Optimal Pricing Scheme Achieving Maximum Revenue For Online Retailers

by

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Abstract

As e-market is becoming more popular, setting a proper price to maximize profit is vital for retailers on trading platforms. Most of the online retailers choose traditional pricing methods such as average pricing and markup pricing to set their prices. These traditional methods set prices based on the costs and the profit gain only, failing to consider the demands, the consumer personal preferences, and the inter-seller competitions. This motivates us to develop a proper pricing method that solves the above problems for online retailers.

In this thesis, we propose an optimal pricing scheme (OPS) which enables the online retailers to achieve maximum revenue by recommending best prices. We applied the market share, the linear weight buyer model, and the most competitive sellers to address the above problems. Based on these platforms, we construct the revenue equations and find the best price and maximum revenue for sellers at different levels. The results for both simulated market and real market show that our proposed pricing scheme achieves higher revenue than traditional methods.

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Table of Contents

List of Tables

List of Figures

List of Symbols

S_i	skyline seller i
S^{set}	seller set including all skyline sellers
S^{set}_{Lower}	lower subset of skyline market
S^{set}_{Upper}	upper subset of skyline market
p_i	the price of seller i
r_i	the reputation of seller i
U_i	the utility of seller i
$E_n(x_n)$	the revenue of the target seller S_n
$\mathcal{P}(x_n)$	the probability seller S_n being chosen as best choice
w	the weight buyer assigns to price
x_i	the normalized price for seller i
y_i	the normalized reputation for seller i
α_u	the upper bound for weight w
α_l	the lower bound for weight w
S_U	the most competitive seller in the upper seller set
S_L	the most competitive seller in the lower seller set

List of Abbreviations

Acronyms	Definition
BC	Best Choice
BEP	Break Even Point
BP	Best Price
CP	Crossing Point
LCS	Less Competitive Seller
MB	Market Basket
MCS	Most Competitive Seller
MP	Market Position
OPS	Optimal Pricing Scheme
SM	Skyline Market
SS	Skyline Seller
TS	Target Seller

Chapter 1

Introduction

1.1 E-commerce

Electronic-commerce emerged and prospered quickly in recent years. It is changing every aspect of our lives, leading people to spend hundreds of billions of dollars online. The definition of electronic-commerce is given by International Organization for Standardization standards/ International Electrotechnical Commission (IOS/IEC) as [3]: "From a commercial, legal and standardization perspective, one can view electronic commerce as: a category of business transactions, involving two or more persons, enacted through electronic data interchange, based on a monetary and for-profit basis. Persons can be individuals, organizations, and/or public administrations". Meanwhile, Levine broadly categorized e-commerce into the following cases [4]: business to business, business to public administration, individual to business, individual to public administration, and public administration to public administration. Rania described e-commerce with more detailed categories, such as business to business (B2B), business to consumer (B2C), business to employee (B2E), business to government (B2G), business to manager (B2M), consumer to business (C2B), consumer to consumer (C2C), government to business (G2B), government to citizen (G2C), government to employee (G2E), and government to government (G2G) [5].

In this thesis, we focus on business to consumers (B2C) platforms which include all manners of business such as products or services selling to customers. Moreover, business to consumers platforms have grown to provide online travel services, online housing sites, and online auctions, etc. Some well-known business to consumers platforms such as Amazon, eBay and Taobao, are becoming the main streams of online purchasing today. So a detailed study of these platforms and how retailers can survive and benefit from these platforms are vital.

1.2 Motivations and Contributions

E-commerce has become a major form of trading today, so the study of online trading platforms is more important and necessary than ever. In this thesis, we focus on the electronic markets on eBay, where retailers set their own prices for products, and buyers pick the products that suits their needs. Buyers choose products by comparing many different aspects such as the seller's price, reputation, quality of service, time for shipping, etc.

Many works were done trying to establish the online shopping strategy [6] [7] [8] [9], especially from buyers' perspective regarding the recommender systems. For example, Feng modeled a online shopping scenario entitled Multi-Attribute Probabilistic Selection (MAPS) framework [10]. This model was able to provide a high accuracy buyer recommendations by considering three factors simultaneously: the inter-attribute tradeoffs, the inter-item competition, and the user preference. Feng addressed the inter-attribute tradeoffs by using the visual angle model, which transformed the market sellers into points on a 2-D coordinate. Based on Feng's model, Gong [11] further increased the recommendation accuracy by borrowing the concepts of indifference curves (IC) and marginal ratio substitution (MRS) from book Microeconomics [12] to model buyer preference and to estimate the probability that each seller being chosen by buyers. However, little work is done from the retailers' perspective, which motivates us. We adopt the visual market model used in MAPS to to study the problem of how to maximize the retailers' revenue by setting an optimal price.

The necessity of finding the optimal price for online retailers is obvious in many aspects. First, a good price is vital because it is the most crucial attribute for customers to evaluate the products. Second, price plays a key role in determining the seller's $market\ position$ (MP), a seller's ranking in an organized e-market. For example, high reputation sellers often place high prices to suggest high quality or high market position. Third, for the sellers, setting the too high price will scare away most buyers, but the too low price will lose profit. It is clear that a proper price will help low reputation retailers improve competitiveness, while high reputation retailers utilize their advantages to gain more revenue.

Without the knowledge of the competitions from other retailers and the demand curves, online retailers usually choose traditional pricing methods (average pricing and mark-up pricing) to set their list prices. The traditional pricing methods display many drawbacks. First, these types of methods set prices based on the cost and profit margin only, failing to consider the demand curve and behaviors of different buyers. However, it is the combination of price and demand to determine the total revenue. The second drawback is that these methods fail to consider the competitions from other sellers in the same e-market. Then the third drawback is that these methods do not consider the $market\ positions$ (MP) of different sellers. In this case, if the low reputation seller adopts traditional pricing, his/her probability being chosen will drop sharply due to the high price. However, if the high reputation seller adopts traditional pricing, it will cause unnecessary profit loss.

This thesis aims to address the problems in the existing works, and develop a more realistic pricing strategy for online retailers. First, in the problem formulation, we choose the linear weight user model as our buyer model because it is simple and mathematically tractable [13]. As the beginning work in this direction, we use linear weight model to gain insights into the problem, and we will extend to more complicated and realistic model in the future. To address the fact that different users have different preferences, we let the weight w follow a probabilistic distribution but not a fixed number. Second, the thesis calculates the $market\ share$ to address the problem of the unknown demand curves. $Market\ share$ is defined as the firm's percentage of the total market. Without loss of generality, we regard the whole market as one, so the market share is also referred to as the probability the seller will be chosen by buyers. Third, this work considers market position as an important factor in the pricing process. For different market positions, our pricing scheme recommends different prices. Also, competitions from other sellers which is an inevitable factor during actual transactions is investigated in the thesis. By introducing the concept of the $most\ competitive\ sellers$, the thesis incorporates the inter-seller competitions into our pricing scheme.

In this paper, we propose an optimal pricing scheme (OPS), enabling the online retailers to achieve global maximum revenue with recommended best prices. The contribution and novelty of this thesis can be concluded as follows:

- This thesis takes the buyer personal preferences into consideration during problem formulation, and let the weight w follow a probalistic distribution rather than a fixed number to address the differences among different buyers.

- This thesis calculates the $market\ share$ to address the problem of the unknown demand curves.

- Unlike traditional pricing methods (the average and mark-up pricing methods), our proposed pricing methods recommends different sellers with different prices.

- The inter-seller competitions are investigated, and the $most\ competitive\ sellers$ are introduced to address this issue.

- Our proposed pricing scheme is experimented on both simulated and real markets. Three observed rules are drawn from large amount of real data calculations, which showed great potential to be used as general business pricing guidance for online retailers.

1.3 Outline

The thesis is organized as follows. Chapter 2 presents literature reviews on existing pricing methodologies and consumer behavior models. Chapter 3 gives the problem definition, and an example is analyzed to better explain our proposed method. Then Chapter 4 shows the numerical results of the simulated market, and analyzes relevant parameters that may affect the results. Furthermore, calculations on real market are conducted and three observed rules are concluded from the experimental results. Finally, Chapter 5 draws the conclusions and gives future research directions.

Chapter 2

Literature Review

2.1 The Role of Pricing

$Price$ is defined as the economic sacrifice a customer must make in exchange for a product or service from a seller [1]. In Bearden's book, the authors mentioned the price in different names. For example, the price paid in universities is called $tuition$, the price charged for professional services is named fee, the price for accommodation is $rent$ and there are many others such as $salaries$, $taxes$, and $donations$, etc. In this thesis, we refer the price as $list\ price$, which is the price set before any promotions or discount. Here, we consider $list\ price$ or $price$ as an important attribute of the online retailers, and we assume customers will pay exactly the same price listed without any promotion.

The importance of price strategy is also discussed in Bearden's book [1]. Setting a price does not require advertising, developing products, or changing distribution channels. Therefore, it becomes the most effective and fastest method to realize a company's maximum profit. Fig. 2.1 [1] demonstrates that price may directly influence the total revenue of a firm. Setting a price too low will forge profit and cause the firm's market position in a low level. But setting a price too high will directly lose customers. So finding an optimal price is crucial and will achieve maximum revenue.

Many factors may influence price, and the price will reversely impact demand

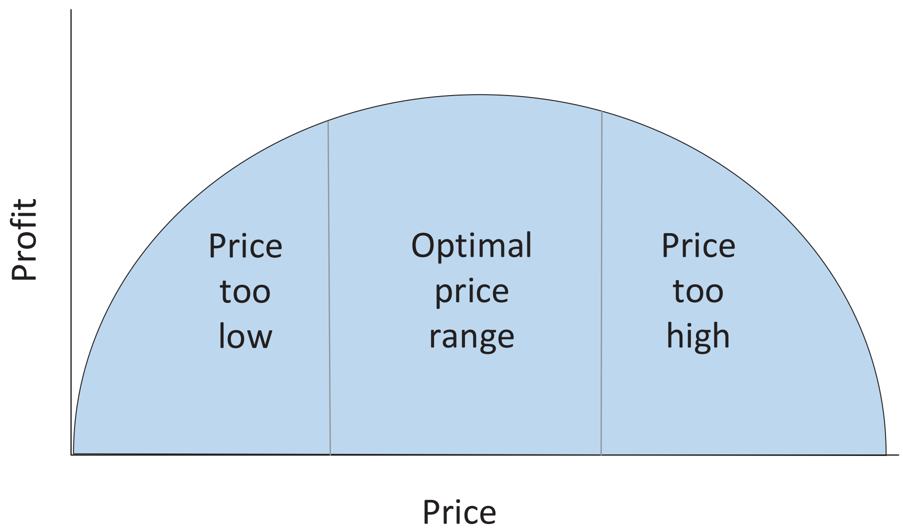


Fig. 2.1. Optimal Pricing Decisions [1]

and profit. Five factors that influence the prices were drawn in Bearden's book [1]. They include $cost$, determining the floor price of a product; $customers$, deciding how they perceive and are willing to pay for the product; $channels\ of\ distribution$, making sure the margin other channels can earn; $competitions$, influencing the price in price war; and, $compatibility$, meaning the price must suits the firm's overall objectives.

2.1.1 Demand Curves

Price and demand together impact the total revenue, and their relationship can be expressed using the $demand\ curves$ in Fig. 2.2 [1] [2]. For the rational buyers, an increase in price will decrease the sales, while a reduction in price will promote the sales. So how to balance the price and demand to reach a maximum revenue is a critical issue to be addressed in our thesis.

2.1.2 Price Elasticity of Demand

The perception of customer towards a product is depending more on the comprehensive attributes than absolute price, which can be represented by using $price\ elasticity\ of\ demand$ [14]. It can be calculated using the equation: Price Elasticity Of Demand $= \frac{\text{percentage change in quantity demanded}}{\text{percentage change in price}}$.

$Elastic\ demand$ happens when a small change in price causes a great change in demand, see Fig. 2.2 (a). When price decreases from P_2 to P_1, the sale quantities largely increase from Q_2 to Q_1, this is elastic demand. In contrast, $Inelastic\ demand$ exists when a small change in price does not significantly affect quantities like Fig. 2.2 (b).

2.2 Business Pricing Strategies

In the real business world, there are many ways to set list price. Perreault's work in [2] classified these approaches into $cost - oriented$ and $demand - oriented$ based on many key factors: cost, demand, pricing objectives, competitions from other sellers, geographic terms, and legal environment, etc..

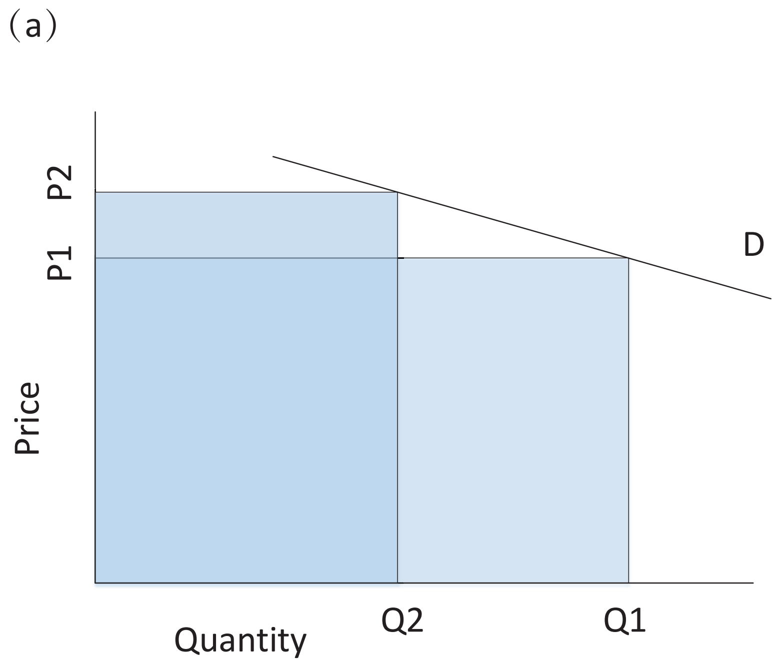
2.2.1 Cost-Oriented Pricing Strategy

Cost-Oriented pricing is the most common strategy in business. Marketers first calculate the total costs, then add the expected profit margin to obtain the price [14]-[15]. The most commonly used cost-oriented pricing method are described in the following [16].

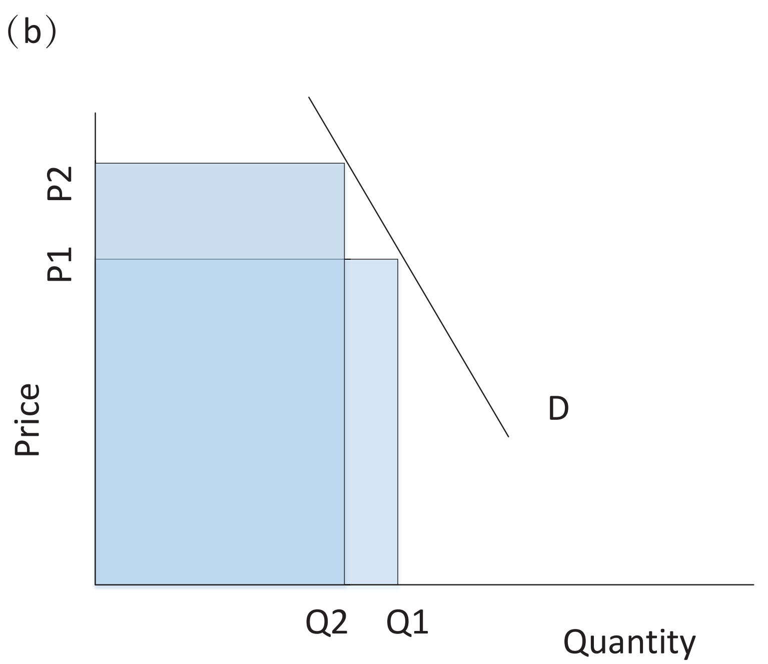
$a.\ Markup\ Pricing$

Most wholesale and retailers choose this traditional pricing method to set their list price [15]. $Markup$ is defined as the amount of money added to the cost of an product to get the price, expressed as percentage. Both equations can represent the markup percentage,

Markup Percentage Of Selling Price $= \frac{\text{markup}}{\text{selling price}} = \frac{\text{selling price}-\text{cost}}{\text{selling price}}$,



(a) Elastic Demand



(b) Inelastic Demand

Fig. 2.2. a. Elastic Demand b. Inelastic Demand [1] [2]

Markup Percentage Of Cost $= \frac{\text{markup}}{\text{cost}} = \frac{\text{selling price}-\text{cost}}{\text{cost}}$.

Many conventional retailers think that higher markup will bring higher profit, without considering the influence on demand. High markup will lead the price to a level that customers will no longer accept. Therefore the retailer will loss the market share, and further lose profit. In this thesis, we used α to represent the markup percentage of cost.

b. Cost Plus Pricing

Cost plus pricing put a reasonable markup on to the average cost for each product. The average cost is calculated using the total cost divided by the total quantity sold in the past. However, this method can be dangerous when the quantity sold in this year is less than the past years.

The disadvantage leads us to consider different types of *cost*. *Fixed cost* is the money spent on facilities, rents and so on, which is unrelated to the quantity sold. But *variable cost* means the wages, material charge, etc., and these things are closely related to the output of the product. The *total cost* is the two cost added together. Our thesis only consider the online micro agents who do not have rental or inventory needs, so we assume the *fixed cost* is zero, and the *total cost* is just the *variable cost*, and we assume the cost for every product sold is fixed.

c. Break Even Analysis

Break even analysis is a useful guide for setting prices. The *break even point* (BEP) is where the seller's total revenue equals to cost (see Fig. 2.3). Loss occurs at the left area of BEP, while profit is achieved at the right area. To reach BEP, the required quantity of the product sold is given by, $\mathrm{Qty}_{\mathrm{BEP}} = \frac{\text{total fixed cost}}{\text{fixed contribution per unit}}$. *Fixed contribution per unit* means the selling price per unit minus the variable cost per unit.

Break even analysis is helpful but cannot be regarded as a pricing solution [2]. First, The profit curve in the figure grows endless beyond the BEP which is unrealistic. Second, the total revenue curve is drawn based on the assumption that these quantity of products can be sold with the assumed price which is also not true in real situations.

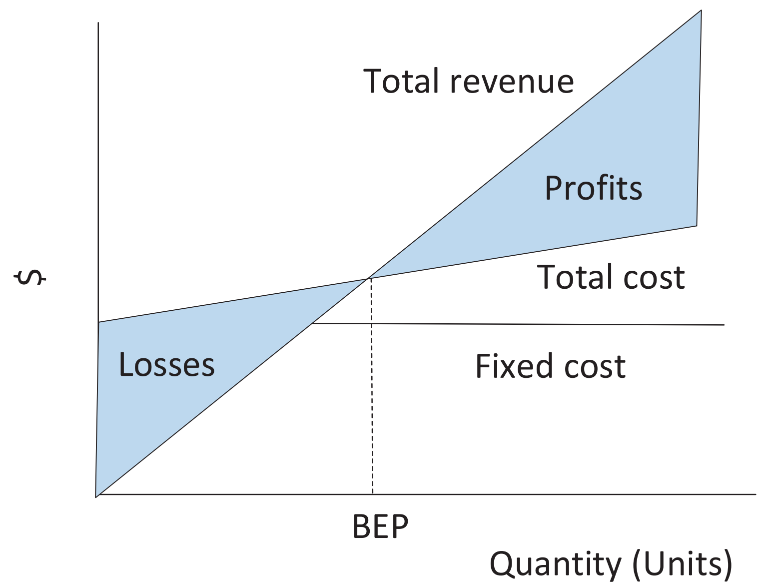


Fig. 2.3. Break Even Analysis [1]

2.2.2 Demand-Oriented Pricing Strategy

Demand-oriented pricing method sets the price by estimating the product value. [2] explained that customers usually have a *reference price* for each product and different customers may have different reference prices for the same product. In this case, if the seller sets the price below the customers' reference price, the demand will increase. Various demand-oriented strategies are used in business world, and we discuss on following four typical cases [16].

a. Leader Pricing

For leader pricing method, low prices are set on specific products called *leader items* in order to attract customers. The leader items are daily use products like milk, ice cream, coffee etc.. Customers usually have specific reference prices towards these items, so they would recognize the promotion. The strategy usually not only expect to increase the sales of the leader products, but also other daily products as well. Leader pricing examples are used everywhere such as "Daily special", "Deals of the day", etc..

b. Bait Pricing

Bait pricing is something similar to leader pricing by setting a really low price on a particular product to attract customers. The difference is that bait pricing expects to increase the sales of the high price products rather than the reduced price product. A standard way of doing this is that after the customers are attracted by the promotions, the employees will specify the drawbacks of the reduced price product, and at the same time, offer a higher quality and more expensive substitution for the customers.

Bait pricing has been criticized as unethical, especially extreme sellers use bait pricing on items they will not sell. The Federal Trade Commission has banned this pricing method in interstate commerce [2].

$c. Odd - even\ Pricing$

Odd-even price is common as well, which simply sets the price that ends with certain numbers. For example, many items are sold at $\$0.99, \$99.99, \$299.99$, etc. Some marketers believe that customers perceive these prices better because these prices seem to be lower than the next price level.

d. Prestige Pricing

Prestige pricing is often used on luxury brands, jewelries, and services industries. The method places a high price on the product to set a high market position. This pricing method is aimed at customers who emphasize on high quality and reputations. Usually, for this type of customers, if the price is set too low, they will doubt about the quality of the item sold and the market position of the company.

There are many other demand-oriented pricing methods not specified here such as demand-backward pricing, price lining, full-line pricing, complementary product pricing and so on.

2.3 Consumer Theory

2.3.1 Consumer Behavior

Understanding the behavior of the customers is fundamental in economics. The microeconomics [12] said that the consumer behavior is understood in three steps: consumer preference, budget constrain, and consumer choices. With these three steps, the consumer theory generalized how the consumers use their personal preferences and limited income to make a purchasing decision.

$a. Utility$

Microeconomic indicates that consumers can provide relative rankings of $market\ baskets$ (a list with specific quantities of one or more goods). So, "$Uitlity$ refers to the numerical score representing the satisfaction that a consumer gets from a market basket" [12]. $Utility\ function$ is a formula to assign the specific score to each market basket. Details will be introduced in the consumer behavior modeling.

2.3.2 Customer Behavior Modeling

Stanford Research Institute had developed values and lifestyles (VALS) system that segments the population into eight different purchasing classes according to their life style, cultural, and demographic aspects [17]. So, with so many diverse human behaviors, personal preference modeling is a challenging problem. Researchers study the user behavior either explicitly or implicitly [6][18]. Explicit user modeling requires users manually input preference or answer questionnaires [19][20]. Implicit user modeling use different techniques to obtain data from users' profiles and purchasing histories. Generally, researchers model user behaviors by clustering buyers into different behavior groups [21] or dynamically learning user behaviors [22][23].

Online retailers who do not have enough knowledge of demand curves and user profiles need implicit user modeling. The work in [10] divided the buyers into four groups: price threshold buyers pick the seller whose price is below the price threshold and has the largest reputation as best choice; dynamic reputation threshold buyers and fixed reputation threshold buyers pick the sellers whose reputations are larger than the reputation threshold and have the lowest prices as their best choices; the extreme buyers who only consider the price or the reputation while neglecting the other attribute when making a purchasing decision. The work in [11] divided the buyers into more groups: the Cobb-Douglas buyers, the price and the reputation threshold buyers, and the extreme buyers. Combining their works, we can conclude the following buyer behavior models.

$a.\ Linear\ Weight\ Buyer\ Model$

Linear weight buyers calculate the utility value of each retailer and choose the retailer with the largest utility score as their best choice. The most widely used utility function that combines multiple attributes with diverse weighting values is shown below [24][13]:

$$U(S) = \sum_{i=1}^{m} w_i * H_i(a_i), \text{with} \sum_{i=1}^{m} w_i = 1. \tag{2.1}$$

$U(S)$ is the utility of the seller, a_i is the attribute of the seller and w_i is the matching weight buyer assigns to this attribute. H_i is the transform function for attribute, which takes different forms, for example, $H(x) = x$, $H(x) = log(x)$, $H(x) = \frac{x}{\sqrt{x*x+\beta}}$, etc.[25][7][26].

$b.\ Cobb - Douglas\ Buyer\ Model$

Cobb-Douglas buyer model is another typical model used in Microeconomics [12] [11] as production function. Later economists adopted it as utility function to model buyers' personal preferences. The equation is shown below,

$$U(S) = \prod_{i=1}^{n} x_i^{a_i}(n \in N, a_1, a_2, ..., a_n > 0). \tag{2.2}$$

$U(S)$ is the utility of the seller, x_i is each attribute considered during the calculation, n is the total number of attributes of item S, and a_i is the weight buyer assigns to each attribute. For example, if only two attributes were concerned, then equation (2.2) can be further simplified into $U(S) = x^{\alpha}y^{\beta}(\alpha, \beta > 0)$. Where x, y are two conflicting attributes like price and reputation, and α, β indicate how much buyer emphasize these attributes.

$c.\ Threshold\ Buyer\ Model$

For these type of users, they filter the products by setting a threshold first [11] [10]. We can divide these buyers into two subgroups: Price-Threshold Buyer and Reputation-Threshold Buyer.

- Price-Threshold Buyer - Buyers of this type set the threshold on $PRICE = \bar{P}$. Among all the items whose prices are lower than $\bar{P}$, the buyers choose the one with the highest reputation.

- Reputation-Threshold Buyer - Buyers of this type set the threshold on $Reputation = \bar{R}$. Among all the items whose reputation are higher than $\bar{R}$, the buyers choose the one with the lowest price.

$d.\ Extreme\ Buyer\ Model$

Extreme buyers only consider one attribute while completely ignore the other when making a purchasing decision. They either buy product with the lowest price or the highest reputation level [11] [10].

The works in [24][13] specified that Multi-attributes Utility Theory (MAUT) has been widely applied in e-commerce to integrate various user preferences. When we take log on both sides of the utility function of Cobb-Douglas Buyer, we found it can be transformed into Linear Weight Buyer. Threshold and extreme buyer models can only be considered as specific cases, they cannot be served as the general buyer behavior model. We choose the linear weight user model a typical MAUT method[25][19] as our buyer model because it is simple and mathematically tractable. We use linear weight model to gain insights into the problem, and we will extend to more complicated and realistic model in the future. To address the fact that different users have different preferences, we let the weight w follow a probabilistic distribution but not a fixed number.

Chapter 3

Optimal Pricing Scheme

In this chapter, we elaborate how our proposed optimal pricing scheme is processed. Our proposed method contains three important parts: determining the most competitive sellers (MCSs) in different price ranges, deriving the maximum revenue equation of the target seller and finding the global optimal solution to the revenue equation. Detailed steps are introduced separately in the following subsections. First, in Section 3.1, we explain the market model, the buyer model and the problem formulation. Then, in Section 3.2, we introduce the concept of the most competitive sellers (MCSs) and discuss how to find them. Furhtermore, in Section 3.3, we give the derivation of the revenue equation for the target seller S_n. Finally, in Section 3.5 we show how to achieve the global maximum revenue for the target seller through an example.

3.1 Problem Definition

3.1.1 Formulation of Seller Set

First, we formulate an e-market with M sellers $S_i(i = 1, 2, ..., n-1, n, n+1, ..., M)$ selling the same item. These sellers form the $Market$ or the $seller\ set$, denote as $S^{set} = \{S_1, S_2, ..., S_{n-1}, S_n, S_{n+1}, ..., S_M\}$. The $Market$ is arranged in order, and the subscript i indicates the $market\ position$ (MP) of each seller. $Market\ position$

is the seller's rank in a sorted market. Also, we denote S_n as the $target\ seller$ (TS), who tries to reset price to gain more revenue. Each $seller$ has abundant properties such as price, custom service quality, item quality, reputation, etc. In this thesis, two key attributes are considered: the price p and the seller reputation r. Price is set by the seller while reputation is determined by the feedbacks the seller received from previous transactions. Then each seller can be denoted as $S_i = (p_i, r_i), i = (1, 2, ..., M)$.

The thesis assumes all buyers are rational, who only prefer items with lower price and higher reputation. So, we only consider $skyline\ sellers$ [27] as competitive ones to form the $Skyline\ Market$ (SM). In other words, no seller will be "worse" than another. The skyline seller means if seller S_i has a lower price than seller S_j ($p_i < p_j$), then the reputation of S_i must also be lower ($r_i < r_j$). Therefore, the $Skyline\ Market$ should satisfy the following: $p_1 < p_2 < ... < p_{n-1} < p_n < p_{n+1} < ... < p_M$ and $r_1 < r_2 < ... < r_{n-1} < r_n < r_{n+1} < ... < r_M$, $\forall p_i > cost, i = (1, 2, ..., M)$. The $cost$ mentioned in our thesis is the $total\ cost$ which is the sum of the $fixed\ cost$ and the $variable\ cost$. Because the online sellers do not have rental, facility or inventory need, so we assume the $fixed\ cost$ is zero. Therefore, the $cost$ in our thesis equals to the $variable\ cost$, and we assume it is fixed for every product. More background knowledge about $cost$ can be found in the literature review 2.2.1.

If S_n resets price p_n, the skyline market may change accordingly due to the skyline seller constraint. Assume the reputations are fixed, and S_n adjusts his price p_n within range $cost < p_n < p_{n+1}$ to gain more revenue, then the following n scenarios may happen.

- $Case\ 1 : p_{n-1} < p_n < p_{n+1}$, all M sellers remain in the skyline market, and $S^{set} = \{S_1, S_2, .., S_{n-1}, S_n, S_{n+1}, .., S_M\}$.
- $Case\ 2\ : p_{n-2} < p_n \leq p_{n-1}$. In this case, the price of S_n is smaller than S_{n-1} while the reputation is larger than S_{n-1}, which contradicts the skyline seller property. Following the rules of $skyline\ market$, S_n masks

S_{n-1}. So, a total of $M - 1$ sellers stay in the skyline market $S^{set} = \{S_1, S_2, .., S_{n-2}, S_n, S_{n+1}, .., S_M\}$.

- $Case\ 3 : p_{n-3} < p_n \leq p_{n-2}$. In this case, both S_{n-1} and S_{n-2} are masked by S_n to ensure all sellers in the market are skyline sellers. $M - 2$ sellers are left in the skyline market $S^{set} = \{S_1, S_2, .., S_{n-3}, S_n, S_{n+1}, .., S_M\}$.
- ...
- $Case\ n : cost < p_n \leq p_1$, S_1 to S_{n-1} are masked by S_n, only $M - n + 1$ sellers are left in the skyline market $S^{set} = \{S_n, S_{n+1}, .., S_M\}$.

Target seller S_n can only mask sellers from S_1 to S_{n-1}, but cannot mask sellers from S_{n+1} to S_M. It is because sellers S_{n+1} to S_M have larger reputations than S_n, and according to the rule of the skyline market, seller S_i cannot be masked by seller S_j if S_i has a higher reputation than S_j.

3.1.2 Normalization

This thesis uses the two dimensional visual model proposed in MAPS [10] to model the skyline market because it captures the inter-attributes tradeoffs among sellers. In order to show the sellers in a two dimensional visual map, we need to normalized all the parameters into the same range, and $[0, 1]$ is used in our thesis. Also, after normalization, a higher normalized value should represent a higher buyer satisfaction level. For rational buyers, the higher reputation and lower price, the better. Therefore, equation (3.1) is adopted to normalize the-higher-the-better reputation, and (3.2) is used to normalize the-lower-the-better price.

$$y_i(r_i) = \frac{r_i}{\sqrt{r_i * r_i + B_0}} \tag{3.1}$$

$$x_i(p_i) = 1 - \frac{p_i}{\sqrt{p_i * p_i + B_1}}. \tag{3.2}$$

In the above equations, p_i is the price of seller S_i and r_i is the reputation of seller S_i. $B_0 = B_1$ are system-level parameters, once they are set, all the sellers will use the same value [10]. In this thesis, we set $B_0 = B_1 = 10^6$. After the normalization, each seller can be expressed by $S_i = (x_i, y_i), i = (1, 2, ..., M)$, and $x_1 > x_2 > ... > x_M$, $y_1 < y_2 < ... < y_M$, so each seller corresponds to one point (x, y) in the 2-D coordinate.

3.1.3 Buyer Model Formulation

Buyers choose the best seller by considering the tradeoffs between various attributes according to their own personal preferences, and the seller chosen here is called the *best choice* (BC). The buyers pick the seller with the highest utility value to be their best choice. In this thesis, the transform function is $H_i(x) = x$, and the utility function is shown below,

$$U_i = w * x_i + (1 - w) * y_i, i = 1, 2, ..., M. \tag{3.3}$$

U_i is the utility value, x_i and y_i are the normalized price and reputation, and w is the weight buyer assigns to the price. Considering different people have different purchasing behaviors, we assume w follows a distribution rather than a fixed value. Denote $f(w)$ as the distribution function of w, as the beginning of this work along this direction, in this thesis, we assume sellers have perfect knowledge of the distribution, and we will study how to model and estimate this distribution in our future work.

3.1.4 Problem Formulation

The problem focuses on setting the best price for S_n to maximize his/her profit. Denote $\mathcal{P}$ as the probability that the next coming buyer chooses S_n as *best choice*, and E_n as the revenue for the target seller S_n. Also, we use (3.2) to normalize the *cost* and let x_{cost} be the normalized cost. The problem can be written as the following mathematic form,

$$\max E_n(p_n) = (p_n - cost) * \mathcal{P}(p_n). \tag{3.4}$$

$E_n(p_n)$ is the revenue for S_n, p_n is the price of S_n, and $\mathcal{P}(p_n)$ is the probability that the next coming buyer chooses S_n as the best choice. So, finding the best price that maximizes the revenue of S_n is equivalent to solving (3.4). With the normalized price and the normalized reputation, (3.4) can be translated into the following form,

$$\min E_n(x_n) = (x_n - x_{cost}) * \mathcal{P}(x_n). \tag{3.5}$$

x_n is the normalized price, and x_{cost} is the normalized cost, and $\mathcal{P}(x_n)$ is the probability that the next coming buyer chooses S_n as the best choice, which is the same as $\mathcal{P}(p_n)$.

Instead of finding the maximum value of $E_n(p_n)$, equation (3.5) finds the minimum value of $E_n(x_n)$. So, the problem becomes solving (3.5), and the found x_n is the normalized $Best\ Price$ (BP) that the system recommends to S_n.

To solve (3.5) and to find the best price for our target seller S_n, solving the probability that the target seller is chosen as best choice becomes a critical issue. The probability that S_n is chosen as best choice is equal to the probability that S_n has higher utility value than all other sellers in the $Market$.

$$\begin{aligned} \mathcal{P}(x_n) &= \mathcal{P}(S_n \text{ be best choice}) \\ &= \mathcal{P}(U_n > U_1, ..., U_n > U_{n-1}, U_n > U_{n+1}, ..., U_n > U_M). \end{aligned} \tag{3.6}$$

U_n is the utility of the target seller S_n, and U_i is the utility of seller S_i.

Instead of comparing the utility of S_n with the utilities of all other $M-1$ sellers in the $Market$. Its better to compare it with one largest utility in the $Market$. So, the most competitive sellers are introduced, and it is a way to address the problem of the inter-seller competitions in the $Market$. With the idea of the most competitive sellers, we only need to compare S_n with two sellers in the subsets. So (3.6) can be further written as the following,

$$\mathcal{P}(x_n) = \mathcal{P}(U_n > U_L, U_n > U_U), (S_L \in S^{set}_{Lower}, S_U \in S^{set}_{Upper}). \tag{3.7}$$

S_L and S_U are the two most competitive sellers, U_L and U_U are the utilities of the two most competitive sellers, and S^{set}_{Lower} and S^{set}_{Upper} are two subsets of the $Market$. The definition of the most competitive sellers and the detailed process of how to find the most competitive sellers are described in the next section.

In this thesis, for the linear weight buyer model, we consider a simple scenario where weight w follows the Gaussian distribution with mean μ and variance σ^2. We will consider in our future work other distributions and how this distribution affects the target seller's revenue. Therefore, we calculate the probability with the following equation,

$$\mathcal{P}(x_n) = \int_{\alpha_l}^{\alpha_u} f(w)\, dw. \tag{3.8}$$

α_l and α_u are the lower and upper boundaries of the possible range of weight w, and they are closely related to the most competitive sellers. How to find these two parameters will also be introduced below.

3.2 The Most Competitive Sellers

Finding the most competitive sellers is a critical process when calculating the probability that S_n be chosen as best choice, and first we divide the market into three subsets: S^{set}_{Lower}, S^{set}_{Upper} and S_n. We can divide the 2-D coordinate into two parts with line $y = y_n$, the sellers on the lower right side of S_n form S^{set}_{Lower}, and the sellers on the upper left side of S_n form S^{set}_{Upper}. So we have, $S^{set}_{Lower} = \{S_1, S_2, ..., S_{n-1}\}$, with $x_i > x_n, y_i < y_n$, and $S^{set}_{Upper} = \{S_{n+1}, S_{n+2}, ..., S_M\}$, with $x_j < x_n, y_j > y_n$.

$The\ most\ competitive\ seller$ (MCS) in each subset is the seller with the largest utility value. As described in subsection 3.1.3, the utility value can be calculated through equation (3.3). U_i is the utility value, x_i and y_i are the normalized price and reputation, and w is the weight buyer assigns to the price. Denote $the\ most\ competitive\ seller$ in S^{set}_{Lower} as S_L, which has $U_L > U_i, S_i \in S^{set}_{Lower}, i \neq L$. Also denote S_U as $the\ most\ competitive\ seller$ in S^{set}_{Upper} which has $U_U > U_j, S_j \in S^{set}_{Upper}, j \neq U$. Other sellers in the subsets are called $the\ less\ competitive\ sellers$ (LCSs).

We can find the most competitive sellers S_L, S_U using Theorem 1.

Theorem 1. For all linear weight buyers, to ensure the target seller S_n has the largest utility, let k_{in} be the slope of the line connecting S_i and S_n, and seller $S_i \neq S_n$. For all $S_i \in S_{Lower}^{set}$, the seller with the largest k_{in} is the most competitive seller, $S_L = S_i$ if $k_{in} = max\{k_{1n}, k_{2n}, ..., k_{(n-1)n}\}$. For all $S_i \in S_{Upper}^{set}$, the seller with the smallest k_{in} is the most competitive seller, $S_U = S_i$ if $k_{in} = min\{k_{(n+1)n}, k_{(n+2)n}, ..., k_{Mn}\}$.

Proof. Fig. 3.1 shows the proof of how to find the most competitive seller S_L in S_{Lower}^{set}, while Fig. 3.2 shows the proof of how to find the most competitive seller S_U in S_{Upper}^{set}. If the upper or the lower subset is empty, then the most competitive seller is empty too. If the subset contains only one seller, then the only seller is the most competitive seller. So we demonstrate how to find S_L and S_U when each subset has more than one sellers.

A. Lower subset

First we assume there are two sellers in S_{Lower}^{set}, then we will extend the case to more sellers. The purpose of finding the most competitive sellers is to find the probability that $U_n > U_i, i = \{1, ..., (n-1)\}$. The utility of the target seller can be written as $U_n = w*x_n + (1-w)*y_n$ and the utilities of other sellers is $U_i = w*x_i + (1-w)*y_i$. When $U_n > U_i$, we have,

$$w * x_n + (1-w) * y_n > w * x_i + (1-w) * y_i$$

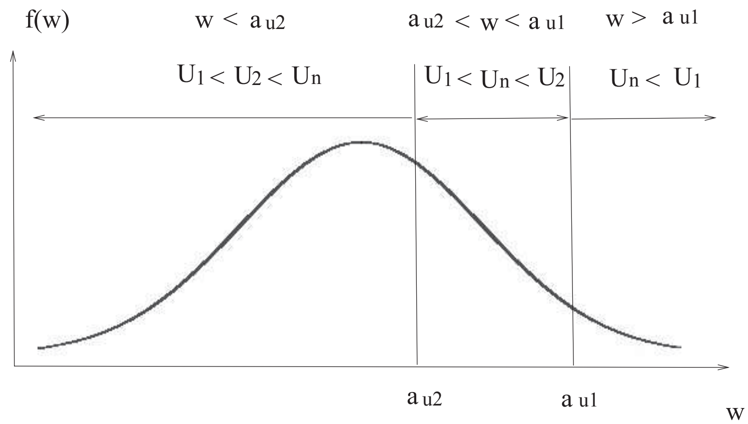
$$\text{or equivalently,} w * (x_n - y_n - x_i + y_i) > y_i - y_n, i = \{1, ..., (n-1)\}. \quad (3.9)$$

Because S_i is in subset S_{Lower}^{set}, so $x_n < x_i, y_n > y_i$, then $x_n - y_n - x_i + y_i < 0$. Hence the above inequality is true only if w satisfies

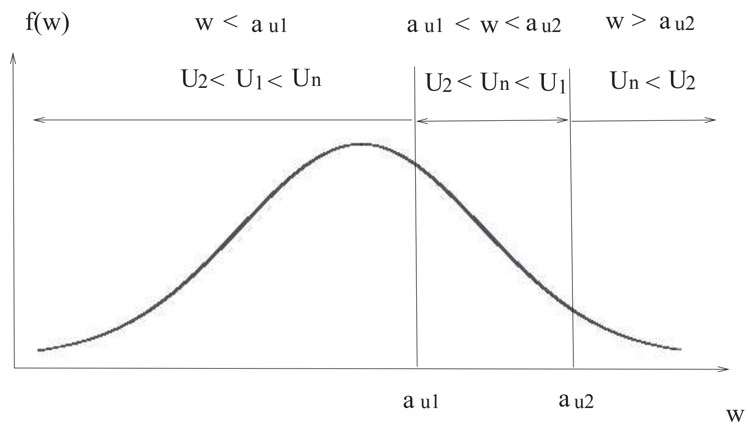
$$w < \frac{y_i - y_n}{x_n - y_n - x_i + y_i}, i = \{1, ..., (n-1)\}. \quad (3.10)$$

Define the above bound of w as $\alpha_{ui} = \frac{y_i - y_n}{x_n - y_n - x_i + y_i}, i = \{1, ..., (n-1)\}$.

Assume there are two sellers in $S_{Lower}^{set} = \{S_1, S_2\}$, and we need to find which



(a) Proof of the most competitive seller in S^{set}_{Lower} when $\alpha_{u1} > \alpha_{u2}$.



(b) Proof of the most competitive seller in S^{set}_{Lower} when $\alpha_{u1} < \alpha_{u2}$.

Fig. 3.1. (a) Proof of the most competitive seller in S^{set}_{Lower} when $\alpha_{u1} > \alpha_{u2}$, (b) Proof of the most competitive seller in S^{set}_{Lower} when $\alpha_{u1} < \alpha_{u2}$.

one has a larger utility value. By using the above inequations (3.9), we can derive two inequations. First, if $U_n > U_1$, then we have $w < \alpha_{u1}$, and if $U_n > U_2$, we have $w < \alpha_{u2}$. Then we can discuss the process in two cases: when $\alpha_{u1} > \alpha_{u2}$ and when $\alpha_{u1} < \alpha_{u2}$.

a. $\alpha_{u1} > \alpha_{u2}$

In this case, with the mathematical transformation, we can transform $\alpha_{u1} > \alpha_{u2}$ into the following expressions,

$$\frac{y_1 - y_n}{x_n - y_n - x_1 + y_1} > \frac{y_2 - y_n}{x_n - y_n - x_2 + y_2}$$
$$k_{1n} < k_{2n}, i = \{1, ..., (n-1)\}. \tag{3.11}$$

So, if $\alpha_{u1} > \alpha_{u2}$, then $k_{1n} < k_{2n}$. Also, from the utilities inequations, we can derive three different cases for the possible range of the weight distribution, shown in Fig. 3.1 (a).

- $w > \alpha_{u1}$: $U_n < U_1$.
- $\alpha_{u2} < w < \alpha_{u1}$: $U_2 > U_n > U_1$.
- $w < \alpha_{u2}$: $U_n > U_2 > U_1$.

Therefore, to ensure that S_n has the largest utility value, w must be in range $w < \alpha_{u2}$, and the corresponding utility relationship is $U_n > U_2 > U_1$. So, if $\alpha_{u1} > \alpha_{u2}$, or $k_{1n} < k_{2n}$, we have $U_2 > U_1$. In this case, S_2 has a larger utility value, denoted as the most competitive seller $S_L = S_2$.

b. $\alpha_{u1} < \alpha_{u2}$

Same as $\alpha_{u1} > \alpha_{u2}$, in this case, with some mathematical transformation, we can transform $\alpha_{u1} < \alpha_{u2}$ into the following expressions,

$$\begin{gathered} \frac{y_1 - y_n}{x_n - y_n - x_1 + y_1} < \frac{y_2 - y_n}{x_n - y_n - x_2 + y_2} \\ k_{1n} > k_{2n}. \end{gathered} \tag{3.12}$$

So, if $\alpha_{u1} < \alpha_{u2}$, then $k_{1n} > k_{2n}$. We have the following, see Fig. 3.1 (b)

- $w > \alpha_{u2}$: $U_n < U_2$.
- $\alpha_{u1} < w < \alpha_{u2}$: $U_1 > U_n > U_2$.
- $w < \alpha_{u1}$: $U_n > U_1 > U_2$.

Therefore, to ensure that S_n has the largest utility value, w must be in range $w < \alpha_{u1}$, and the corresponding utility relationship is $U_n > U_1 > U_2$. So, if $\alpha_{u1} < \alpha_{u2}$, or $k_{1n} > k_{2n}$, we have $U_1 > U_2$. In this case, S_1 has a larger utility value, being defined as the most competitive seller $S_L = S_1$.

To conclude, if $k_{1n} < k_{2n}$, $S_L = S_2$, and if $k_{1n} > k_{2n}$, $S_L = S_1$, or we can say that for $S^{set}_{Lower} = \{S_1, S_2\}$, the seller with the larger k_{in} is the most competitive seller.

If there are more than two sellers in S^{set}_{Lower}, then follow the steps above, compare the third seller with S_L, reset S_L, and repeat the steps for all the other sellers. We can proof that for all $S_i \in S^{set}_{Lower}$, the seller with the largest k_{in} is the most competitive seller, $S_L = S_i$ if $k_{in} = max\{k_{1n}, k_{2n}, ..., k_{(n-1)n}\}$.

B. Upper subset

Same method can be applied on the upper subset to proof Theorem 1. To ensure that $U_n > U_j, j = \{(n+1), ..., M\}$ (M is the number of skyline sellers in the $Market$). The utilities of the target seller and other sellers can be written as $U_n = w * x_n + (1-w) * y_n$ and $U_j = w * x_j + (1-w) * y_j$. When $U_n > U_j$, we have,

$$\begin{aligned} w * x_n + (1-w) * y_n &> w * x_j + (1-w) * y_j \\ or, w * (x_n - y_n - x_j + y_j) &> y_j - y_n, j = \{(n+1), ..., M\}. \end{aligned} \tag{3.13}$$

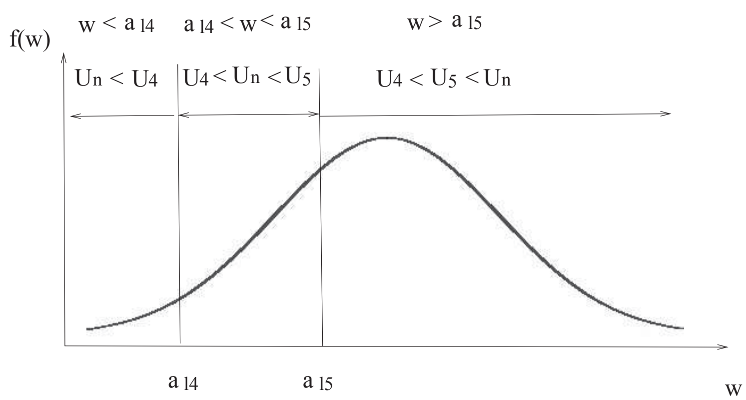
Because S_j is in subset S^{set}_{Upper}, so $x_n > x_j, y_n < y_j$, then $x_n - y_n - x_j + y_j > 0$. Hence the above inequality is true only if w satisfies

$$w > \frac{y_j - y_n}{x_n - y_n - x_j + y_j}, j = \{(n+1), ..., M\}. \tag{3.14}$$

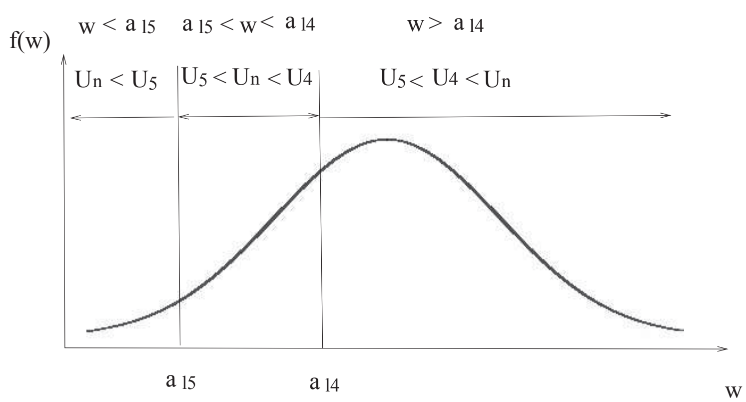
Define the lower bound of w as $\alpha_{lj} = \frac{y_j - y_n}{x_n - y_n - x_j + y_j}, j = \{(n+1), ..., M\}$.

Assume there are two sellers in $S^{set}_{Upper} = \{S_4, S_5\}$, and we need to find which one has a larger utility value. By using the above inequations (3.13), we can derive two inequations. First, if $U_n > U_4$, then we have $w > \alpha_{l4}$, and if $U_n > U_5$, we have $w > \alpha_{l5}$. Then we can discuss the process in two cases: when $\alpha_{l4} < \alpha_{l5}$ and when $\alpha_{l4} > \alpha_{l5}$.

a. $\alpha_{l4} < \alpha_{l5}$



(a) Proof of the most competitive seller in S_{Upper}^{set} when $\alpha_{l4} < \alpha_{l5}$.



(b) Proof of the most competitive seller in S_{Upper}^{set} when $\alpha_{l4} > \alpha_{l5}$.

Fig. 3.2. (a) Proof of the most competitive seller in S_{Upper}^{set} when $\alpha_{l4} < \alpha_{l5}$, (b) Proof of the most competitive seller in S_{Upper}^{set} when $\alpha_{l4} > \alpha_{l5}$.

In this case, we can transform $\alpha_{l4} < \alpha_{l5}$ into the following expressions,

$$\frac{y_4 - y_n}{x_n - y_n - x_4 + y_4} < \frac{y_5 - y_n}{x_n - y_n - x_5 + y_5}$$
$$k_{4n} > k_{5n}. \tag{3.15}$$

So, if $\alpha_{l4} < \alpha_{l5}$, then $k_{4n} > k_{5n}$. Also, as shown in Fig. 3.2 (a),

- $w < \alpha_{l4}$: $U_n < U_4$.
- $\alpha_{l4} < w < \alpha_{l5}$: $U_5 > U_n > U_4$.
- $w > \alpha_{l5}$: $U_n > U_5 > U_4$.

Therefore, to ensure that S_n has the largest utility value, w must be in range $w > \alpha_{l5}$, and the corresponding utility relationship is $U_n > U_5 > U_4$. So, if $\alpha_{l4} < \alpha_{l5}$, or $k_{4n} > k_{5n}$, we have $U_5 > U_4$. In this case, S_5 has a larger utility value, denoted as the most competitive seller $S_U = S_5$.

b. $\alpha_{l4} > \alpha_{l5}$

We can transform $\alpha_{l4} > \alpha_{l5}$ into the following expressions,

$$\frac{y_4 - y_n}{x_n - y_n - x_4 + y_4} > \frac{y_5 - y_n}{x_n - y_n - x_5 + y_5}$$
$$k_{4n} < k_{5n}. \tag{3.16}$$

So, if $\alpha_{l4} > \alpha_{l5}$, then $k_{4n} < k_{5n}$. We have, see Fig. 3.2 (b),

- $w < \alpha_{l5}$: $U_n < U_5$.
- $\alpha_{l4} < w < \alpha_{l5}$: $U_4 > U_n > U_5$.
- $w > \alpha_{l5}$: $U_n > U_4 > U_5$.

Therefore, to ensure that S_n has the largest utility value, w must be in range $w > \alpha_{l5}$, and the corresponding utility relationship is $U_n > U_4 > U_5$. So, if $\alpha_{l4} > \alpha_{l5}$, or $k_{4n} < k_{5n}$, we have $U_4 > U_5$. In this case, S_4 has a larger utility value, denoted as the most competitive seller $S_U = S_4$.

To conclude, if $k_{4n} > k_{5n}$, $S_U = S_5$, and if $k_{4n} < k_{5n}$, $S_U = S_4$, or we can say that for $S_{Upper}^{set} = \{S_4, S_5\}$, the seller with the smaller k_{jn} is the most competitive seller.

If there are more than two sellers in S_{Upper}^{set}, same method could be applied. We can proof that for all $S_j \in S_{Upper}^{set}$, the seller with the smallest k_{jn} is the most competitive seller, $S_U = S_j$ if $k_{jn} = min\{k_{(n+1)n}, ..., k_{Mn}\}$.

□

As we mentioned in seller set formulation that there are n cases when the target seller resets his/her price at different price ranges. So even for the same $Market$, S_{Lower}^{set} and S_{Upper}^{set} are different for different cases. So, we need to find the most competitive sellers for each case separately.

3.3 Derivation of The Revenue Equation

After we find the most competitive sellers in each subset, we can then calculate $\mathcal{P}(x_n)$, and also derive the revenue equation. With the definition of the lower and upper boundaries of weight w, we can write (3.8) in the following equation,

$$\mathcal{P}(x_n) = \int_{\frac{y_U - y_n}{x_n - y_n - x_U + y_U}}^{\frac{y_L - y_n}{x_n - y_n - x_L + y_L}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(w-\mu)^2}{2\sigma^2}} \, dw. \tag{3.17}$$

Substitute (3.17) into (3.5), we get

$$\begin{aligned} E_n(x_n) &= (x_n - x_{cost}) \textstyle\int_{\alpha_l}^{\alpha_u} f(w) \, dw \\ &= (x_n - x_{cost}) \textstyle\int_{\frac{y_U - y_n}{x_n - y_n - x_U + y_U}}^{\frac{y_L - y_n}{x_n - y_n - x_L + y_L}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(w-\mu)^2}{2\sigma^2}} \, dw, x_n < x_{cost}, \alpha_l < \alpha_u. \end{aligned} \tag{3.18}$$

To obtain the minimum point, we let the first order derivative of $E_n(x_n)$ equal to 0,

$$\frac{\partial E_n(x_n)}{\partial x_n} = \mathcal{P}(x_n) + (x_n - x_{cost}) \frac{\partial \mathcal{P}(x_n)}{\partial x_n} \tag{3.19}$$

$$= \textstyle\int_{\frac{y_U - y_n}{x_n - y_n - x_U + y_U}}^{\frac{y_L - y_n}{x_n - y_n - x_L + y_L}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(w-\mu)^2}{2\sigma^2}} \, dw + (x_n - x_{cost}) \frac{\partial \mathcal{P}(x_n)}{\partial x_n}, x_n < x_{cost}, \alpha_l < \alpha_u.$$

we have

$$\frac{\partial \mathcal{P}(x_n)}{\partial x_n} = f(\alpha_u)\frac{\partial \alpha_u(x_n)}{\partial x_n} - f(\alpha_l)\frac{\partial \alpha_l(x_n)}{\partial x_n}$$
$$= \frac{1}{\sqrt{2\pi}\sigma}[e^{-\frac{(\alpha_u-\mu)^2}{2\sigma^2}}\frac{(y_n-y_L)}{(x_n-y_n-x_L+y_L)^2} - e^{-\frac{(\alpha_l-\mu)^2}{2\sigma^2}}\frac{(y_n-y_U)}{(x_n-y_n-x_U+y_U)^2}]. \tag{3.20}$$

Take (3.20) into (3.19), we have the first derivative of the revenue for the target seller S_n,

$$\frac{\partial E_n(x_n)}{\partial x_n} = \int_{\frac{y_U-y_n}{x_n-y_n-x_U+y_U}}^{\frac{y_L-y_n}{x_n-y_n-x_L+y_L}} \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(w-\mu)^2}{2\sigma^2}}\,dw + (x_n - x_{cost})$$
$$*\frac{1}{\sqrt{2\pi}\sigma}[e^{-\frac{(\alpha_u-\mu)^2}{2\sigma^2}}\frac{(y_n-y_L)}{(x_n-y_n-x_L+y_L)^2} - e^{-\frac{(\alpha_l-\mu)^2}{2\sigma^2}}\frac{(y_n-y_U)}{(x_n-y_n-x_U+y_U)^2}]. \tag{3.21}$$

Letting the first derivative equal to 0, minimum $E_n(x_n)$ can be found, and the corresponding x_n is the normalized $best\ price$ for seller n.

The closed-form solution of (3.21) is difficult to find, and in our work, we use numerical methods to find the solution.

3.4 Overall Optimal Pricing Scheme

To demonstrate how to use the numerical method to find the solution to the revenue equation, we conclude our overall optimal pricing scheme in the following.

Algorithm 3.1 Overall Optimal Pricing Scheme

Require: Market; Target Seller S_n; cost;
Ensure: best price, maximum revenue for the target seller S_n
1: Filter the market with only skyline sellers to form the skyline market
2: Construct buyer model
3: Determine $M - n + 1$ Cases for the target seller S_n
4: **for** Case 1: $M - n + 1$ **do**
5: a. Filter the skyline market again
6: b. Find the most competitive sellers S_L, S_U
7: c. Construct revenue equation for S_n
8: d. Find the local best price and maximum revenue for S_n
9: Compare $M - n + 1$ cases, find the global solution

Seller ID	Price	Reputation
S_1	\$100	200
S_2	\$173	384
S_3	\$315	450
S_4	\$400	576
S_5	\$524	734

TABLE 3.1
SKYLINE MARKET WITH FIVE SELLERS

3.5 Numerical Methods to Find Best Price

In this section, we use an example to explain our proposed scheme. We find the local maximum revenue in each price range using traversal method, and then obtain global optimal maximum revenue and the corresponding best price for the target seller.

The example includes 5 skyline sellers $S^{set} = \{S_1, S_2, S_3, S_4, S_5\}$ in Table 3.1. The target seller is S_3, so $S_n = S_3$.

We first normalize the data in Table 3.1 using (3.1) and (3.2), and display in Fig. 3.3. From the visual market model in this figure, we can see intuitively that if S_n changes price, with a fixed reputation value, S_n will move along line $y = y_n$. As we mentioned in seller set formulation, $M-n+1 = 5-3+1 = 3$ cases if S_n sets price in different price ranges. Also, because the reputation of S_n is smaller than S_4, the price of S_n should also be lower than S_4 which is $cost < p_n < p_4$ $(x_4 < x_n < x_{cost})$. We will discuss each case separately because when S_n sets his/her price in different price ranges, the most competitive sellers S_L, S_U are different.

Case 1: $p_2 < p_n < p_4 (x_2 > x_n > x_4)$.

In this case, five skyline sellers remain in the skyline market: $S^{set} = \{S_1, S_2, S_n, S_4, S_5\}$, and the $Market$ can be divided into two subsets $S^{set}_{Lower} = \{S_1, S_2\}$, $S^{set}_{Upper} = \{S_4, S_5\}$ and $S_n = S_3$. Then the revenue of the target seller S_n

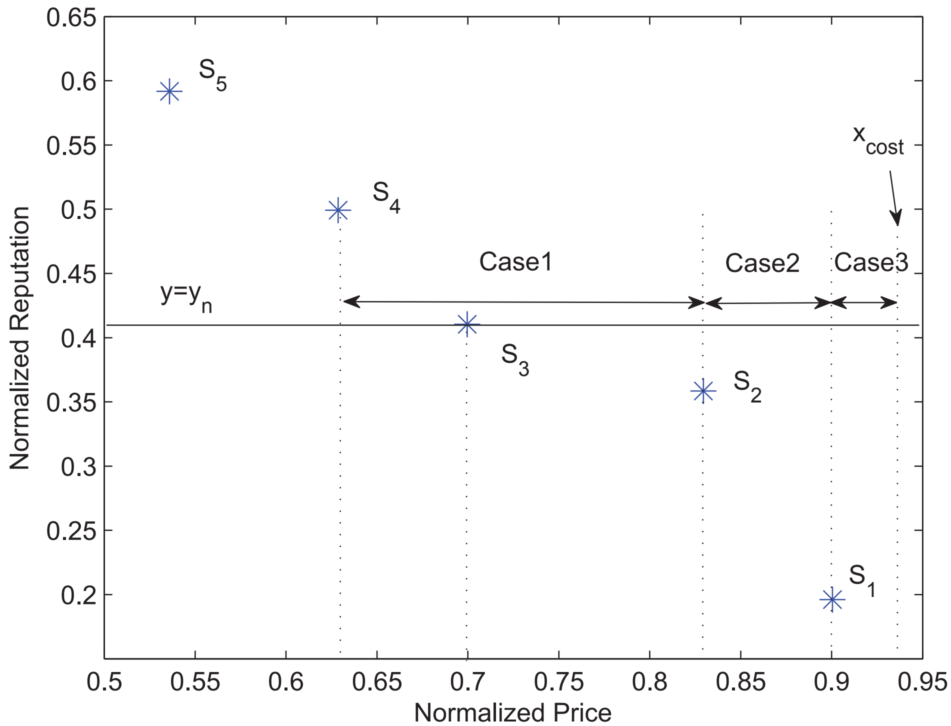


Fig. 3.3. Visual market model with five skyline sellers.

can be written as:

$$
\begin{aligned}
En(x_n) &= (x_n - x_{cost})\mathcal{P}(x_n) \\
\text{where,}\mathcal{P}(x_n) &= \mathcal{P}(U_3 > U_1, U_3 > U_2, U_3 > U_4, U_3 > U_5) \\
&= \mathcal{P}(U_3 > U_L, U_3 > U_U).
\end{aligned} \tag{3.22}
$$

U_i is the utility of seller S_i, U_L is the utility of the most competitive seller in the lower subset S_L, U_U is the utility of the most competitive seller in the upper subset S_U. Then we need to find the most competitive sellers for the target seller.

If the relative positions of the five sellers vary, four different scenarios will happen in determining the most competitive sellers as shown in Fig. 3.4. We use $Line_{ij}$ to denote the line connecting seller S_i and S_j. Also we use $Crossing\ Point$ (CP) to denote the intersection point of $Line_{ij}$ and $y = y_n$. More specifically, we

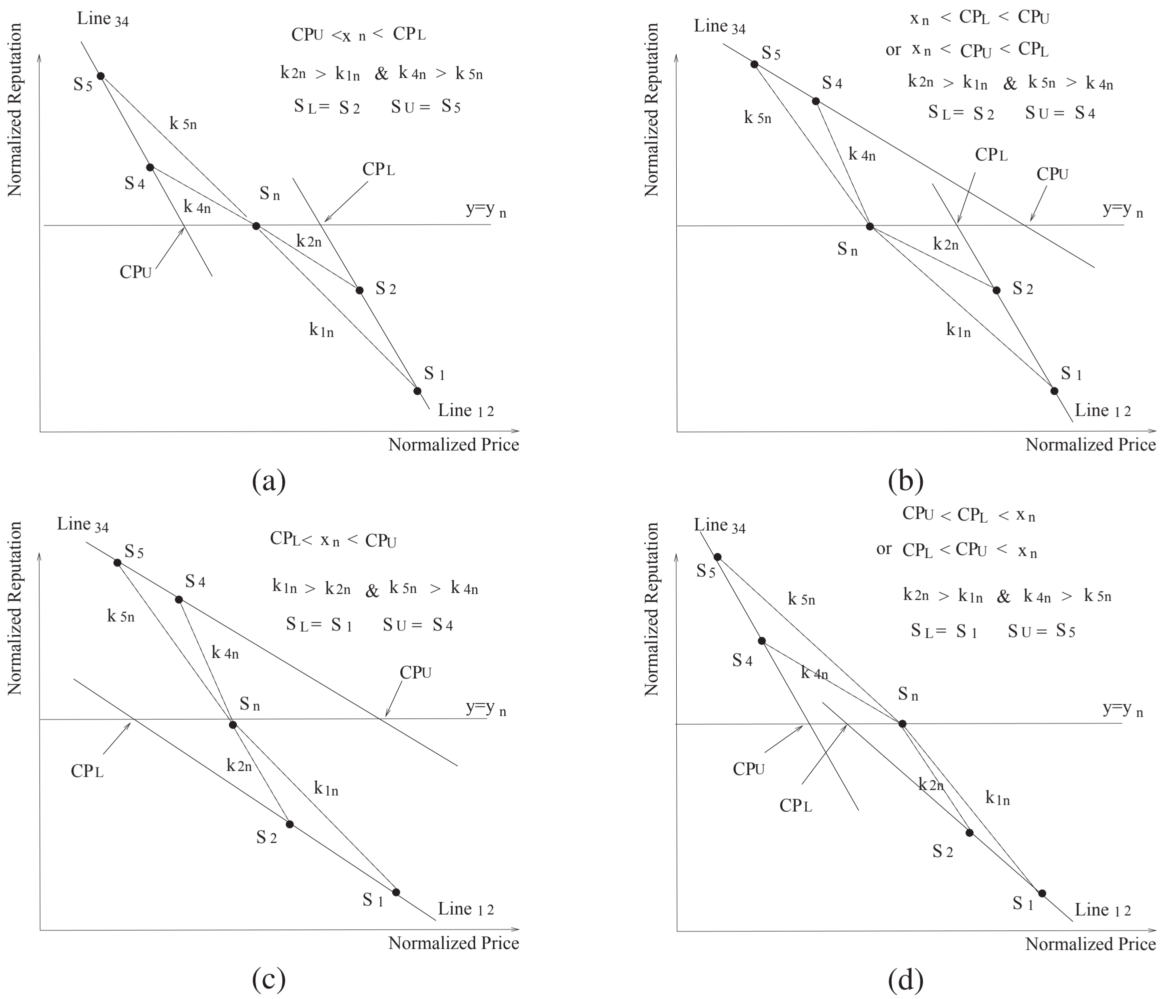


Fig. 3.4. (a) The most competitive sellers as $CP_U < x_n < CP_L$ for Case 1, (b) The most competitive sellers as $x_n < CP_L < CP_U$ or $x_n < CP_U < CP_L$ for Case 1, (c) The most competitive sellers as $CP_L < x_n < CP_U$ for Case 1, (d) The most competitive sellers as $CP_L < CP_U < x_n$ or $CP_U < CP_L < x_n$ for Case 1.

use CP_L to represent the intersection point of $Line_{12}$ and $y = y_n$, and CP_U to represent the intersection point of $Line_{45}$ and $y = y_n$. k_{in} is the slope of the line connecting seller S_i and S_n. We can see which sellers are the most competitive sellers directly from the visual map.

In Fig. 3.4 (a), $CP_U < x_n < CP_L$. We can see that $k_{2n} > k_{1n}$ and $k_{4n} > k_{5n}$, S_n is at the left side of $Line_{12}$, and at the right side of $Line_{45}$. It is exactly the case for example 1. From Theorem 1, we know that for the lower subset, the seller with a higher slope is the most competitive seller, so $S_L = S_2$, while for the upper subset, the seller with a smaller slope is the most competitive seller, so $S_U = S_5$. Then the probability of S_n be chosen as best choice is $\mathcal{P}(U_3 > U_2, U_3 > U_5)$, and the revenue equation (3.4) can be derived,

$$\begin{aligned} En(x_n) &= (x_n - x_{cost}) \int_{\alpha_{l5}}^{\alpha_{u2}} f(w)\, dw \\ &= (x_n - x_{cost}) \int_{\frac{y_5 - y_n}{x_n - y_n - x_5 + y_5}}^{\frac{y_2 - y_n}{x_n - y_n - x_2 + y_2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(w-\mu)^2}{2\sigma^2}}\, dw. \end{aligned} \tag{3.23}$$

Then we traverse the price of the target seller S_n from $p_n = \$173$ to $p_n = \$400$ with step 1. We substitute x_n with the normalized p_n in equation (3.23), and derive the revenues corresponding to each price. Then the largest revenue denote as $MaxRev_{case1} = \$89.67$ and the corresponding price denote as $bp_{case1} = \$197$ can be found in this price range. Same method is applied on Case 2 and Case 3 to find the local maximum revenue $MaxRev_{case2}, MaxRev_{case3}$ and the corresponding local best price bp_{case2}, bp_{case3}. By comparing the maximum revenues from these three price ranges, we can find the global maximum revenue and the global best price denoted as $MaxRev$ and bp.

If the relative positions of the five sellers changes, then the most competitive sellers will also change. Fig. 3.4 (b)-(d) show the other three scenarios in determining the most competitive sellers.

In Fig. 3.4 (b), $x_n < CP_L < CP_U$ or $x_n < CP_U < CP_L$. In is case, $k_{2n} > k_{1n}$ and $k_{5n} > k_{4n}$, and S_n is at the left side of both $Line_{12}$ and $Line_{45}$. Then the most competitive sellers are $S_L = S_2$ and $S_U = S_4$. The probability is $\mathcal{P}(U_3 > U_2, U_3 >$

U_4), and the revenue equation is,

$$\begin{aligned} En(x_n) &= (x_n - x_{cost}) \int_{\alpha_{l4}}^{\alpha_{u2}} f(w)\, dw \\ &= (x_n - x_{cost}) \int_{\frac{y_4 - y_n}{x_n - y_n - x_4 + y_4}}^{\frac{y_2 - y_n}{x_n - y_n - x_2 + y_2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(w-\mu)^2}{2\sigma^2}}\, dw. \end{aligned} \tag{3.24}$$

In Fig. 3.4 (c), $CP_L < x_n < CP_U$. In is case, $k_{1n} > k_{2n}$ and $k_{5n} > k_{4n}$, and S_n is at the right side of $Line_{12}$, and at the left side of $Line_{45}$. Then the most competitive sellers are $S_L = S_1$ and $S_U = S_4$. The probability is $\mathcal{P}(U_3 > U_1, U_3 > U_4)$, and the revenue equation is,

$$\begin{aligned} En(x_n) &= (x_n - x_{cost}) \int_{\alpha_{l4}}^{\alpha_{u1}} f(w)\, dw \\ &= (x_n - x_{cost}) \int_{\frac{y_4 - y_n}{x_n - y_n - x_4 + y_4}}^{\frac{y_1 - y_n}{x_n - y_n - x_1 + y_1}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(w-\mu)^2}{2\sigma^2}}\, dw. \end{aligned} \tag{3.25}$$

In Fig. 3.4 (d), $CP_L < CP_U < x_n$ or $CP_U < CP_L < x_n$. In is case, $k_{1n} > k_{2n}$ and $k_{4n} > k_{5n}$, and S_n is at the right side of both $Line_{12}$ and $Line_{45}$. Then the most competitive sellers are $S_L = S_1$ and $S_U = S_5$. The probability is $\mathcal{P}(U_3 > U_1, U_3 > U_5)$, and the revenue equation is,

$$\begin{aligned} En(x_n) &= (x_n - x_{cost}) \int_{\alpha_{l5}}^{\alpha_{u1}} f(w)\, dw \\ &= (x_n - x_{cost}) \int_{\frac{y_5 - y_n}{x_n - y_n - x_5 + y_5}}^{\frac{y_1 - y_n}{x_n - y_n - x_1 + y_1}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(w-\mu)^2}{2\sigma^2}}\, dw. \end{aligned} \tag{3.26}$$

Case 2: $p_1 < p_n \leq p_2 (x_1 > x_n \geq x_2)$

In this case, the price of S_n is smaller than S_2, but the reputation of S_n is larger than S_2. So S_2 is masked by S_n when S_n resets his/her price in this price range. Four skyline sellers are left in the skyline market: $S^{set} = \{S_1, S_n, S_4, S_5\}$, and the $S^{set}_{Lower} = \{S_1\}$, $S^{set}_{Upper} = \{S_4, S_5\}$. Because S_1 is the only seller in the lower subset, so $S_L = S_1$, and CP_L does not exist. The revenue is

$$En(x_n) = (x_n - x_{cost})\mathcal{P}(x_n)$$

$$\text{where,} \mathcal{P}(x_n) = \mathcal{P}(U_3 > U_1, U_3 > U_4, U_3 > U_5)$$

$$= \mathcal{P}(U_3 > U_1, U_3 > U_U). \tag{3.27}$$

There are two scenarios for the most competitive sellers in the upper subset.

In Fig. 3.5 (a), $CP_U < x_n$, and $k_{4n} > k_{5n}$, S_n is at the right side of $Line_{45}$. The most competitive seller in the upper subset is $S_U = S_5$. (3.5) can be further written as

$$\begin{aligned} En(x_n) &= (x_n - x_{cost}) \int_{\alpha_{l5}}^{\alpha_{u1}} f(w)\, dw \\ &= (x_n - x_{cost}) \int_{\frac{y_5 - y_n}{x_n - y_n - x_5 + y_5}}^{\frac{y_1 - y_n}{x_n - y_n - x_1 + y_1}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(w-\mu)^2}{2\sigma^2}}\, dw. \end{aligned} \tag{3.28}$$

Similar with Case 1, we traverse the price of S_n from $p_n = \$100$ to $p_n = \$173$. Then calculate the corresponding revenues using equation (3.28), substitute x_n with normalized p_n. So, the largest revenue $MaxRev_{case2} = \$85.56$ and the corresponding best price $bp_{case2} = \$173$ in this range are found.

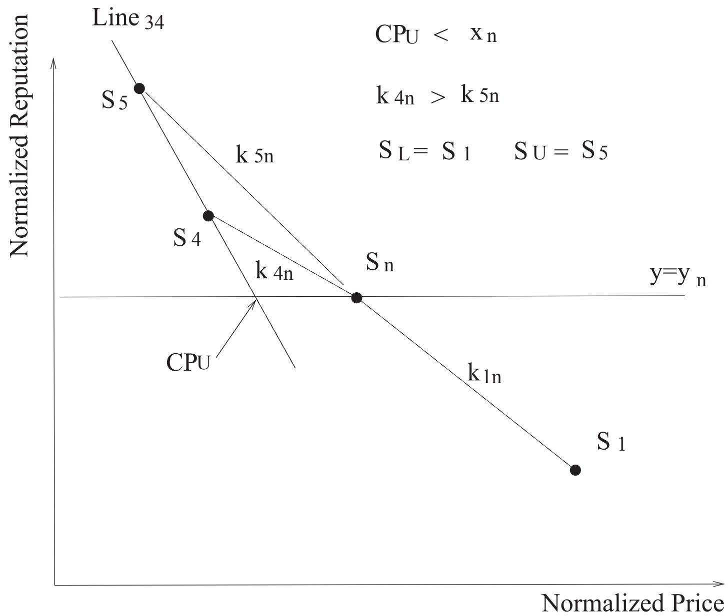
Fig. 3.5 (b) shows another scenario, where $x_n < CP_U$, and $k_{5n} > k_{4n}$, S_n is at the left side of $Line_{45}$. The most competitive seller in the upper subset is $S_U = S_4$. (3.27) can be further written as

$$\begin{aligned} En(x_n) &= (x_n - x_{cost}) \int_{\alpha_{l4}}^{\alpha_{u1}} f(w)\, dw \\ &= (x_n - x_{cost}) \int_{\frac{y_4 - y_n}{x_n - y_n - x_4 + y_4}}^{\frac{y_1 - y_n}{x_n - y_n - x_1 + y_1}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(w-\mu)^2}{2\sigma^2}}\, dw. \end{aligned} \tag{3.29}$$

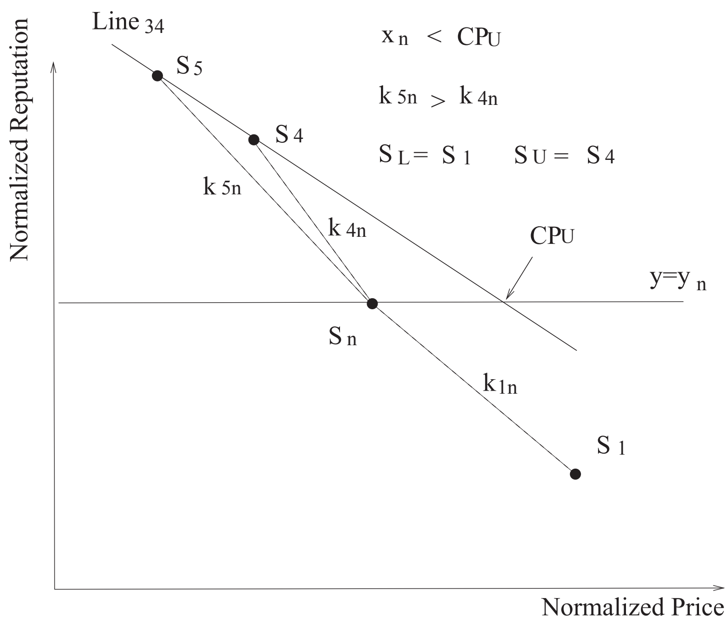
Case 3: $cost < p_3 \leq p_1 (x_{cost} > x_n \geq x_1)$

When S_n sets price in this price range, the price of S_n is smaller than both S_1 and S_2, but the reputation of S_n is larger than both S_1 and S_2. So, both S_1 and S_2 are masked by S_n, only three skyline sellers are left in the skyline market: $S^{set} = \{S_n, S_4, S_5\}$. The lower subset is empty $S^{set}_{Lower} = \{\varnothing\}$, so the most competitive seller S_L does not exist. The upper subset is $S^{set}_{Upper} = \{S_4, S_5\}$. The revenue is

$$En(x_n) = (x_n - x_{cost})\mathcal{P}(x_n)$$
$$\text{where,} \mathcal{P}(x_n) = \mathcal{P}(U_3 > U_4, U_3 > U_5)$$



(a) The most competitive sellers as $CP_U < x_n$ for Case 2.



(b) The most competitive sellers as $x_n < CP_U$ for Case 2.

Fig. 3.5. (a) The most competitive sellers as $CP_U < x_n$ for Case 2, (b) The most competitive sellers as $x_n < CP_U$ for Case 2.

$$= \mathcal{P}(U_3 > U_U). \tag{3.30}$$

Similar with case 2, there are two scenarios for the most competitive sellers in the upper subset.

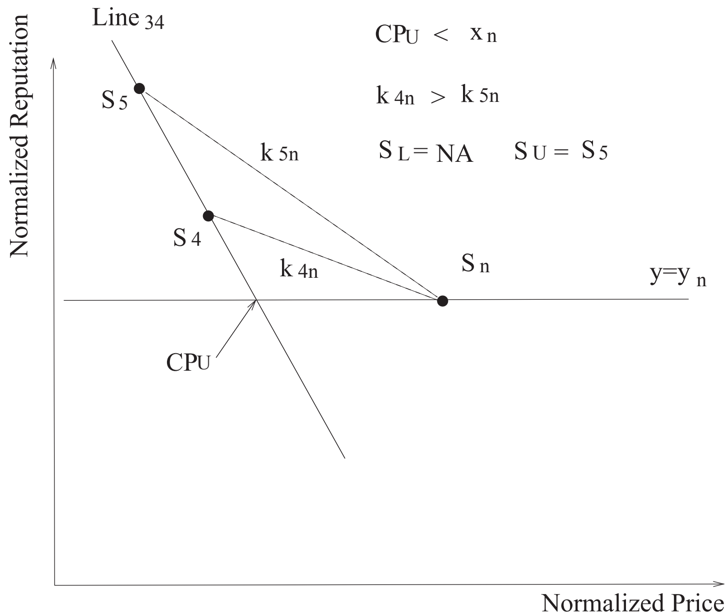
In Fig. 3.6 (a), $CP_U < x_n$, and $k_{4n} > k_{5n}$, S_n is at the right side of $Line_{45}$. The most competitive seller in the upper subset is $S_U = S_5$. (3.30) can be further written as

$$\begin{aligned} En(x_n) &= (x_n - x_{cost}) \int_{\alpha_{l5}}^{1} f(w)\, dw \\ &= (x_n - x_{cost}) \int_{\frac{y_5 - y_n}{x_n - y_n - x_5 + y_5}}^{1} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(w-\mu)^2}{2\sigma^2}}\, dw. \end{aligned} \tag{3.31}$$

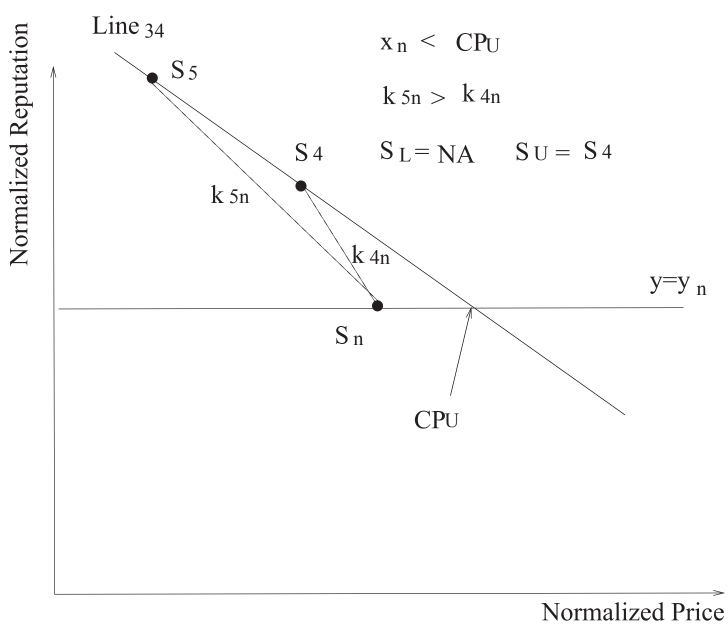
Then we traverse the price of the target seller S_n from $p_n = \$50$ to $p_n = \$100$, by using revenue equation (3.31), we can find the largest revenue $MaxRev_{case3} = \$42.6$ and the corresponding best price $bp_{case3} = \$100$ in this price range. Finally, by comparing the maximum revenue in each price range, we find the global maximum revenue $MaxRev = MaxRev_{case3} = \$89.67$, and the best price $bp = bp_{case3} = \$197$ for target seller $S_n = S_3$.

Fig. 3.6 (b) is another scenario when the positions of S_4, S_5 change. In this figure, $x_n < CP_U$, and $k_{5n} > k_{4n}$, S_n is at the left side of $Line_{45}$. The most competitive seller in the upper subset is $S_U = S_4$. (3.30) can be further written as

$$\begin{aligned} En(x_n) &= (x_n - x_{cost}) \int_{\alpha_{l4}}^{1} f(w)\, dw \\ &= (x_n - x_{cost}) \int_{\frac{y_4 - y_n}{x_n - y_n - x_4 + y_4}}^{1} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(w-\mu)^2}{2\sigma^2}}\, dw. \end{aligned} \tag{3.32}$$



(a) The most competitive sellers as $CP_U < x_n$ for Case 3.



(b) The most competitive sellers as $x_n < CP_U$ for Case 3.

Fig. 3.6. (a) The most competitive sellers as $CP_U < x_n$ for Case 3, (b) The most competitive sellers as $x_n < CP_U$ for Case 3.

Chapter 4

Results and Discussion

In this chapter, we apply the revenue equations on simulated and real markets respectively to obtain the maximum revenue and best price. In the first section, we analyse the simulated e-market (the example given in Chapter 3), and discuss how different parameters may affect the best price and maximum revenue of the target seller. The parameters discussed includes the reputation of the target seller, the price and the reputation of the competitive sellers. Then in the second section, we calculate the revenue and best price on the real market data crawled from eBay. In order to prove that our proposed scheme applies to products at different price ranges, three typical items are selected: coffee maker, usually selling at $\$100$, defined as low price product; Itouch, selling around $\$200$, the median price level product; and Cannon camera, mainly selling at $\$1800$ the high price product. We compare our proposed scheme with traditional average pricing and mark-up pricing methods, and the results showed that our proposed scheme achieves higher revenue than these traditional methods for all skyline sellers. We firstly elaborate the simulation process, then give numerical results.

4.1 Numerical Analysis for Simulated e-Market

In this section, we perform our proposed scheme and Monte Carlo simulation on simulated market given in Chapter 3. The skyline market is presented in Table 3.1

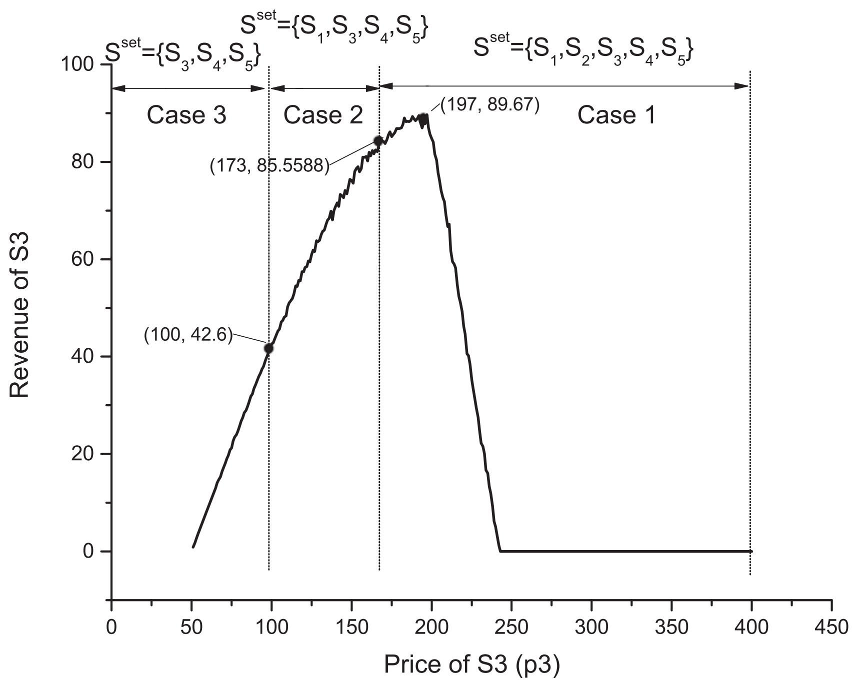
with five skyline sellers sorted in the ascending order of price. Assume $cost = \$50$, the target seller S_3 tries to reset his/her price. The price range for this market is from \$100 to \$524, while the reputation range is $[200, 734]$.

As we analysed in section 3.5, we traverse the price of the target seller from $p_n = \$50$ to $p_n = \$400$. There are three cases, we calculate the revenue for S_3 with (3.23), (3.28) and (3.31) by substituting x_n with normalized p_n and depict Fig. 4.1. The figure shows how the revenue and the probability of S_3 being selected by buyers are influenced by the target seller's price. From Fig. 4.1 (a), we see as the price of S_3 increases, the revenue increases first, then after reaching the highest point, it decreases, which agrees with Fig. 2.1. The maximum point $MaxRevOPS = \$89.67$ is reached at price $p_3 = \$197$. Then the revenue becomes zero when $p_3 > \$243$ because the probability S_3 be chosen becomes zero in this range.

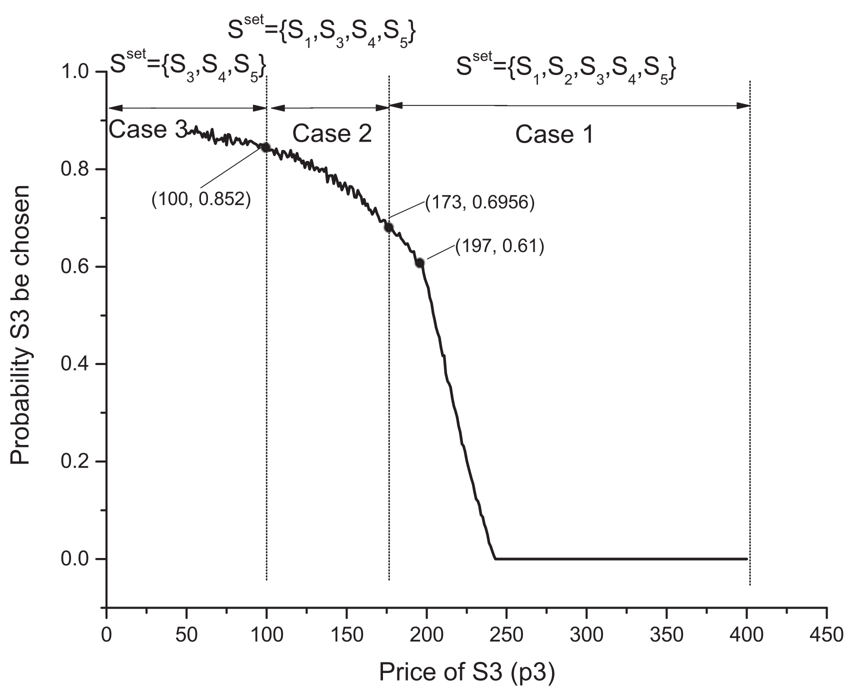
Fig. 4.1 (b) demonstrates how the probability S_3 being chosen as $best\ choice$ changes when p_n increases. We notice that the probability decreases at different rate in different price ranges, and finally drops to zero. The probability decreases because as the price increases, S_3 becomes less competitive. Also, the reason that probability decreases at different rate is due to the number of sellers left in the skyline market. When more sellers join the competition in the skyline market, the target seller S_3 achieves a smaller market share, that is, the percentage of the market S_3 gets smaller regarding the whole market as one. Also, in this example, we have $CP_L = 197, CP_U = 294$, based on the analyse in section 3.5, we can further discuss the skyline market at different price ranges.

- First range $p_n \in [50, 100]$.

  It is Case 3 in the example, where $S^{set} = \{S_3, S_4, S_5\}$, S_3 masks seller S_1 and S_2, and S_L does not exist, $S_U = S_5$. The probability S_3 being chosen as best choice is high and drops slowly as there are fewer competitors in the market. We observe from Fig. 4.1 (a) that S_3's expected revenue is increasing as p_3 increases, and the local maximum revenue is achieved at the boundary when $p_3 = \$100$, and $MaxRev_{case3} = \$42.6$.



(a) The revenue for S_3 as the price of S_3 increases



(b) The probability that S_3 being chosen as best choice as the price of S_3 increases

Fig. 4.1. (a) Revenue change of S_3 as p_3 increases (b) Probability change of S_3 as p_3 increases

- Second range $p_n \in [100, 173]$.

  It is Case 2 in the example, where $S^{set} = \{S_1, S_3, S_4, S_5\}$, S_3 masks seller S_2 only, and $S_L = S_1, S_U = S_5$. The probability S_3 being chosen as best choice drops a little faster than Case 3. Comparing with Case 3, S_1 joins the competition, so the $market\ share$ for all sellers decreased. In this range, as the price increases, S_3's revenue continues to increase and the local maximum revenue is achieved at $p_3 = \$173$, $MaxRev_{case2} = \$85.56$.

- Third range $p_n \in [173, 400]$.

  It is Case 1 in the example, where $S^{set} = \{S_1, S_2, S_3, S_4, S_5\}$. In this case, when $p_3 \in [173, 197]$, $S_L = S_1$, when $p_3 \in [197, 400]$, $S_L = S_2$, and $S_U = S_5$. Comparing with Case 3 and Case 2, all sellers join the competition, so the $market\ share$ for S_3 drops dramatically. We observe that the probability drops sharper at $p_3 = \$197$, and it is because the most competitive seller S_L changes from S_1 to S_2. S_2 is more competitive than S_1, so when the most competitive seller changes to S_2, the $Market\ Share$ for S_3 is even less. The local maximum revenue is achieved at $p_3 = \$197$, $MaxRev_{case1} = \$89.67$. Also, the probability becomes zero when $p_3 > \$243$. This is because when p_n increases, for (3.8), the upper bound of the weight w decreases, while the lower bound increases until $\alpha_l > \alpha_u$, which means S_n cannot have a larger utility than both most competitive sellers, so the probability of S_3 being chosen becomes zero.

When we traverse the price of the target seller S_n from $p_n = \$50$ to $p_n = \$400$ with step 1. We substitute x_n with the normalized p_n in the corresponding revenue equations, and derive the local maximum revenues and best price. The maximum revenue and best price are listed in Table 4.1. After finding the suboptimal solution in each price range, we compare these suboptimal solutions, and pick the largest revenue as the global optimal solution. In our example, the $MaxRev = \$89.67$, and the best price is $p_n = \$197$.

Item	Case 1	Case 2	Case 3
$Maximum\ Revenue$	\$89.67	\$85.56	\$42.6
$Best\ Price$	\$197	\$173	\$100

TABLE 4.1
MAXIMUM REVENUE AND BEST PRICE IN EACH PRICE RANGE

In order to evaluate the result of our proposed scheme, the standard Monte Carlo simulation was applied on the same data.

$Monte\ Carlo$: For each skyline market in the simulation, we simulate 5000 linear weight buyers where their weight w follows Gaussian distribution with mean $\mu = 0.5$, and variance $\sigma^2 = 0.5/3$. These buyers pick the seller with the largest utility value as their best choice, so for each buyer, by calculating the utilities of all sellers, we can determine which seller is chosen as his/her best choice. Then the probability that the target seller S_3 is chosen as $best\ choice$ when $p_n =$ \$197 is calculated through equation, $\mathcal{P}_{MC}(x_n) = \frac{\text{number of buyers selected} S_3}{\text{total number of buyers}} = \frac{2523}{5000} = 0.5046$. The result shows that S_3 will achieve maximum revenue $MaxRevMonteCarlo =$ \$74.17. Large amount of experiments demonstrate that our proposed pricing results are close to the actual result calculated by Monte Carlo simulation.

4.1.1 Discussion of Different Parameters

This section discusses the impact of system parameters on the target seller's probability of being chosen and the target seller's revenue. We will focus on the following factors: the reputation of the target seller S_n, the price/reputation of the competitive sellers including the MCSs and the LCSs (S_1, S_2, S_4, S_5 in this example).

$a.\ Reputation\ of\ the\ Target\ Seller$

We keep the price and the reputation of other sellers unchanged, and only change the reputation of the target seller S_3 from 384 to 734. We obtain Fig. 4.2 by conducting our proposed pricing method and Monte Carlo simulation. Fig. 4.2

shows the selected optimal price (a) and predicted maximum revenue (b) increase as r_3 increases.

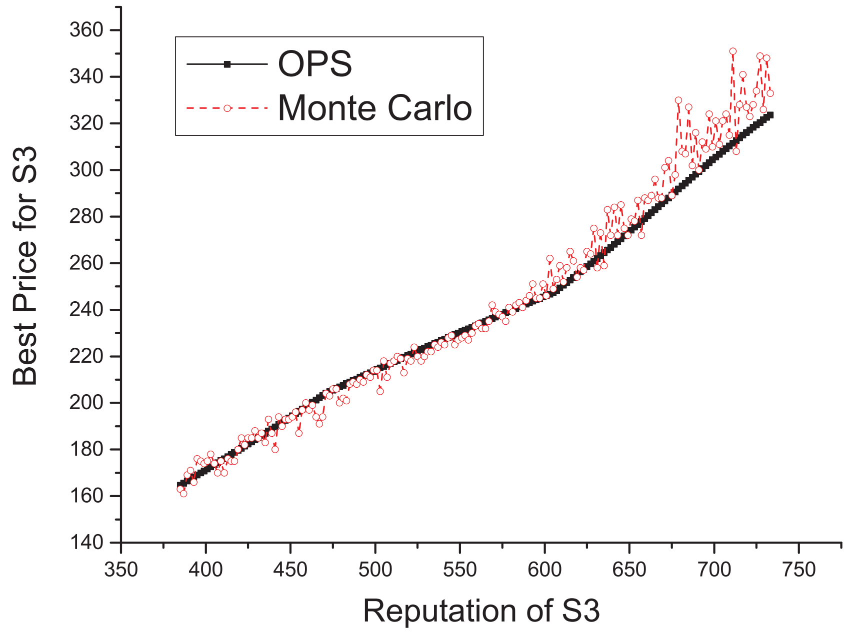
As the reputation of S_3 increases, S_3 becomes more competitive in the market, so we can see that the best price and the maximum revenue increase in both figures. Also, we observe the best price displays an obvious biphase, and it increases faster in the second phase when fewer sellers are left in the market. When $r_3 \in [384, 576]$, the skyline market is $S^{set} = \{S_1, S_2, S_3, S_4, S_5\}$. When r_3 in $r_3 \in [576, 734]$, the skyline market becomes $S^{set} = \{S_1, S_2, S_3, S_5\}$, and S_3 masks S_4.

$b. Price\ and\ Reputation\ of\ Competitive\ Sellers$

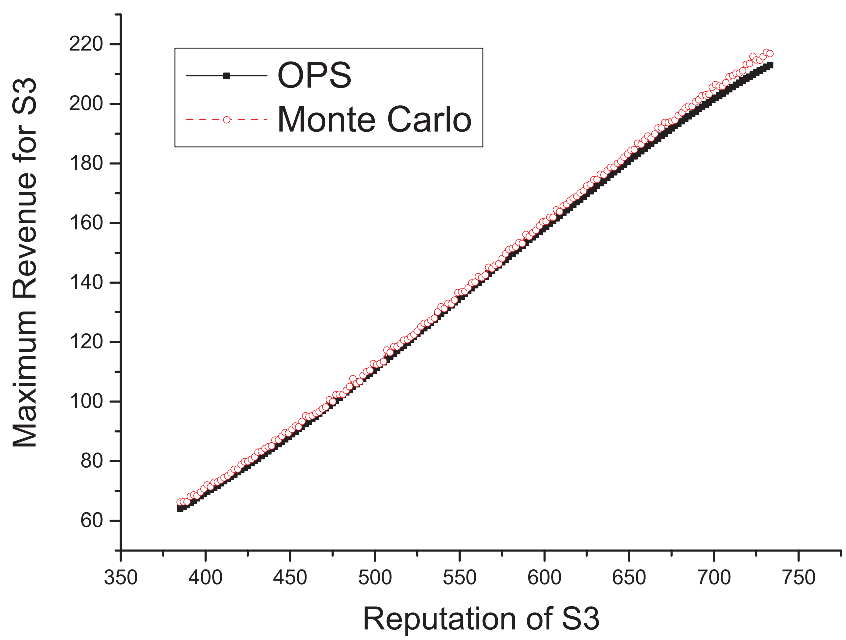
Fig. 4.3 demonstrates how best price and maximum revenue are influenced by the price/reputation of seller 1. We keep the reputation of the target seller r_3, the price and the reputation of S_2, S_4, S_5 fixed as in Table 3.1. Then, we keep the reputation of S_1 fixed, and increase the price of S_1 from \$50 to \$173. The best price and the maximum revenue for S_3 calculated based on our proposed pricing scheme and Monte Carlo simulations as plotted in Fig. 4.3(a)(c). The curves obtained with our proposed scheme and Monte Carlo simulation are nearly identical, confirming our proposed scheme is correct and applicable. To determine the most competitive sellers, in Fig. 4.3 (a), we also plot the CP_L, which is the intersection point of $Line_{12}$ and $y = y_n$, as in Fig. 3.4 to Fig. 3.6. In this case, $CP_U = 294$, and $S_U = S_5$ which stays the same as we change price/reputation of S_1. r_3 is fixed, so as p_3 changes, S_3 moves along line $y = y_n$. When the target seller $S_n = S_3$ is at the right side of the CP_L (below line CP_L), $S_L = S_1$, but when $S_n = S_3$ moves to the left side of the CP_L (above line CP_L), $S_L = S_2$.

From the Fig.4.3 (a)(c) , we observe that as p_1 increases, the best price of S_3 increases first, then drops and finally keeps unchanged afterwards. We can separately discuss the three parts.

- $p_1 \in [50, 100]$. In this case, the best price is below line CP_L, S_1 is the most competitive seller. When the most competitive seller increases his/her price, it increases the target seller S_3's market share, and to gain more revenue, the best responding strategy for S_3 is to increase his/her own price.



(a) Best price for S_3 as the reputation of S_3 increases



(b) Maximum revenue for S_3 as the reputation of S_3 increases

Fig. 4.2. (a) Best price for S_3 as r_3 increases (b) Maximum Revenue of S_3 as r_3 increases

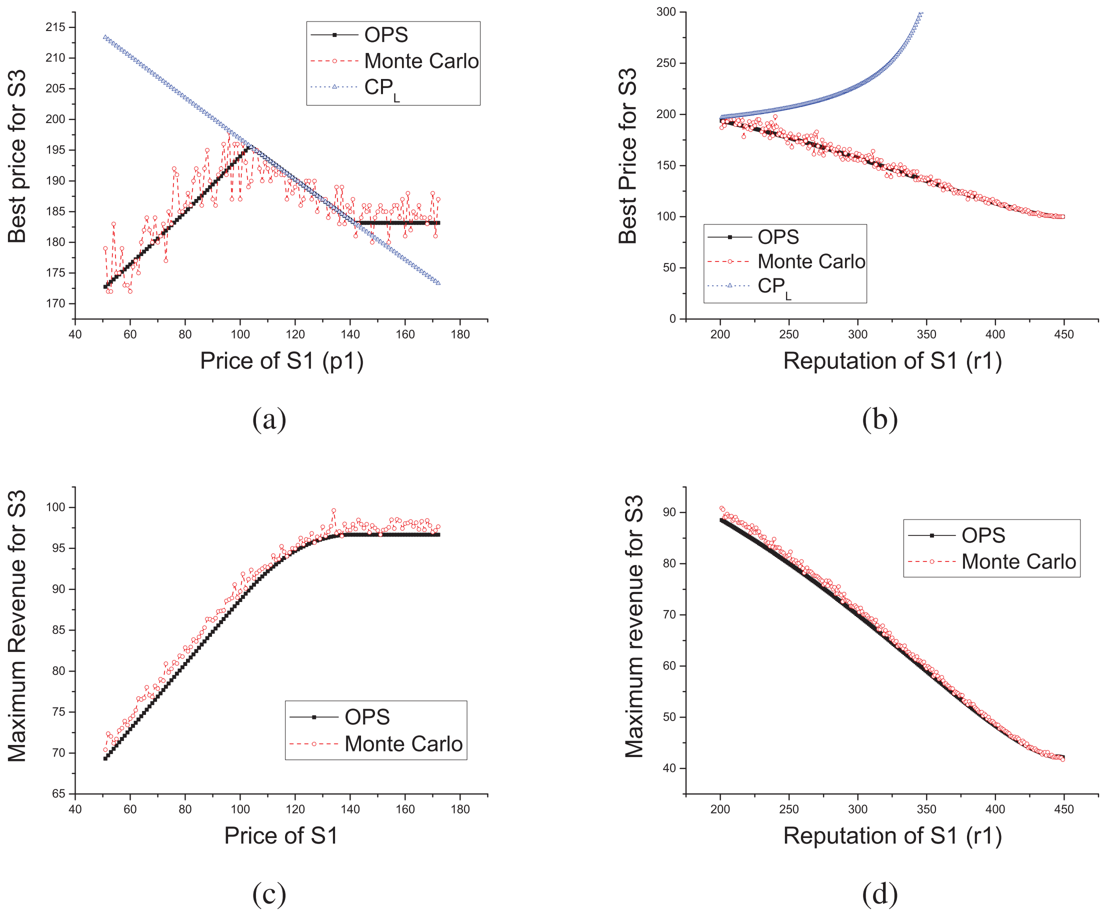


Fig. 4.3. (a) Best price for S_3 as S_1 increases price (b) Best price for S_3 as S_1 increases reputation (c) Maximum revenue for S_3 as S_1 increases price(d) Maximum revenue for S_3 as S_1 increases reputation

- $p_1 \in [100, 140]$. In this range, the best price is achieved at the point where the most competitive seller S_L changes from S_1 to S_2. It is because when the most competitive seller changes to S_2, the probability that S_3 be chosen as best choice drops more sharply, as a result, the expect revenue for S_3 drops sharply. So, the maximum revenue is achieved just before the most competitive seller changes to S_2. As p_1 increases, CP_L drops, and the best price is at point CP_L, so the best price drops in this range.

- $p_1 \in [140, 173]$. In this range, the best price is above the line CP_L, which means S_2 is the most competitive seller. In this case, the price change in S_1 will not affect the best price and maximum revenue of S_3.

To summarize, as the price of the most competitive seller increases, the best price of the target seller increases at first, then reached at the crossing point CP_L, where the most competitive seller changes, and the corresponding maximum revenue increases. As expected, the price of the less competitive seller will not affect the best price and maximum revenue of the target seller.

Fig. 4.3(b)(d) show how the reputation of S_1 influences the target seller's best price and maximum revenue. We keep the price of S_1 fixed, and increase the reputation of S_1 from 200 to 450, and plot the best price and the maximum revenue for S_3. We see that the best price of S_n decreases in the whole reputation range. It is because in the whole reputation range, the best price is below line CP_L, and S_1 is the the most competitive sellers, so when S_1 further increase the reputation, S_1 becomes even more competitive. The target seller S_n should decrease price to make sure his utility is larger than S_1, and as a result, the maximum revenue decreases.

So, as the reputation of the most competitive seller increases, the best price and the corresponding maximum revenue of the target seller decrease. For other sellers, if we change their prices and reputations, we observe the same trend and draw the same conclusion.

In Fig. 4.4, we keep the reputation of the target seller r_3, the price and the reputation of S_1, S_4, S_5 unchanged. Then for Fig. 4.4(a)(c), we keep the reputation

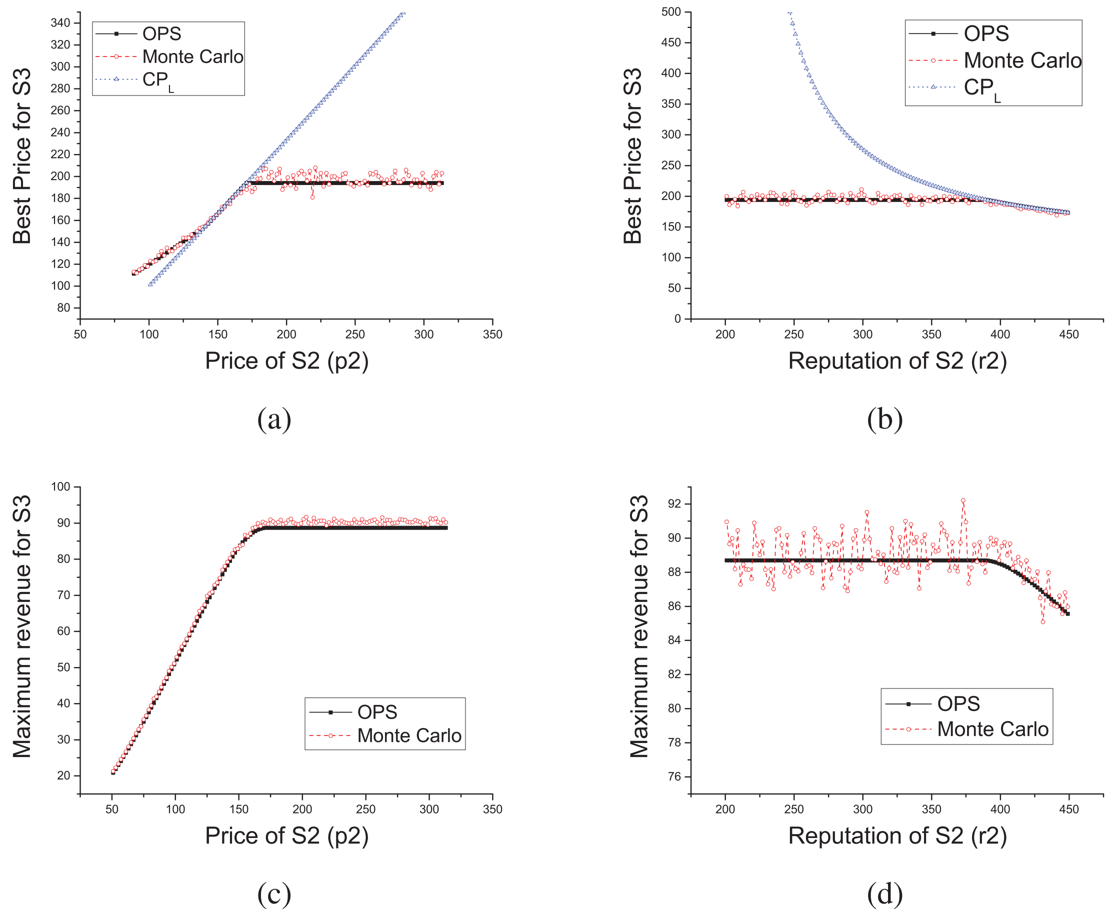


Fig. 4.4. (a) Best price for S_3 as S_2 increases price (b) Best price for S_3 as S_2 increases reputation (c) Maximum revenue for S_3 as S_2 increases price(d) Maximum revenue for S_3 as S_2 increases reputation

of S_2 fixed, and increase the price of S_2 from \$100 to \$315, and plot the best price and the maximum revenue for S_3 using our proposed pricing scheme and Monte Carlo simulation. For Fig. 4.4(b)(d), we keep the price of S_2 fixed, and increase the reputation of S_2 from 200 to 450, and plot the best price and the maximum revenue for S_3 using our proposed pricing scheme and Monte Carlo. In Fig. 4.3 (a)(c), we also plot the $Crossing\ Point$ of Line1 CP_L to determine the S_L.

Fig. 4.4(a)(c) show how the price of S_2 influences best price and maximum revenue for target seller S_3. As p_2 increases, the best price of S_3 increases first, then reached at CP_L, and finally the stays unchanged. At first the best price is above line CP_L, S_2 is the most competitive seller, then after the best price is below line CP_L, S_1 becomes the most competitive seller. So, as the most competitive seller S_2 increases price, the best price and maximum revenue increase. But when S_1 becomes the most competitive seller, p_2 will not affect best price and maximum revenue. Fig. 4.4(b)(d) show how the reputation of S_2 influences the best price and maximum revenue. The best price is below line CP_L at first, then above line CP_L, which means S_L changes from S_1 to S_2. So as r_2 increases, the best price and the maximum revenue keep unchanged first and then decrease.

To conclude Fig. 4.4, as the most competitive seller increases the price, the best price of the target seller S_n increases first, then stays the same with crossing point CP_L, and the maximum revenue increases. As the most competitive seller increases the reputation, the best price and the maximum revenue decrease. The price or the reputation change of the less competitive seller will not influence the best price and maximum revenue of the target seller.

In Fig. 4.5, we keep the reputation of the target seller r_3, the price and the reputation of S_1, S_2, S_5 unchanged. Then for Fig. 4.5(a)(c), we keep the reputation of S_4 fixed, and increase the price of S_4 from \$315 to \$524, and plot the best price and the maximum revenue for S_3 using our proposed scheme and Monte Carlo simulation. For Fig. 4.5(b)(d), we keep the price of S_4 fixed, and increase the reputation of S_4 from 450 to 830, and plot the best price and the maximum revenue for S_3 using our proposed pricing scheme and Monte Carlo simulation. In Fig.

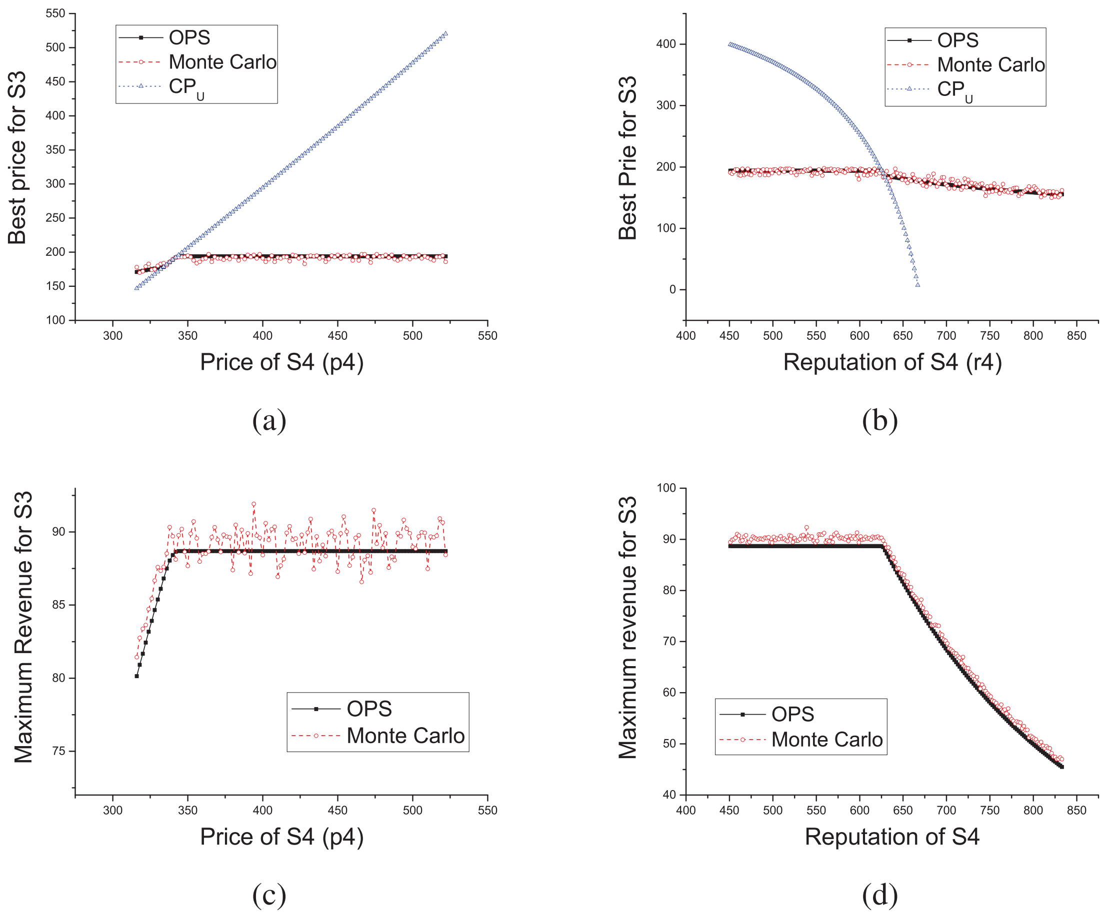


Fig. 4.5. (a) Best price for S_3 as S_4 increases price (b) Best price for S_3 as S_4 increases reputation (c) Maximum revenue for S_3 as S_4 increases price (d) Maximum revenue for S_3 as S_4 increases reputation

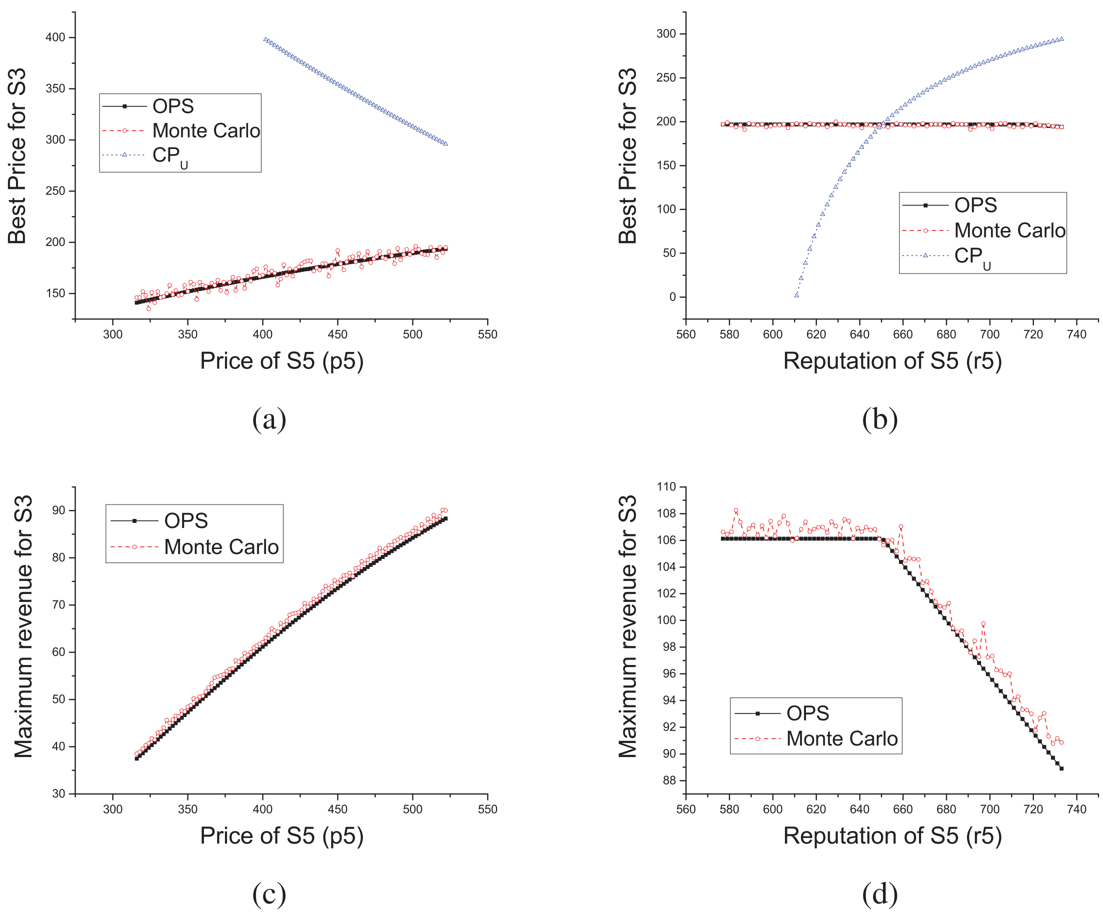


Fig. 4.6. (a) Best price for S_3 as S_5 increases price (b) Best price for S_3 as S_5 increases reputation (c) Maximum revenue for S_3 as S_5 increases price (d) Maximum revenue for S_3 as S_5 increases reputation

4.5 (a)(c), we also plot the $Crossing\ Point$ of Line2 CP_U to determine the most competitive seller S_U. When the target seller $S_n = S_3$ is at the right side of the CP_U (below line CP_U), $S_U = S_5$, but when $S_n = S_3$ moves to the left side of CP_U (above line CP_U), $S_U = S_4$ instead.

In Fig. 4.6, we keep the reputation of the target seller r_3, the price and the reputation of S_1, S_2, S_4 fixed. Then for Fig. 4.6(a)(c), we keep the reputation of S_5 fixed, and increase the price of S_5 from \$315 to \$524, and plot the best price and the maximum revenue for S_3 using our proposed pricing scheme and Monte Carlo simulation. For Fig. 4.6(b)(d), we keep the price of S_5 fixed, and increase the reputation of S_5 from 576 to 740, and plot the best price and the maximum revenue for S_3 using our proposed pricing scheme and Monte Carlo simulation. In Fig. 4.6 (a)(c), we also plot the $Crossing\ Point$ of Line2 CP_U to determine S_U.

Fig. 4.5 and Fig. 4.6 are in the same token, show how price/reputation of the competitive sellers influence the best price and maximum revenue of the target seller. When S_4 is the most competitive seller, as p_4 increases, the best price and maximum revenue of S_3 increases, but as r_4 increases, the best price and maximum revenue of S_3 decreases. When S_5 is the most competitive , and as p_5 increases, the best price and maximum revenue of S_3 increases, but as r_5 increases, the best price and maximum revenue of S_3 decreases.

From these figures, we have two conclusions. First, only the most competitive sellers influence the best price and maximum revenue of the target seller. As the price of the most competitive sellers increases, the best price and maximum revenue for the target seller S_n increase, and the best price is often achieved at the point where the most competitive seller changes. Then, as the reputation of the most competitive seller increases, the best price and maximum revenue for the target seller S_n decrease. Second, the less competitive sellers will not affect the best price and maximum revenue calculated through our pricing scheme.

4.2 Real Market Simulation

Next, we use eBay to extract real items on sale to validate our proposed pricing method. During the process, five steps are performed consecutively. First, we crawl data from eBay, second, we filter and generate the $skyline\ market$, third we implement our proposed pricing scheme, then we implement traditional average pricing and mark-up pricing methods, and finally we compare and discuss the results.

- $Crawl\ Data$: The real market data is crawled from eBay using $ROST\_DetailMinner$ developed by Wuhan University. During the process, we obtain a table containing the following information: seller ID, price, and reputation. We select coffee maker, Itouch, and Cannon camera selling at relative low, medium, and high prices as our examples.

- $Filter\ Skyline\ Seller$: Users are assumed rational in this thesis, so they will not choose the seller with high price and low reputation. In this case, the raw list we get from crawling usually contains hundreds of sellers, each selling at their own price and reputation. So filtering the market with only skyline sellers is necessary. Then after the filtering, we assort the remaining sellers by price from low to high to form the $skyline\ market$.

- $Implementation\ of\ OPS$: Input the $skyline\ market$, our proposed scheme outputs the best price and the maximum profit. As an example, we let the weight w follows Gaussian distribution with assigned parameter $\mu = 0.5$, and variance $\sigma = 0.5/3$, and we observe the same trend for other values of mean and variance. We assume that sellers have perfect knowledge of the distribution parameters, and we plan to investigate the impact of the weight distribution and the parameter estimation error in our proposed further work.

- $Implementation\ of\ traditional\ pricing\ methods$: Two traditional pricing methods are also implemented for comparison with our proposed scheme.

- Average pricing: $p_n = 1/M \sum_{i=1}^{M} p_i, i \in [1, 2, ..., M]$.
- Mark-up pricing: $p_n = (1 + \alpha) * cost$, α is the percentage sellers want to gain. For retailers, it is usually between $0.3 - 0.5$. In our simulations, we set $\alpha = 0.3, 0.4, 0.5$ respectively. $cost$ is assumed the same for all sellers in the same skyline market.

4.3 Numerical Results for Real Data

In this subsection, we show the numerical results obtained from the real eBay data. By implementing our proposed pricing scheme, we obtain the optimal price and the corresponding maximum revenue for the target seller S_n.

During the calculation, we use IncrPerc to show the increased percentage of the maximum revenue achieved by our proposed scheme compared to the revenue achieved by other pricing methods for target seller n. The equation can be written as,

$$\mathrm{IncrPerc} = \frac{|\mathrm{MaxRev_{OPS}} - \mathrm{MaxRev_{OtherMethods}}|}{\mathrm{MaxRev_{OtherMethods}}}. \tag{4.1}$$

$\mathrm{MaxRev_{OPS}}$ represents the maximum revenue calculated by our proposed scheme, and $\mathrm{MaxRev_{OtherMethods}}$ represents the maximum revenue calculated using average or markup pricing methods. Then we use $'+'$ to represent an increase in revenue, and $'-'$ to represent a decrease. For example, if we want to compute the increased percentage of the maximum revenue computed by our proposed scheme compared to the original eBay market when $n = 3$ shown in Table 4.2, we have $\mathrm{IncrPerc} = \frac{40.15 - 39.62}{39.62} = 0.01337$, then use $'+'$ to shown the improvement in revenue, and get $+1.33\%$.

4.3.1 Coffee Maker

Coffee maker is selling at a relative low price, and after filtering the raw data, the skyline market is shown in Table 4.2, with 6 skyline sellers arranged in price from

Seller ID	Price	Reputation
S_1	\$75.99	7
S_2	\$89.5	6446
S_3	\$89.95	13484
S_4	\$97.4	15054
S_5	\$109.2	15757
S_6	\$121.13	30636

TABLE 4.2
COFFEE MAKER SKYLINE MARKET

low to high. From the table, we see that skyline market has the price ranged from $\$75.99$ to $\$121.13$, while reputation from 7 to 30636. Assume the $cost$ is \$50.

Table 4.2 shows that for the original market, S_1 has $r_1 = 7$ which is too low to be considered by buyers. S_2's price is close to S_3's, but S_2's reputation is much lower. S_3, S_4, S_5, S_6 have reputations large enough, but S_3 has a relative lower price. Therefore, S_3 is much more competitive than all other sellers in the market. In this market as shown in Table 4.2, $S3$ has probability 0.9919 to be chosen by buyers, and we call this type of seller the $dominating\ seller$. More specifically, in this thesis, we define the $dominating\ seller$ as the seller who has a probability larger than 0.85 to be chosen by buyers.

For the market in Table 4.2, we try to find the best price for the target seller in the market to maximize his/her revenue. During calculation, once the target seller is chosen, all other sellers' reputations and prices as well as the target seller's reputation are fixed. We calculate the maximum revenue and the corresponding best price the target seller could achieve using the original market price, average pricing, markup pricing ($\alpha = 0.3, 0.4, 0.5$), and our proposed pricing method. Then by using equation (4.1), we obtain the increased percentage in maximum revenue $\mathrm{IncrPerc}$ with our proposed method comparing to other pricing methods. We performed $n = 1, 2, 3, 4, 5, 6$ respectively and obtained Table 4.3.

From Table 4.3, we observe that our optimal pricing scheme (OPS) realized the highest maximum revenue for all skyline sellers ($n = 1, 2, ..., 6$) compared to other pricing methods. From $\mathrm{IncrPerc}$, we see that by using our proposed pricing

n=1						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$75.99	$97.2	$65	$70	$75	$61.63
Max Revenue	$0.01	$0	**$0.02**	**$0.02**	$0.01	**$0.02**
IncrPerc	>+100%	inf	0	0	>+100%	-
n=2						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$89.5	$97.2	$65	$70	$75	$75.9
Max Revenue	$0.06	$0	$13.74	$17.31	$19.06	**$19.27**
IncrPerc	>+100%	inf	+40.24%	+11.32%	+1.10%	-
n=3						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$89.95	97.2	$65	$70	$75	$91.21
Max Revenue	$39.62	$0	$14.94	$19.91	$24.89	**$40.15**
IncrPerc	+1.33%	inf	>+100%	>+100%	+61.31%	-
n=4						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$97.4	$97.2	$65	$70	$75	$89.98
Max Revenue	$0.01	$0.02	$14.94	$19.93	$24.89	**$39.7**
IncrPerc	>+100%	>+100%	>+100%	+99.19%	+59.5%	-
n=5						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$109.2	$97.2	$65	$70	$75	$89.98
Max Revenue	$0	$0.15	$14.95	$19.93	$24.91	**$39.72**
IncrPerc	inf	>+100%	>+100%	+99.30%	+59.45%	-
n=6						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$121.13	$97.2	$65	$70	$75	$90.06
Max Revenue	$0.24	$2.6	$14.96	$19.95	$24.93	**$39.87**
IncrPerc	>+100%	>+100%	>+100%	+99.85%	+54.93%	-

TABLE 4.3
COFFEE MAKER SKYLINE MARKET PRICING METHODS COMPARISON, N=1,2,3,4,5,6

scheme, all skyline sellers achieve revenue increase at different levels. Comparing our proposed scheme with the original data crawled from eBay, we observe that the revenue increases 1.33% for the dominating seller S_3, and over 100% for other skyline sellers. Also, our proposed scheme achieves more than 100% revenue increase compared to average pricing method. Furthermore, our proposed scheme achieves 0 to more than 100% revenue increase compared to 0.3, 0.4, 0.5 markup pricing.

Then, from Table 4.3, we also observe that as long as the cost is the same, average pricing and markup pricing suggest the same price for all skyline sellers. The reason is because these methods only consider profit gain, but neglect user personal preference and inter-seller competitions. Having the same price may cause sellers with low reputation missing the chance to be chosen by buyers, while sellers with high reputation lose profit. In contrast, our proposed pricing scheme recommends different sellers with different prices according to their own market positions.

We observe the following patterns of the price recommended by our proposed method. First, only the $dominating\ seller$ is suggested an increase in price, while all other sellers are suggested to lower their prices. It is reasonable because in the original market, over 85% of the buyers prefer the $doninating\ sellers$. So for other sellers, their best strategy is to decrease the price and increase their competitiveness in the market. Second, S_1 with extremely low reputation should further lower price to attract customers that emphasis on price only. In our coffee maker example, our proposed method recommends S_1 to set price to $p_1 = \$61.13$. Third, for other sellers in the market, our proposed method recommends prices that are close to that of their most competitive sellers S_L. This is because if S_n lower his/her price to a level close to the price of S_L, S_n with a higher reputation will be even more competitive than S_L. Therefore, buyers may switch from S_L to S_n, which increases S_n's market share and his/her revenue. In the coffee maker case, the most competitive seller for S_2 is $S_L = S_1$, and our proposed method recommends S_2 set price to $p_2 = \$75.9$ which is very close to the price of its most

Seller ID	Price	Reputation
S_1	\$189.95	318
S_2	\$205.88	1368
S_3	\$209.95	3501
S_4	\$235.99	8707

TABLE 4.4
ITOUCH SKYLINE MARKET

competitive seller $p_1 = \$75.99$. Also, the most competitive seller for S_4, S_5, and S_6 is $S_L = S_3$, our proposed method recommends these sellers to set prices to $p4 = \$89.98$, $p5 = \$89.98$, and $p6 = \$90.06$ which are very close to the price of S_3 where $p_3 = \$89.95$.

4.3.2 Itouch

Itouch is the medium price product we selected from eBay. After filtering the raw data, we get Table 4.4, and all skyline sellers are arranged in price from low to high. There are only four sellers left in the skyline market, and the price range is $[189.95, 235.99]$, the reputation range is $[318, 8707]$. Assume the $cost$ is \$150. Same as the coffee maker example, once the target seller is chosen, all other sellers' reputations and prices are fixed as well as the target seller's reputation. By calculating the best price, the maximum revenue and $\mathrm{IncrPerc}$ using the original market price, the average price, the markup prices, and our recommended price, we obtain Table 4.5 for target seller $n = 1, 2, 3, 4$ respectively.

From Table 4.4, we observe that there is no $dominating\ seller$ in the itouch skyline market. However S_1, S_2 has close prices with S_3, S_4, but rather low reputations, which means S_1, S_2 are less competitive than S_3, S_4.

Table 4.5 shows the revenue gain compared among different pricing methods, and our proposed method obtains the highest revenue. For S_1, S_2, their original revenue are nearly zero, but our proposed method slightly increased their revenue. Similar with the coffee maker example, our proposed method achieves the highest maximum revenue among all pricing methods for all skyline sellers. More

n=1						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$189.95	$210.44	$195	$210	$225	$169.7
Max Revenue	$0.04	$0	$0.03	$0	$0	**$0.05**
IncrPerc	+25.15%	inf	+66.67%	inf	inf	-

n=2						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$205.88	$210.44	$195	$210	$225	$189.95
Max Revenue	$0	$0	$0.21	$0	$0	**$1.46**
IncrPerc	inf	inf	>+100%	inf	inf	-

n=3						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$209.95	$210.44	$195	$210	$225	$194.23
Max Revenue	$20.25	$19.79	$27.25	$20.23	$3.88	**$27.27**
IncrPerc	+34.67%	+37.80%	+0.07%	+34.80%	>+100%	-

n=4						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$235.99	$210.44	$195	$210	$225	$221.38
Max Revenue	$56.63	$60.22	$44.87	$59.78	$65.78	**$66.48**
IncrPerc	+17.39%	+10.40%	+48.16%	+11.21%	+1.06%	-

TABLE 4.5
ITOUCH SKYLINE MARKET PRICING METHODS COMPARISON, N = 1,2,3,4

specifically, comparing with the original market, our proposed method achieves revenue increase ranged from $17.39\% - inf$, comparing with average pricing method, our scheme increases revenue from 10.40% to inf, then comparing with markup pricing, our scheme achieves revenue increase from 0.07% to inf. Results indicate that our proposed method could elevate the profits for all skyline sellers.

Same with the coffee maker market, average pricing and markup pricing provide the same prices for all skyline sellers. From the $\mathrm{IncrPerc}$, we observe that our proposed method achieves higher maximum revenue than average and markup pricing.

Different from the coffee maker market, the Itouch market do not have a $dominating\ seller$, so our proposed method suggests everyone to lower their prices to become more competitive in the market. From Table 4.5, we observe that the patterns concluded above still comply. Our proposed method recommends S_1 further lower his/her price to $p_1 = \$169.7$ to attract extreme users. Also, our proposed method recommends S_2 reduce price to $p_2 = \$189.95$ to mask his/her most competitive sellers $S_L = S_1$, S_3 reduce price to $p_3 = \$194.23$ to mask his/her most competitive seller $S_L = S_2$, and S_4 decrease price to $p_4 = \$221.38$ to make his/her most competitive seller $S_L = S_3$ less competitive.

4.3.3 Cannon Camera

Canon camera can be considered as high price product, using the data crawled, we obtain Table 4.6. There are five skyline sellers with price ranged in $[1729, 2662.5]$, and reputation ranged in $[674, 34388]$. Assume the $cost$ is \$1500. Once the target seller is chosen, other sellers' reputations and prices are fixed as well as the target seller's reputation. We perform our proposed scheme and average and markup pricing for target seller $n = 1, 2, 3, 4, 5$, we can obtain Table 4.7.

If we compare S_3 with S_1, S_2 in Table 4.6, we found that the prices are similar but the reputation of S_3 increased dramatically. So, S_1, S_2 are unlikely to be chosen by customers as best choice. Then if we compare S_3 with S_4, S_5, we found that even though S_4, S_5 have high reputation values, S_3 with a much lower price and a

Seller ID	Price	Reputation
S_1	$1729	674
S_2	$1769	696
S_3	$1789	12465
S_4	$2199	16100
S_5	$2662.5	34388

TABLE 4.6
CANNON CAMERA SKYLINE MARKET

competitive reputation are more likely to be preferred by customers. Therefore, S_3 is the $dominating\ seller$ in the Cannon camera skyline market.

From Table 4.7, we observe that using our proposed pricing method, skyline sellers S_1, S_2, S_4, S_5 improve their maximum revenues by more than 100% compared to original, average and markup pricing. The dominating seller S_3 improve maximum revenue from 2.74% to more than 100% compared to other pricing methods.

Table 4.7 shows that average and markup pricing methods recommend prices regardless of the seller's market position. Unlike the traditional way, our proposed pricing method gives reasonable prices accordingly, and the revenue gained are higher than other pricing methods.

The upper rules can still be applied to this market. Our proposed pricing method suggests the $dominating\ seller$ S_3 to increase the price by 18.89% to maximize his advantage in the market. Then our proposed pricing method recommends S_1 drop his/her price to $\$1602.4$ to achieve higher revenue, S_2 decrease price to $p_2 = \$1729$ to mask his/her most competitive seller $S_L = S_1$ to gain more market share, S_4 reduce price to $p_4 = \$1789.18$ to make his/her most competitive sellers $S_L = S_3$ less competitive, and S_5 reset price to $p_5 = \$1790$ to make his/her most competitive sellers $S_L = S_3$ less competitive.

n=1						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$1927	$2029.7	$1950	$2100	$2250	$1602.4
Max Revenue	$0.12	$0	$0	$0	$0	**$0.24**
IncrPerc	>+100%	inf	inf	inf	inf	-

n=2						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$1769	$2029.7	$1950	$2100	$2250	$1729
Max Revenue	$0	$0	$0	$0	$0	**$1.04**
IncrPerc	inf	inf	inf	inf	inf	-

n=3						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$1789	$2029.7	$1950	$2100	$2250	$2127
Max Revenue	$287.6	$523.66	$446.55	$586.36	$0	**$602.43**
IncrPerc	>+100%	+15.04%	+34.91%	+2.74%	inf	-

n=4						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$2199	$2029.7	$1950	$2100	$2250	$1789.18
Max Revenue	$0	$0.4	$1.13	$0	$0	**$288.03**
IncrPerc	inf	>+100%	>+100%	inf	inf	-

n=5						
Pricing	Original	Average	30%Markup	40%Markup	50%Markup	OPS
Best Price	$2662.5	$2029.7	$1950	$2100	$2250	$1790
Max Revenue	$1.93	$3.86	$6.43	$3.42	$2.3	**$288.95**
IncrPerc	>+100%	>+100%	>+100%	>+100%	>+100%	-

TABLE 4.7
CANNON CAMERA SKYLINE MARKET PRICING METHODS COMPARISON, N=1,2,3,4,5

4.4 Three Observed Rules

To summarize, from our simulations, we observe the following rules for sellers to set their prices in the skyline market.

- Rule 1: The seller with the lowest reputation in the skyline market should further lower the price to attract more extreme buyers who put too much emphasis on price while ignoring reputation. In our buyer model, these type of customers put too much weight w on price, so the extreme low price will draw their attention and bring more market share.
- Rule 2: Dominating seller, whose probability be chosen by buyers are larger than 0.85. Our proposed pricing method recommends these sellers to raise their retail prices to further increase their revenue. Due to the obvious advantage, a proper increase in price will gain more profit without decreasing the demand.
- Rule 3: For less competitive sellers, our proposed method suggests these sellers reduce prices so they can be more competitive than their most competitive seller S_L. Usually, the price our proposed method recommends is a little lower or comparable to the price of S_L. With the suggested price, the target seller will gain more market share which results in a higher revenue.

Chapter 5

Conclusion and Future Work

This chapter summarizes the contributions in the thesis. The future works that can be done are also introduced.

5.1 Conclusion

As e-market is becoming an essential form of trading today, surviving and making more profit for online retailers in the e-market is vital. In this thesis, we propose an optimal pricing scheme that suggests the target seller with an optimal price in order to make maximum revenue.

Our work followed the works of MAPS and MRS. But they focused on improving the accuracy of product recommendations for buyers, we put emphasis on trying to reset sellers' price to ensure the maximum revenue. Our proposed method consists of three main parts: determining the most competitive sellers; constructing the maximum revenue equation; and finding the global optimal solution to the revenue equation.

In the first part, with the founded theory, we identify the most competitive sellers in both lower and upper sets of the skyline market to address the problem of the inter-seller competitive within each price range. In the second part, with the upper and lower boundaries founded using the most competitive sellers, we derive the revenue equation. Finally, by comparing the suboptimal solutions found in different price ranges, we find the global optimal solution.

By conducting calculation on both simulated and real markets, we found that our proposed method achieves higher revenue than average and markup pricing. Also, through analysing different factors, we discovered that only the most competitive sellers have impact on the best price and maximum revenue of the target seller. Also, we concluded three observed rules that will apply to market pricing for all skyline sellers. First, the seller with the lowest reputation should lower the price to attract more extreme buyers. Second, the dominating seller should properly raise his/her price to expand the advantage and gain more revenue. Third, the less competitive sellers should reduce prices to a level close to or lower than the price of the most competitive seller, so the target seller could get a higher market share and gain more revenue.

5.2 Future Work

Based on our proposed pricing scheme, there are a few directions for further research, and they are listed as follows.

- 1. Estimation of the weight distribution for user model

  This thesis assume the sellers have perfect knowledge of the user behavior, and assume the weight w follows the Gaussian distribution with mean and variance. Future work could focus on how to find the estimation of the two parameters.

- 2. Pricing scheme for all skyline retailers simultaneously

  This thesis proposed an optimal pricing scheme for a single seller to reset his price to maximized his own revenue. Further research could assume that all skyline sellers in the e-market are trying to reset their own prices to achieve maximum revenue simultaneously. Then for this dynamic e-market, game theory is often used to reach an equilibrium that each seller could gain the most possible revenue.

- 3. Estimation of reputation

  Our scheme is conducted with a precondition that the reputation of each skyline seller in the e-market is fixed which is true because it is based on the comments of previews transactions. However, there is a phenomenon that sellers buy reputation from the system to increase its possibility be chosen by buyers. Assume the cost of each reputation unit is fixed, then how much reputation should a seller buy to obtain the maximum revenue can be analysed.

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