Optimal forest harvest age considering carbon sequestration in multiple carbon pools: a comparative statics analysis

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Abstract

We present an analytical model for determination of the economically optimal harvest age of a forest stand considering timber value, and the value of carbon fluxes in living biomass, dead organic matter, and wood products pools. Through comparative statics analysis, we find that consideration of timber value and fluxes in biomass carbon increase harvest age relative to the timber only solution, and that the effect on optimal harvest age of incorporating fluxes in the dead organic matter and wood products pools is indeterminate.

We also present a numerical example to examine the magnitudes of these effects. In general, incorporating the dead organic matter and wood products pools have the effect of reducing rotation age. Perhaps more interestingly, when initial stocks of carbon in dead organic matter or wood products pool is relatively high, consideration of these pools can have a highly negative effect on net present value.

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1. Introduction

Concerns about climate change associated with increasing concentrations of greenhouse gases (GHGs) has led to addition of carbon sequestration to the list of ecosystem services provided by forests with potential economic value. Forests may mitigate the effects of greenhouse gas (GHG) induced change, since trees remove substantial amounts of CO_2 from the atmosphere through photosynthesis (IPCC, 2000). It is important to remember, however, that they also release CO_2 to the atmosphere through the processes of respiration and decomposition.

The choice of harvest age is the fundamental decision in an even-aged silvicultural system. Generally, delaying the harvest age allows stands to increase in volume, thereby storing more carbon (Harmon and Marks, 2002). Forests managed on a longer harvest cycle accumulate more dead organic matter (DOM) and, on average, store more carbon than forests managed on a shorter harvest cycle (Krankina and Harmon, 2006). The choice of harvest age will also affect the stock of carbon stored in wood products. If wood products decompose at a slower rate than DOM in the forest, there may be an advantage to choosing a harvest age that provides a larger mean annual increment (MAI), and therefore more wood products. There may also be some benefits of substituting wood products for more GHG intensive construction materials such as concrete and steel.

Hoen (1994), van Kooten et al. (1995), Hoen and Solberg (1997), and Stainback and Alavalapati (2002) have investigated the impact of carbon tax and subsidy schemes on the optimal harvest age for even-aged management. In these models landowners are paid a subsidy for periodic carbon uptake in living biomass and taxed when carbon is released through harvest or decay. The models developed in the above studies are essentially variations on the Hartman (1976) model, which includes non-timber benefits related to the standing forest. These models demonstrate that carbon taxes and subsidies will affect the optimal forest harvest age and, consequently, the carbon stored in forests. Englin and Callaway (1995) examined the impact of carbon price on the optimal harvest age of Douglas-fir and concluded that rotation age increases with increasing carbon price. Plantinga and Birdsey (1994) used an analytical model in the framework of Hartman (1976) to show that with carbon benefits only in the analysis, the harvest age was infinite in most cases and with carbon and timber, the harvest age would be between the carbon only and the timber only harvest age. Enzinger and Jeffs (2000) showed that the optimal rotation for *Eucalyptus* spp. plantations is longer when carbon payments are considered than when they are not.

These studies do not account for carbon stored in dead wood, the forest floor, soil, or wood products. A substantial proportion of the total carbon stored in forest stands is dead organic matter. The choice of harvest age can have a substantial effect on soil carbon stocks (Aber et al., 1978; Kaipainen et al., 2004). In Asante et al. (2011), we demonstrated that the optimal harvest age can differ substantially between cases where carbon in the DOM pool is considered and where it is not. It may also be important to consider carbon stored in the wood product pool. The amount of carbon in the wood product pool is affected by the choice of harvest age as it directly affects harvest volume. The harvest age also affects the size and distribution of logs in the stand and therefore the product mix (Krankina and Harmon, 2006).

The study presented here differs from the previous studies because it presents a matheatical model of the economics of forest carbon sequestration suitable for a comparative statics analysis, considering timber values and carbon values for biomass, dead organic matter, and wood product pools. The effect of changing carbon prices and the inclusion of different pools in the analysis is considered. A numerical example is also presented.

2. The Analytical Model

The analytical model assumes that the landowner wishes to determine the optimal harvest age, T^*, that maximizes the net present value (NPV) of timber and carbon sequestration values of an area of bare forest land. To simplify this analysis, it is also assumed that the forest is managed under a single cycle establishment, growth, and harvest. We make this assumption because as we showed numerically in (Asante et al., 2011), the optimal harvest age may be dependent on the carbon stocks in the DOM pool and this will change over time: it is possible that the optimal harvest age for any given rotation is different from the optimal harvest age for subsequent rotations (see Fig. 1). A correctly formulated infinite time horizon model should incorporate the possibility for multiple rotation ages: unfortunately, our mathematical skills are not up to that challenge. Nonetheless, we believe something interesting can be learned from a single rotation model.

A reviewer suggested that we could use a generalized Faustmann model such as that used by Chang (1998) and by Armstrong and Phillips (1989) to account for the value of future stands. This technique is used to find the optimal harvest age for the first rotation, given an exogenous specification of the land expectation value (LEV) of the stand at the beginning of the second rotation. The generalized Faustmann model is a useful way of thinking about the optimal forest rotation problem, given the near certainty that "market conditions, government policies, regeneration technology, management activities, and timber stand growth can change the land expectation value from timber crop to timber crop" (Chang 1998, p.654). This technique would work well if the future LEV can be viewed as exogenous. However, in the model we present here, future LEV is related directly to the harvest age chosen for the current stand, as that decision affects the initial dead organic matter and product pool stocks for subsequent rotations, and consequently affects the LEV. For the purposes of this paper, we believe a correct specification of a single period model is more useful than an incorrect specification for an infinite time horizon model.

2.1. Timber only

We will begin by building the timber only model. All prices and costs are expressed in Canadian dollars (CAD). The NPV of one rotation of a stand of timber is

$$\mathrm{NPV}_t = ((P^w - C^v)\, V[T] - C^a)\, e^{-\rho t} - C^e \tag{1}$$

where P^w is the price of timber (CAD/m^3), C^v is the harvest cost per unit volume (CAD/m^3), C^a is the fixed harvest cost per unit area (CAD/ha), C^e is the stand establishment cost (CAD/ha), and ρ is the real discount rate. The volume of timber (m^3 ha^{-1}) standing on the forest at any age t is given by the timber yield function $V[t]$. The timber harvest volume of a stand harvested at age T is given by $V[T]$.

The first order condition for maximization is

$$\mathrm{NPV}_t'[t] = \rho\,(((P^w - C^v)\, V[T]) - C^a) - (P^w - C^v)\, V'[T] = 0 \tag{2}$$

which can be rearranged to become: Find T such that

$$\rho = \frac{(P^w - C^v)\, V'[T]}{(P^w - C^v)\, V[T] - C^a} \tag{3}$$

The right hand side of Eq. 3 will be referred to as the relative value growth rate (RVGR). The harvest age should be chosen such that the RVGR at the harvest age is equal to the discount rate. We will find it useful to create an alias for the numerator, $X^t = (P^w - C^v)\, V'[T]$, and the denominator, $Y^t = (P^w - C^v)\, V[T] - C^a$. Using the timber yield and financial parameters from (Asante et al., 2011), the decision rule is illustrated in Fig. 2.

We will use the simple model in Fig. 2 as the basis for the comparative statics analysis of the effect of including different carbon pools in the optimization. If the RVGR curve shifts upward, the optimal harvest age is older. Conversely, if the RVGR curve shifts downward, the optimal harvest age is younger.

2.2. Timber plus biomass pool

Suppose there exists a market where a forest landowner is paid for an increase in the mass of carbon stored in her trees and pays for a decrease. The NPV of biomass for carbon sequestration in one timber rotation can be written as

$$\mathrm{NPV}_b = \int_0^T e^{-\rho t} P^c B'[t]\, dt - e^{-\rho T} P^c B[T] \tag{4}$$

where P^c is the price of carbon (CAD per tonne of carbon (tC henceforth)), and $B[t]$ expresses the mass of carbon in living trees (biomass, tC/ha) as a function of stand age, t (years). The term to the left of the minus sign expresses the NPV of carbon stored over the life of the stand; the term to the right expresses the payment that must be made when the stand is harvested, as the living biomass is assumed to be set to zero at the time of harvest.

The NPV of timber harvest and carbon sequestration services from biomass can be calculated as

$$\mathrm{NPV}_{tb} = \mathrm{NPV}_t + \mathrm{NPV}_b \tag{5}$$

The first order condition for maximization is

$$\mathrm{NPV}'_{tb} = e^{-\rho t}\left(-C^a\rho + P^c\rho B[T] + (C^v - P^w)\left(\rho V[T] - V'[T]\right)\right) = 0 \tag{6}$$

which can be rearranged to become: Find T such that

$$\rho = \frac{(P^w - C^v)\, V'[T]}{(P^w - C^v)\, V[T] - C^a - P^c B[T]}. \tag{7}$$

If we substitute in the aliases for the numerator and denominator from the timber only model, we get

$$\rho = \frac{X^t}{Y^t - P^c B[T]}. \tag{8}$$

$B[T]$ is assumed to be increasing with T. For positive carbon prices, the right hand side of Eq. 8 is always larger than the right hand side of Eq. 3. This means that the relative growth rate curve for timber and biomass is always above the relative growth rate curve for timber only. For any discount rate, the optimal harvest age will be older when timber and biomass are considered as compared to the timber only case. One way of interpreting Eq. 8 is that the optimal harvest age is older in order to delay the penalty associated with removing the biomass pool at harvest.

We will find it useful to create an alias for the denominator of Eq. 8, $Y^{tb} = Y^t - P^c B[T]$, so that we can compare solutions considering other carbon pools to the timber plus biomass solution.

2.3. Timber plus biomass and dead organic matter (DOM) pools

As we did in Asante et al. (2011), we assume that the DOM pool in the model decays at a constant rate, α. The living biomass contributes to the DOM pool through a process we call litterfall. A fixed proportion, β, of the biomass pool is added to the DOM pool. The change in the DOM pool is described as a differential equation

$$D'[t] = \beta B[t] - \alpha D[t]. \tag{9}$$

If we solve this differential equation for $D[t]$ given an initial DOM stock of $D[0]$, the DOM stocks at any stand age can be written as

$$D[t] = e^{-t\alpha}\left(D[0] + \int_0^t e^{k\alpha}\beta B[k]\,dk\right). \tag{10}$$

The rate of change in DOM stock, given $D[0]$ is

$$D'[t] = \beta B[t] - e^{-t\alpha}\alpha\left(D[0] + \int_0^t e^{k\alpha}\beta B[k]\,dk\right). \tag{11}$$

It is important to note that this rate of change is dependent on the initial stock, $D[0]$, the the DOM pool.

We assume that the market rewards accumulation of carbon in the DOM pool and penalizes reductions. The NPV of the DOM pool is

$$\mathrm{NPV}_d = \int_0^T P^c D'[t] e^{-\rho t}\,dt + P^c(B[T] - \gamma V[T])e^{-\rho T} \tag{12}$$

The term to the left of the plus sign shows the contribution to NPV resulting from the DOM decay and litterfall processes. The term to the right of the plus sign indicates the value of the pulse of input to the DOM pool associated with harvest. The parameter, γ is a conversion factor used to convert timber volume (m^3) to mass of carbon (tC). All biomass, except that removed as merchantable volume, is transferred to the DOM pool at the time of harvest.

The NPV of timber plus the biomass and DOM pools is

$$\mathrm{NPV}_{tbd} = \mathrm{NPV}_t + \mathrm{NPV}_b + \mathrm{NPV}_d \tag{13}$$

and the corresponding harvest age decision rule, with the aliases for the numerator and denominator from the timber only solution substituted in, is

$$\rho = \frac{X^{\mathrm{t}} + P^c \left(B'[T] - \gamma V'[T]\right) + P^c D'[T]}{Y^t - P^c \gamma V[T]}. \tag{14}$$

The incorporation of $-P^C \gamma V[T]$ will have the effect of lengthening the rotation to delay the loss of the harvested wood volume to the system. However, the direction of change implied by the numerator is indeterminate. The term $P^c \left(B'[T] - \gamma V'[T]\right)$ will generally be positive, as it represents the difference between the rate of growth of biomass and the biomass equivalent of merchantable volume (one possible exception to this general statement could occur at a stand age when many of the trees in reach a merchantability threshold: at this age the rate of increase in merchantable biomass good be greater than the rate of increase of total biomass). However, $P^c D'[T]$, could be positive or negative depending on stand age, decay rate, litterfall rate, and initial DOM stocks, $D[0]$. All other things being equal, a higher $D[0]$ will result in a lower rotation age.

When we substitute in timber plus biomass solution,

$$\rho = \frac{X^{\mathrm{t}} + P^c \left(B'[T] - \gamma V'[T]\right) + P^c D'[T]}{Y^{\mathrm{tb}} + P^c \left(B[T] - \gamma V[T]\right)}, \tag{15}$$

the discussion relating to the numerator remains the same, so the conclusions about an indeterminate direction must hold. However as compared to the timber plus biomass solution, the denominator will always be larger, which results in a shorter rotation relative to the timber plus biomass solution.

The numerator for Eq. 15 will be aliased as $X^{tbd} = X^{\mathrm{t}} + P^c \left(B'[T] - \gamma V'[T]\right) + P^c D'[T]$ and the denominator by $Y^{tbd} = Y^{\mathrm{tb}} + P^c \left(B[T] - \gamma V[T]\right)$ for subsequent discussions.

2.4. Timber plus biomass, DOM, and wood product pools

Carbon is stored in wood products, and the amount of wood products produced is related to the volume of trees harvested, and volume is assumed to be increasing with stand age. Wood products are considered in the forest carbon offsets protocol developed by California Climate Action Registry (2009). In the California Climate Action Registry protocol, offset credits are increased with increasing wood product production and decreased with decreasing wood product production.

Including the wood product pool in the calculation of carbon offsets paid to a forest landowner is difficult to justify, in our minds. It is unlikely that the landowner has any custody of the wood once it leaves her forest, or perhaps her mill. The choice of final end use for the products is out of the hands of the landowner. However, there is at least one existing protocol that incorporates forest products, so for completeness, we will include forest products pool in this analysis.

Because our analysis is at the stand level, we will define a forest products pool that will represent the stock of forest products harvested from the land area currently occupied by the timber stand. For our purposes we will assume that a proportion of the harvested wood volume becomes a relatively long-lived product (say lumber in housing) and that the remainder becomes a very short-lived product (say toilet paper) and waste. We assume that the long-lived product decays at a rate, θ, and that the carbon in the short-lived product and waste is released to the atmosphere immediately upon harvest. We make this assumption so that we can deal with a single pool representing products. We will use $Z[t]$ to represent the stock of carbon (tC/ha) in the long-lived product pool at time t: $Z[0]$ will represent the initial stocks.

$$Z[t] = e^{-t\theta} Z[0] \tag{16}$$

The amount of carbon emitted as a result of decay from the product pool is

$$Z'[t] = -\theta Z[0] e^{-\theta t} \tag{17}$$

At the time of harvest, a portion of the merchantable volume, $\gamma\lambda V[t]$ is transferred to the product pool. Our landowner is paid for additions to product pool stocks and pays for reductions. The constant λ represents the proportion of merchantable volume that enters the product pool: γ is a conversion factor used to convert merchantable volume (m^3 to mass of carbon (tC)).

The NPV of the additions to and removals from the product pool is calculated as

$$\mathrm{NPV}_z = \gamma\lambda P^c V(T) e^{-\rho T} + \int_0^T P^C e^{-\rho t} \frac{\partial Z(t)}{\partial t}\, dt \tag{18}$$

$$= e^{-\rho T} P^c \gamma\lambda V[T] - \frac{\left(1 - e^{-T(\theta+\rho)}\right) P^C \theta Z[0]}{\theta + \rho} \tag{19}$$

The RVGR for timber plus biomass, DOM, and product pools is shown below, with the numerator and denominator for the timber plus biomass and DOM pools substituted in.

$$\rho = \frac{X^{\text{tbd}} + P^c \left(e^{-t\theta} Z[0] + \gamma\lambda V'[T]\right)}{Y^{\text{tbd}} + P^c \gamma\lambda V[T]} \tag{20}$$

Including the product pool has the effect of making the denominator larger, therefore leading to a tendency to shorten the rotation. However the numerator is also larger (and dependent on the initial stocks of the product pool) which would tend to lengthen the rotation, relative to the timber plus biomass and DOM model. The overall direction of the change in optimal harvest age relative to the timber plus biomass and DOM pools is indeterminate.

3. Numerical example

We present a numerical example to illustrate further the points made in the comparative statics analysis. Most of the parameter values for this example are from Asante et al. (2011) representing a lodgepole pine [(*Pinus contorta* Dougl. var. *latifolia* Engelm. (Pinaceae)) stand in northeastern British Columbia, Canada. All costs and revenues are expressed in Canadian dollars (CAD). We refer the interested reader to Asante et al. (2011) for detailed description of the parameter derivation: the timber yield, price, and cost information is appropriate for a lodgepole pine stand managed for lumber production in northeastern British Columbia at the time this work was done.

Merchantable timber volume (m^3/ha) grows according to a Chapman-Richards growth function

$$V[t] = 500.4(1 - e^{-0.027t})^{4.003}. \tag{21}$$

Biomass (tC/ha) also grows according in a Chapman-Richards function

$$B[t] = 198.6(1 - e^{-0.0253t})^{2.64}. \tag{22}$$

We use $\gamma = 0.2$ to convert between merchantable wood volume and carbon mass. This is consistent with a carbon content of wood of approximately 200 kg m^{-3} (Jessome, 1977). We use the parameter, $\lambda = 0.67$, to express the proportion of merchantable stand volume that is converted to lumber. This was calculated on the basis of a lumber recovery factor of 250 board feet of lumber per cubic metre of roundwood input.

The price of timber, P^w is set at 89.40 CAD/m^3; volume based harvest cost, C^v, is set to 47.55 CAD/ha; and area based harvest cost, C^a, is set to 6 250 CAD/ha. Instead of setting establishment costs, C^e, to 1 250 CAD/ha, as in Asante et al. (2011), we use $C^e = 0$ to avoid tangential discussion about the negative NPV that would arise in the timber only case. For the purposes of the example, we use carbon offset prices ranging from 1 to 50 CAD/tCO$_{2e}$, which equates to 3.67 to 183 CAD/tC, using 3.67 as the ratio of the molecular weight of carbon dioxide to the atomic weight of carbon.

We use a continuous discount rate, $\rho = 0.05$, a decay rate $\alpha = 0.00841$, a litterfall rate $\beta = 0.01357$, and a product decay rate $\theta = 0.00578$. The derivation of the numerical values for α and β are discussed in detail in Asante et al. (2011). These parameters were estimated using a non-linear least squares estimation procedure to find the decay and litterfall rates that leads to the best approximation of the projection of sum the non-living carbon pools generated by the CBM-CFS3 model developed by the Canadian Forest Service (Kurz et al., 2009), for a lodgepole pine stand in northeastern British Columbia following the timber yield curve shown above. The product decay rate, θ, was estimated using non-linear regression to find the estimate of θ which resulted in the best fit to data tabulated by Kurz et al. (1992) showing the proportion of orginal carbon remaining in lumber over a 100 year time horizon in 20 year steps.

3.1. Timber only

The optimal harvest age considering only timber production for a single rotation is 68.3 years and the associated NPV is 140 CAD/ha using the decision rule in Eq. 3.

3.2. *Timber plus biomass pool*

Table 1 presents the optimal harvest age and associated NPV for a range of carbon offset prices considering timber and the biomass carbon pool calculated using the decision rule shown in Eq. 7. The optimal harvest age increases with increasing carbon offset price: at 50 CAD/tCO_2e the optimal decision is to never harvest. NPV increases with increasing carbon offset prices from 140 CAD/ha with a carbon price of 0 CAD/tCO_2e to 4390 CAD/ha with a carbon price of 50 CAD/tCO_2e. Note that all NPVs in Table 1 are greater than in the timber only case.

3.3. *Timber plus biomass and DOM pools*

Tables 2 and 3 present the optimal harvest ages and associated NPVs for combinations of carbon price and initial DOM stocks. The optimal harvest age increases with increasing carbon price, and decreases with increasing initial DOM stocks. It is interesting to examine the joint effects of carbon price and initial DOM stocks on NPV (Table 3). When initial DOM stocks are low, NPV increases with carbon price; when DOM stocks are high, NPV decreases with increasing carbon prices. This is due to the payment required for declining DOM carbon stocks due to decomposition (Fig. 1).

All the NPVs presented in Table 3 for initial DOM stocks of 0 or 100 tC ha^{-1} are greater than in the timber only case. For initial DOM stocks of 200 tC ha^{-1} or greater, the NPVs are lower. For initial DOM stocks of 300 tC ha^{-1} or greater NPVs decline with increasing carbon price. For initial DOM stocks of 200 tC ha^{-1}, the behaviour is more complicated, as the NPV declines with increasing carbon price up to 20 CAD/tCO_2e, but shows an increase at 50 CAD/tCO_2e.

This behaviour will be explained using Fig. 3 Between prices of 24 and 30 CAD/tCO_2e, the NPV function switches from having a maximum at a finite rotation age to having a maximum at an infinite rotation age.

3.4. Timber plus biomass, DOM, and product pools

Tables 4 and 5 present the optimal harvest ages and associated NPVs for combinations of carbon price and initial DOM and product pool stocks. In comparison to the model where timber, biomass, and DOM are considered, optimal harvest ages are younger: substantially so for carbon prices greater than 20 CAD/tCO_2e. The optimal harvest ages for carbon prices of 50 CAD/tCO_2e are older, but finite, in contrast. Optimal harvest ages and NPVs decline with increasing initial stocks of the product pool.

4. Conclusions

We developed an analytical model of the economically optimal harvest age of a timber stand considering timber value, and the value of CO_2 capture and emissions considering biomass, dead organic matter, and wood product pools. The model presented here considers just a single rotation. We do this because, as we show in the analysis, optimal harvest age depends on initial stocks of carbon in the DOM and product pools, and these will change from one harvest cycle to another. We also apply the model in a numerical example using parameters from Asante et al. (2011).

The results from our comparative statics analysis show that

1. as compared to the timber only model, the timber plus biomass model results in older optimal harvest ages,

2. when changes in the stock of carbon in DOM are valued, a higher initial stock of DOM will generally result in a younger optimal harvest age,
3. the effect on optimal harvest age of incorporating biomass and DOM pools relative to the timber only solution or the timber plus biomass solution is indeterminate, and
4. the effect on optimal harvest age of incorporating biomass, DOM, and product pools relative to the timber only solution or the timber plus biomass and DOM solution is indeterminate.

There are some interesting observations to be made about our numerical example. In this example,

1. participation in the market when initial DOM stocks are 200 tC ha^{-1} or higher would be unattractive to a landowner as the NPV is lower than the timber only case. Combinations of high carbon prices and high initial DOM stocks can result in negative NPVs,
2. in our example, including biomass and DOM stocks always resulted in an older rotation age than the timber only model, and a younger rotation age than the timber plus biomass model, and
3. in our example, incorporating the product pool leads to a reduction in the optimal harvest age.

In our minds, the most important conclusion to draw from this study is that the optimal harvest age for a stand of timber will depend on which carbon pools are being accounted for. Considering only the living biomass in a forest stand will generally lead to an older optimal rotation than if changes in the carbon stored in snags, the forest floor, and soil (what we referred to dead organic matter) is taken into account.

One result from this study that surprised us was that NPV always decreased with increasing initial stocks of carbon in the DOM pool or the product pool. In retrospect, this should not have been a surprise Because of decay, the non-living carbon pools represent a source of greenhouse gases. In our model, the proportional decay rate is constant: increased stocks of carbon in the DOM and product pools would result in greater absolute emissions of greenhouse gases, all other things being equal. In a carbon market where a decision maker pays for reductions in the size of the carbon pools under control, stocks of carbon become a liability. Pardoxically, there is a benefit associated with increasing carbon stocks, and a cost associated with holding the carbon stocks. This cost is what drives the shorter rotations found when carbon stocks in the non-living pools are considered. It is important to keep this paradox in mind when developing policies or markets related to the sequestration of carbon in forests.

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Table 1: Optimal harvest age and net present value by CO_2 offset price when the biomass pool is considered.

Carbon price (CAD/tCO_{2e})	Optimal harvest age (yr)	NPV (CAD/ha)
1	70.0	211
2	71.8	283
5	77.9	507
10	91.6	902
20	235	1 750
50	∞	4 390

Table 2: Optimal harvest age by CO_2 offset price and initial dead organic matter stocks when the biomass and dead organic matter pools are considered.

Carbon price	Initial DOM stocks ($tC\,ha^{-1}$)				
CAD/tCO_{2e}	0	100	200	300	400
1	69.6	69.5	69.3	69.2	69.1
2	70.9	70.6	70.4	70.1	69.8
5	75.1	74.4	73.7	73	72.3
10	83.3	81.8	80.3	78.8	77.2
20	108	104	99.9	95.9	91.9
50	∞	∞	∞	∞	∞

Table 3: Optimal net present value by CO_2 offset price and initial dead organic matter stocks when the biomass and DOM pools are considered.

Carbon price	Initial DOM stocks ($tC\,ha^{-1}$)				
CAD/tCO_{2e}	0	100	200	300	400
1	236	184	132	80.6	28.7
2	333	229	125	21.3	−82.5
5	627	366	106	−154	−415
10	1 130	605	82.0	−441	−963
20	2 170	1 120	63.4	−989	−2 040
50	5 400	2 770	125	−2 510	−5 150

Table 4: Optimal harvest age by CO_2 offset price, initial dead organic matter stocks, and initial product pool stocks when the biomass, DOM, and product pools are considered.

Carbon price	Initial DOM	Initial product pool stocks (tC ha^{-1})				
CAD/tCO_{2e}	stocks (tC ha^{-1})	0	50	100	150	200
1	0	69.5	69.4	69.4	69.3	69.2
	100	69.3	69.3	69.2	69.2	69.1
	200	69.2	69.1	69.1	69.0	69.0
	300	69.1	69.0	69.0	68.9	68.8
	400	68.9	68.9	68.8	68.8	68.7
2	0	70.6	70.5	70.4	70.3	70.2
	100	70.4	70.2	70.1	70.0	69.9
	200	70.1	70.0	69.9	69.8	69.6
	300	69.8	69.7	69.6	69.5	69.4
	400	69.6	69.4	69.3	69.2	69.1
5	0	74.2	74.0	73.7	73.4	73.1
	100	73.6	73.3	73.0	72.7	72.4
	200	72.9	72.6	72.3	72.0	71.7
	300	72.2	71.9	71.6	71.3	71.1
	400	71.5	71.2	70.9	70.7	70.4
10	0	80.9	80.3	79.7	79.1	78.5
	100	79.5	78.9	78.3	77.7	77.1
	200	78.1	77.5	76.9	76.3	75.7
	300	76.7	76.0	75.4	74.8	74.2
	400	75.2	74.6	74.0	73.3	72.7
20	0	97.8	96.4	94.9	93.5	92.1
	100	94.6	93.1	91.7	90.3	88.9
	200	91.3	89.9	88.4	87	85.6
	300	88.0	86.6	85.1	83.7	82.2
	400	84.6	83.2	81.7	80.3	78.8
50	0	281	268	257	241	227
	100	261	247	232	218	204
	200	239	220	209	194	180
	300	214	198	183	169	156
	400	186	171	157	144	133

Table 5: Optimal net present value by CO_2 offset price, initial dead organic matter stocks, and initial product pool stocks when the biomass pool is considered

		Z0				
PCO2	DOM0	0	50	100	150	200
1	0	238	219	201	182	164
	100	186	167	149	130	112
	200	134	116	97.0	78.4	59.8
	300	82.3	63.7	45.2	26.6	7.96
	400	30.5	11.9	−6.71	−25.3	−43.9
2	0	336	299	262	225	187
	100	232	195	158	121	83.5
	200	129	91.4	54.1	16.9	−20.3
	300	24.8	−12.5	−49.7	−86.9	−124
	400	−79.0	−116	−153	−191	−228
5	0	634	541	448	354	261
	100	374	281	187	93.8	0.460
	200	114	20.3	−73.0	−166	−2 510
	300	−147	−2 310	−333	−426	−5 110
	400	−407	−4 910	−593	−686	−779
10	0	1 140	953	765	577	389
	100	617	4 210	242	54.5	−133
	200	94.7	−92.8	−280	−468	−655
	300	−428	−615	−802	−989	−1 180
	400	−949	−1 140	−1 320	−1 510	−16 100
20	0	2 180	1 800	1 420	1 050	668
	100	1 130	750	373	−4.90	−382
	200	77.1	−300	−678	−1 050	−1 430
	300	−973	−1 350	−1 730	−2 100	−2 480
	400	−2 020	−23 100	−2 770	−3 150	−3 530
50	0	5 400	4 460	3 510	2 560	1 610
	100	2 770	1 820	865	−84.8	−1 030
	200	125	−825	−1 770	−2 720	−3 670
	300	−2 510	−3 460	−4 410	−5 360	−6 310
	400	−5 150	−6 100	−7 050	−8 000	−8 950

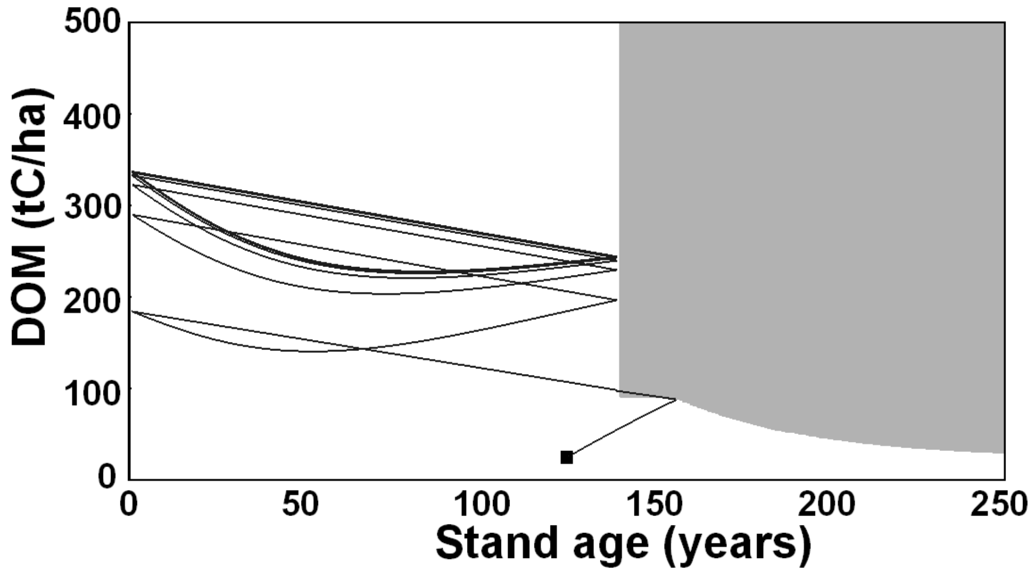

Figure 1: Optimal harvest decision rule and trajectory of DOM. Reprinted from Asante et al. 2011.

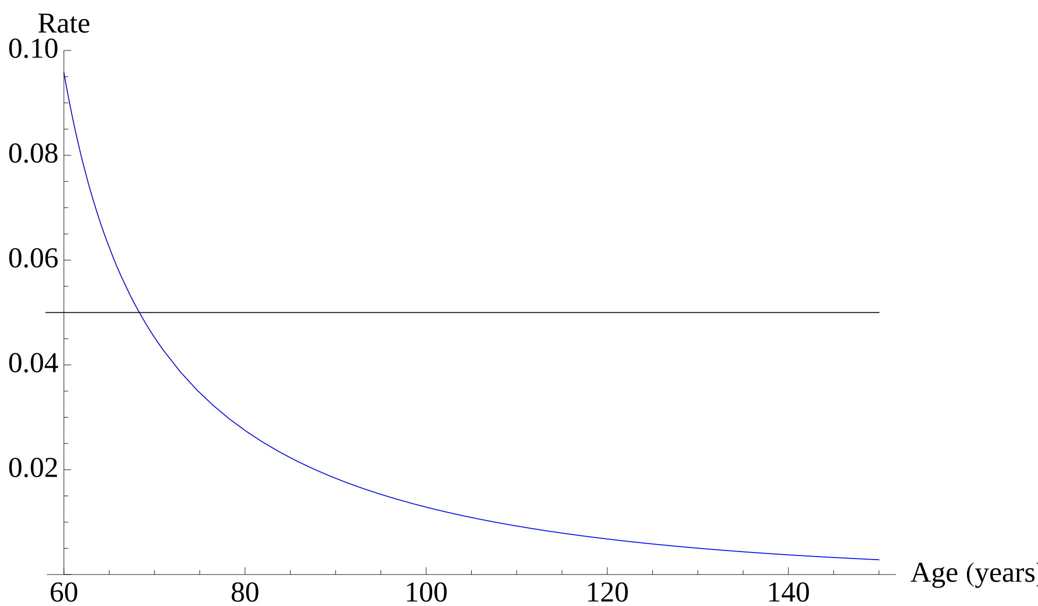

Figure 2: First order condition for optimization of harvest age. The horizontal line represents a discount rate of 5%. The curve is the relative value growth rate. The intersection of the two represents the optimal harvest age, 68.3 years, in this example.

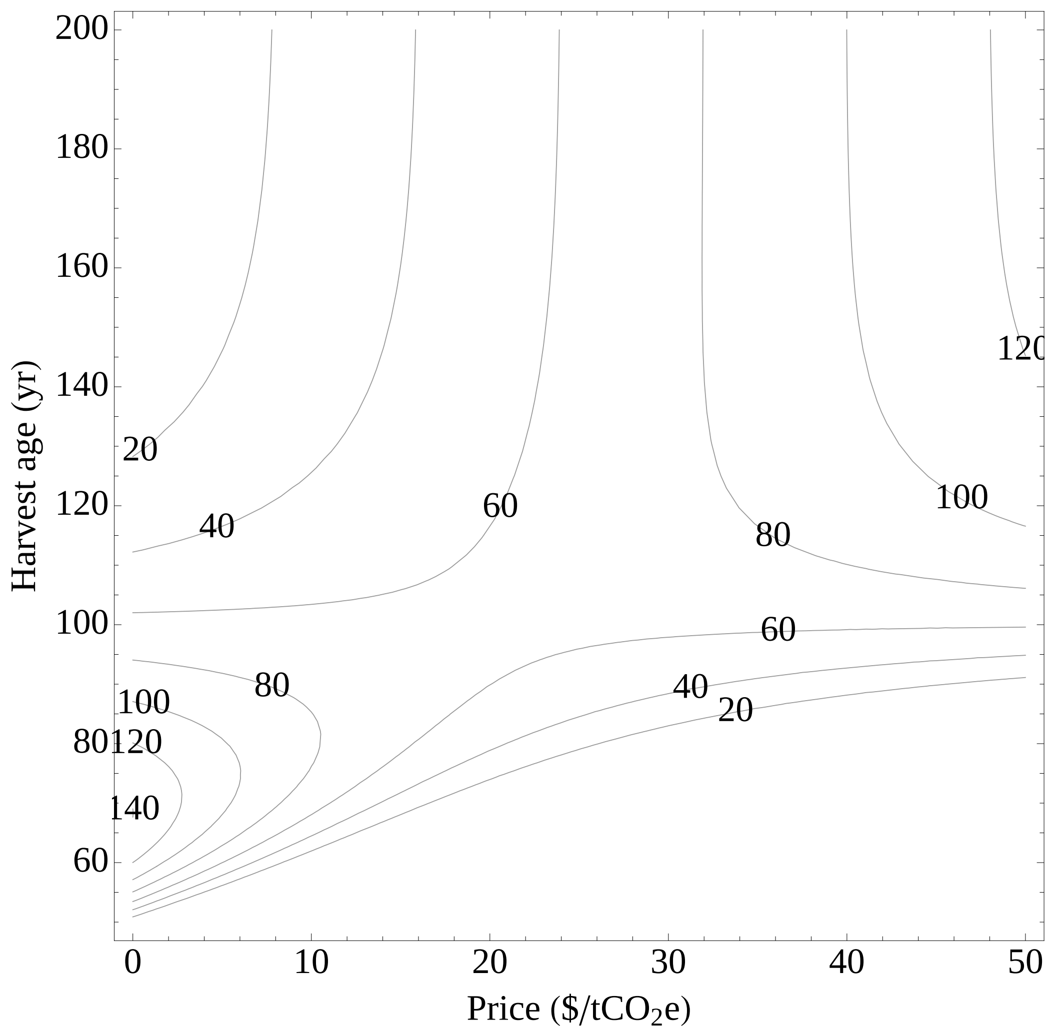

Figure 3: Contour plot of net present value function for timber plus biomass and DOM pools for initial DOM stocks of 200 tC ha^{-1}.