The Alternating Semantics for Default Theories and Logic Programs*

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Abstract

Recent proposals for semantics of default theories are all based on some types of weaker notion of extensions. This is typified in the *well-founded semantics* and the *extension class semantics* for default theories. Although these semantics solve the no-extension problem in Reiter's default logic, they also present a departure from Reiter's original extension semantics, even for default theories which can be completely characterized by the extension semantics. This results in a weaker capability for *skeptical reasoning.*

In this paper we propose a semantics for default theories, based on van Gelder's alternating fixpoint theory. The distinct feature of this semantics is its preservation of Reiter's semantics when a default theory is not considered "problematic" under Reiter's semantics. Differences arise only when a default theory has no extensions, or has only "biased extensions" under Reiter's semantics. This feature allows skeptical reasoning in Reiter's logic to be properly preserved in the new semantics. By the familiar, natural translation from logic programs to default theories, the semantics proposed for default theories provides a natural extension to the stable model semantics of normal logic programs.

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1 Introduction

Default logic, introduced by Reiter [13], forms one of the most important formalisms for nonmonotonic reasoning. One of the problems with this default logic is that a reasonable default theory may not always have extensions. In such a case, the semantics of the default theory is not defined. There are also default theories for which there seem to be reasonable (partial) extensions that could not be captured in Reiter's extension semantics (from now on, the extension semantics).

The aim of this paper is to define a semantics for default theories that is faithful to the extension semantics for those default theories whose extension semantics yields (arguably) perfect extensions. Arguably, if a default theory is not considered problematic under the extension semantics, one expects to get the same conclusions from any semantics that "corrects" its problem. This semantic preservation is important for preserving both choice reasoning and skeptical reasoning, as advocated originally by Reiter on the use of default logic.

Various semantics have been proposed in an attempt to resolve this problem. Marek and Truszczynski [8] introduced the concept of *weak extensions* of default theories and showed that weak extensions of a default theory correspond to expansions of suitably translated formulas in autoepistemic logic. A default theory however is not guaranteed to possess a weak extension. The well-founded semantics for logic programs has been extended to default theories (see, for example, [12]). Although the well-founded approach is computationally more attractive, it nevertheless pays the price for being unable to draw skeptical conclusions that are implied under Reiter's extensions semantics. Based on the concept of *stable class* for logic programs, Baral and Subrahmanian [1, 2] proposed the extension class semantics for default theories. This semantics is still considered weak in that there exist default theories with (arguably) perfect extensions; but its extension class semantics could not draw any useful conclusion. This phenomenon has a significant impact on identifying intended stable classes for logic programs. As a matter of fact, as we will show, some obvious unintuitive conclusions can be implied by the stable class semantics for logic programs.

Van Gelder, on the other hand, has shown that the well-founded semantics of logic programs can be defined by the alternating fixpoint theory [5]. The basic idea is that if an operator T is anti-monotonic, i.e., for E_1 and E_2, from $E_1 \subseteq E_2$ we have $T(E_2) \subseteq T(E_1)$, then T^2, the function that applies T twice, is monotonic, and thus has at least one alternating fixpoint.

The main result of this paper is to show that the alternating fixpoint theory can be used to define a semantics that corrects the problem of the extension semantics without affecting those default theories whose extension semantics have been considered appropriate. The next section recalls default logic, and reviews various semantics for default logic. Section 3 discusses drawbacks in existing semantics. Then in Section 4 we present our new semantics for default theories, with Section 5 showing that this semantics automatically applies to normal logic programs. Section 6 discusses some of problems which are still not satisfactorily solved by the recent proposals of semantics for default theories.

2 Default Logic and its Semantics

We assume a propositional language L consisting of the usual well formed formulas over an alphabet $\mathcal{B}$. A *default* is a triple $d = \{p(d), j(d), c(d)\}$, where $p(d)$ and $c(d)$ are formulas of L, and $j(d)$ is a finite subset of L. $p(d)$ is called the *prerequisite* of d, $j(d)$ the *justification* of d, and $c(d)$ the *consequent*, or *conclusion* of d. Default d is usually denoted by $\frac{p(d):j(d)}{c(d)}$. A default theory is a pair (D, W), where $W \subseteq L$, and D is a collection of defaults.

2.1 The Extension Semantics of Reiter

Given a default theory (D, W) and a set E of formulas (called *context*), we first define an operator $R_{E,D}$ which maps sets of formulas to sets of formulas in the following way:

$$R_{E,D}(S) = Cn(S \cup \{c(d) | d \in D, p(d) \in S, \neg j(d) \cap E = \emptyset\}$$

where Cn denotes the familiar Tarskian consequence operator and $\neg j(d) = \{\neg\alpha | \alpha \in j(d)\}$. The operator was initially defined by Reiter [13] and paraphrased in [1]. In this paper we follow the notations used in [1].

We now define an operator T that transforms a given set E of formulas into a set $T(E)$ of formulas as follows.

Definition 2.1 ([13, 1]) Suppose (D, W) is a default theory, and E is a set of formulas, called the context. Assume

$$\begin{array}{rcl} R^0_{E,D} & = & Cn(W) \\ R^{n+1}_{E,D} & = & R_{E,D}(R^n_{E,D}(W)) \\ R^\infty_{E,D} & = & \bigcup_{i=0}^\infty R^i_{E,D}(W) \end{array}$$

Then an operator $T_{D,W}(E)$ is defined as

$$T_{D,W}(E) = R^\infty_{E,D}(W).$$

When there is no confusion, we also use $T(E)$ to denote $T_{D,W}(E)$. $\Box$

Lemma 2.1 ([2]) T is anti-monotonic, that is $T(E_1) \subseteq T(E_2)$ if $E_2 \subseteq E_1$. $\Box$

Given a default theory (D, W) and a set of formula $E = Cn(E)$, E is said to be a *fixpoint* of (D, W) if and only if $E = T(E)$. E is said to be an *alternating fixpoint* of (D, W) if and only if $E = T(T(E))$. Since T is anti-monotonic, T^2 is monotonic and therefore, (D, W) has at least one alternating fixpoint.

The extension semantics of a default theory is determined by its extensions.

Definition 2.2 ([13, 1]) E is an extension of default theory (D, W) if and only if E is a fixpoint of $T_{E,D}$. $\Box$

Example 2.1 Suppose (D, W) is given by

$$W = \{a \leftarrow c\}, \qquad D = \{\frac{:\neg b}{c}, \frac{:\neg c}{d}, \frac{a:\neg d}{p}\}$$

and $E = Cn(\{a, c, p\})$ is the context. Then:

$$\begin{array}{rcl} R^0_{E,D} & = & Cn(\{a \leftarrow c\}) \\ R^1_{E,D} & = & Cn(\{a \leftarrow c, c\}) \\ R^2_{E,D} & = & Cn(\{a \leftarrow c, c, p\}) = R^\infty_{E,D}(W) = Cn(E) \end{array}$$

Since $T(E) = E$, E is an extension of (D, W). $\Box$

According to Reiter, there are two reasoning modes in using default logic: each arbitrarily chosen extension can be seen as an acceptable set of beliefs, or the truth of a formula is determined by whether it is contained in all extensions. The former is called *choice reasoning* and latter *skeptical reasoning*.

However, a default theory may have no extensions at all.

Example 2.2 Consider the default theory (D, W), where $W = \{p\}$, and $D = \{\frac{:\neg q}{q}\}$. The two possible candidates for extensions are:

$$E_1 = Cn(\{p\}) \qquad \text{and} \qquad E_2 = Cn(\{p, q\})$$

But $T(E_1) = E_2 \neq E_1$ and $T(E_2) = E_1 \neq E_2$. Hence the default theory have no extensions at all. $\square$

To address the no-extension problem, various semantics for default theories have been proposed.

2.2 The Extension Class Semantics

Baral and Subrahmanian have proposed the structure of *extension class* to represent the meaning of default theories [1, 2]. The basic idea behind the concept of extension class is that T may not always have fixpoints; however, there may exist a collection of points so that T circles around this collection of points.

Definition 2.3 ([1]) Let (D, W) be a default theory. A family, $E = (E_i)_{i \in \mathcal{A}}$ of sets of formulas is an *extension class* of (D, W) if and only if

1. $E = \{T(E_i) \mid E_i \in E\}$, and

2. no proper subset of E satisfies the above condition.

$\square$

As defined above, an extension class is a set of *sets of formulas*. A formula F is assigned *true* (resp. *false*) by an extension class $E = (E_i)_{i \in \mathcal{A}}$ of a default theory (D, W) iff F is true (resp. false) in each E_i, $i \in \mathcal{A}$.

Example 2.3 Consider the default theory (D, W) in Example 2.2 again. This theory has exactly one extension class

$$\{Cn(\{p\}), Cn(\{p, q\})\}$$

Since p is true in both sets of this extension class, we say p is implied by the extension class semantics of this default theory, and q is *unknown* as it can be said to be neither true nor false, based on the extension class. $\Box$

It has been shown that every default theory has at least one nonempty extension class [1].

2.3 The Well-Founded Semantics

The well-founded semantics, introduced by Van Gelder, Ross, and Schlipf [6], is one of the most prominent semantics for logic programs, which can be redefined in terms of alternating fixpoints as below.

A logic program is a set of clauses of the form

$$A \leftarrow B_1, \ldots, B_n, \neg C_1, \ldots, \neg C_m$$

where $m, n \geq 0$ and A, B_i's, and C_j's are atoms.

Let $\mathcal{P}$ be a program and I a (two-valued) Herbrand interpretation of $\mathcal{P}$. Then the Gelfond-Lifschitz transformation of $\mathcal{P}$ with respect to I is the logic program P^I obtained from $\mathcal{P}$ as follows:

1. eliminating from $\mathcal{P}$ each clause whose body contains the negation of an atom in I;

2. from the body of each remaining clause in $\mathcal{P}$, delete all negative literals.

Recall the transformation $\mathbf{T}_P(I)$, called the *immediate consequence operator for a definite program*, whose output is a set of atoms such that $a \in \mathbf{T}_P(I)$ if and only if a is the head of some clause in $\mathcal{P}$ all of whose literals in the body are in I. P^I is a definite program and hence has a unique least model which is given by $\mathbf{T}_{P^I} \uparrow \omega$.

We define $\mathbf{S}_P(I) = \mathbf{T}_{P^I} \uparrow \omega$. A *fixpoint* of $\mathbf{S}_P$ is an interpretation of $\mathcal{P}$ such that $I = \mathbf{S}_P(I)$, and an *alternating fixpoint* of $\mathcal{P}$ is an interpretation I of $\mathcal{P}$ such that $I = \mathbf{S}_P(\mathbf{S}_P(I))$. It has been shown that I is a stable model of $\mathcal{P}$ if and only if I is a fixpoint of $\mathbf{S}_P$ [7, 5].

Let $\mathbf{A}_{\mathcal{P}}(I)$ be $\mathbf{S}_P(\mathbf{S}_P(I))$. Since $\mathbf{S}_P$ is anti-monotonic, $\mathbf{A}_{\mathcal{P}}$ is monotonic and its least fixpoint exists.

Proposition 2.1 ([5]) Let T be the least fixpoint of $\mathbf{A}_{\mathcal{P}}$, $F = \{a | a \notin \mathbf{S}_P(T)\}$. Then $< T, F >$ is the well-founded model of $\mathcal{P}$. $\Box$

Baral and Subrahmanian extend the well-founded semantics to default theories in the following way [2].

Since T is anti-monotonic, T^2, the function that applies T twice, is monotonic. Let $lpf(T^2)$ be the least fixpoint of T^2 and $gfp(T^2)$ be the greatest fixpoint of (T^2). Then the well-founded semantics is defined as follows.

Definition 2.4 ([2]) Let (D, W) be a default theory and F be a formula. Then F is true in the well-founded semantics of the theory if F is true in $lfp(T^2)$. F is false in the well-founded semantics of the theory if F is false in $gfp(T^2)$. $\Box$

Przymusinski also gave a well-founded semantics for default logic [12], based on a three-valued formalism for autoepistemic logic and the natural correspondence between default theories and autoepistemic theories. Both well-founded semantics, by Przymusinski and by Baral and Subrahmanian respectively, reduce to the well-founded model semantics for normal logic programs.

2.4 Stable Class Semantics for Logic Programs

Baral and Subrahmanian [1] also introduced the concept of *stable class*. A stable class for a logic program $\mathcal{P}$ is a set S of interpretations such that $S = \{\mathbf{S}_P(I) | I \in S\}$. It has been shown that M is a (strict) stable class of $\mathcal{P}$ if and only if $E = \{Cn(M_i) \mid M_i \in M\}$ is an extension class of the default theory translated from $\mathcal{P}$ [1, 2].

Baral and Subrahmanian have realized that some stable classes do not make positive contribution to capturing intended semantics and therefore defined a preference relation among all stable classes and then defined the stable class semantics for logic programs as the union of all *minimal strict stable classes* of $\mathcal{P}$ [1]. (Note that the extension class semantics for default theories is defined by the set of all extension classes.)

3 Recent Semantics Considered Too Weak

The original extension semantics for default theories suffers from the no-extension problem, that is, some default theories may not have extensions at all, as demonstrated by Example 2.2

The recently proposed semantics to address this problem do not preserve Reiter's semantics for "non-problematic" default theories.

Example 3.1 Let (D, W) be the default theory, where

$$W = \{c \leftarrow a, c \leftarrow b\} \qquad D = \{\frac{: \neg b}{a}, \frac{: \neg a}{b}\}$$

There are two extensions $E_1 = Cn(\{a, c\} \cup W)$ and $E_2 = Cn(\{b, c\} \cup W)$, which seem to capture exactly the intuitive meaning of the theory. It then seems desirable for a new semantics that intends to "correct" the no-extension problem to preserve the semantics of Reiter for "non-problematic" default theories. $\square$

Under the extension class semantics, the theory has three extension classes, viz.

$$\begin{aligned} \mathcal{E}_1 &= \{Cn(\{a, c\} \cup W)\} \\ \mathcal{E}_2 &= \{Cn(\{b, c\} \cup W)\} \\ \mathcal{E}_3 &= \{Cn(\{a, b, c\} \cup W), Cn(W)\}. \end{aligned}$$

In the well-founded semantics by Baral and Subrahmanian, as well as in the three-valued approach by Przymusinski, all the atoms have the unknown value. This yields a weaker semantics, and the arguably intuitive meaning under Reiter's semantics is not preserved. As a result, neither choice reasoning nor skeptical reasoning under the original semantics can be preserved.

It appears that in order to preserve Reiter's semantics, all one needs to do is to identify the desired extension classes, i.e., $\mathcal{E}_1$ and $\mathcal{E}_2$ for the preceding example. Baral and Subrahmanian have studied different orderings over extension classes and proposed to use the so called *Smyth ordering*. As a matter of fact, the set of all minimal extension classes under this ordering defines the stable class semantics for logic programs. For the default theory above, this ordering indeed isolates $\mathcal{E}_1$ and $\mathcal{E}_2$ as desired. Thus, it seems that the problem can be resolved by eliminating undesirable extension classes. However the following example shows that the approach used in the stable class semantics may yield unreanonable semantics for logic programs.

Example 3.2 Let $\mathcal{P}$ be given by

$$a \leftarrow \neg a$$
$$b \leftarrow \neg b$$
$$c \leftarrow a, \neg a$$
$$c \leftarrow b, \neg b$$

$\mathcal{P}$ has two strict stable classes, viz. $C_1 = \{\{a, b, c\}, \emptyset\}$, and $C_2 = \{\{a, c\}, \{b, c\}\}$, but only C_2 is minimal by Smyth ordering. Therefore, the stable class semantics of $\mathcal{P}$ is determined by C_2, which implies c is true.

Since the premises for deriving c can never be satisfied in any circumstance, c shall not be true in any reasonable semantics.

If the above program is expressed as a default theory by the familiar translation, by applying Smyth ordering, one gets the unintuitive extension class that corresponds to C_2 above.

□

4 The Alternating Semantics for Default Theories

In this section, we define the alternating semantics for default theories and demonstrate that the alternating semantics provides a satisfactory solution to the problems discussed in the previous section.

The basic idea behind the alternating semantics is that given a default theory (D, W), $T_{D,W}$ may not always have fixpoints, which is the source of the no-extension problem for the extension semantics of default logic. However, since T is anti-monotonic, T^2 is monotonic and therefore, the least fixpoint of T^2 does exist. By considering fixpoints of T^2, that is the alternating fixpoints, instead of extension classes, we are able to define a desired semantics.

Definition 4.1 Let (D, W) be a default theory, and E be a set of formulas. E is said to be an alternating point of (D, W) if and only if $E = T(T(E))$. □

Example 4.1 Consider the default theory in Example 3.1 again. The theory has four alternating fixpoints, viz. $I_1 = Cn(\{a, c\} \cup W)$, $I_2 = Cn(\{b, c\} \cup W)$, $I_3 = Cn(\{a, b, c\} \cup W)$, and $I_4 = Cn(W)$. $\Box$

An alternating fixpoint is a set of formulas, and a formula F is true in an alternating fixpoint if and only if F is contained in F and F is false in an alternating fixpoint if and only if $\neg F$ is contained in the point. Since T is anti-monotonic and T^2 is monotonic, every default theory has at least one alternating fixpoint. The following theorem shows that the well-founded semantics is characterized by the set of all alternating fixpoints of a default theory.

Theorem 4.1 A formula F is true (resp. false) in the well-founded semantics of a default theory (D, W) if and only if it is true (resp. false) in the set of all alternating fixpoints of the theory.
Proof: It directly follows from Definition 2.4 and the fact that for each alternating fixpoint I, we have $I \subseteq gfp(T^2)$ and $lfp(T^2) \subseteq I$. $\Box$

The well-founded semantics is determined by the set of all alternating fixpoints of the theory. However, not every alternating fixpoint makes positive contribution to the semantics. Consider the theory in Example 4.1 again. (D, W) has four alternating fixpoints, viz I_1, I_2, I_3, and I_4. But the intuitive meaning of the theory is characterized by the first two alternating fixpoints. The challenge here is how to eliminate those undesirable alternating fixpoints.

Let (D, W) be a default theory and I be an alternating fixpoint of the theory. Then $J = T(I)$ is also an alternating fixpoint, $\{I, J\}$ is an extension class, and I is a fixpoint if and only if $I = J$. This simple fact tells us that each fixpoint is closely attached to another alternating fixpoint. Let I and J be such two alternating fixpoints. Then $\{I, J\}$ can be used to represent the meaning of the theory without conflict only if one is a subset of another, i.e., either $I \subseteq J$ or $J \subseteq I$. This observation leads to the following definition.

Definition 4.2 Let (D, W) be a default theory, and I be an alternating fixpoint of the theory. Then I is said to be

1. a *max-alternating fixpoint* of the theory if and only if $T(I) \subseteq I$,

2. a *minimal max-alternating fixpoint* of the theory if I is a max-alternating fixpoint of the theory and there exists no man-alternating fixpoint J such that $J \subset I$, and

3. a *normal* alternating fixpoint of the theory if either I is a minimal max-alternating fixpoint or $T(I)$ is a minimal max-alternating fixpoint of the theory.

The alternating semantics of the default theory is then defined by the set of all normal alternating fixpoints of the theory. $\square$

Example 4.2 Consider the default theory in Example 3.1 and 4.1 again. I_1, I_2 and I_3 are max-alternating fixpoints, but I_4 is not. Furthermore, only I_1 and I_2 are minimal max-alternating fixpoints, and therefore, I_1 and I_2 are the only normal alternating fixpoints of the theory. $\square$

The following theorem shows that every default theory has at least one normal alternating fixpoint.

Theorem 4.2 Every default theory has at least one normal alternating fixpoint.
Proof: It is sufficient to show that every default theory has at least one max-alternating fixpoint. Since T is anti-monotonic, $lfp(T^2) \subseteq gfp(T^2)$ and $lfp(T^2) = T(gpf(T^2))$. Therefore, $gpf(T^2)$ is a max-alternating fixpoint of the theory. $\square$

Example 4.3 Consider the default theory corresponding to the logic program in Example 3.2 as follows.

$$W = \emptyset, \qquad D = \{\frac{:\neg a}{a}, \frac{:\neg b}{b}, \frac{a:\neg a}{c}, \frac{b:\neg b}{c}\}$$

The theory has four alternating fixpoints, viz. $I_1 = Cn(\{a, b, c\})$, $I_2 = Cn(\emptyset)$, $I_3 = Cn(\{a, c\})$, and $I_4 = Cn(\{b, c\})$.

Of four alternating fixpoints, only I_1 is a max-alternating fixpoint, and therefore, I_1 and I_2 are the only normal alternating fixpoints of the theory. Hence the alternating semantics of the theory is the same as its well-founded semantics. $\square$

There are also default theories for which there seem to be reasonable (partial) extensions that could not be captured in Reiter's extension semantics.

Example 4.4 Consider the default theory (D, W) given by

$$W = \emptyset, \qquad D = \{\frac{:\neg a}{b}, \frac{:\neg b}{a}, \frac{:\neg a}{p}, \frac{:\neg p}{p}\}$$

This default theory corresponds to the following logic program $\mathcal{P}$

$$a \leftarrow \neg b$$
$$b \leftarrow \neg a$$
$$p \leftarrow \neg a$$
$$p \leftarrow \neg p$$

The default theory has three normal alternating fixpoints: $I_1 = Cn(\{b, p\})$, $I_2 = Cn(\{a, p\})$, and $I_3 = Cn(\{a\})$. I_1 corresponds to one extension class and the other extension class consists of I_2 and I_3. Thus, The alternating semantics of (D, W) coincides with the stable class semantics of $\mathcal{P}$. However, under the extension semantics we have exactly one extension I_1, which seems biased. $\square$

Note that the alternating semantics is not a proper extension of the stable class semantics, since a stable class may not correspond to any alternating fixpoints.

Example 4.5 Consider the default theory:

$$W = \emptyset, \qquad D = \{\frac{:\neg a}{b}, \frac{:\neg b}{c}, \frac{:\neg c}{a}\}$$

There are two extension classes:

$$E_1 = \{Cn(\{a, b, c\}, Cn(\emptyset)\}$$
$$E_2 = \{Cn(\{a\}), Cn(\{b\}), Cn(\{c\}), Cn(\{a, b\}), Cn(\{a, c\}), Cn(\{b, c\})\}$$

The alternating semantics is determined by the two normal alternating fixpoints, $Cn(\{a, b, c\})$ and $Cn(\emptyset)$. If we apply Smyth ordering, the extension class E_2 will be chosen to represent the semantics of theory. Indeed, the stable class semantics of the corresponding logic program is determined by the stable class $C = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, which corresponds to E_2 $\square$

5 The Alternating Semantics for Logic Programs

By the familiar, natural translation from logic programs to default theories [8], the alternating semantics we propose here for default theories automatically provides a natural semantics for normal logic programs.

A logic program clause

$$A \leftarrow B_1, \ldots, B_n, \neg C_1, \ldots, \neg C_m$$

can be translated to a default

$$\frac{B_1 \wedge ... \wedge B_n : \neg C_1, ..., \neg C_m}{A}$$

Then an alternating fixpoint of a program $\mathcal{P}$ is a (two-valued) Herbrand interpretation, and normal alternating fixpoints of $\mathcal{P}$ can be identified in exactly the same way as for default theories.

We argue that this semantics is a faithful extension of Gelfond and Lifschitz's stable model semantics, as it assigns the same semantics for those logic programs whose stable models are never questioned or faulted in the literature. Further, for logic programs that have no stable models, it provides a satisfactory extension.

6 Final Remarks

We comment that, like the well-founded semantics and the extension class semantics, the alternating semantics proposed in this paper tackles the problem of no-extension and biased extension, it however does not resolve the problem of *inconsistency*. Inconsistency can arise because a given default theory is inconsistent in the sense of traditional logic, or because seemingly independent justifications lead to contradictory consequents. The second case is more interesting in the context of nonmonotonic reasoning.

Let us first consider the default theory given by

$$W = \{a\}, \qquad D = \{\frac{: b}{d}, \frac{: c}{\neg d}\}.$$

There are two alternating fixpoints, $Cn(\{a\})$ and the set of all formulas, both of which are normal. Now suppose we have one more default, i.e.,

$$W = \{a\}, \qquad D' = \{\frac{:b}{d}, \frac{:c}{\neg d}, \frac{:p}{q}\}.$$

Its alternating fixpoints are exactly the same as before. However, it is intuitively desirable to derive q, independent of the fact that some contradictory beliefs could also be derived. (For the two examples we discussed here, the alternating semantics coincides with the well-founded and extension class semantics.)

Consistency-based default logics, like J-default logic [4], Cumulative Default Logic [3], and THEORIST [10], avoid the contradiction in the above example by branching into two extensions, one containing $\{a, d, q\}$ and the other containing $\{a, \neg d, q\}$. This indeed solves the problem at hand. However, these approaches also depart from Reiter's semantics when a default theory is not considered problematic. For example, the default theory below

$$W = \emptyset, \qquad D = \{\frac{:\neg a}{b}, \frac{:\neg b}{c}\}$$

has exactly one extension in Reiter's logic; but it has two extensions in consistency-based logics. Thus, these logics do not preserve Reiter's semantics for "non-prolematic" theories. As a matter of fact, the unique extension in Reiter's logic is not only reasonable but is also intuitive and desirable, as it presents the familiar notion of preference (particularly in the context of logic programming). Indeed, the intuitive reason that the first default has higher priority is because it provides "evidence" against assuming $\neg b$. The problem of identifying preferred extensions has recently caught great attention in the field of nonmonotonic reasoning (see, for example, [11]).

It is then interesting to see whether the problem of inconsistency can be properly treated while still preserving Reiter's semantics for "non-problematic" default theories.[1]

[1]We should mention that the problem of handling inconsistency of this type has been considered for the well-founded semantics [9].

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