Reply to discussion by O. Hungr
of

"Momentum transfer and friction in the debris of rock avalanches"

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Hungr raises 3 points

1) Hungr argues that the kinetic energy of the slide mass as it enters the accumulation zone is $\frac{1}{2} M v_0^2$ where M is the slide mass and v_o the entry velocity. This assumes that the velocities of all the particles in the rock avalanche system are the same and equal to v_o.

Consider a system of 2 particles each of mass M/2. If the particles each move with velocity, v_o, the kinetic energy of the system is

$$\frac{1}{2}\left(\frac{M}{2} v_o^2 + \frac{M}{2} v_o^2\right) = \frac{1}{2} M v_o^2$$

If one particle has a velocity $v_o/2$ and the second particle has a velocity, $3 v_o/2$, their kinetic energy is

$$\frac{1}{2}\left(\frac{M}{2}\frac{v_o^2}{4} + \frac{M}{2}\frac{9v_o^2}{4}\right) = \frac{5}{4} \cdot \frac{1}{2} M v_o^2$$

25% higher than the system with 2 particles moving at the same velocity. Notice, however, that the linear momentum and the mean velocity of the two systems are the same.

The velocity of the system of n particles discussed in our paper is the velocity, $\bar{v}$, defined in equation 1 of the paper

$$\bar{v} = \Sigma M_i v_i / \Sigma M_i$$

The velocity v_E in Hungr's energy balance is

$$v_E = \left(\Sigma M_i v_i^2 / \Sigma M_i\right)^{\frac{1}{2}}$$

The distribution of velocities in the system of particles controls by how much V_E exceeds $\bar{v}$.

Particles in our system may also have rotational energy, a possibility not open to Hungr's single sliding block. If a sphere rolls down an incline without sliding its kinetic energy is $1.2(\frac{1}{2}Mv_o^2)$ with, again, the same linear momentum and mean velocity.

To summarize our views on this point, we agree with Hungr that the change in kinetic and potential energy of the system of particles should equal the total work done by external forces in the accumulation zone. However, an independent estimate of the kinetic energy of our system on its entry to the accumulation zone requires a knowledge of the distribution of velocities in the system and the processes by which the particles are moving. Their kinetic energy will certainly be substantially greater than $\frac{1}{2} M v_o^2$

2) Hungr argues that our application of Newton's second law to the behaviour of rockslides is in error. He extends our analogy with a rocket and suggests that as rockslides have no rocket engine, "u, the velocity of the gasses expelled from the rocket engine relative to the rocket itself ..... (is) equal to zero". His equation (3) therefore becomes

$$M(s)\frac{dv}{dt} = F \qquad (1)$$

whose integration, Hungr claims, leads to

$$L = -v_0^2/2g\,(\sin\theta \;-\; \mu\cos\theta) \tag{2}$$

We do not agree. Perhaps, a careful indentification of the variables in our system will clarify this.

M(s) is the mass of the moving system after travelling a distance s into the accumulation zone. L is the distance travelled by the centre of gravity of the total mass (Figure 1).

As in Equation 1, ds/dt = v and M(s) g (sinθ - μcosθ) = F, so

$$M(s)\,v\,\frac{dv}{ds} = M(s)g\,(\sin\theta \;-\; \mu\cos\theta)$$

and

$$v^2 = 2g\,(\sin\theta \;-\; \mu\cos\theta)s + v_0^2 \tag{3}$$

with $v = v_o$ at $s = 0$. As $v = 0$ at $s = s_f$

$$s_f = \frac{-\,v_0^2}{2g\,(\sin\theta \;-\; \mu\cos\theta)} \tag{4}$$

Assuming that $M(s) = \frac{s_f - s}{s} \cdot M$, as in equation (7) of our paper, we have, for a linearly decreasing mass,

$$L = \frac{S_f}{2} = \frac{-v_0^2}{4g\,(\sin\theta \;-\; \mu\cos\theta)} \tag{5}$$

The value of L in equation (5) is half that predicted by Hungr's equation (2) and is clearly incorrect. We suggest that this error results from Hungr's incorrect assumption, u = 0.

Hungr's equation (1) is a statement that the change of linear momentum of a system of particles is equal to the external forces on the system (Beer and Johnson, 1987, pp. 633-634), u is the relative velocity of the particles expelled from the system with respect to the system velocity, v. In our paper, the expelled particles are those at rest. They are no longer part of the rock avalanche which is moving with average velocity v. Thus u = -v. As particles are expelled, their contribution to the change in linear momentum is counted. Once the particles are at rest they don't contribute to the changing momentum, they are outside the system. Hungr may have been misled by our rocket analogy into the assumption that his equation 1 applies only to systems with engines. We also drew attention in our paper to Timoshenko and Young's (1948, p. 113-118) comments on the motion of a balloon discarding ballast and this may be more helpful. Of course, the masses of rock moved by the rock avalanches we considered are several orders of magnitude larger than those of balloons or rockets. We apologize for the confusion this seems to have caused.

3) Finally, we did not expect that our paper "would revoke the existing consensus" because we do not think there is a consensus. Recent publications (Campbell, 1989, for instance) confirm that there is a lively debate in progress. Further, we do not share Hungr's view that our discipline advances by "revoking consensus" as in so-called

scientific revolutions (Kuhn, 1970). We prefer the approach of Popper and Lakatos (Lakatos, 1970) and regard our paper as proposing a research programme which will, for the first time, concentrate some attention on the depositional profiles of rock avalanches. From this perspective, we welcome Hungr's comments as they have prompted us to clarify our model. Given the depositional profile and the friction angle of the displaced material, we can now estimate the mean velocity at the entry to the accumulation zone of a dry rock avalanche.

References

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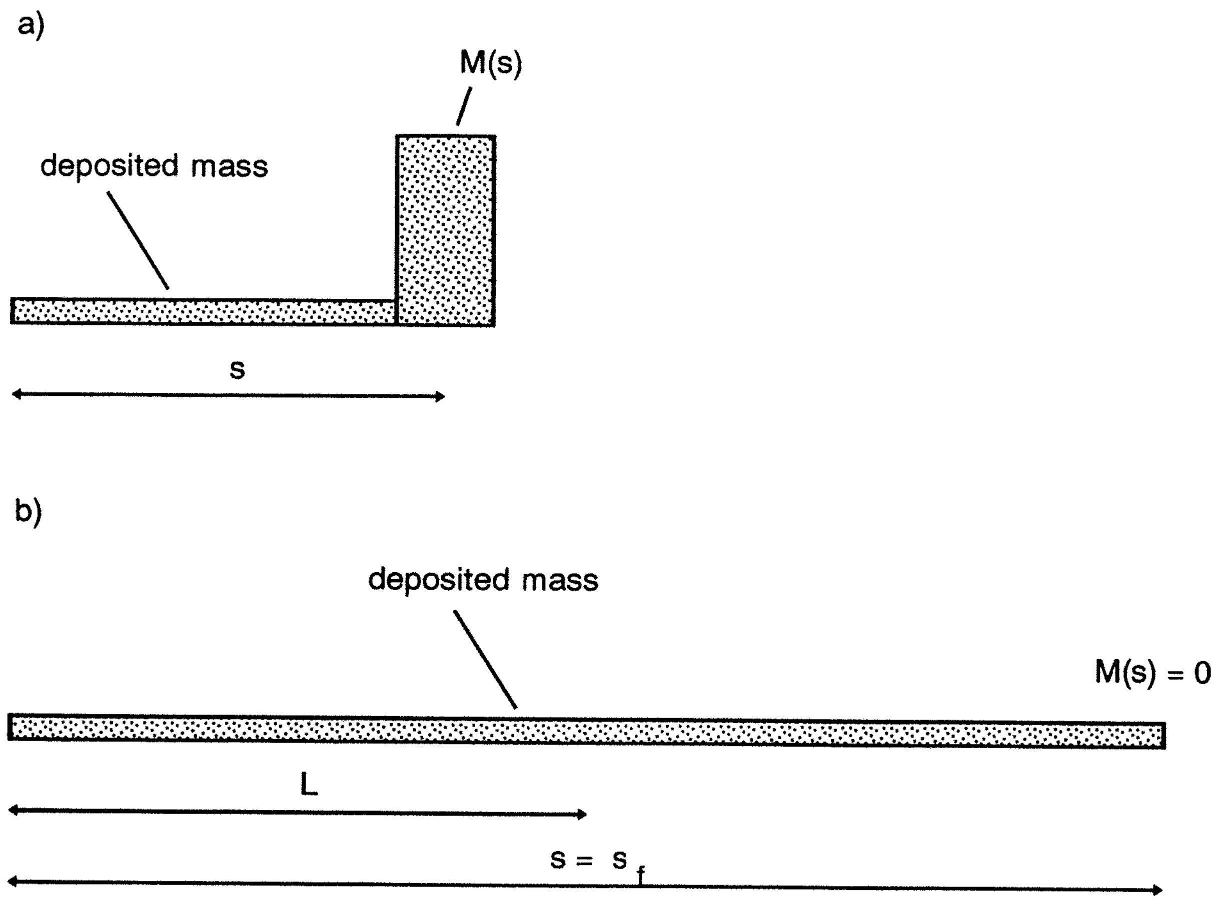

Figure 1 Definition of terms a) avalanche in motion b) avalanche deposited

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