An elastic band is adjusted by the subject at the new water level. The ball is removed and returned to its support. The band remains in place and in view. Ball and doughnut are covered.

If I put the "doughnut" into this glass of water, will the level of the water be the same as it is for the ball (pointing), or will it be lower, or will it be higher?

Test items: Level of complexity II

Three test items at level of complexity II are based on one deformation of each subject performed by the experimenter: a "snake" and small cubes of plasticine. Each is placed on a display support, the cubes being distributed over a four square inch area. Each is covered by a display supports.

Test item IV, conservation of substance

> Is there the same amount of plasticine in the snake (pointing) as in the pieces (pointing) or is there more in the pieces or is there less plasticine in the pieces?

Test item V, conservation of weight

> If we put the snake on this pan of the balance scale and the pieces on this pan, will the pieces weigh the same as the snake or will the pieces weigh less than the snake or will they be heavier than the snake?

Test item VI, conservation of volume

> Two glasses containing water are presented on two display supports. The weight of the glasses of water and the levels of the water are determined to be equal by the subject. "Snake" and cubes remain covered.
>
> Suppose we put the "snake" into this glass of water (pointing) and the cubes into this glass of water (pointing). Will the level of the water go higher for the "snake" than for the pieces, or will the level be lower for the "snake" than for the pieces or will the level be the same for both?

Test items: level of complexity III

Two glasses containing water are each centered on a display support. The weight and the level of the water in each glass are determined (or adjusted) to be equal by the subject. Sugar cubes are placed near the supports. Preliminary questions are presented:

> Suppose I put these sugar cubes into this glass of water, what will happen? What else will happen?

Sugar cubes are placed in the water. The subject marks the level of the water with an elastic band.

What is happening? What is happening to the sugar? What has happened to the water level? Suppose we put each of these glasses on the balance scale now. Will the sugar water weigh more, or will it weigh the same or will it weigh less than the pure water? Why is that?

<u>Test item VII, conservation of substance</u>

If we come back tomorrow, will there be a sweet taste left in this water (pointing)?

<u>Test item VIII, conservation of weight</u>

Tomorrow, when we come back, suppose we put this glass of sweet water on one pan and the glass of pure water on the other pan of the balance scale (pointing). Will this glass of sweet water be heavier than the pure water or will it be lighter or will it be the same weight as the pure water?

<u>Test item IX, conservation of volume</u>

Tomorrow, will the level of the sweet water (pointing) be higher than the level of the pure water (pointing) or will it be at the same level or will it be lower?

Protocols

Protocols representing explanations at levels zero, I, II are presented below. The age of the subject is in brackets following the protocol number. The range of scores for each response, including the decision, is 0 to 3.

<u>Responses presenting explanations in support of non-conservation</u>

Scores assigned for the following non-conserving decisions and explanations are:

Decision (non-conserving)	0
Explanation level I	0
Explanation level II	0

34 (9:9) The pancake was lighter; it was all flattened out.

14 (9:4) The water will go higher for the ball 'cause it's thick and the doughnut has a hole in it.

6 (9:7) No, there'll be no sweet taste tomorrow because. uh. because it's all dissolved .. It went into the air. Maybe .. maybe vibration blew it up.

E. What does "dissolved" mean?

Dissolved means evaporated, like clouds. There'll be no sweet taste 'cause it just disappears into mid-air.

10 (9:9) The ball will weigh more because it's not stretched.

9 (9:9) Heavier. The big chunks are heavier. When you have it in small pieces they're lighter because they're easier to carry.

31 (9:6) I think it would be lighter (pancake on the head) because it would feel lighter (than the jack-in-the-box). It was all flattened out.

<u>Responses presenting non-logical explanations in support of conservation</u>

Scores assigned for decisions for conservation followed by non-logical (perceptual or moral) explanations are:

Decision (conserving)	1
Explanation level I	0
Explanation level II	0

24 (9:10) The same amount. They still look the same. (Snake and cubes.)
E. How can you be sure they are the same?
I just looked at them.

20 (9:5) The same level. Because if you put anything in the water, the water will go up.
E. Why does the water go up?
The weight makes it go up. I "could" say that it pushes down on the water, so the water rises as soon as it hits.

14 (9:7) The same, all the birds got the same because they were all hungry and cold and the snow was over everything.

<u>Protocols presenting explanations level I in support of conservation</u>

Explanations at level I assert the equality of the objects after deformation and infer the conservation of the initial relation on the basis of the Principle of Transitivity. The protocols are assigned the following scores:

Decision (conserving)	1
Explanation level I	1
Explanation level II	0

The arguments below are also presented in symbolic form. (P: premise; I: inference; a and = actions of deforming.)

89 (9:11) They have around the same ($A_1 = A_2$) because he just cuts it into smaller pieces (A_1 and A_1). He took an equal piece (A) and another equal piece (A_2) and then he cut them into smaller pieces.

A_1'	$=$	A_2'	P	
$A_1' + a$	$=$	A_1''	P	Assertion
$A_2' + a$	$=$	A_2''	P	Assertion
$A_1' + a$	$=$	$A_2' + a$	P	Assertion
A_1''	$=$	A_2''	I	Transitivity

<u>Protocols presenting explanations level II in support of conservation</u>

The following protocols present an explanation at level II for a decision asserting conservation. The premises include a general proposition in the form of a tautological implication and a synthetic proposition. The subject states (with evidence) that a praticular event is included in the class of events indicated in the general proposition. The deductive inference follows and is expressed in terms of necessity. The scores assigned are:

Decision (conserving)	1
Explanation level I	1
Explanation level II	1

In a number of the protocols the evidence presented referred to one but not to both conditions in the addition - subtraction proposition, for example the statement, "unless this one took some from that one and it didn't". If either or both of the addition-subtraction conditions are mentioned (+, or -, or both) the explanation is, in this respect, considered to be at level II and is presented in the symbolic representation of the argument as $A \pm 0$.

19 (10:1) The same amount .. $A_1 = (A_1' \ \& \ A_1'') = A_2'$ 'cause he never lost any ($A_2' \pm 0$) and he had to make the doughnuts out of the same stuff as the ball.

		A_1	P	
A_1	=	$(A_1' \ \& \ A_1'')$	P	
$(A_1' \ \& \ A_1'')$	=	A_2'	P	
$A_2' \pm 0$			P	Synthetic
A_1	=	A_2'	I	Modus Ponens

9 (9:9) There's the same amount, because you never took any out .. or put any in .. you just left it how it was and it was exactly the same. So it still would be the same amount (snake and cubes).

25 (10:3) More .. no .. the same amount, yes, because it comes down (the man on stilts) but you still don't lose any .. and the muff's just as big .. none of it's on the ground .. it won't look the same but the stilts are just longer. Sure .. they're the same.

APPENDIX B

TESTS OF CLASSIFICATION

The Ducks Arrive in Spring

Every spring the prairies become a fly-way for the birds on their way north for the summer.

The first birds to arrive are ducks, and the first ducks are the pintail. There will be ice on the ponds and lakes and some snow still on the fields when the pintail fly in in April. But these ducks can live off the land. They eat the seeds they find in the yellow stubble of the wheat fields until the ice melts. The pintail come in flocks of hundreds, long black lines of ducks against the blue prairie sky. They circle, then drop into the snowy fields.

The next ducks to arrive are the pond-feeders. They need weedy pools that are not very deep. These are the mallard, teal and shoveller. They swim on the surface of shallow ponds and bob their heads under the water to feed. These ducks must wait for the ice to melt on the shallow pools and ponds. As soon as the ice is melted, the pintail will leave the fields and swim about on the pools with the mallard and teal. They are pond-feeders.

The last ducks to arrive are the diving ducks. These are canvas backs, red-head, and golden eyes. Diving ducks must wait for the ice to go out on the lakes and rivers. They dive into deep water to get their food.

A City of Long Ago

Many thousands of years ago, there lived a people in India who built beautiful cities. Their streets were straight, like ours, and the streets met at corners like ours. But these people made a long curve at the corner of their streets and built a house there. So this house on the corner belonged to both streets. It looked up one street and it looked down the other. It belonged to both streets.

People came from far away to live on the streets of this beautiful city. Some families came down the river on rafts from their villages in the mountains. Some families travelled on foot for many days through the forests. Every family who came on foot carried some small treasure to remind them of their old homes.

In the strange new city families from the same village liked to live near one another, to be near their friends. So it happened that everyone along one street came from the same village. They had travelled together for many days through the forest. And everyone along the other street had come together down the river on rafts from their village in the mountains.

The forest people set out their treasures in front of their houses. These were treasures they had carried with them from their old homes. Every treasure was painted a bright yellow color to show how happy the family was to reach the great city. There was an old church bell in front of one house. It was painted bright yellow. An old axe was in front of another house. It was bright yellow, too. In front of another house there was a tall post carved with the strange signs the people used for letters in those days. It was yellow, too. Every house had its bright yellow treasure set out in front for all to see.

In front of every house on the other street was the family's old waterlogged raft. They had come down the river on that raft and they treasured it. They wanted everyone to see it, just as it was. All along this street was a row of river rafts.

There was only one house that had no treasure set out in front for all to see. That was the house at the corner which belonged to both streets. It looked up the street with all the yellow treasures and down the street with all the rafts. What could this family put out for everyone to see? They wanted to show that they belonged to both streets, because they lived where the two streets met. No one could think how this could be done. No one could think what to put out in front of the house at the corner where the two streets met.

Can you tell the family who lived on the corner what to put out in front of their house?

Stories Tests of Classification

Inclusion Relations

<u>Preliminary questions, "The Ducks"</u>

This is a story about ducks that fly to the prairie in the spring; it's about the birds that come to the lakes and ponds of the prairie in the spring.

What kind of birds come back first in the spring?
What kind of ducks arrive first?
What is the weather like when the pintail arrive?
How do the pintail get their food when they arrive?
Why do they feed off the wheat fields?
Where do they feed when the ice melts?
Which ducks come next after the pintail?
Where do the mallard and teal get their food?
How do they get their food?
What kind of ducks arrive after the mallard and teal?
Why do the canvas-back and red-head come last?
Where do they get their food?

<u>Test item I</u>

Are there more ducks or more pintail here on the prairie in the summer?
How did you know that? Why is that?
How else can you be sure of it? Are pintail ducks?

<u>Test item II</u>

Are there more ducks or more birds on the prairie in the summer?
How did you know that? Why is that?
How else can you be sure of it? Are ducks birds?

<u>Test item III</u>

Are there more animals or more birds in the world?
How did you know that? Why is that?
How else can you be sure of it? Are birds animals?

Test item IV

If all the birds left the prairie and flew into the far north would there be some ducks here on the prairie?

How did you know that? Why is that?
How else could you know that?

Predicates.

Test item V

I want you to tell me the "kinds" of ducks that come to the prairie.
Put the ducks you read about into two different kinds or lots. You can do this without using their names. Describe them.

What would be two kinds of ducks that come in the spring? Describe them.

If the response is to list names, the examiner replies:

Can you describe them? What kinds of ducks come back in the spring? Put them into "two" lots.

Test item VI

Tell me another way to describe the kinds of ducks that come back in the spring. Put the ducks that come to the prairie into two different kinds or lots in another way and describe them.

If the response is a name rather than a predicate, for example "mallard", the examiner replies:

Can you describe them?
What are the kinds of ducks that come back in the spring?

Test item VII

There is still another way to describe the kinds of ducks that come back in the spring. Put the ducks into two lots in another way, and describe them.

Multiplicative Classes

Test item VIII and IX

You're driving along the highway and you see shallow ponds along the road and you see a lake not far away.

What "kinds" of ducks will live in this neighbourhood?

Class of Intersection

Preliminary questions, "A City of Long Ago"

This is a story about a city that was built by people in India a very long time ago. It was a beautiful new city and many people came to live there. Some people travelled down the river.

How did the people travel who came down the river?
What is a raft?
When the people who travelled down the river on rafts reached the city, where did they put the rafts? Why did they do that?

Other people came on foot through the forest.
What reasures did they carry with them?
Where did these people put their treasures?
What color did they paint them?
Why did they paint them yellow?

So we have two streets: the street with the rafts, and the street with the yellow treasures (gesture on the table of a right angle). These two streets meet at the corner. (Repeat above gesture).

Test item X

You remember the family that live at the corner where the two streets meet. Now what is the right thing for this family to set out in front of their house. They want to show that they belong to both streets: the street with the rafts and the street with the yellow treasures.

They want to put out "one thing": one thing to show that they belong to the street with the rafts and the street with the yellow treasures.

What should they put out?

Concrete Tests of Classification

The Construction of Classes.

Test item 1, two classes

The collection consists of

10 red rounds
3 white rounds
3 red rounds
3 white squares.

The items are identified and the collection is covered.

I am going to ask you to put these counters (pointing) to the covered collection) into two lots. Everything in one lot is to belong together and everything in the other lot is to belong together. Everything in each lot is to belong together.

What will you put here (pointing to the left display support)?
And what will you put here (pointing to the right display support?
The cover is removed from the collection and the examiner says:
Go ahead and do it.
What have we here? (pointing).
What have we here? (pointing).

Test item II, four classes

I am going to ask you to make four lots from these two lots of counters, two from this lot (pointing to the covered set on the left) and two from this lot (pointing to the set on the right). Everything in each lot must belong together.

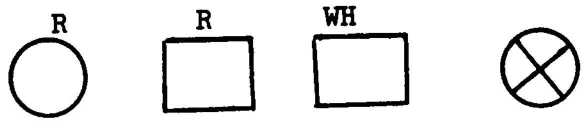
How will you do this?
What will you put in this lot(pointing to the left empty support)? And what will you put in the other lots (gesturing along the row of four empty supports)?

Go ahead and do it.
What have we here? (The question is repeated for each of the four classes).

Class Inclusion Relations

The examiner states that the white rounds are to be removed.

They are placed at the upper right, and in view.



Test Item III

The classes are covered.
Are there more red ones or more round ones? Why is that?

Test Item IV

In these lots (gesturing), are all the square ones, red?
Why is that?

Test item V

In these lots (gesturing), are all the white ones square?
How did you know that?

Test item VI

In these lots, are some of the red ones round?
Why do you say (not say) some of them are round?

Test item VII

I am going to give you a square one.
Will it have to be red? Why is that?

Test item VIII

I am going to give you a red one.
Will it have to be round? Why is that?

Test item IX

I am going to give you a white one.
Will it have to be square? Why is that?

Test item X

I am going to give you a round one.
Will it have to be red?

The Construction of Predicates.

<u>Test item XI</u>

Pattern I is presented. The categories are size and color.

This is a design a boy/girl made. He has put together what belongs together to make a pattern. Take this rod and put it on the pattern to show how he has put together what belongs together. What pattern does the rod help you see?

How do these belong together?
How do these belong together?

<u>Test item XII</u>

Is there another way to lay the rod to show a pattern? Put the rod another way to show how the boy(girl) put together what belongs together to make a pattern.

How do these belong together?
How do these belong together?

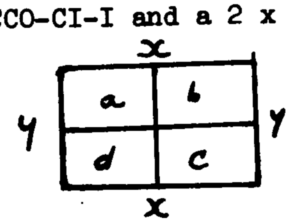
<u>Test item XIII</u>

Pattern II is presented. The category is shape.
The instructions for test item XII are repeated.

Multiplicative Classes

<u>Test item XIV</u>

The material consists of a random collection of counters as for test item CCO-CI-I and a 2 x 2 matrix:



The preliminary questions presented are:

How many sections are there?
When I pick up this rod (X---x) how many sections are there?
The rod (x---x) is replaced.
When I pick up this rod (y---y) how many sections are there? The rod (y---y) is replaced.
And how many sections are there?

The test questions are:

I am going to ask you to put these counters pointing to the covered collection of counters) into these four sections: Everything in each section must belong together. Put them so that if I pick up this rod (x---x) is lifted and replaced) they are in a good order; and if I pick up this rod (y---y) is lifted and replaced) they are in a good order.

How will you do this?
What will you put in this section? (pointing to the upper left section, above. In this section (pointing to section b?). In this section? In this section? (The examiner points to c and d).

The collection is uncovered.
Go ahead and do it.

<u>Test item XV</u>

Are they in a good order when I pick up this rod? The rod (x---x) is lifted and replaced immediately. Why is that? Are they in a good order when I pick up this rod? The rod (y---y) is lifted and replaced. Why is that?

The subject may recognize that the classes as he has arranged them are diagonally positioned. Two attempts to correct the order are permitted. The questions, "Are they in a good order?" are repeated after each attempt.

Class of Intersection.

<u>Test item XVI</u>

Pattern III is presented:

This is a pattern a boy/(girl) made. He didn't finish it. He didn't put anything here (pointing to empty square at the point of intersection). We want to put something here; one thing. It must belong to this row (gesturing along the row of large squares) and it must belong to this row (gesturing along the row of small rounds). We want to choose one thing. It must belong to this row and it must belong to this row (gestures repeated). Just one thing.

What will you put here?

If the subject suggests two objects for the intersecting class the examiner repeats: "Just one thing".
If the subject's choice is incorrect the examiner asks"
How does it belong to this row?
How does it belong to this row?
Three minutes are allowed for finding a solution.

Test item XVII

A collection of counters in a random pile is uncovered.
The collection contains:
2 white rounds, 2 white squares, 1 black square (4/5")
3 white squares, 1 red, 1 black square (2/5")
3 white rounds, 1 red round (2/5")

The subject selects a counter.

Why is this the right thing to put here?
How does it belong to this row?
How does it belong to this row?

Two attempts are permitted. The collection of counters is covered for test item XVIII.

Test item XVIII

There is another thing we could put in that space which would do just as well: "one thing". It would belong to this row and it would belong to this row (gestures). What else could we put in this space that would belong to this row and belong to this row? Procedures for test item XVII, above, are replaced.

Test item IX

The collection is uncovered. The subject selects a counter from the collection to represent the class of intersection. Procedures for test item XVII, above, are followed.

Protocols

The following responses have been selected from the protocols of the Concrete tests (CCO-CI) and the Stories tests (SCO-CI) of classification to illustrate those to which a score 0 was assigned, and those to which a score 1 was assigned.

<u>SCO-CI to IV</u>	Score	
7 (8:10)	1	More ducks, because there's more than two different kinds of ducks. Yes, pintail's one kind. (P D).
	1	More birds 'cause there's more bird than ducks because ducks are birds. (D B)
	0	More animals ... , well, there's more animals because there's more kinds of animals. E. Are birds animals? No ... because animals all have four feet.
49 (9:7)	0	More pintail 'cause it said in the they story come in flocks of hundreds. They come first.
17 (10:1)	0	More ducks, because ducks come in the spring. E. Are ducks birds? No, they're better. They swim and fly.
15 (9:1)	0	There will be more animals because birds don't get as many babies as animals. E. Are birds animals? No, they don't get babies like animals.
	0	Yes, there'll be ducks here because ducks like to fly around the prarie and the pools.
<u>SCO-CI-V to VII</u>		
50 (9:7)	1	Well, there's the first kind that stay on the land till the ice is gone on the shallow pools. They come first. And there's the other kind that wait till the ice is gone off the rivers and lakes.
	1	Category time of arrival recognized above.
	0	There's ... well .. the mallard and teal and canvas back ... The category "feeding habits" not recognized.

SCO-CI-VIII-IX	Score	
78 (9:7)	1	There'll be ducks that eat in the shallow water, and ducks that dive under the water to get food.
3 (9:7)	0	There'd be pintail ... mallard ... and .. uhm .. E. Instructions repeated. I don't know.

SCO-CI-X

9 (19:9)		They could put out two things. E. Just one thing.
	1	They could put out maybe just a raft and color it bright yellow because one of the streets had to have bright yellow treasures and the other had to have just rafts.
52 (19:3)		Paint half the raft yellow. Half yellow and half weather color, etc.
79 (9:6)	0	They didn't set out anything. E. Instructions repeated. No, I don't know.

CCO-CO-1-11

37 (9:6)	0	Divide it into two parts, half in one, half in the other. E. Instructions repeated. Seven in one and seven in the other. E. Explanation provided.
	0	Put three white circles, three red circles, three white squares, three red squares. E. Instructions repeated. Put three white squares, one red circle, one white square, one red square. E. Explanation provided. (This subject obtained an "IQ Estimate" in the range 111-115 on PMA.)

CCO-CI-III-VI		More R's or more O's?
98 (10:4)	1	More red ones because there's two groups of red ones and only one group of circles. Some reds are square too.
		Are all 's .... R's?
	1	No, because one group of squares are white.
		More R's or more O's?

Score

75 (9:6) 0 More round ones. Because when I put them on the table .... I picked up more round ones.

<u>CCO-CI-VII-X</u>

78 (9:7) 1 No, it could be red or white.

If x is white must it be square?

0 No, because some are square and some are round.

<u>CCO-CI-XI to XIII</u> Pattern 1

4 (10:7) 1 Red together ... white together.

0 This is a red circle and this is a white one. The rod is laid below successive pairs of counters, and each is described by shape and color. Categories size and shape not recognized.

<u>CCO-CI-XV to XIX</u> Class of Intersection

93(9:1) 1 A large white circle 'cause it's large and 'cause it's a circle.

1 A small white square. Well ... it's small. It's a square.

83(9:7) 0 A large white circle ... No, there's no way, these are little circles, it doesn't really belong. You can't put squares and circles together.

0 No, you can't. You could only do it with two things.

APPENDIX C

TESTS OF DEDUCTION

The Prairie Blizzard

Jimmy's father was a doctor in the early days of settlement on the prairie. Many of the doctor's patients lived across the river on lonely farms far from the village. There were no bridges across this river. In summer the people crossed by ferry; in the winter they crossed on the ice.

One cold winter night there was a loud knock on the doctor's door. When he opened it, a man wrapped in a heavy buffalo robe stumbled into the room. He was covered with snow.

"Could you come right away, doctor", he said. My neighbour's wife is having a baby. They need you."

The doctor wakened Jimmy.

"We must go", he said. You'll have to help with the horses. There's a terrible blizzard."

Jimmy put fresh straw in the big sleigh for warmth and helped hitch up the horses. They reached the river and started across. In fifteen minutes they should be on the other side. But it might be hard to find the road on the other side since the storm had covered all the tracks.

Jimmy knew the banks on both sides of this river very well. In summer he fished and in the fall he hunted prairie chicken up and down the banks. In winter he hunted rabbits.

"The only way to get to the road on the other side of the river is to find the cut in the river bank", he said. Everywhere else the banks are too high for the horses and the sleigh. But if we can find the cut we can go through it. That cut stays open even in a blizzard. Then we will come to a farm house. The road to the neighbour's house runs right past this farm house.

By this time the blizzard was in full force and visibility was down to zero. There was some danger that the horses would turn with the wind and circle back home again. But they let the horses take the lead and hoped they would find the way across to the other side.

Then they would look for the cut in the riverbank and try to reach the farm house at the far end of it. That would mean they had found the road on the other side of the river and the doctor could reach the neighbour's house by following that road.

Stories Tests of Deduction

Preliminary questions: "The Ducks"

Preliminary questions for this story were administered prior to test items SCO-Classification. The test items below, also based on the story "The Ducks", are administered after test item V,

Test item I, Modus Ponens

There's no ice on the rivers and lakes. Are there ducks on the prairie? How did you know that?

$$\sim I \,\&\, (\sim I \longrightarrow D) \longrightarrow D.$$

Test item II, Undetermined

The lakes and ponds are frozen over.
Would there be any ducks on the prairie?
Why is that?

If the answer is, "There could be, the pintail come ...", the examiner asks, "Could there "not" be? Why is that?

$$\left[(\exists F) : (F \longrightarrow D)\right] \sim \left[(F) : (F \longrightarrow D)\right].$$

Test item III, Undetermined

There are ducks on the prairie.
Are the ponds and lakes frozen? Why is that?

$$\left[(\exists D) : (D \longrightarrow \sim F)\right] \sim \left[(D) : (D \longrightarrow F)\right].$$

Test item IV, Modus Tollens

It's the first of April, but there are no ducks on the prairie.
Has the ice melted on the ponds?
How did you know that?

$$\sim D \;\&\; (IM \longrightarrow D) \longrightarrow \sim IM.$$

Test item IV, Modus Tollens

It's the first of April, but there are no ducks on the prairie.
Has the ice melted on the ponds?
How did you know that?

$$\sim D \quad \& \quad (IM \longrightarrow D) \longrightarrow \sim IM.$$

Test item V, Principle of Transitivity

This test item is presented typed on a card. It is read aloud by the examiner as the subject reads silently. The subject then reads the item aloud. He reads:

If the mallard are back, the teal are back.
If the teal are back, the shoveller are back.
The mallard are back. Are the shoveller are back?

The examiner then asks:

What about the shoveller, would the shoveller be back?
How can you tell? Can you be sure? How can you be sure?

The problem may be symbolized:

$$M \dashrightarrow T$$
$$T \dashrightarrow Sh$$
$$M \dashrightarrow Sh.$$

Preliminary questions, "The Blizzard"

Why did Jimmy and his father let the horses take the lead?
What is a blizzard like?
Jimmy and his father were afraid the horses would turn and go home. Why might the horses do that?
There was only one way to get to the road on the other side of the river. What was it?
If they find the cut in the river bank and then go through the cut, how will they know they've reached the road?
If they find the farm house and find the road, then where will they go?
Why is it important for them to reach this patient?

Test item VI, Possible

Is it possible to go through the cut in the river bank and reach a farm house? How is that possible (not possible)?

$$M(^{C}/_{FH}).$$

Test item VII, Not-Possible

Is it possible to reach this farm house and not go through the cut in the river bank?
How is that (possible) not-possible?

$$\sim M(\sim C/_{FH}).$$

Test item VIII, Not-Possible

Is it possible to find the cut and not get to the farm house?
How is that (possible) not-possible?

$$\sim M(C/_{\sim FH}).$$

Test item IX, Possible

Is it possible to not find the cut and not get to the farm house?
How is that possible (not possible)?

$$M(\sim C/_{\sim FH}).$$

Test item X, Possible

Would it be possible to find a road and not go through the cut in the river bank?
How do you explain that?

$$M(Rd/_{\sim C}).$$

Test item XI, Modus Ponens

Jimmy and his father have gone through the cut in the river bank. Is there a farm house near them?
How can you know that?

$$C \,\&\, (C \longrightarrow FH) \longrightarrow FH.$$

Test item XII, Modus Tollens

Jimmy and his father have "not" gone through the cut in the river bank.
Have they reached the farm house?
How can you know that?

$$\sim C \,\&\, (FH \longrightarrow C) \longrightarrow \sim FH.$$

Test item XIII, Modus Ponens

Jimmy and his father have reached the farm house.
Have they gone through the cut in the river bank?
How did you know that?

$$FH \; \& \; (FH \longrightarrow C) \longrightarrow C.$$

Test item XIV, Modus Tollens

Jimmy and his father have not reached the farm house,
Have they gone through the cut?
How did you know that?

$$\sim FH \; \& \; (C \rightarrow FH) \rightarrow \sim C.$$

Test item XV, Undetermined

Jimmy and his father have reached "a road".
"Must" they have gone through the cut in the river bank? How would you explain that?

$$\left[(\exists Rd): (Rd \rightarrow C)\right] \sim \left[(Rd): (Rd \rightarrow C)\right].$$

Test item XVI, Undetermined

There is a house in front of them. Is it "certain" that they have gone through the cut in the river bank? How would you explain that?

$$\left[(\exists H):(H \rightarrow C)\right] \sim \left[(H):(H \rightarrow C)\right].$$

Test item XVII, Principle of Transitivity

This test item is presented typed on a card. It is read aloud by the examiner as the subject reads silently. The subject then reads the item aloud.
He reads:

If they find the farm house, they will find the road.
If they find the road, the doctor will reach the patient's house.
If they find the farm house, will the doctor be certain to reach the patient?
How did you know that? Can you be sure of it?
Explain how you knew that?

The problem may be symbolized:

$$(FH \rightarrow Rd) \; \& \; (Rd \rightarrow PH) \quad (FH \rightarrow PH).$$

Concrete Tests of Deduction

"Village" Tests.

Preliminary instructions

This is a village. These are houses. This is the church (pointing). What is this? Yes, this is the Post Office. To come into this village you must come by the road (pointing). No one is allowed to walk on the fields around the village.

This is a man (pointing) who lives in another village. To go to the Post Office he must pass in front of this house (pointing) by the side of the road.

(The figure of the man is lifted by the examiner and moved slowly along the road, past the house, into the village square, and to the Post Office. The figure is then returned to its original position on the road outside the village.)

Test item I, Modus Ponens

The man has been to the Post Office (PO).
Did he pass in front of the house (H)?
How did you know that?

$$PO \;\&\; (PO \longrightarrow H) \longrightarrow H.$$

Test item II, Undetermined

The man has "not" been to the Post Office ($\sim$PO).
Did he pass in front of the house (H)?
Can you explain that?

$$\left[(\exists \sim PO):(\sim PO \longrightarrow \sim H)\right] \sim \left[(\sim PO);(\sim PO \longrightarrow \sim H)\right].$$

Test item III, Undetermined

The man has passed in front of the house.
Has he been to the Post Office?
How is that? Can you explain that?

$$\left[(\exists H):\sim H \longrightarrow PO\right] \sim \left[(H):\ (H \longrightarrow PO)\right].$$

Test item IV, Modus Tollens

The man has not passed in front of the house (H).
Has he been to the Post Office? How did you know that?

$$\sim H \;\&\; (PO \longrightarrow H) \longrightarrow \sim PO.$$

Test item V, Possible

Is it possible that one day the man will pass in front of the house and will have been to the Post Office?
How is that? Would you explain that?

$$M(^{H}/_{PO}).$$

Test item VI, Not - Possible

Is it possible that he will "not" have passed in front of the house and will have been at the Post Office? Explain how you knew that.

$\sim M(^{\sim H}/_{PO})$.

Test item VII, Possible

Is it possible that he will have passed in front of the house and that he will "not" have been at the Post Office? Explain how you knew that.

$M(^{H}/_{\sim PO})$.

Test item VIII, Possible

Is it possible that he will "not" have passed in front of the house and that he will "not" have been at the Post Office? How is this (not) possible?

$M(^{\sim H}/_{\sim PO})$.

"Light" Tests

Preliminary instructions

This is an apparatus with lights that can be worked by a switch. These are windows which open and shut. (The operation of the windows is demonstrated, then both windows are closed).

There is a red light (the window is opened to show the red light on, then closed).

There is a green light. (This is demonstrated as above. The electric switch is operated by the examiner from a position not visible to the subject).

These lights can not be turned on just any way. There is a rule to say how they work. The rule is: If the red light is on, the green light is on. Tell me the rule. The tule is also repeated by the examiner. Say the rule again. Yes that's the rule. We have just one rule. If the red light is on, the green light is on.

If I work this switch by the rule in all the ways it will work, and if I keep these windows open so you can see, tell me all the ways these lights will work, tell me what you will see. Yes, what else will you see?

The possibilities are red and green and both on; red and green both off; the green light may be on alone. The subject is not informed of these possibilities.

Test item IX, Possible

Is it possible to have the red light on (R), and the green light on (G)?

Why is this possible (not-possible)?

$$M(^{R}/_{G}).$$

Test item X, Not-Possible

Is it possible to have the red light "on" and the green light "out"? Why is this not possible (possible)?

$$\sim M(^{R}/_{\sim G}).$$

The subject again repeats the rule.

Test item XI, Possible

Is it possible to have the red light out and the green light on? Would you explain that? Tell me how you knew that?

$$M(^{\sim R}/G).$$

Test item XII, Possible

Can you have the red light out and the green light out? Now is this possible by the rule?

$$M(^{\sim R}/\sim G).$$

Test item XIII, Modus Ponens

The red light is on (window open).
What about the green light? How did you know that?

$$R \ \& \ (R \longrightarrow G) \longrightarrow G.$$

Test item XIV, Undetermined

Both windows are closed.
The red light is out (window open).
What about the green light?

If the subject fails to answer or the decision or the explanation is incorrect, the examiner asks once, "What is the rule?" and adds, "Yes, that is the rule and repeats, "If red light is on the green light is on".

The red light is out (pointing). And the green light.....?
How did you know that?

$$[(\exists \sim R):(\sim R \longrightarrow \sim G] \sim [(\sim R):(\sim R \longrightarrow \sim G)].$$

Test item XV, Undetermined

Both windows are closed.
What is the rule?
The green light is on (window open).
What about the red light?
Why is that? How did you know that?

$$[(\nexists G) : (G \longrightarrow R] \sim [(G) : (G \longrightarrow R)].$$

Test item XVI, Modus Tollens

Both windows are closed.
What is the rule? Say the rule again.
The green light is out (window open).
What about the red light?
How did you know that?

$$\sim G \ \& \ (R \longrightarrow G) \longrightarrow \sim R.$$

Test item XVII, Principle of Transitivity

Preliminary instructions accompany the construction of the first two lines of the argument by the examiner.
The syllogism is of the form:

$$C_1 \longrightarrow C_2$$
$$C_2 \longrightarrow C_3$$
$$C_1 \longrightarrow C_3.$$

Protocols

The following responses have been selected from the protocols of Concrete tests (CCO-D) and the Stories tests (SCO-D) of deduction, to illustrate those assigned a score 0, and those assigned a score 1. The responses have been grouped for test items assessing Necessary, Undetermined, and Possible, Not-Possible inferences.

Necessary inferences

CCO-D		Score	Modus Ponens
I	48(9:5)	1	Yes, because he has to go by the house to get to the Post Office.
XIII	61(9:8)	1	It's on. If the red one's on the green one's gotta be on too.
SCO-D			
I	97(9:2)	1	Yes, because they're near the water so they can swim.
XI	70(9:3)	0	Yes, 'cause they had to. They had to get to the patient.

CCO-D			Modus Tollens
XVI	14(9:4)	1	It's out (R). If the red's on the green's on. The green's not on so the red's not on.
SCO-D			
IV	94(9:5)	0	No, the story said it hadn't melted.
	99(9:1)	0	No, not all of it. It's too chilly yet for it to start to melt in April.

CCO-D			Principle of Transitivity
XVII	52(9:9)	1	A red square (C_3). Well, like the rule... if the big white one can have the little one ... and the little one ... can have the red one, then the big one can too.
	74(9:7)	0	A red square. It looks best to have another red one here, it matches.

Score

<u>SCO-D</u>

XVII 23(9:6) 0 They're not back ... because the shoveller come with the teal and teal come with the mallard.
E. What about the shoveller?
They won't come.

<u>Undetermined inferences</u>

<u>CCO-D</u>

XV 41(8:1) 1 It could be on or off. 'Cause if the green light's on it doesn't matter. The red can be on by the rule but it can be off.

8(9:7) 0 The red's on too. Because you have to have the red light on to have the green light on.
E. The rule? [Repeated correctly]
E. The green light is on, what about the red one?
It's on too, because if the green light's on the red light's on.

<u>SCO-D</u>

III 98(10:4) 1 Well, they can be and they can't. Some of them come back when the ponds and lakes are frozen and some when they're not frozen.

III 22(10:3) 0 Yes, they're still frozen, the pintail come back before any other birds.

<u>Inferences Possible, Not-Possible</u>

<u>CCO-D</u>

VVI 82(10:7) 1 Yes, he could have been at the house and not be at the Post Office ... yet.

$$M(^{H}/_{\sim PO}).$$

XI 24(9:10) 1 Yes, well when the red light's on the green is on so it should be possible to have the red light off and green light on.
E. By the rule?
Yes. You can have the red on and green one, and you can have just the green on.

$$M(^{\sim R}/G).$$

Score		
XII 21(9:10)	1	Yes, you can pull the plug. E. By the rule, can you have the red out and the green out? Oh, sure, put the green out and the red's got to go too.

<u>SCO-D</u>

VII 100(9:1)	1	No, because the banks were too steep, it said in the story.

$$\sim M(^{FH}/_{\sim C}).$$

IX 23(9:6)	0	Yes, they might go another way but most likely they froze. The horses would break a leg and they'd freeze out there in the blizzard.

$$M(^{\sim C}/_{\sim FH}).$$

APPENDIX D

TESTS OF INDUCTION

The Cave

One summer Bob and Jack discovered a secret cave. It was on the far side of a mountain that rose hundreds of feet above the valley where they lived. People climbing the mountain had not seen the cave. It looked just like another crack in the rock wall. There were many wide cracks in the rocks made by the ice and melting snow. They all looked much like this one. But Bob and Jack found an opening running off from the crack. They scrambled through this opening and came to a huge dark cave. The cave seemed to go on and on deep under the mountain.

The boys were much too frightened to explore the cave right away. They went in only a little way, just a few steps, the first day. Next day they came back and went in a little further. But they still just peered into the deep darkness of the cave and backed out again very quietly.

The third day they had a real shock. They were only a little way inside the cave when hundreds of bats flew screaming out of the cave over their heads. Not one of the bats collided with the boys, but how they could miss them was a miracle. The air was black with bats. The noise of their screaming echoed from the walls of the cave.

Now the first time this happened it was very frightening. But the boys got used to it. Every day when they came to the cave hundreds of bats flew out of it over their heads. The boys began to enjoy the excitement. They would imitate the wild shrill calls of the bats and listen to the echoes from inside the cave.

Slowly, now, the boys began to explore the cave. A little way inside they found a tiny waterfall.

"This cave would make a good den for a wild animal", they said to each other. But then they remembered that there were no animals that lived in caves on this mountain. There were only rabbits and squirrels and chipmunks. These animals dug burrows in the ground. Besides, they lived further down the mountain where there was food for them.

The boys pushed a little further each day into the cave. It got darker and darker as they went in. They decided to bring candles so they could see their way.

For a month the boys visited the cave every day. Every day the bats flew out over their heads. The boys waited until the bats had gone, then lit their candles and started to explore.

Cont'd . . . .

One day they came to the cave and shouted as usual for the bats to fly out. But nothing happened. The silence was terrible. The boys threw some stones and shouted to make a noise. All was silent and still. They waited and listened. There was no sound from the cave. They took one step inside the opening and lit their candles. The candles went out. They lit them again. The candles sputtered and went out again. The boys turned slowly and stepped back out of the cave.

When they got out into the sunlight they sat down on the stones at the door of the cave to think. A chipmunk ran up to them. They tossed him a peanut. He stuffed it in his cheek and headed for the cave. But he turned back as soon as he reached the opening and ran to the top of the rock behind the boys.

There was no sight or sound of the bats. The whole mountain was silent and still. An eagle soared in the blue sky over the valley. When the chipmunk saw the shadow of the eagle on the rocks he scooted for cover under a stone.

Was there really something strange here or were they just imagining it? That's what the boys were wondering. That was their problem as they sat on the rocks in front of the cave to think.

Stories Tests of Induction

Preliminary questions, "The Cave"

What frightened the boys when they first went exploring into the cave? But the boys soon stopped being frightened of the bats. They got used to them. Why did the boys bring candles? One day when the boys came to the cave they found some things that were different. What seemed different to them on this day? What else surprised them? What else did they notice that seemed different? When they sant down on the rocks to think things over, what did they notice? What else? What happened when they gave a chipmunk a peanut?

Test item 1, recognition

Were things really different or were the boys just imagining it? Why do you think so?

Test item II, a hypothesis

What do you think would be going on here? Can you find out? Go ahead and see if you can figure it out. That's what the boys were trying to do. What do you think may have happened? How would that explain what they boys noticed?

Test item III, an inference supported by evidence

What do you think would account for all the strange happenings at the cave on this day?
What does your explanation account for?
Why do you think it is a good explanation?

$$(A):(A_x \rightarrow B_x).$$

Test item IV, an inference supported by a law

What sort of thing could have happened in the cave?
How do you explain such different things happening as no bats and the candles going out?

$$\sim B \; \& \; (A \; \& \; T \rightarrow B) \rightarrow \sim A.$$

Concrete Tests of Induction

<u>Test item 1, recognition</u>

Where will the spinner stop? Why did you choose these colours? Where will it stop next time? Why did you choose these colours? The questions are repeated until the subject recognizes the regularity.

<u>Test item II, hypothesis classificatory</u>

What do you think is going on here? Can you find out? What can you do to find out? Go ahead.

<u>Test item III, hypothesis propositional</u>

Is there something more you can do to find out about this? How can you find out? Go ahead.

<u>Test item IV, an inference supported by evidence</u>

Why do you think you have the right explanation? Could there be another explanation? Can you show that some other explanation of the way the spinner acts would not be a good one? What did you show?

$$(A):(\sim B_x \rightarrow \sim A_x).$$

<u>Test item V, an inference supported by evidence</u>

Have you found out what makes the spinner act like it does? Show me how you can be sure about what explains the way the spinner acts? How else can you show that this is likely to be the right explanation?

$$(A):(A_x \rightarrow B_x).$$

<u>Test item VI, an inference supported by association with a law</u>

Is there an explanation for why "this block" stops the spinner? Is there a way to show that? What about the other blocks?

$$B \ \& \ (A \ \& \ T \rightarrow B) \rightarrow A.$$

Protocols

The following responses selected from the Stories (SCO-I) and Concrete (CCO-I) tests of inductive reasoning illustrate responses scored 0 and responses scored 1.

SCO-I		Score	Recognition of a constant
I	11(9:2)	0	Oh, the boys are just imagining it.
	15(9:4)	1	It was really different in the cave.
			Hypotheses
II	94(9:5)	0	The boys just likely got to the wrong cave.
	99(9:1)	0	They knew they shouldn't have come. They were told not go in caves.
	37(9:6)	1	Maybe a big wind came in through a crack in the cave and blew out the candles and maybe the bats didn't like the wind. E. What about the silence? The wind could be stopped. E. And the chipmunk? Well that would have to be the eagle or something.
			Inductive inference
III	98(10:4)	1	Well, what made the boys back out was the candles going out ... that never happened before. Something made them go out. And I think the bats were dead. It couldn't be just nothing was wrong because ... Well, them two things were wrong.
			$(A):(\sim B_x \rightarrow \sim A_x).$
	83(9:7)	0	Oh, I think it's quite different, because candles usually don't keep on going out. Maybe the water splashed on them. Maybe the bats didn't see the boys. Maybe they went to another cave earlier. Maybe it was windy in the cave. Maybe somebody is in there. I could have been. But I don't know. A lot of things seemed different.

		Score	
IV	9(9:9)	1	Well, if the candles went out it could be because it was a certain kind of air, if you light a match it goes out in a bottle. If the candles went out the air wouldn't be O.K. for the bats either. E. Could it be just a wind blowing out the candles? No, a wind wouldn't blow out the bats and make it all silent. I read about some kind of gas.
<u>CCO-I</u>			Recognition of a constant
I	13(9:1)	0	Tries to guess various points for spinner to stop (beyond five experiments).
	29(8:9)	1	It's stopping on the same one again! (alert attention; begins to manipulate the blocks.)
			Hypotheses
II	6(9:7)	1	One was heavy when I put them on. I'll see if that one does it.
			A∨~A.
	8(9:3)	1	It's a magnet. These two are magnets. I'll touch them to the spinner and see.
			B∨~B.
	17(9:8)	0	Someone is pushing a button. E. How could that be? I don't know. Up there somewhere I guess (looks at the ceiling) How can you find out? I don't know. (No indication of an intention to investigate).
		0	The board has got worn down with all the spinning. E. Could you find out? I could if I had a level. E. Could you find out in this room now? No, I don't see how. (No alternative suggestions or hypotheses).

		Score	Inductive inferences
IV	53(9:6)	1	It's these two stop it. Look, if I take these two away it won't just stop here I bet. [Experiment]. It's these two all right. E. Both of them? Why do you think they stop it? I don't know. Something inside them maybe. But this one feels like wood. E. So what do you think? It's these two. The others don't do anything.
	8(9:7)	0	It's a medium heavy and a heavy one beside it. E. Can you show that these two stop it? No, you can't do that.
V	24(9:10)	1	It's been stopping there for a long time. They could both be pulling on it or that could be a wooden one on the blue and a steel one on the white. The steel one could be a magnet and the wooden one could repel. I'm not really sure. I'll find if this is a magnet. [Experiment]. The steel one turns the spinner and the wooden one doesn't do much. I'll try the two blocks on the black and orange now .... $(A):(A_x \rightarrow B_x)$.
VI	14(9:4)	1	These two are magnets. [Touches each to spinner and then to each other]. No, this one doesn't do anything. Just this one is the magnet. E. Can you be sure just this one is the magnet? [Removes "the magnet" and touches each of the other seven blocks to the spinner]. They don't pull it. This one's the magnet, it pulls the spinner. $B \,\&\, [A \,\&\, T \rightarrow B] \rightarrow B$.
		0	It's a magnet, one of these is a magnet. Can you prove this? Can you show that one of these is a magnet? No you couldn't. But it's a magnet all right, because we learned about magnets in school.

APPENDIX E

TESTS OF PROBABILITY

Home From School

Mary got off the school bus and walked along the gravel road that led to her house. Suddenly she tripped and fell. Her books were scattered all over the road. She covered with sand, and her hands hurt. She got up, brushed herself off, picked up her books, and walked on.

In a few minutes a small green snake scuttled across the road in front of her. It disappeared in the tall grass at the side of the road.

As she passed the tall pine tree a flock of black crows settled noisily in its branches.

A few minutes later, Mary opened the kitchen door, and there was mother. She had baked a lovely pan of muffins.

Stories Tests of Probability

Preliminary questions, "Home from School"

What happened when Mary got off the bus?
What happened next when she tripped and fell?
What happened next as she walked along the road?
What happened after the snake scuttled across the road?
What happened next?

Test item I

Is it possible that each of these things will happen again just like this "tomorrow" when Mary gets off the bus?

Test item II

Why is that? Why do you think it is likely (not likely) these things will happen again like this tomorrow?

Test item III

Can you explain why things like this are likely (not-likely) to happen again like this the next day?

Test item IV

Is it possible that each of these things will happen again just like this one day "next week" when Mary gets off the bus?

Test item V

Why do you think it is likely (not likely) that these things will happen again just like this one day next week when Mary gets off the bus?

Test item VI

Can you explain why things like this are likely (not likely) to happen again like this one day the next week?
Why is that?

Concrete Tests of Probability

Random Distribution

Preliminary Questions

What color are the beads?
What color are they on this side of the partition?
This box can be teerered. It can be tilted this way and back again this way (the procedure is demonstrated once). If I teeter the box three times will I get the beads sorted into the two colors, one color on this side of the partition and one color on this side of the partition? Why is that?

Test item I

If I teeter the box ten times, am I likely to get a sorting of the beads into two colors, one color on each side of the partition? Why is that?

Test item II

If I continue to teeter the box for a long time, say, 1,000 times, am I likely to get a sorting of the beads into two colors, one color on each side of the partition? Why is that?

Quantification of Chance

Test item III (one dark and 2 light beads)

If I put the beads back in the box after each draw and shake the box how often would you expect to draw a "dark bead" in three tries? In six tries? Why is that?

Test item IV (one light and three dark beads)

If I shake the box (demonstrated) what color would you guess you would draw on the first try? Why did you guess that color? What chance have you of getting a light bead in one draw?

Uniform Chance Distribution

Preliminary questions

Where will the bar stop as you spin it?
Experiment: the subject turns the spinner. Were you right?
Where will the bar point next time? Why did you choose
________ and _________?
The questions and experiments continue until the subject indicates he cannot predict the colors at which the bar will stop, that is, until he recognizes that he is guessing.

Test item V

Five items assessing an understanding of uniform change distribution are presented. Four of the five items answered correctly is scored 1.

a) Can the bar stop "twice" in succession at the same color? Why is that?
b) Can the bar stop "ten" times in succession at the same color? Why is that?
c) If you played the game all afternoon could you learn to guess correctly where the bar will point each time when the spinner stops?
d) How could you do that (if the answer is "yes") Why is that (if the answer is "no").
e) Will the bar stop at all the colors as you keep spinning? Why do you think it will stop (not stop) at all the colors?

Test item VI

Will the spinner be "more likely" to stop the same number of times on all the colors if we spin it 16 times or 1,600 times? Why did you say _______ times?

Protocols

The following responses have been selected from the Concrete (CCO-P) and Stories (SCO-P) tests of probability reasoning to illustrate the scores assigned (1 and 0).

SCO-P		Score	"Tomorrow"
I,II,III	68(9:2)	1	No, because she was likely to remember about the rocks (laughs). E. What about the other things that happened? Um-m- it would be a hard thing for animals to do the same thing they did the next day. E. So, what do you think? Are things like this likely to happen next day the same way or not?
		0	M-m-m - they could. E. Why is that?
		0	Well, she could have forgotten about the rock.
	17(9:3)	1	No, because the snake might not cross the road and she might not fall, and the crows might not come, and mother might not bake muffins. E. Why is it these things might not happen again tomorrow?
		1	Well, because the snake will go away and the crows will not come.
		0	Well, I don't know -- like I said, the snake will go away.

SCO-P			"Next Week"
IV,V,VI	21(9:8)	1	No, because it would be very funny if the same things happened over again.
		1	Not very many things happen like that if it ever does.
		1	Well, uh, I don't think, well you get up and things are a little different -- and well -- every day's different a bit.
	59(9:6)	0	It might. I dont' know. The snake might be in the green grass ... she might trip again on the same thing. E. Why do you think it might happen like this again?
		0	Well, she could just start again and then .... E. Can you explain why it could happen again?
		0	The snake could've come back, and the crows ...

CCO-P			Irreversibility
I	59(1:6)	1	No, because some light beads will go with dark colored beads, they get mixed. They go down to the other end and they can come up either one side or the other.
II		0	1,000 times ... well, it will go down and just come back any way and it may take 1,000 times before it happens.

III	Score	Quantification
P (a/h):1/3 31(9:6)	1	Once out of three. Well there's only one dark bead in the lot. E. There are two light ones. Yes, but that's three.
P (a/h):2/6	0	Three times in six tries. E. Why is that? You have twice as many chances.
P (a/h):1/4		A dark blue, because there's more dark blues. E. What chance of a light blue in one draw?
	0	Not much of a very good chance.
<u>CCO-P</u> 31(9:6)		E. Where will it stop? Green and yellow. I just took a guess. E. Where next? Red and orange. E. Know for sure? (Experiment) No, Oh, it stopped there twice. E. (a) "twice on same color in succession? Yes, it did it before. (b) May .. be. It would be pretty hard! E. Can it stop ten times on the same color? Not really. (c) If you played it all day .. because there aren't very many colors? (d) and it could go on a different color and keep on going? (e) Yes, it can stop anywhere.
VI	1	1,600 times! 'Cause you'd have more tries and it would have to land on the same ones every time .. well not every time .. just every one.

APPENDIX F

PROTOCOL #45

PROTOCOL # 45

STORIES TESTS

Age of Child: 10:1

PMA "I.Q. Estimate": 122-133

CONSERVATION Funny Race	Score	
What did boys make with the wax?		Funny figures, etc.
Complexity 1 Substance 1		The same amount ... They said that they had the most wax.
That's what the children said but I don't really want to know what the children wondered because they may have been wrong. You, David, what do you think? (Repeat S-1).	0 0 0 0	I think it would be less than he had in the ball . Because the ball doesn't have a hole in it and the donut would have a whole in it.
Weight 2	1 1	About the same weight as the others. Because . . it would be about the same size as the other one.
The muff would be about the same size as the ball is that the reason?		Uhm hum .
Any other reason?	0	I can't think of any other reasons.
	1	I think about the same . Because the muff is pretty thick and the other . the two donuts would be about the same size.
Volume 3		
What makes water go up when you put something into it? Like donuts . or like wax? Yes . and the water goes up around it eh?		When you put something in the water. it just has to make room for it cause . the thing's harder than the water. Yes.
Would the two donuts take up the same amount of room as the muff? How could you be sure they would take up the same amount of room?	1	The two donuts would be about as thick as the muff and about as high.
O.K. Since they are so close in shape and size . is that it?	0	Yes.
Complexity 11		

Substance 1	1	The same amount of wax . If you rolled them up in a ball they'd both be about the same.
Would you give me the reason again that they were the same?	1	If you rolled them both up again they would be about the same size.
I see. And if you could put them back to the same size there would be the same amount of wax?		Yes.
Is there a reason you could think of that they would have the same amount of wax?	0	No.
Weight 2	0	I think it was lighter .
	0	Well because it was flat and everything and there was . it wasn't so thick.
Can you explain it another way? (pancake and jack-in-the-box)	0	Well, you couldn't feel it hardly on your head.

<u>Jimmy Feeds the Birds</u>

Preliminary Q.		Well . . . the birds that were around the cabin . . sparrows . . and chickadees and . .
Canada Jays		Yes. Jays.
How?		He cuts it up for the birds. All same.
Why?		Well so as to give the birds the same to eat.
Chickadees?		One big chunk and he nailed it to the tree.
Sparrows?		Little bits . . . on a stump of an old tree. On the ground.
Jays? What size?		Middle chunks, bigger than the sparrows bits.
And the crow?		He got half and Jimmy kept half.
Complexity III Substance I	1	I think about . the same . Because the pieces were the same but they were just cut up into little bits. and the other were just cut into middle-sized pieces.
How come they stayed the same when you cut them up into different sizes? You told me one reason was they started out the same. Is there any other reason?	0	I can't think of any other.
Weight 2	0	It'll get heavier.
	0	Because there would be a big piece and the others will be . in littler places.
	0	
Volume 3		I think it'd have less food than Jimmy 'cause Jimmy got his popped and there is more when it's popped and the crows just had the seeds.

Is there more actual food to eat when it's popped?		Yes.
How come? Where does the more food come from . It's bigger, isn't it?		Well the seed pops up into bigger pieces? Uhm hm.
Is there more food in it?	1	Uh . no.
Why?		Because it's the same thing . and uh . uh I don't know.
I asked if there was more food in it when it was popped . it looked bigger . but I said is there any more food there and you said "no" and why?	1	Well, it comes from the same thing but when it's popped it couldn't have more food . it would just be bigger.
Excellent.		
Complexity II		
Volume 3	0	Uh . it would be partly full. Because the pieces would be smaller and . than the others.
	0	and . it would be smaller pieces.
So it would just partly fill the box?	0	Yes.
Score conservation	11/27	

<u>CLASSIFICATION Ducks in Spring</u>

<u>Preliminary</u>		Pintsil, redheads, mallards. The pintail come in flocks of hundreds.
Pintail?		Oh . it's still pretty cold, there's ice still. They go to the fields . Yes . . but when the ice, it melts, then they can swim . ducks like to swim and dive.
How?		Oh . they get in on sloughs and ponds . . mallards . . teal.
Canvas back?		They wait for the ice to melt on the lakes.
How?		They dive.
<u>Hierarchical</u>		
Inclusion 1	0	More pintail . Because they are the first ones to arrive.
Inclusion 2	0	More ducks . Because ducks come before birds.
Are ducks birds?		Yes.
So the ducks come first?		Yes.
And so there are more of them?		Yes. uh no, there's not more ducks but they come back first. They come back before the other kind. the other birds.

Are there more ducks or more birds that fly back in the summer?	0	More ducks.
Why?	0	I don't know.
Inclusion 3		More animals. Because there is more different kinds of animals . insects and man an all those different kinds of animals and there's not that many kinds of birds.
Are ducks birds or not?	1	Yes.
Are birds animals?		Yes.
And there are more animals than birds?		Yes.
Why is that again?	1	Because there's more kinds of animals than birds and more kinds of mammals than birds, I guess.
More kinds of mammals than birds.		Uhm hm.
Would insects be animals?		Yes.
Quantification	0	No. Ducks are birds. They'd go too.
So?		
Predicates		
1		
What kinds?		Pintail . uh uhm mallard . I can't remember the other one. Oh, lots of kinds, I know now . . red-heads
Shoveller?		and teal and . . . yes . . . shoveller.
2	1	The pintail comes first . . . then the others come . . the mallard and teal.
Describe?		
3	0	Yes. The pintail and the mallard.
two lots?		
Matrix		
1		
What kinds		All kinds.
	1	Well . . . pintail on the ponds for sure.
and?	1	red-head and . . . shoveller . on the lake.
2		
So what kinds?		
He glanced at the story quickly and he had it.	1	The dive ducks . . . uh and pond feeders.

MULTIPLICATION City Long Ago.

What did people from river put out?		They put rafts in front of their house.

So you can go a little further and explain it? You've got it I think. because?		With the mallards back the . uhm . . shovellers have to be back because the teal and uhm the teal's backs too . the teals after the mallard and the shovellers after the teal. so the shovellers has to come after the mallard if the teal comes 'cause
so?		the teal has to come with the mallard.
and the mallard? the mallard?	1	The mallard are the first. If the mallard are back the teal are and if the teal are back the shovellers are back.
Very good. I think he made it - it was a hard one.		

DEDUCTION The Blizzard

Why let horses take lead?		Because they couldn't find the road.
Will the horses find it?		They might, but they might turn and go home.
They were pretty sure they couldn't and they thought the horses might. What way to reach the other side?		There was a cut.
If they go through the cut how know reach road?		They will see a farmhouse, and a road and get to the patient . Yes. The path doesn't ever . close or anything . even in a blizzard.
P 1	1	Yes. It's right at the end of the cut.
1 2	1	No. Because the banks are too steep.
1 3	1	No. Because if you find the cut the farmhouse will be right at the end of it.
P 4	1	Yes. Because there's so much snow round it . and ice around it so .
P 5		They may not find it.
MP 7	0	No. Because there's no bridge across the river.
MT 8	1	Yes. The farmhouse is right at the end of the cut.
MP 9	1	No. Because they have to go through the cut to reach the farmhouse.
MP 10	1	Yes. Because they can't get to the house without going through the cut.
MT 11	1	No. Because they have to go through the cut to reach the farmhouse.

UND 11	0	Yes. Because the road is on the other side of the cut.
UND 12	0	Yes. Because the houses are on the other side of the cut.
TRANS.13		Yes. Because if they find the farmhouse they find the road and the . and if they find the road they find the patient.
Yes. So?	1	
Score deduction	13/17	

<u>PROBABILITY HOME from School</u>

What happened when Mary got off bus?		She tripped and fell.
What see?		She saw cows.
No, a snake scuttled across road. Then she walked along and what did se see?		Cows.
No they were crows.		Oh.
Then got home and saw mother.		Baking muffins.
1 Why not likely?	0	Yes, it's possible . No-o . not likely. Maybe . maybe she just tripped and fell and scattered books in the road . and she might not see a snake next day . uh or the crows.
2 Why is it not too likely?	1	They don't usually happen at the same time . or in the same place.
3 Do you think it could happen? Can you explain it?	0	Some of them could. She could be careless and trip again. And the snake could maybe go the other way. But not just the same.
4 Next week?	0	Oh . no . Well it's possible.
5 Do you think it likely?		No.
6 Why not likely next week?	1	She wouldn't keep tripping every day or anything. all the time and scatter her books all over and see the same snake and the same crows or anything.
	0	
Score probability	3/6	

<u>INDUCTION: The Cave</u>

What frightened boys?		Well there was bats coming out . they were scared of the bats.

Why bring candles?		So they could explore where it's darker.
What difference this day?		Well the bats . nothing came out of the cave . it was silent . and they tried to light the candles but they just went out.
The chipmunk? (1)		Yes - s . . he wouldn't go in. I think . they were just imagining it.
Do you? Give me your explanation of the bats not coming out.		I think they just went away from the cave.
What about the candles going out? (2)	0	I don't know about the candles.
How do you explain that?	0	I don't know.
He can't explain the candles. He thinks the boys are just imagining it.		
Score induction	0/4	
Total score Stories	34/64	

CONCRETE TESTS

CONSERVATION

Complexity 1

Substance 1

	1	The same amount. Well. when you have them both rolled up on a ball and you put them on the scales they both weighed the same.
And you think they are still the same? What makes you think they are still the same after I make one of them into a donut?	1	
Weight 2		
Supposing I put them on the scales <u>now</u> will they still weigh the same or will one weigh more than the other?	1	Yes.
Why?		Well they still got the same amount of plasticine in them.
	1	I think they will still weigh the same.
Volume 3	1	Because there's the same amount of plasticine in each thing an . there's no less and no more.
	1	And if you put the same amount of things on there they will weigh the same.
	1	I think about the same. Because
	1	they both got the same amount of plasticine in them.

What makes water go up when I put something like a ball of plasticine to it?		Well, the plasticine is harder than the water and the water has to make room for it.
I see. So what . .		uhm uh well it's harder than the water and when you put it in it makes the water go up . cause the water . cause there's no room for water in the space where
That's very good. Thank you.		it goes in.
Complexity II		
Substance 1	1	There's about the same amount.
	1	If you cut it up and didn't take
It would stay the same, very good.	1	anything out of it.
Weight 2	1	I think that will be the same too.
	1	They were the same and you didn't
Volume 3	1	take anything out of it.
Very good.		
Complexity III	1	Well. they'll both be the same.
	1	Because they both got the same
Substance 1	1	amount of plasticine in.
Well it will be in the water still . it won't be the same. No.	1	No. yes . Because the little lumps of sugar will dissolve and . it will be in the water still but it will be .
	1	It won't.
		But there'll still be little crystals and uh.
And the sweet taste will be as sweet as it is now for instance.	1	uh. Yes.
Weight 2		It will be a bit . heavier.
It will be heavier tomorrow than it is right now?		Oh no, not
		Yes.
Will it be the same weight	1	
as we have right now?	1	Uhm . yes.
Why?	1	Because it's still got the same amount of . sugar in it.
Volume 3	1	It'll stay up the way it is.
	1	It will still have the little salt . <u>sugar</u> crystals in it.
And they'll keep it up?	1	Yes.
Very good.		
Score conservation	27/27	

<u>CLASSIFICATION: Hierarchical Structure</u>

	1	All squares and all circles.
Construction 1		

Construction 2	1	You'll put all the red squares in one . all the red circles in one . all the white squares in one and all the white circles in one.
Do that.		
Class Inclusion		
1	0	There's more round red ones.
E repeats 1		More round ones . Because there's more round ones under here than there is . uh square ones.
2	1	No . These three under here are white and the other ones are red.
3	1	Yes. You took the circles away.
4		Yes. Most of them are not some of them are . most of the red ones . are circles.
Yes . and so I . some of them are circles to . Did you say some of them were?		Yes.
Why could you say some of them were?	1	Because there's some red circles and some red squares.
5	1	No. Because there's white ones too.
6	1	No. There are squares red ones too.
7		Yes. Because the . a red one?
Repeats question No. 6		No. Because there's square red ones too.
Repeats question No. 7	1	No. because . oh yes!! It will! Because you took the circles away.
8	1	Yes. Because the white round ones are gone.
Predicates		
1		The small whites and the two small reds and the two large whites and the two large reds.
So?		He's got them in a good order.
How do these belong together?	1	Both red small ones.
These are?		Large ones.
2	1	All the white ones are together and all the red ones are together.
Pattern II		
3	1	You got all the circles on one side and all the squares on one side.
Very good.		
Matrix		
1	1	Square red ones square white ones round red ones round white ones.

Go ahead. o.k. what have you got now? You've got - -		red squares white squares red round white rounds
2	1	Yes. Cause all the white ones are on one side and all the red ones are on one side.
3		Yes. Because all the red squares and the white squares are on one side . and all the circles . all the red circles and all the white circles are on the other side.
Very good.		

<u>Class of intersection</u>

David has chosen a large white circle. Why?	1	Because the large white circle will go . will go with the large squares. and it will go with the circles.
Show me another thing that's just as good as that. That one's correct. He's chosen a small white square. How does it go with these?	1	It's a square.
And this one?	1	They're all small.
That was correct.		
Score classification	18/19	

<u>DEDUCTION: "Village" tests</u>

MP 1	1	Yes, cause that's the only road to take and that house is right in front of the road.
UND 2	0	No, cause he didn't go on the road then.
UND 3	1	He might have went to the church or school or something. The post office isn't the only place there in the village.
MT 4	1	No, cause he has to go past the house to get into the village.
P 5	1	Yes, he has to pass the house to get there.
NP 6	1	No, because they're not allowed to walk on the fields around the village and the roads the only place to take to the village and he has to go past the house to get there.
P 7	1	Yes, cause there's more than one building in the village.
P 8	1	Yes, because he has to go past the house to go into the village.

"Lights"

Say the rule:			If the red light's on the green light's on.
	MP 9	1	It's on. Because the rule says if the red light's on the green light's on.
	UND 10	1	It's out. Because the red light's . oh it could be on too. Cause it doesn't say the red light would be on if the green light was on. So maybe the green light could be on. maybe it isn't on.
	UND 11	1	It could be on and it could be off, because it doesn't say if the green light's on the red light's on.
Very good.			
	MT 12		It's out. Because if the red light's on the green light's on too.
But?			But if the green light's on the red light doesn't have to be on.
Now just a minute. The green light's out. What about the red one . you said it was?			Out . uh it could be out . maybe it's out and maybe it isn't.
Think it over again. Go back to the rule.			
Uhm hm. so?		1	Uh . it's out! Because if the red light's on the green light has to be on too.
Very good. Why?			So the . red light's not on.
	P 13		The . green light's not on.
It does mean that if the red light's on the green light's on . that's all it means.		1	Yes. Because the rule says if the red light's on the green light's on. That means both lights are on at the same time.
	NP 14	1	No. Because the rule says that if the red light's on the green light has to be on.
	P 15	1	No. Because if the red light's on the green light has to be on and . the green light won't be on when the red light's not on.
No-o . that's not the rule quite . What's the rule again?			When the red light's on the green light's on.
That's all the rule I have. Now can I have the red one off and the green one on?			No . . Yes!
How come?			Because it doesn't say that . uh if the green one's on the red one's on.
o.k. with out this one? Very good.			You could have the green light on just.
	P 16		Yes.

ah . are they both off? 1

Yes. Because uhm . well if the green light . the red light's on the green light has to be on but if the red light's off the green light's off too and . yes.

That's good . that's the reason.

TRANS. 17

What David has chosen is the red square.
Could you explain why you chose that?

I've already got two . a large white one and a small white one together and a medium sized red and a small white. and uh it's a large white medium sized red.
Yes.

Will go together?
You're doing well on that David.
Take you time . I'd like to have it all on tape all your thinking about it . Now say it again, please. I want to know why you chose the red one . as clearly as possible. Explain fully. 1

Well - I because these . because there's a large white one a small white and a small white and a red medium-sized.
So here the large white one can have a medium-sized red one.

Very good. I think David's getting that.

Score deduction. 16/17

PROBABILITY: Random distribution

If I teeter the box 3 times will I get one color on each side of the partition?

No because the beads wouldn't all go over to one side just because you're pushing it back and forth, they'd mix up again.

10 times 1 1

No, the same reason as before . if you keep doing it they can't all go to one side, they'll keep going to both sides, different colors.

1000 times 2 0

In 1000 teeters . because well . . um . m . a . 'cause the more times it goes back and forth you can get more the same color of beads each side.

QUANTIFICATION

Two light and one dark . which?

A light blue . cause more light blue than dark blue ones.

3

One dark and two light?
How often a dark one in 3 tries?

About once.

6 tries? 1

I think . . . maybe about two times.

What did the people put out who came through the forest?

They put out treasures from their old house.

All kinds of things. What color did they paint them?

Bright yellow.

1

That's two things. They want to put out one thing. What?

They could put a yellow treasure and a raft.

They are not so interested in showing where they came from, what they want to show is that they belong to the street with the rafts and that they belong to the street with yellow treasures now that they're here. How could they do that?

If they came from across the river they should put a raft and if they came from . through the forest they could put one of those bright yellow treasures.

Well - I . .
No . . .

Well he says he doesn't know.

David is suggesting that they set out something depending on where they came from and that isn't so good because they want to show they belong to the two streets.
How can they show that, etc.? 0

No. I don't see how. They'd have to put out two things..

No, he can't get that one.

Score classification 6/11

<u>DEDUCTION: Ducks in Spring</u>.

1 Modus Ponens 1

Yes. Because that's when most of the ducks are here. Because the last duck comes when the ponds aren't frozen or anything.

2 Undetermined 0

Uhm . yes. Because the pintails come when there is still snow . it says here.

3 Undetermined 1

Uhm . it depends on what kinds of ducks.

What do you mean?

Well if there's just pin-tails there wouldn't be . it would still be frozen . and if there is some pond-feeders . uhm . dive ducks they wouldn't be frozen.

Very good.

4 Modus Tollens 1

No. Because some kinds of ducks don't come when . uh . there's still snow or ice.

5 Transitivity

Yes . Well, 'cause the birds that come after the mallard . the bird that comes before the shovellers . uhm . I mean yah . comes before the birds that comes before the . uh . uh . mallards.

4		
One light and three dark.		A dark blue, there's dark ones
Which draw?		than light.
What chance of a light red?	0	About 1000 to 1.
Uniform distribution		
5		
Where stop?		On orange and red.
Where next?		Black and purple.
Where next?		Black and purple.
Can you tell where stop?		No.
a) Twice		Yes.
b) Ten?		I guess so, cause it can stop wherever it slows down.
c) All day?		I don't think so, you don't know where it stops you might when it starts slowing down, but when
d) Why?		you just spin it you couldn't.
e) All colors	1	Mm. I guess it will . yes . I think so.
6		16 times. Well it couldn't stop on same color 1600 times!
Examiner gives explanation.	1	Oh . 1600 times. You'd have a
Repeat 6		better chance.
Score probability	4/6	

INDUCTION

Where stop?		I think it's going to stop on black and purple.
Right?		No . red and orange.
Where next?		- and -
Right?		No it stopped on red and orange again.
Where next?	1	Red and orange.
Why choose red and orange?	1	Well, there's a magnet there.
Why think so?	1	(David picked up the block on the red). The spinner follows the block. The spinner's a magnet if it turns when I move the block.
What prove?	1	The spinner turns when block turns. Look. Both ways it turns.
Can you be sure?		(He picks up blocks on green and yellow).
	1	The spinner doesn't go around with them. The spinner <u>isn't</u> the magnet. The <u>block's</u> the magnet.
What about the others?	1	(He tests all the other blocks against the spinner). Just the block on the red is the magnet. The rest don't do a thing.
Score induction	6/6	
Total score Concrete	71/75	

APPENDIX E

Cooperative Sequential Tests of Educational Progress, Reading, Form 4B

Form 4 B

Cooperative

Sequential Tests of Educational Progress

Reading

Cooperative Test Division • Educational Testing Service • Princeton, N.J. • Berkeley 4, Calif.

General Directions

This is a test of some of the understandings, skills, and abilities you have been developing ever since you first entered school. You should take the test in the same way that you would work on any new and interesting assignment. Here are a few suggestions which will help you to earn your best score:

1. Make sure you understand the test directions before you begin working. You may ask questions about any part of the directions you do not understand.
2. You will make your best score by answering *every* question because your score is the number of correct answers you mark. Therefore, you should work carefully but not spend too much time on any one question. If a question seems to be too difficult, make the most careful guess you can, rather than waste time puzzling over it.
3. If you finish before time is called, go back and spend more time on those questions about which you were most doubtful.

DIRECTIONS FOR PART ONE

Each of the questions or incomplete statements in this test is followed by four suggested answers. You are to decide which one of these answers you should choose.

You must mark all of your answers on the separate answer sheet you have been given; this test booklet should not be marked in any way. You must mark your answer sheet by blackening the space having the same letter as the answer you have chosen. For example:

0 Which one of the following is an animal?
- **A** Bed
- **B** Dog
- **C** Chair
- **D** Box

Since a dog is an animal, you should choose the answer lettered **B**. On your answer sheet, you would first find the row of spaces numbered the same as the question—in the example above, it is **0**. Then you would blacken the space in this row which has the same letter as the answer you have chosen. See how the example has been marked on your answer sheet.

Make your answer marks heavy and black. Mark only one answer for each question. If you change your mind about an answer, be sure to erase the first mark completely.

PART ONE

When Ben Franklin was a boy, he raced home one evening very late for supper. It was a holiday, and he had gone swimming with a friend.

Bursting into the big kitchen where his father was reading, he cried, "Father, I had a rare good time today. When I tired of swimming I made the wind tow me across the pond."

Ben's father asked smiling, "How did you persuade the wind to help you, my son?"

"First," explained the boy eagerly, "I flew a big kite, tied to a big stick. When it was pulling strongly on the stick, I threw myself into the water on my back and held the stick in both hands behind my head. The kite was a sail and I was a boat. The wind took me clear across the pond."

That game was the first of Ben Franklin's many experiments, inventions, and discoveries. They were all like games because he made them only for enjoyment. He always said he wasn't a scientist. Yet when he proved that lightning was the same force as electricity, every scientist in the world honored him. Neither did he think of himself as an inventor. He made a great many new things, but he never dreamed of making money from the things he created.

1 What did the wind do?

- **A** Made lightning come.
- **B** Made Ben get tired.
- **C** Took the stick out of Ben's hand.
- **D** Took Ben across the pond.

2 Why did Ben have a chance to go swimming that day?

- **E** His father was reading.
- **F** He wanted to fly his kite.
- **G** It was a holiday.
- **H** It was a warm day.

3 Ben said that he and his kite were like

- **A** a father and son
- **B** a boat and a sail
- **C** a stick and a string
- **D** the wind and the water

4 The writer wants us to know that, even as a boy, Ben Franklin was

- **E** inventing things
- **F** reading to his father
- **G** tired of swimming
- **H** interested in water

5 People are likely to enjoy reading this story because

- **A** it shows how Ben Franklin's father helped him to become an inventor
- **B** it proves that lightning is electricity
- **C** it shows us how Ben Franklin became rich
- **D** it shows us how Ben Franklin enjoyed inventing

"Hello, Johnny!" Lono greeted. "Ready to see the Barking Sands?"

"Hi, Lono!" replied Johnny. "Is there really a strange beach that barks like a dog when you step on it? Lono, I think you are joking with me!"

Lono laughed. "Just wait and see, Johnny." Lono liked to show his beautiful beach to visitors from the United States.

They walked through the trees and passed the sugar mill. Then they climbed a hill. From the top of the hill they could see the sandy beach below them.

"There it is, Johnny," Lono proudly said, "the beach of the Barking Sands. It will bark like a dog when you step on it."

"It looks just like the other beaches to me, Lono," Johnny said as they ran down the hill to the edge of the Barking Sands.

Lono's brown eyes were bright with happiness. "Step out on the sand and see for yourself, Johnny," he told him.

"Oh-h-h-h, no!" Johnny replied. "This is too strange for me; besides, I think you're joking!"

Lono laughed because he could see that Johnny was a little frightened of the Barking Sands.

"All right, Johnny," he said. "I'll go first. Now, listen and tell me what you hear."

Lono stepped out upon the beach, but the sand did not make a sound!

"I don't hear a thing, Lono," Johnny said.

6 Johnny's home is
- **E** in the United States
- **F** on the beach
- **G** in the same house as Lono
- **H** on a farm

7 Why does Lono tell Johnny about the Barking Sands?
- **A** Lono is afraid of the beach.
- **B** Lono is brave enough to step on the barking sands.
- **C** Lono is sure the sands will bark.
- **D** Lono is playing a trick on Johnny.

8 Why does the writer tell us that Johnny was NOT willing to step on the beach?
- **E** To prove that Lono was brave
- **F** To show that Johnny was a little frightened
- **G** To show that Johnny did not expect the beach to bark
- **H** To make us think that the sand would bite

9 Does the story make you sure that the sands really bark?
- **A** No, because Johnny wouldn't walk on the sands.
- **B** Yes, because both Lono and Johnny thought so.
- **C** Yes, because Lono seemed so sure that the sands would bark.
- **D** No, because the sand did not make a sound.

10 The writer tells you this story
- **E** mostly by talking himself
- **F** by writing you a letter
- **G** mostly by letting the boys talk
- **H** in a poem

Go on to the next page.

The Clinker

When the ship drives on through the tumbling sea,
And speeds through the darkest night,
With the steady wash of turning screws,
That drive her in her flight;
And you, in your bunk or up on deck,
Have naught to do but ride,
Do you ever think of the watch below,
Have you ever thought what drives her so,
Or have you never tried?

Do you ever picture the turning wheels
Or flashing rods of steel,
Or hissing steam, or scorching heat,
'Way down there near the keel;
Do you ever think of the black stoke-hold,
And its sweating, straining crew;
Do you ever think of the flaming bed,
In that gaping maw that must be fed,
Or is it strange to you?

Do you ever picture the dusty "heave"
Who toils in the bunkers' gloom,
Where the air is dead and clogged with dust,
'Mid silence of the tomb?
Do you pity the clinker who struggles along
With no complaining sound?
Well, if you do, don't say it aloud,
He wants no pity—the boy is proud,
He's making the wheels go 'round.

11 What does the clinker do?

A Steers the ship.
B Makes the wheels go 'round.
C Scrubs the deck.
D Plans the trip.

12 The words "air is dead" mean

E the hold is quiet
F there is no light
G the bunkers were silent as a tomb
H there is no movement of the air

13 The poet wants you to

A understand how the ship is run
B know how many men work below the deck
C go down in the hold and thank the stoker
D think about the work of the men below

14 How does the poet try to keep you interested?

E By describing the sea
F By telling an exciting story
G By asking many questions
H By telling what the captain does

15 Which of these lines tries to make the ship's passenger feel ashamed of himself?

A "That drive her in her flight" (line 5)
B "Or have you never tried" (line 10)
C "Do you ever picture the turning wheels" (line 11)
D "He's making the wheels go 'round" (line 28)

There are several ways to score in a football game, but the main idea is to make a touchdown by carrying the ball over the other team's goal line. That's worth six points.

Now, to start a game, one team kicks off. Let's say Terry holds the ball for Bunky. He sets one end on the ground and holds the top end to steady it. Bunky takes a run and kicks the ball as far as he can.

You're on the other team, Peter, and you catch it. You run back toward Terry and Bunky as fast as you can. You try to dodge them and run all the way for a touchdown. But probably somebody tackles you and gets you down.

The next play starts from where you went down. And you have four chances to gain ten yards.

The job of the players in the line is to bump the tacklers out of the way, without using their hands, and clear a path for the player in the backfield, who is carrying the ball.

There are three ways to gain ground:

By running through the line,
By running around the ends of the line,
By throwing a pass to one of the players in the backfield or to one of the players on the ends of the line.

If you can't make ten yards in four plays, the other team gets the ball.

16 Why did the writer first tell about making touchdowns?

E Everyone makes touchdowns in football.
F That is the main idea of the game.
G That is the way the game starts.
H That is the way the game ends.

17 Bunky is playing

A on the same team as Terry
B on the same team as Peter
C against Terry
D against Peter and Terry

18 The job of a player in the line is to

E tackle the runner without using his hands
F throw passes to players in the backfield
G bump tacklers out of the way of the runner
H carry the ball

19 How does the writer help us learn about gaining ground?

A He tells about three ways to gain ground.
B He has Terry gain ground.
C He says it is easy to gain ground.
D He shows how the tacklers gain ground.

20 The writer does NOT tell you

E the job of the players in the line
F the number of players on a team
G the ways to gain ground
H the job of the players in the backfield

Any cut or scratch should be kept perfectly clean so that it will heal rapidly. For a small cut or scratch, pure water is a good thing to use. Put the cut hand or foot in a basin of cool water or under a faucet and let the water wash out the wound. Running water from a faucet is best. If the water is not pure, it should be boiled before it is used on any open wound.

If the wound is large and there is not a very great flow of blood, it is best to let the cut bleed for a little while. The flowing blood will help wash out dirt and germs.

To dress or bind up the wound, cover it at once with sterile gauze or a clean white cloth. If nothing else can be found, use a clean handkerchief. For a small wound, tear off a long strip one to two inches wide. Roll this strip around the hand or foot three or four times and then around the wrist or nearest joint to keep the bandage from slipping.

Even after a wound has been washed, there may be germs on it. A good way to check the growth of germs is to paint the wound with mercurochrome or with a mild solution of iodine, and then bind it. If iodine is used, let it dry before the wound is bound, in order to prevent blistering.

If a wound still has dirt in it after it has been washed, the safest thing to do is see a doctor at once.

21 This story is mostly about

A how to take care of a cut
B the dangers of cuts
C how to bind a cut
D how to stop bleeding

22 If you let some cuts bleed for a minute or two, the blood

E is hard to stop
F helps clean the cut
G cannot be stopped
H may carry germs into the cut

23 The writer makes taking care of a cut seem

A like something only a doctor should do
B too hard for most people to do
C like something only a grownup could do
D important for you to know about

24 In treating a small cut or wound, you should first

E clean it as well as possible
F put on medicine and a bandage
G bind it with strips torn from a handkerchief
H cover it at once with sterile gauze

25 The writer does NOT tell you

A anything about large cuts
B how to take care of small cuts
C how to stop the bleeding of a large cut
D how to keep the bandage from slipping

Go on to the next page.

A law to provide for moving the Indians away from their own land was introduced in Congress. This law was hotly debated by the congressmen. Davy Crockett listened to the statements made by both sides. He got to his feet and demanded the right to be heard.

"From what I have heard here I am led to believe that some of you do not like Indians. I have good cause for not liking them myself. My grandfather, David Crockett, was killed and scalped in a fight with the Indians. I fought against the Indians in the Creek War. I was just as determined then as any man that the power of the Indians should be broken. But times have changed. The Indians have changed, too. I know them. Many of them are now my friends. They are friendly to the whites, and they lead peaceful lives. They have adopted the ways of the white man. I am against removing them from their homes."

"You are not representing the wishes of your people," a congressman shouted.

"Maybe not," said Davy, "but I am representing my own conscience. There is a question of right or wrong in this law. I stand for what I know to be right. My vote against the law may cost me my seat in Congress. But I will vote against the law as it now stands.

"Let me tell you about the Indians from whom you want to take the land. Let me tell you about the land to which you want to send them.

"These Indians are no longer hunters and warriors. They have become farmers. The land on which they now live is fertile and can be farmed. You want to take them from this fertile, easily farmed land and send them to the western plains. If you will give them land as good for farming as the land you intend to take from them, I will vote for this law. If you plan to give them poorer land, I will vote against the law."

26 Before he spoke, Davy
- **E** heard what both sides had to say
- **F** voted against moving the Indians
- **G** talked the problem over with other white men
- **H** talked the problem over with the Indians

27 What is the main thought of the second paragraph?
- **A** Davy didn't like Indians because they had killed his grandfather.
- **B** Davy liked Indians because they needed a friend.
- **C** Davy used to dislike Indians, but now he wanted to help them.
- **D** Davy didn't want the Indians to remain on their own lands.

28 This story makes us think that Davy Crockett
- **E** was a great Indian fighter
- **F** wanted the right thing to be done
- **G** had always liked Indians
- **H** wanted to repay the Indians for wrongs done them

29 "I am representing my own conscience" (lines 23 and 24) means
- **A** "I don't care about what you think"
- **B** "I am doing what I think is right"
- **C** "I think this bill is a bad one"
- **D** "I have my own ideas on this subject"

30 Why did some congressmen want to move the Indians?
- **E** The Indians were hostile.
- **F** The Indians were farmers.
- **G** The Indians were living on poor land.
- **H** The story doesn't tell us, but we can guess.

Go on to the next page.

My dear Mary,

I am very glad you like the new copy of *Alice's Adventures,* and I should like very much to come and see you all again, and "Snow-drop," if I could find the time, which I can't at present. But, by the bye, it's your turn to come and see me now. I'm sure I called last. My room is very easy to find when you get there, and as for distance, you know—why, Oxford is as near to London as London is to Oxford. If your geography book doesn't tell you that, it must be a wretched affair, and you'd better get another.

Now I want to know what you mean by calling yourself "naughty" for not having written sooner! Naughty, indeed! Stuff and nonsense! Do you think I'd call myself naughty, if I hadn't written to you, say for 50 years? Not a bit! I'd just begin as usual "My dear Mary, 50 years ago, you asked me what to do for your kitten, as it had a toothache, and I have just remembered to write about it. Perhaps the toothache has gone off by this time—if not, wash it carefully in hasty pudding, and give it 4 pincushions boiled in sealing-wax, and just dip the end of its tail in hot coffee. This remedy has never been known to fail." There! That's the proper way to write!

I want you to tell me the last name of those cousins of yours (I think they were) that I met one evening at your house. Mary and May were their first names. Also please tell your Papa I have read Alec Forbes, and am delighted with it, and I very much want to meet Annie Anderson in real life. Where does she live?

With kindest regards to your Papa and Mama and best love to your brothers and sisters, I remain

Your loving friend,
Charles L. Dodgson

31 The writer of this letter says that

A he wants Mary to visit him next
B he is coming to see Mary soon
C he visited Mary a few days ago
D it is his turn to visit Mary

32 In lines 8 to 13 the writer is

E angry at Mary for not visiting him
F joking
G giving Mary directions to find his room
H telling Mary her geography book is not good

33 In line 30 the writer put the words "I think they were" between these marks ( ) to show that

A he wasn't sure that Mary and May were really Mary's cousins
B Mary and May are first names
C Mary and May are the real names of Mary's cousins
D Mary and May are last names

34 Why does the writer choose "50 years" in line 20?

E Because Mary is 50 years old
F So that Mary will know she waited too long to answer
G So that Mary will know he is joking
H Because the writer is 50 years old

35 If the writer of this letter were to write a story for children, what kind do you think it would be?

A A very serious story
B A story about geography
C A funny or fanciful story
D A story about explorers

STOP **Stop. If you finish before time is called, check your work on this part. Do *not* go on to Part Two until you are told to do so.**

DIRECTIONS FOR PART TWO

Part Two contains the same kind of material as Part One. Mark your answers in the same way.

PART TWO

One day at playtime it was raining. We were playing inside. Dick was looking at the eggs. "Look!" he cried, "something is coming out of our eggs."

"What are they, Miss Parker?" we cried.

"What do you think they are?" asked Miss Parker.

"Worms," said Dick.

"Caterpillars," said Jimmy.

"They are called silkworms," said Miss Parker. "They are really caterpillars."

"Do caterpillars like these make silk?" asked Nancy. "Was the silk in my dress made by silkworms?"

"Yes," said Miss Parker. "If we feed these caterpillars, they will grow. They will make silk."

"We must feed them," said Susan. "What will they eat?"

"They eat white mulberry leaves," said Miss Parker.

"I'll get some mulberry leaves," said Dick.

"May I have some of the worms for my own?" asked Jimmy. "I would like to take some of them home in a box."

Then Miss Parker said, "You may have these. Be sure to feed them plenty of mulberry leaves."

1 Silkworms eat
- A caterpillars
- B insects
- C silk
- D mulberry leaves

2 Who said that silkworms are really caterpillars?
- E Miss Parker said so.
- F Jimmy said so.
- G Dick said so.
- H No one in the story said so.

3 When something came out of the eggs, the children
- A knew they were silkworms
- B did not care what they were
- C did not know they were silkworms
- D were afraid to go near the silkworms

4 The children asked questions about the worms. This shows that they
- E liked Miss Parker
- F wanted to know more about the worms
- G wanted to play outside
- H wanted to take the worms home

5 The writer has Nancy ask about her dress so we can find out that
- A all dresses are made by worms
- B every girl should have a silk dress
- C Miss Parker had on a silk dress
- D some dresses are made of silk from worms

Dear Henry,

I got your letter yesterday, and it must be fun playing in the snow. I miss it because we don't have any snow here. But guess what I did yesterday. We went swimming in the ocean. Can you beat that—swimming in the middle of winter?

Jimmy and Bob went with me. They are two friends I met in school. The school I go to here has a much bigger playground than the one at Westview. It is all dirt and grass, and not concrete. It has swings, slides, rings, bars and seesaws, but I just go on the swings. The little children like the slides and seesaws best because they don't know how to pump when they swing, so someone has to push them.

What have you and Don been doing? I miss you and all the kids because we used to have fun together. My mother and dad say that I might be visiting you during the summer. Write soon and give me the news.

Your friend,

Al

6 What does Al tell us he did yesterday?
- E Played in the snow
- F Went on the swings at school
- G Went to school with Jimmy and Bob
- H Went swimming in the ocean

Go on to the next page.

7 From what we are told, where would you guess that Henry lives?

A He lives on a farm.
B He lives in the South.
C He lives in the North.
D He lives in the same place as Al.

8 What kind of letter is this?

E A business letter
F A friendly letter
G A thank-you letter
H A letter of invitation

9 Al is writing this letter because

A he is lonesome
B he and Don have gone swimming
C his mother told him to
D Henry wrote to him

10 Al tells about the swings, seesaws, and slides because he wants Henry to know that

E his school has a nice playground
F he is a big boy
G he doesn't like to swing
H little children don't like swings

NEW BOOKS FOR SPRING READING

A new book by Mary and Conrad Buff always means a book so beautiful that it is worth owning or giving as a gift to a friend. *Hurry, Skurry, and Flurry,* by Mary and Conrad Buff is illustrated by Conrad Buff (Viking Press, $2.75). Hurry, Skurry, and Flurry are three baby squirrels who were born in the spring when everything in the woods was fresh and new. For seven weeks they grow and play on the branches of the tree. Then summer comes and they go out into the forest and meet other forest folk. One of them is Dash, the great handsome buck, the king of the forest. It is the same little Dash you knew as a fawn in *Dash and Dart* by Mary and Conrad Buff. There are beasts that are not so friendly, such as the bear, the hawk, and the mountain lion.

Fall means golden leaves on the black oak trees and plenty of food for the forest folk. Hurry finds a new friend in a young squirrel named Silver. They spend the winter in a warm, snug hole high up in a pine tree. There they wait for the coming of spring and their new family.

11 Which animal was friendly to the squirrels?

A The bear
B The buck
C The lion
D The hawk

12 What does the story tell us about the squirrels' mother?

E She is not in the story.
F She spanked them when they were naughty.
G She went away and left them.
H She taught them to jump.

13 The writer tells us about the life of the squirrels

A till they go out into the forest
B from spring to summer
C till the time Hurry met Silver
D from one spring to the next

14 Why does the writer tell you about this book?

E She likes squirrels.
F She likes to read children's books.
G She thinks it is a good book for you.
H She liked reading the story.

15 Hurry, Skurry, and Flurry are good names for squirrels because squirrels

A usually run fast
B are friendly
C play on the branches
D live in the forest

Go on to the next page.

JONATHAN BING

Poor old Jonathan Bing
Went out in his carriage to visit the King,
But everyone pointed and said, "Look at that!
Jonathan Bing has forgotten his hat!"
(He'd forgotten his hat!)

Poor old Jonathan Bing
Went home and put on a new hat for the King,
But up by the palace a soldier said, "Hi!
You can't see the King; you've forgotten your tie!"
(He'd forgotten his tie!)

Poor old Jonathan Bing,
He put on a beautiful tie for the King,
But when he arrived an Archbishop said, "Ho!
You can't come to court in pajamas, you know!"

Poor old Jonathan Bing
Went home and addressed a short note to the King:
"If you please will excuse me I won't come to tea;
For home's the best place for all people like me!"

16 Jonathan Bing did NOT forget
- **E** his hat
- **F** his tie
- **G** his trousers
- **H** the tea

17 How do you think the poet felt about Jonathan?
- **A** He was angry with him.
- **B** He was bothered by Jonathan.
- **C** He felt sorry for him.
- **D** He disliked Jonathan.

18 Jonathan's biggest problem was
- **E** getting a carriage
- **F** remembering things
- **G** passing a soldier
- **H** passing an Archbishop

19 The note that Jonathan wrote
- **A** does not tell what happened to him that day
- **B** will make the King angry at the Archbishop
- **C** was too long
- **D** asked for another invitation

20 Why does the poet tell you about Jonathan's forgetting so many things?
- **E** To show that he felt sorry for Jonathan
- **F** To make a sad story
- **G** To show you how mixed up Jonathan was
- **H** To make a longer poem

Go on to the next page.

There are some boys and girls with whom everyone likes to play. It is fun to play with them because they have a good time whether they win or lose. If you win a game from them, they want to play against you again to see if they can defeat you the next time. They are called "good losers."

Not long ago Stanley and his baseball team played a game with Richard and his team. The teams were quite evenly matched, but Richard's team made two home runs and won the game. Richard and his team enjoyed the game so much that he asked Stanley to play another game with them the next week.

Stanley said he did not want to play again. He did not think the game had been a fair one because Richard had some larger boys on his team, and they had practiced more than his team had.

Richard and his team left. They knew that what Stanley had said was not fair, since the two boys who were larger were no older than the other boys. As for practicing more, of course they had practiced so that they would be able to play well against Stanley's team and other teams.

The next week, Stanley talked with Frank and arranged a game.

Frank's team was badly defeated, but before they left, Frank got his team together and they gave a good cheer for Stanley's winning team. Frank got a promise from Stanley that he would bring his team to the Lincoln School grounds the next week for a return game.

"That was a good game!" exclaimed Frank. "Wouldn't it be great if we could win next week?"

21 A "good loser" is someone who

A doesn't like to win
B always loses
C doesn't care if he wins or loses
D has a good time whether he wins or loses

22 The writer does NOT tell you

E why Frank wanted to play again
F how Richard would feel about losing
G how Richard would feel about winning
H why Richard's team won

23 The writer tells this story because he wants you to

A be a good sport
B practice before playing a game
C play harder to win
D be like Richard

24 Why did Stanley's team lose to Richard's team?

E All of Richard's team were bigger boys.
F Richard's team practiced more.
G Stanley was a poor player.
H Stanley was a poor loser

25 The writer shows you what a good loser is by showing the difference between which two boys?

A Frank and Richard
B Stanley and Richard
C Frank and Stanley
D None of the above

Go on to the next page.

FATHER: Bill, what's in that box?

BILLY: A new hat for you, Dad. Say, you've found your old hat!

FATHER: Yes, it was at the top of the closet.

BILLY: Oh, well, then I can take this one back.

MOTHER: You can do no such thing. (*Taking box from Billy*) Why, it's a beautiful hat.

FATHER: But I don't like it. You shouldn't have brought it home.

MABEL: But, Dad, you simply can't wear that old hat now. It's all dusty.

FATHER: Dusty? I don't see any dust. It's as good as new.

MOTHER: Now, John, try on your new hat.

FATHER: That's not my hat.

MOTHER: Don't get so excited. (*Mother dents the top of the new hat, leaving the brim down in front.*) Now try it this way. You're more the sporty type.

FATHER: (*He puts it on, and then turns.*) Well, here I am all ready for the races.

BILLY: Gee, Dad, you look awful.

FATHER: Son, I look fine. It's this hat that looks awful.

MABEL: Oh, Dad, let me try. Some of the boys at school wear their hats this way.

FATHER: Very well. (*Puts hat on; then turns with a silly grin*) Even I have to laugh at this one. I look like a gibbering idiot. Any more suggestions?

MOTHER: I just don't know what's wrong. We've tried creasing it in every possible way.

FATHER: Every possible way except the right way.

MOTHER: What do you mean by that?

FATHER: I'll show you. (*He pushes the sides together and squeezes the hat between his knees. The final result looks just like his old hat.*) How's that?

JANE: Why, Dad, you look like yourself again.

MABEL: I know, it's the casual look.

MOTHER (*Smiling*): Well, whatever it is, it's very nice.

BILLY: Gosh, Dad, it looks just like your old hat.

26 What do you think happened before this play began?

- **E** Father lost his hat.
- **F** Billy found Father's hat.
- **G** Father bought a hat.
- **H** Mother threw Father's hat away.

27 How did Father feel about the two hats?

- **A** He liked his new hat better.
- **B** He asked Billy to buy him a new hat.
- **C** He still liked his old hat.
- **D** He didn't like the two hats.

28 The play tries to make you feel that

- **E** the family likes Father the old way after all
- **F** everyone likes Father's new hat
- **G** Father likes his new hat
- **H** only the children want Father to wear a new hat

29 At the end of the play, the writer wants you to believe that Father

- **A** was angry about the new hat
- **B** knew how he looked best
- **C** needed a new hat
- **D** was the sporty type

30 Why does the writer put the name of a person at the beginning of almost each line?

- **E** To show you what each person does
- **F** To let you know what each person is thinking
- **G** To tell you who is in the play
- **H** To tell you who is talking

Go on to the next page.

(1) Edison was one of the greatest inventors who ever lived. He is best known for his invention of the electric light, the phonograph, and moving pictures. You probably will be surprised to learn that Edison also invented waxed paper and gummed paper tape for wrapping packages.

(2) It has been said that Edison's greatest "invention" was really a method of inventing. He was the first to see the value of having many trained engineers working together like a team on scientific problems. With the money he received from businessmen and from his own inventions, Edison was able to hire large staffs of scientists and to direct their work toward new discoveries. Today there are hundreds of such laboratories in the United States where trained scientists are working together in the cooperative way that Edison pioneered.

(3) What was the secret of Edison's great inventive genius? He himself once answered this question by saying, "Genius is simply hard work—ninety-eight per cent perspiration and two per cent inspiration." Whether this is true or not, there is no doubt about the tireless speed at which Edison drove himself. But perhaps the real secret of his boundless energy was the fact that he enjoyed his work so much it was almost the same as play for him.

31 The main idea of paragraph 1 is that
- **A** Edison invented the electric light
- **B** Edison was one of the greatest inventors
- **C** Edison was a genius
- **D** Edison invented the Ediphone

32 Which of these would be the best heading for paragraph 2?
- **E** How to Be an Inventor
- **F** Edison's Greatest Contribution
- **G** Edison—Wizard of Light
- **H** Edison's Secret

33 The writer feels that Edison's main contribution was
- **A** his invention of the electric light
- **B** his pioneer spirit
- **C** his method of inventing
- **D** his ability to train a staff of scientists

34 The writer begins the part of the story which tells about the secret of Edison's inventive genius by
- **E** asking a question
- **F** telling about laboratories
- **G** telling a story about Edison's childhood
- **H** quoting a famous writer

35 The writer does NOT mention
- **A** Edison's secret of inventive genius
- **B** Edison's best known inventions
- **C** Edison's speed and energy
- **D** Edison's poor hearing

If you finish before time is called, you may check your work on Part Two. Do not go back to Part One.

APPENDIX H

PILOT STUDY

PILOT STUDY

A pilot study was conducted to test the clarity and completeness of the materials and of the instructions presented for the test items, to obtain an indication of the time required to administer the tests, and to determine the appropriateness of the criteria for scoring the responses. Some of the modifications indicated were described in Chapter III. A trial run of the data was planned in order to examine the general characteristics of the information likely to be provided by the tests.

The Concrete and Stories tests and STEP Reading, Form 2B, were administered to seventeen subjects, boys and girls age nine-and-ten-years, attending regular Grade four classes in one school in the City of Edmonton, Alberta. The subjects were selected by the Principal of the school as representative of pupils in these classes. The interviews with the children were tape-recorded, typed and scored.

Adjustments in the tests indicated by the pilot study

An examination of the protocols suggested the need for certain changes in both the Concrete and Stories tests.

Errors in the responses indicated that relevant information in the Stories should, in a number of instances, be made more specific. This could be done without altering the condition that the solutions to the problems presented in the test questions should not be "given" in the text. In Story 1 (see Appendix A), the phrase "every ball the same size" was added in line 7. In Story II, an incident describing a light fall of snow covering the pieces of suet was removed. Children considered the possibility (quite rightly) that in brushing away the snow, Jimmy could also brush away pieces of the suet. Later in the story the phrase "half a cup" replaced a less definite expression "half for himself." Some of the subjects in the pilot study seemed unfamiliar with the names "shoveller", pintail", "red-head". As anticipated, however, subjects in the main study, living closer to the prairie, were more familiar with these names. They were pronounced for a subject who had difficulty in decoding them. The names were retained.

In the story "The Cave" (see Appendix D), paragraph six was added. A number of subjects in the pilot study assumed that a "wild animal" had entered the cave, without considering the need to explain the candles going out.

In a number of instances sentences were shortened and substitutions were made for unfamiliar words to obtain a lower readability grade score.

The preliminary questions for each story were increased in number and an attempt was made to improve these questions to ensure as far as possible that all the items of information required in solving the problems were correctly read and recalled by the subject.

Three additional test items assessing conservation of substance, weight and volume were added to the Stories tests of conservation to improve the comparability of corresponding test items Stories and Concrete. Test items based on the Principle of Transitivity were added to the Stories and Concrete tests of deduction (see Appendix C). Two test items were removed from the Concrete tests of probability. The items had required the subject to draw the trajectories of the beads. These items were difficult to score objectively and it was also desirable to reduce the time required in administering the tests. A further reduction in time was effected by reducing the instances in which the subject manipulated the materials. Initial relevant manipulations were retained; later manipulations were performed by the investigator. For example, the subject continued to match the two balls of clay and the water in the two glasses, by weighing these on the balance scales; the investigator made the deformations in the material, placed the balls of clay and the sugar lumps in the water, etc. The activities assigned to the subject and to the investigator are indicated in the description of the tests (see Appendixes A to E).

On a first attempt to score the protocols of the pilot study it was clear that adjustments were required in the original criteria for scoring. In some cases the criteria were not applicable to the responses obtained (see Chap. VII). In other cases, the variety of the responses indicated the need for more specifically stated criteria. This was the case for scoring the levels of explanation for defending decisions in conservation and explaining decisions in the test of probability (see Chaps. IV and VIII). The protocols were re-scored on the basis of the adjusted criteria and a trial run of the data was obtained.

Trial run of the pilot study data

The distribution of scores on STEP Reading, Form 4B, for the 17 subjects in the pilot study was noted. Converted scores ranged from 237 to 295, with 8 subjects scoring at and above the converted score 260.

The percentile bands for the converted scores obtained ranged from 25-46 to 99.5 - 100. Eleven subjects scored in percentile bands at or above 76 - 88.

Product-moment correlations between scores on the Concrete and Stories variables and between these variables and STEP Reading total scores are presented in Table 31.

TABLE 31.-Product-moment correlations between variables of the Concrete and Stories tests and between these variables and STEP Reading total scores. Pilot study. (N = 17).

Variable	2	3	4	5	6	7	8	9	10	STEP Reading
CCO-Con 1	61**	72**	23	-02	27	02	37	30	-19	42
CCO-Cl 2		67**	33	-07	39	32	49*	57*	02	58**
CCO-D 3			48*	38	53*	29	62**	40	-09	56*
CCO-P 4				27	42	44	60**	02	-07	39
CCO-I 5					40	59**	56*	24	09	12
CCO-total										52*
SCO-Con 6						72**	51*	54*	52*	74**
SCO-Cl 7							72*	34	23	57*
SCO-D 8								29	31	62**
SCO-P 9									-02	52*
SCO-I 10										44
SCO-total										78**

* $\gamma = .47$ $p < .05$
** $\gamma = .59$ $p < .01$
[a]Decimal points and diagonal elements are omitted.

The patterns of relations within each set of variables, Concrete and Stories, shown in Table 31, were similar in that for each set, the highest correlations appeared to involve the three variables conservation, classification, and deduction.

The relations between the Concrete and Stories variables and STEP Reading totals were also of interest in the pilot study. In the Concrete data, classification and deduction appeared to be significantly related to scores on STEP Reading ($r = .58$, $r = .56$, $p < .05$). The Stories tests, with the exception of Stories induction were significantly related to STEP ($p < .01$).

Correlations between subscores CCO-Con-S-W-V and SCO-Con-S-W-V for the pilot study data are shown in Table 32.

TABLE 32.-Product-moment correlations between subscores CCO-CON-S-W-V and SCO-CON-S-W-V and between these subscores and STEP Reading. Pilot Study. (N = 17).

Variable		2	3	4	5	6	STEP Reading
CCO-Con-S	1	62**	-12	06	23	28	49*
CCO-Con-W	2		-07	21	06	26	22
CCO-Con-V	3			02	14	03	16
SCO-Con-S	4				54*	60**	59**
SCO-Con-W	5					81**	60**
SCO-Con-V	6						63**

* $r = .47$, $p < .05$
** $r = .59$, $p < .01$
[a]Decimal points and diagonal elements are omitted.

Correlations between scores on subtests of classification CCO and SCO, and their relation to STEP Reading total scores are shown in Table 33.

TABLE 33.-Product-moment correlations between scores on subtests of classification, CCO and SCO, and between these scores and STEP Reading total scores. Pilot Study. (N = 17).

Variable		2	3	4	STEP Reading
CCO-Cl-hierarchical	1	63**	19	29	43
CCO-Cl-multiplicative	2		14	32	57*
SCO-Cl-hierarchical	3			46*	51*
SCO-Cl-multiplicative	4				61**

$r = .47$, $p < .05$*
$r = .59$, $p < .01$**
[a]Decimal points and diagonal elements are omitted.

Tests of deduction consisted of five subtests on the pilot study (the test based on the Principle of Transitivity was included in the main study). The correlations between scores on these subtests are presented in Table 34.

It may be seen in Table 34 that three of the significant relations between subtests of the Concrete and Stories tests of deduction involve the inference Undetermined.

The results of a trial run of the data of the pilot study suggested that a statistical analysis of data obtained on a random sample of a Grade four population might reveal relationships important for an understanding of cognitive operations in reading.

TABLE 34[a].-Product-moment correlations between scores on subtests of deduction CCO and SCO and between these scores and STEP Reading total scores. Pilot Study. (N = 17).

Variable		2	3	4	5	6	7	8	9	10	STEP Reading
CCO-D-Modus Ponens	1	72**	16	25	20	33	28	28	23	15	25
CCO-D-Modus Tollens	2		28	25	63**	51*	28	38	19	36	33
CCO-D-Undetermined	3			51	19	29	11	71**	-06	64**	32
CCO-D-Possible	4				09	40	09	52*	-10	19	54*
CCO-D-Not-Possible	5					56*	26	25	43	34	48*
SCO-D-Modus Ponens	6						57*	32	32	31	47*
SCO-D-Modus Tollens	7							06	51*	30	35
SCO-D-Undetermined	8								-15	47*	39
SCO-D-Possible	9									10	48*
SCO-D-Not-Possible	10										66**

* r = .47 p< .05

** r = .59 p< .01

[a]Decimal points and diagonal elements are omitted.

APPENDIX I

ADDITIONAL STATISTICAL DATA

TABLE 35.--Percentage of correct responses for the inclusion of classes. Material: animals (Piaget).

Inclusion relation	8 (n=17)	9 (n=22)	10 (n=14)	11 (n=17)	12-13 (n=47)
A ⊂ B	43	50	50	46	67
B ⊂ C	38	66	62	82	75
Both questions	25	27	42	46	67

TABLE 36.--Distribution of I.Q. Estimates for 95 of the 100 subjects in sample, based on scores on SRA Primary Mental Abilities Tests, Ages 7 - 11 - Form AH, administered November, 1965. Mean 109.7 (N = 95)

Number of subjects in range	I.Q. Estimate based on formula 2 V + R
6	134 and up
20	122 - 133
16	116 - 121
18	111 - 115
17	105 - 110
17	91 - 104
1	90 and under

TABLE 37.--Distribution of the scores available for subjects in the sample on the <u>Iowa Arithmetic Group Test</u> (N=92), the <u>Iowa Language Group Test</u> (N=94), and the <u>Gates Reading Survey</u> (N=95).

Grade Intervals	Iowa Arithmetic May, 1965 N = 92	Iowa Language November, 1965 N = 94	Gates Reading Survey May, 1965 N = 98
7.0 - 7.9	0	1	0
6.5 - 6.9	0	0	1
6.0 - 6.4	0	4	1
5.5 - 5.9	0	5	10
5.0 - 5.4	1	10	10
4.5 - 4.9	8	12	11
4.0 - 4.4	39	23	26
3.5 - 3.9	33	17	33
3.0 - 3.4	10	9	6
2.5 - 2.9	1	13	0

T A B L E 38[a].--Product-moment correlations between scores on the Concrete tests, scores on the Stories Tests and STEP Reading total scores. (N = 100).

TABLE 38 - (continued)

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
1. Sex		183	244	088	194	233	175	042	089	086	138	116	043	020	140	039	124	013	183	3
2. CCO-Con-S			720	544	854	909	799	049	229	182	034	210	056	237	017	031	218	187	114	-0
3. CCO-Con-W				642	900	901	749	218	196	271	052	141	203	189	037	-017	242	218	125	0
4. CCO-Con-V					854	630	550	132	151	185	002	002	123	152	097	024	163	195	013	0
5. CCO-Con						926	796	155	219	244	032	128	148	219	061	015	237	230	093	0
6. Explanation Level I							989	162	250	270	045	196	157	227	040	003	259	228	135	0
7. Explanation Level II								126	224	229	134	149	162	139	163	-012	246	221	126	0
8. CCO-Cl-Hierarchical									168	764	-010	115	137	342	045	-190	253	314	055	2
9. CCO-Cl-Multiplicative										765	054	126	-031	194	-121	042	096	054	081	0
10. CCO-Cl-Total											029	158	070	351	-050	-097	228	241	089	1
11. CCO-D-Modus Ponens												087	060	134	185	-117	265	207	198	-0
12. CCO-D-Modus Tollens													195	468	047	064	650	598	295	1
13. CCO-D-Undetermined														276	-046	005	646	400	559	1
14. CCO-D-Possible															-005	-042	766	715	359	2
15. CCO-D-Not-Possible																095	305	189	244	0
16. Transitivity																	142	056	-048	2
17. CCO-D-Total																		797	605	2
18. "Lights"																			019	2
19. "Village"																				0
20. CCO-I-Total																				
21. CCO-I-Hypotheses																				
22. CCO-P-Total																				
23. Concrete Total																				
24. SCO-Con-S																				
25. SCO-Con-W																				
26. SCO-Con-V																				
27. SCO-Con-Total																				
28. Explanation Level I																				
29. Explanation Level II																				
30. SCO-Cl-Hierarchical																				
31. SCO-Cl-Multiplicative																				
32. SCO-Cl-Total																				
33. SCO-D-Modus Ponens																				
34. SCO-D-Modus Tollens																				
35. SCO-D-Undetermined																				
36. SCO-D-Possible																				
37. SCO-D-Not-Possible																				
38. SCO-D-Transitivity																				
39. SCO-D-Total																				
40. SCO-I-Total																				
41. SCO-I-Hypotheses																				
42. SCO-P																				
43. Stories Total																				
44. STEP Reading Total																				

[a]Decimal points and diagonal elements are omitted.

TABLE 38 - (continued)

21	22	23	24	25	26	27	28	29	30	31	32
300	193	286	153	-120	131	072	049	170	119	000	112
-028	014	698	418	223	228	376	383	301	209	067	210
067	043	790	370	301	312	397	397	291	238	133	251
006	-063	691	287	283	345	372	320	238	190	100	199
017	-006	835	408	311	364	439	419	316	243	116	253
086	036	831	438	261	299	404	424	324	225	120	236
093	037	709	374	158	250	318	335	312	240	186	265
178	094	466	253	180	179	247	218	106	051	140	078
-027	076	433	121	026	001	059	116	059	071	214	112
098	111	588	244	135	118	200	218	108	080	232	125
-010	055	098	004	-146	003	-052	-082	-074	049	136	075
099	175	331	061	-080	095	035	018	024	106	069	114
230	016	309	138	064	169	153	060	056	127	053	130
195	156	499	393	165	254	330	311	166	048	205	089
050	129	125	013	-006	103	048	-015	043	-037	074	-019
248	-096	050	-015	-080	-024	-046	-020	009	-079	-073	-089
275	167	538	277	059	257	245	170	120	098	172	129
236	121	484	193	013	126	137	110	004	-009	199	034
101	142	260	216	099	271	242	146	194	193	045	190
976	114	355	290	-042	102	145	194	256	-022	062	-007
	122	333	286	-064	084	127	168	237	-005	023	-000
		233	196	146	227	232	204	148	029	193	068
			515	285	404	489	471	359	217	233	253
				574	497	834	878	649	286	267	325
					494	823	844	624	060	189	097
						819	556	459	170	156	193
							912	695	211	246	250
								796	181	288	231
									200	207	231
										193	978
											394

TABLE 38 - (continued)

33	34	35	36	37	38	39	40	41	42	43
058	191	065	045	122	061	137	075	098	042	128
145	169	064	046	241	094	203	095	130	012	363
204	088	-057	-043	264	104	135	134	179	077	386
196	074	031	038	203	147	198	-073	-026	069	348
210	124	014	020	270	134	206	053	102	062	420
163	114	021	007	245	137	-176	107	141	079	396
090	091	001	-021	182	188	-140	092	096	004	316
086	140	114	116	149	100	178	-018	-034	217	274
020	-011	080	232	244	102	168	124	179	172	163
070	019	127	228	257	133	226	069	095	255	286
-139	060	033	072	-122	134	040	-086	-081	114	-043
120	102	088	118	076	154	189	-070	-038	061	105
051	004	-012	081	199	171	111	241	240	075	205
005	179	202	308	236	269	340	048	100	168	374
-058	-112	122	065	094	073	085	-207	-205	-117	-001
109	-007	-182	-060	007	004	-033	-179	-157	009	-079
045	094	135	249	244	291	298	023	062	091	294
068	155	076	155	142	268	249	002	080	144	199
-037	042	166	227	223	139	182	074	033	-037	251
181	134	050	243	120	132	247	-019	-024	212	207
187	106	028	213	122	116	220	-017	-029	190	185
-089	013	143	132	171	083	133	-038	-122	165	236
207	145	111	212	374	241	354	055	107	214	528
212	296	207	362	394	227	468	135	133	255	821
086	082	072	276	175	040	209	054	042	244	680
158	233	089	174	219	191	295	020	030	178	703
186	251	149	325	319	189	394	083	082	272	891
184	260	173	336	321	185	405	123	128	306	845
164	247	135	188	154	136	288	073	035	180	635
263	311	176	014	072	209	282	107	105	028	445
161	095	081	193	134	207	227	189	182	156	367
280	312	182	054	096	240	313	140	137	060	495
	341	120	229	356	251	613	057	063	213	407
		046	256	230	346	635	083	099	172	456
			330	037	128	484	-020	-019	-027	263
				311	210	683	-008	009	070	430
					172	549	258	237	263	473
						575	-017	024	100	355
							011	032	224	655
								925	-095	190
									-092	186
										465

TABLE 39[a].--Matrix of 70 X 70 phi coefficients based on items of STEP Reading. (N = 100).

	2	3	4	5	6	7	8	9	10	11	12
1	229	421	-033	131	-093	-036	185	-085	-118	-059	-014
2		102	-048	-069	250	098	038	-123	155	272	-088
3			-029	037	-082	-124	118	-075	-105	079	-074
4				305	-058	235	082	167	102	208	-164
5					-039	184	150	-093	078	125	014
6						144	300	-060	075	279	057
7							273	191	018	211	-119
8								213	-071	248	-128
9									-115	077	052
10										225	-001
11											097
12											
13											
14											
15											
16											
17											
18											
19											
20											
21											
22											
23											
24											
25											
26											
27											
28											
29											
30											
31											
32											
33											
34											
35											
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											
11											
12											
13											
14											
15											
16											
17											
18											
19											
20											
21											
22											
23											
24											
25											
26											
27											
28											
29											
30											
31											
32											
33											
34											
35											

TABLE 39.-(continued)

	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	-094	-056	-023	-068	025	-111	282	140	012	006	-016	-011	041	029	138	0
2	-136	-007	168	025	127	175	030	204	104	093	214	306	127	042	133	0
3	063	018	021	-060	189	-078	228	-092	170	031	-112	-108	-106	-147	-102	-0
4	117	091	014	221	-062	161	-085	126	-067	-069	092	097	140	048	000	-1
5	-043	013	-005	054	-062	109	138	-073	-016	-028	177	128	199	103	000	0
6	094	004	041	-013	138	107	014	124	036	025	122	134	050	015	-058	0
7	-055	041	038	236	-020	096	-001	-095	-109	-122	-011	003	281	-151	023	0
8	000	115	035	170	063	186	184	-015	030	074	123	028	104	-171	-069	0
9	113	033	-087	-109	-077	-141	-149	079	-093	-022	-056	-048	-007	-135	185	0
10	020	221	225	119	177	-012	-026	224	014	126	127	200	162	252	074	2
11	123	212	112	193	051	140	131	032	292	105	185	153	244	141	000	1
12	214	065	108	-033	-048	044	084	145	-039	-060	005	026	-038	-069	101	-1
13		-027	041	-060	-200	066	112	098	064	187	155	089	057	026	-122	0
14			047	107	099	-011	-124	-004	127	029	035	-033	211	-053	022	0
15				030	099	049	105	072	-005	-028	079	102	-038	157	181	1
16					173	184	077	-035	-042	-049	014	-068	-006	022	-147	-1
17						108	114	020	080	067	-044	-100	-009	-025	055	0
18							255	082	057	187	186	113	264	044	-040	-1
19								096	078	236	164	-040	125	104	046	0
20									-143	188	069	017	071	069	-027	1
21										503	362	380	355	213	208	2
22											523	420	322	352	127	2
23												628	346	360	261	2
24													376	332	241	3
25														239	140	2
26															084	3
27																3

[a]Decimal points and diagonal elements are omitted.

TABLE 39 - (continued)

29	30	31	32	33	34	35	1	2	3	4	5	6	7	8	9
111	046	124	081	072	115	111	-058	056	187	-058	-033	-063	024	421	132
027	067	114	117	105	167	161	-084	-066	102	197	-048	039	035	272	050
078	-102	090	072	043	-026	-025	-052	-130	-042	-052	-029	-056	-043	219	-035
-161	000	-152	050	105	-107	-017	-036	066	-029	-036	490	-039	195	-029	052
-004	052	128	-002	071	039	048	-009	-013	170	101	305	075	038	-096	114
067	-173	-140	142	-066	-014	-107	019	-004	065	262	-058	002	187	212	-099
-232	-023	-059	-002	-026	135	040	131	058	106	036	074	-079	-068	-124	-044
-140	-023	-012	129	036	000	047	146	-013	118	-049	082	113	109	118	184
-107	123	039	031	013	185	017	-093	035	082	036	167	019	-077	-075	068
012	123	043	103	069	074	160	077	049	-105	283	-074	147	085	020	-153
-040	-043	088	173	099	129	047	007	-013	079	188	055	-020	216	079	-026
-125	-101	-153	176	-125	030	247	-134	202	029	206	164	-080	167	029	-008
-123	-082	-074	170	-086	051	-107	-052	027	167	120	117	-016	128	-042	096
113	066	-072	057	-007	-099	-123	-024	166	018	069	091	089	146	018	074
-036	060	006	061	074	050	-008	025	042	123	-059	014	146	095	021	-036
-108	-074	-092	104	062	-129	039	391	-107	-060	-075	042	-081	247	128	-050
-107	000	028	-021	034	014	112	119	022	050	119	-062	094	194	329	-132
066	-081	027	183	059	040	136	115	-034	127	031	161	-006	068	127	063
-028	046	238	063	053	125	204	042	023	344	138	078	-073	186	344	020
153	-027	159	159	053	093	133	-114	180	179	110	-065	-020	114	043	-035
121	156	128	165	180	169	101	-009	-013	037	210	-067	-027	038	170	003
149	229	303	170	088	051	275	-015	028	031	307	-069	-033	-088	161	-069
179	261	181	116	202	095	245	162	017	131	262	092	129	097	131	122
-103	290	210	187	238	205	276	069	033	015	272	097	044	-035	015	038
211	[illegible]	199	103	203	210	219	079	097	098	079	-003	034	-036	098	007
201	338	295	117	305	148	381	085	-133	-039	085	-103	-031	049	177	-189
229	360	140	-032	-021	250	040	000	[illegible]	000	168	000	039	-063	102	064
	381	405	139	355	321	350	-069	016	115	268'	009	-023	065	012	021
	282	336	075	194	161	310	-031	-188	-025	054	-017	006	-068	181	-021
		421	096	314	150	363	000	022	-102	000	000	039	-147	204	-021
			181	443	381	618	-100	028	090	069	-009	023	-107	090	-191
				211	463	268	-180	-057	072	089	050	-154	057	072	-025
					157	532	097	-039	043	009	105	037	-077	150	-065
						413	-189	-011	102	126	071	-059	000	-026	-074
							-031	-055	078	139	127	-073	-025	181	-106
								-161	-052	-064	-036	261	-009	163	-086
									319	210	066	170	039	094	161
										378	335	144	171	219	182
											265	-069	079	163	091
												-039	195	-029	204
													045	-056	224
														064	020
															-143

TABLE 39 -(Continued)

10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
-047	-063	198	-009	074	-016	140	040	101	005	100	056	081	-096	019
272	170	-021	055	107	214	293	250	025	007	306	301	186	047	028
-042	-056	082	-050	127	-112	043	065	-060	151	015	207	156	-086	-153
-029	-039	-053	-035	161	261	-065	-058	-042	032	097	224	-038	140	190
037	-027	147	152	162	053	-004	-114	-042	106	003	217	-180	241	028
212	228	-060	019	049	190	124	252	093	-027	134	250	011	073	-122
-124	010	-155	151	096	256	-155	-116	070	-071	-051	207	046	-129	-034
000	204	071	-071	093	178	046	166	170	059	084	293	012	081	-096
-075	019	053	036	107	018	-085	-149	005	-043	-048	035	092	017	-085
020	051	036	050	087	068	093	075	119	141	023	-059	090	127	125
079	064	077	095	126	236	032	279	114	067	-002	270	037	012	066
132	-080	-010	-069	044	053	037	-001	-033	051	075	068	-004	031	190
167	144	113	076	189	155	043	035	-060	276	089	072	051	057	060
018	-084	-035	-001	078	192	-121	067	188	195	073	020	-033	-040	020
123	067	037	-120	130	079	126	-017	178	060	053	-002	110	014	075
-060	064	-109	079	-039	189	-133	200	049	-085	110	055	-079	-124	163
050	094	-077	-052	053	021	093	138	-028	043	032	022	-061	046	-100
230	231	-079	196	-055	090	-079	281	110	117	210	233	095	192	175
112	284	272	088	164	109	157	155	077	-007	070	274	-065	006	078
315	-020	079	080	135	006	008	201	063	-089	274	-055	126	108	-007
037	-027	-093	045	109	-009	065	261	-042	106	003	217	-018	022	028
161	-033	056	169	290	099	052	319	045	083	174	084	107	153	221
131	129	091	036	186	209	132	259	014	196	111	227	025	103	184
138	044	-048	018	210	226	210	134	110	120	242	139	157	115	105
098	-045	116	263	063	203	018	107	068	128	086	186	036	188	068
-039	-031	060	170	044	-041	181	075	100	032	-075	-087	-062	053	121
-102	-039	000	165	-040	071	186	115	-074	144	097	110	021	028	125
115	-023	-039	077	013	104	161	082	165	192	129	060	016	-003	-038
078	085	017	-030	-107	101	-028	183	039	007	228	-011	-053	-135	-049
-102	-039	123	124	040	071	186	058	000	103	145	-022	021	028	083
090	023	224	088	068	134	053	091	056	178	210	028	108	115	-003
072	-029	-165	120	054	040	-096	142	104	085	032	084	-071	058	197
043	037	077	073	017	-047	-003	054	216	028	238	053	046	073	026
-026	039	-046	278	-060	154	027	-086	147	092	084	044	-021	000	115
078	243	017	178	055	006	079	125	113	089	179	-011	072	034	119
-052	096	036	024	115	-038	-002	141	081	057	069	024	-068	012	-102
207	-003	035	-044	277	174	-055	-131	-107	076	033	-020	216	102	118
479	344	-075	-050	230	010	043	065	-060	-059	138	207	-055	200	060
378	096	036	024	285	162	110	141	-075	143	069	210	106	248	074
335	241	-053	-035	161	092	-065	-058	-042	179	269	224	-186	340	190
144	232	140	-189	073	129	189	002	064	102	044	084	-033	104	039
171	127	052	-030	025	147	003	187	015	-028	015	132	-046	044	105
219	144	082	-050	127	131	179	359	128	046	261	094	051	057	-047
074	058	003	-195	063	071	022	-161	-050	224	-116	208	-079	122	138
	344	082	-050	025	131	179	359	128	046	261	207	-055	200	060
		019	134	073	036	293	228	064	102	138	257	-033	-005	202
			-028	107	091	161	028	118	083	027	035	219	103	044
				-053	-160	-030	078	-149	-008	-082	046	-045	-017	158
					138	028	-009	036	117	113	144	262	192	091
						132	-015	277	148	283	122	173	103	135
							124	063	129	017	121	016	108	049
								-013	-027	203	060	070	-089	058
									067	288	-026	226	-124	-068
										070	122	103	181	084
											193	107	183	004
												-104	040	118
													032	057
														152

TABLE 39 - (continued)

26	27	28	29	30	31	32	33	34	35
229	006	005	-055	040	204	055	076	068	069
333	155	007	065	154	159	151	111	099	101
153	271	-163	-026	065	055	-084	047	039	021
286	102	179	059	-058	038	096	-042	103	014
221	078	266	080	036	072	-136	-137	116	-005
014	-067	032	166	086	-011	146	002	-075	098
090	129	068	-164	-051	-093	115	-107	-067	-097
058	099	107	062	-033	084	087	000	-073	128
-108	-039	-043	019	-149	-219	048	-044	070	-087
295	096	041	291	146	266	186	029	163	175
193	225	155	143	094	140	089	113	176	112
071	-051	-032	073	-001	-205	-029	-064	026	-014
000	-080	151	282	094	034	-018	068	-026	041
099	221	150	190	067	125	143	-066	100	135
101	027	-146	176	098	139	085	092	055	152
-055	-152	067	041	-119	156	038	068	-022	-044
123	244	-013	061	-019	118	116	-014	-147	263
111	087	117	059	-009	113	-059	-133	-002	089
046	198	227	145	155	207	-046	065	-008	105
173	-103	-034	157	201	260	131	062	100	179
156	142	266	305	336	180	145	026	062	413
140	188	240	502	392	421	049	151	-083	433
261	127	294	404	259	270	161	063	091	318
278	023	318	323	342	193	043	097	075	345
390	309	292	181	107	213	078	-152	-028	243
222	045	205	295	197	368	025	142	159	242
150	025	021	022	288	104	108	-042	-042	261
180	-043	151	235	428	233	155	-045	126	356
-010	-137	-035	115	242	220	103	007	129	154
100	074	144	151	231	186	108	083	-042	342
120	190	096	155	322	182	105	129	085	328
016	024	085	236	142	137	110	-131	018	061
105	069	-015	039	115	128	097	061	093	242
063	074	041	173	202	072	205	-115	063	101
141	160	089	115	125	345	103	007	087	235
-021	-027	057	013	-102	155	-104	-074	-085	025
121	-005	212	191	-131	-079	095	204	-053	175
026	020	255	084	-082	161	137	-060	039	-082
189	077	230	377	262	242	169	102	004	110
107	102	179	059	-058	038	096	-190	103	-129
157	147	-059	155	002	033	014	124	031	-012
000	033	058	097	-054	-041	129	-061	173	011
153	020	151	194	-082	266	137	153	-069	226
021	004	006	-057	085	-053	-035	039	-080	-079
026	-105	046	194	-082	161	026	-060	-069	021
059	051	102	155	002	196	184	042	114	146
123	112	020	152	028	163	-085	149	005	223
-021	-001	077	079	138	173	054	-072	047	087
312	285	283	364	223	197	071	287	-087	170
202	127	245	302	190	123	161	212	-009	079
106	-037	075	272	124	205	131	118	-012	179
086	075	091	103	169	347	021	002	107	214
037	-062	-009	121	-013	079	118	-009	056	178
215	343	159	173	209	067	137	087	-032	225
217	-037	120	166	134	143	-062	-054	075	199
231	157	167	143	187	012	047	066	087	131
176	192	-109	111	070	158	156	202	-069	277
210	058	123	263	073	084	-082	-093	006	014
094	176	-002	218	058	159	142	042	-077	034
163	080	-042	163	-015	-035	-031	-032	-212	018
	356	215	205	303	342	119	115	253	201
		141	026	287	215	080	-074	-045	126
			351	209	237	004	002	141	266
				353	382	122	142	025	350
					407	146	183	229	388
						155	143	025	348
							128	021	259
								011	176
									013