Dynamic Fragmentation of Granite for Impact Energies of 6 to 28 J

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Abstract

The dynamic fragmentation of granite at impact energies of 6 to 28 J is examined in this paper. Results indicate that more dense materials, or those with a higher quartz content, produce less fractured mass, and have larger dominant fragment sizes and smaller aspect ratios in their fragment probability distributions. Values of the peaks in fragment size distributions are in agreement with theoretical predictions. Examination of the fracture surfaces reveals information concerning transgranular cracking, crack bifurcation mechanisms and evidence of comminution of sub-micron K-feldspar and plagioclase fragments. Fractal dimensions of the cumulative distribution of fragment sizes were $\simeq 2$, indicating that comminution was a dominant fragmentation mechanism in these tests. Peaks in the probability distributions of sub-micron fragments on fracture surfaces reveal a limit of coherent fragments of approximately 0.60 μm for plagioclase and K-feldspar. The smallest fragments found on the surfaces were approximately 0.30 μm and this is considered to be the comminution limit for these materials.

Keywords: dynamic fragmentation of rock, drop-tests, microscale fracture

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mechanisms, comminution, grinding limit, granite fracture, fractals and fragmentation, brittle fracture, scanning electron microscopy of rock, image analysis of rock fragments

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1. Introduction

The fracture and fragmentation of brittle materials and, in particular, of rock has been an area of continued scientific research since the early works by Mott [1, 2]. Applications include: developing new ceramic-metal shielding systems for military use [3]; in space science where the collision and fragmentation of planetary bodies play important roles in the development of our solar system [4]; in geology research where fragmentation of rock is important in understanding faulting [5], rock slides [6] and associated earthquakes [7]; in exploration drilling for oil, gas and mineral deposits; production drilling and related reservoir fracturing techniques (e.g., shale gas); and in mining and blasting applications, where the safe excavation and transportation of rock is a daily concern [8, 9]. The fragmentation of brittle materials continues to be an active field of materials research [10–22] and is the topic of this paper.

During brittle fragmentation, cracks initiate at stress concentrations, grow and propagate to a fraction of the Rayleigh wave speed [23], and eventually coalesce with one another to form fragments. Energy dissipation during brittle fracture has many forms. Minor dissipation mechanisms include the kinetic energy of moving fragments [1, 2, 24], acoustic emission [7, 25], and thermal dissipation [26]. The primary mechanism of energy dissipation during fracture in brittle materials is the generation of new surface area [11, 13, 15, 17, 18].

Energy dissipated via fragmentation is related to the number of fragments, the size of these fragments, and, more specifically, the generation of new surface area [27]. Past experimental investigations have had to rely on size (or grouped mass measurements) of fragments as small as approximately 500 μm in order to gain insight into the fracture processes due to the difficulty of counting and sizing fragments smaller than this [28]. However, fragments smaller than 500 μm account for a large percentage of the total number of fragments generated during fragmentation and, hence, represent a notable portion of the total energy dissipated during fracture. Fragments as small as 10 μm are considered in the low-speed impact tests conducted in this study.

Analytical models [1, 2, 13, 14, 16–18, 29–32] and numerical models [12, 33, 34] have been developed to predict mean fragment size and distributions. Early work by Rosin and Rammler [29] used an exponential function to describe fragment size distributions. Later, Mott stimulated the theoretical modelling of fragmentation of rapidly expanding rings [1, 2]. Mott's theory of fragmentation suggested that the distribution of fragment sizes is dependent on the propagation of the release waves generated during fracture and the on-going straining over the range of failure strain distributions. He further postulated that unloading of the material proceeds at much lower speeds and made an estimate of the stress-history in a rigid-plastic material model [2, 35]. Grady [17] developed on Mott's theory by considering the instantaneous appearance of fracture and defining the statistical properties of the failure strain [36]. Grady [17] used an energy balance approach to predict the average fragment size under high strain-rates by assuming that all local kinetic energy was converted into fracture energy. He predicted that the average fragment size decreased with increasing strain rate and material brittleness. Glenn and Chudnovsky [31] extended the work by Grady to include the contribution from the elastic potential energy, which dominates at low loading rates. They also predicted a quasi-static average fragment size that is independent of strain rate. Experimental results have shown the energy-balance models of Grady [17] and Glenn and Chudnovsky [31] over-estimate the dominant fragment size [37, 38].

More recently, Zhou et al. [33, 39] and Levy and Molinari [12] have proposed equations for predicting dominant fragment size based on numerical simulation results. Their numerical models account for the evolution of the residual damage, as well as the wave reflections and interactions; something that the energy-based models are not able to accommodate. The energy-based model of Grady [17] has also been modified recently [15], and now includes the concept of entry and exit correlation length scales for the dynamic fragmentation of brittle materials. Over these scales, the fragmentation process is considered scale invariant. The theories proposed by Glenn and Chudnovsky [31], and more recently by Zhou et al. [33, 39], Levy and Molinari [12], and Grady [15] are explored in greater detail in this paper.

The goal of this paper is to investigate the fragmentation (i.e., number, size, and shape of fragments) for three types of granite rock under low-energy impact loadings of 5.9 J to 28.0 J. The experiments were performed three times for each material-impact energy combination in order to investigate variability. Experimental results are compared with theoretical models. Scanning electron microscopy is used to investigate microscale fragmentation mechanisms that govern the fragmentation processes; an approach not typically considered in other works.

2. Experimental Setup and Analysis Methods

The drop-test apparatus shown in Figure 1 was used to study the fragmentation of three types of granite. Tubes over 1 m in length and 32 mm in diameter directed steel impactors onto the targets, which were placed flat on a steel plate to help produce the desired fragmentation. Three steel cylinder impactors, masses of 590 g, 830 g and 1080 g, with a hemispherical end were used (Figure 2). Drop heights and impactors were varied to achieve kinetic energies at impact of 5.9, 8.4, 11.0, 15.0, 19.4, and 28.0 Joules.

All targets were cut approximately 45 mm by 45 mm and 15 mm thick. For simplicity, they are labelled Black, Red and White in this paper. The Black material mainly contained plagioclase (pale in colour), biotite (black), and quartz (translucent and glassy). The Red material primarily contained K-feldspar (pale pink in colour), with less amounts of biotite and quartz. The White samples mainly comprised plagioclase and quartz, with some traces of garnet (red). These targets are massive igneous and metamorphic rocks and are free from obvious weakness planes, such as pre-existing micro-cracks, joints and faults. The densities of each material were 2300 kg/m^3 for Red, 2500 kg/m^3 for Black, and 2900 kg/m^3 for White. Photographs of the materials are shown in Figure 3. The blocks were weighed before impact and fragments larger than 10 mm were collected and weighed after each impact using a Scientech S120 scale with resolution of $\pm 1 \times 10^{-4}$ g. Fragments larger than 10 mm were part of the initial targets, while the remaining fragments were assumed to be part of the fragmentation process. As will be shown later, the smaller fragments represent over 99 $\%$ of the total number of fragments generated, therefore his approach is acceptable from a statistical standpoint. The difference between the original mass and the large fragments was used to define the mass of the fragments generated as a result of impact. In turn, this was used to study the effect of material type and impact energy on the fragmentation of the blocks.

A Hitachi SU-70 analytical Field Emission Scanning Electron Microscope (FESEM) was used to take secondary electron (SE) images of the granitic fragments. The acceleration voltage range was 100 V to 30 kV with a beam current of a few nano-amperes. The resolution of the FESEM is 1.0 nm at 15 kV and 1.4 nm at a landing voltage of 1 kV. The composition of the mineral phases was determined by Energy Dispersive X-ray Spectroscopy (EDS) using an INCAx-act LN2-free Analytical Silicon Drift Detector at 15 kV accelerating voltage and 5 nA beam current, with acquisition times of 100 s for all elements.

2.1. Determination of fragment size and shape distributions

Analysis of the number, size and shape of the fragments was primarily accomplished using a Nikon SMZ-U zoom binocular microscope equipped with a x0.5 ED (extra-low dispersion) plane objective and DS-Fi1 (524 megapixel) digital camera system. Samples were prepared by physically pouring out fragments from a container onto a sheet of black paper and agitating the fragments to spatially distribute them. Smaller fragments were typically re-distributed towards the bottom of the container throughout transportation, therefore initial samples contained larger fragments. Subsequent samples got smaller in size. Samples were categorized as ”large”, ”medium”, ”small” and ”fine” according to their size. Among all the four sample sets, the fine samples have the lowest peak value in their probability density distribution of fragment sizes. Photographs of fragments for various size ranges are shown in Figures 4, 5, and 6 for the Black, Red and White materials, respectively.

Figure 7 illustrates the procedure used to determine the number, size and aspect ratio, or shape, of sample fragments. The Red material at a impact energy of 19.4 J is used as an example (Figure 7). A high-resolution photographic image of a sample was taken (Figure 7a). The image was then converted to a black and white image (Figure 7b) and Matlab [40] was used to determine the major [1] and minor axes [2] (μm), area [3] (μm^2) and shape (major/minor axis) of the fragments. Histogram distributions of the minor and major axes, area and shape of the fragments using a $\log_{10}$ transformation are shown in Figure 7c. Logarithmic transformations were taken to force near-normality of the data for curve fitting. A least squares fit of generalized extreme value (gev) distribution [41] in the form of equation (1) was fit to the data:

$$\mathrm{gev}(x, \xi, \sigma, \mu) = \frac{1}{\sigma}\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi-1}\exp\left(-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right) \quad (1)$$

where ξ is a shape parameter, σ is a scale parameter, and μ is a shift parameter. The solution to equation (1) must satisfy 1+ξ(x-μ)σ>0. Rayleigh [12], Wiebull [42] and log-normal distributions [38] were used elsewhere, but gev-fits are chosen here because of the added parameter to describe the distribution (i.e., μ). Peaks in the minor and major axis, area, and shape distributions occur at 81 μm (1.91), 132 μm (2.12), 11,749 μm^2 (4.07), and 1.51 (0.18), respectively. Log_{10} values are shown in parentheses here and throughout this paper.

Following image collection, the fragments were re-distributed through agitation and another image was taken. This was repeated for a total of three image

[1] Major axis is the largest dimension of a fragment.
[2] Minor axis is perpendicular to the largest dimension.
[3] Area is the total projected area of the fragment on the image.

partners to investigate the variation within a sample. Shown in Figure 7d is the variation in the major axis dimension and shape for the three image partners used in this example. Peaks in the major axis distribution are similar for each image partner, occurring at 126 μm (2.10), 132 μm (2.12), and 132 μm (2.12). Peaks in the shape distribution occur at 1.51 (0.18) for all cases. Only one image partner is considered hereafter since little variation exists.

The variation among the major axis dimension and shape for three comparably-sized samples from the same trial is shown in Figure 7e and f, respectively. This was done to show that multiple samples contain similar peaks in their distributions and, hence, demonstrating that not all fragments need to be counted. Instead, representative samples from each size range may be extrapolated to estimate the total number of fragments generated for each material-impact energy combination. This will be applied later to estimate the total number of fragments. The major axis dimension in Figure 7e varies from 129 μm (2.11) to 135 μm (2.13), and the shape in Figure 7f varies from 1.48 (0.17) to 1.51 (0.18). Again, little variation is found.

3. Experimental Results

Initially, the fragment mass is plotted against impact energy in Figure 8. A least squares fit of a logarithmic function in the form of equation (2) was used to fit the data:

$$\text{fragment mass} = \alpha_{material} \log(\beta_{material} \text{KE}) \tag{2}$$

where KE is the kinetic energy at impact, and $\alpha_{material}$ and $\beta_{material}$ are material coefficients. The logarithmic curve-fits and coefficient values are also shown in Figure 8. The coefficient of determination, R^2, is roughly 0.90 for all cases indicating that a logarithmic curve fit is suitable for this data. The amount of fragment mass generated from impact increases from White to Black to Red. This is the order of decreasing densities, from 2900 kg/m^3 for White to 2300 kg/m^3 for Red; suggesting more dense materials generate less fragment mass during fracture under the loading conditions used in this study. More likely, this result is associated with the material properties (e.g., fracture toughness, yield and shear strength) of the constituent minerals in the types of granite. Since the red has the greatest percentage composition of fracture-prone K-feldspar and plagioclase, then it is likely to fracture the most. Conversely, the White material has the least percentage composition of these minerals, and the highest-percentage composition of quartz and almandine, therefore it fractures the least. These results indicate that the percentage mass of fragments, generated from low-speed impact with a steel backing plate, asymptotically approaches a finite mass, or volume, of fragments as the impact energy is increased.

Next, the total number of fragments, mass of the fragments, and size distributions were estimated by extrapolating measurements from 10 images of samples for each material-impact energy combination. One image was the ”fine” sample, three images were ”small” samples, three were ”medium” samples, and three ”large” samples. An example of this procedure for Red material and the 5.9 J impact is shown in Table 1. For brevity, the three images used for the ”small”, ”medium”, and ”large” samples are combined in the table. Each fragment mass is estimated using:

$$\text{mass of fragment} = \rho \frac{4\pi}{3} \text{ area} \times \text{minor axis} \tag{3}$$

where ρ is the material density, and the area and the minor axis dimension were previously defined. An interesting feature of Table 1 is the estimation of 10,662 total number of fragments for the low impact energy and Red material. Similar estimates of the total number fragments for all material-impact energy combinations are shown in Table 2 and are plotted with power-law curve fits in Figure 9. The number of fragments increase substantially for an increase in impact energy. Fragment estimates as high as 75,000 are obtained.

Major axis dimension histogram distributions of the extrapolated data with gev-fits are shown in Figures 10a, b, and c for each material and impact energy. A $\log_{10}$ transformation is again used to force near-normality for curve fitting. The peak in the major axis distribution decreases signficiantly and the distribution becomes more narrow-band as the impact energy is increased for all materials. The distribution peaks for the minor and major axis dimension, area, shape and the curve-fit parameters σ and ξ from equation (1) are shown in Table 2. The values of ξ increases and the values of σ decreases for increases in impact energy for all materials. These are plotted in Figure 10d.

The trends of the dominant peaks in the probability distributions for the minor and major axis, area, and shape (data from Table 2) are shown in Figure 11. A power-law curve in the form of equation (4) was used to fit the data:

$$\text{fragment size} = a_{material}\ \text{KE}^{n_{material}} \tag{4}$$

where $a_{material}$ and $n_{material}$ are material dependent coefficients. The coefficient values are also shown in the Figures 11a, b and c. For all cases, R^2 is larger than 0.91, confirming that a power-law fit is appropriate for this data. The White material has the largest major axis dimension for all impact energies, followed by Black and then Red (Figure 11b). A similar power-law trend in the area measurements is shown in Figure 11c. For all cases the minor axis, major axis, and area decreases as the impact energy is increased, as expected. The variation in shape (major/minor axis) for each material and impact energy is shown in Figure 11d. The shape for each material remains relatively constant, and does not increase at a statistically significant rate over the range of impact energies studied; with a mean value of 1.60 for Red, 1.53 for Black, and 1.48 for White. Together, these results indicate that more dense materials, or those with higher quartz content, have larger dominant fragment sizes and smaller aspect ratios.

4. Comparison of Results with Theory

The fragment distribution and average fragment size are two important characteristics of fragmentation events. Grady's [15] model to calculate fragment size assumes local kinetic energy is converted to the necessary energy for creating new surfaces. The average fragment size according to Grady [15] can be calculated as:

$$\mathrm{s}_{\mathrm{Grady}} = \left(\frac{48G_c}{\rho\dot{\epsilon}^2}\right)^{1/3} \tag{5}$$

where ρ is the material density (kg/m^3), $\dot{\epsilon}$ is strain rate (s^{-1}), and G_c is the fracture energy (J/m^2).

Glenn and Chudnovksy [31] modified Grady's theory to include a strain energy term and assumed that the stored strain energy and the local kinetic energy are converted to fracture energy following fragmentation. They deduced an equation for the average fragment size:

$$\mathrm{s}_{\mathrm{GC}} = 4\sqrt{\frac{3}{\alpha}}sinh\left(\frac{\phi}{3}\right) \tag{6}$$

where

$$\phi = sinh^{-1}\left[\beta\left(\frac{3}{\alpha}\right)^{3/2}\right] \tag{7}$$

and

$$\alpha = \frac{3\sigma_c^2}{\rho E\dot{\epsilon}^2} \tag{8}$$

$$\beta = \frac{3G_c}{2\rho\dot{\epsilon}^2} \tag{9}$$

where E is Young's modulus (Pa) and σ_c is the compressive strength of the material before failure (Pa).

Zhou et al. [33, 39] proposed the strain-rate dependent fragment size as:

$$\mathrm{s_{Zhou}} = \frac{4.5EG_c}{\sigma_c^2}\left[1 + 0.77\left(\frac{\dot{\epsilon}}{c\sigma_c^3/E^2G_c}\right)^{1/4} + 5.4\left(\frac{\dot{\epsilon}}{c\sigma_c^3/E^2G_c}\right)^{3/4}\right]^{-1} \tag{10}$$

where c is the longitudinal wave speed (m/s) given by:

$$c = \sqrt{E}\rho \tag{11}$$

Levy and Molinari [12] proposed the average fragment size be calculated as:

$$\mathrm{s_{LM}} = t_0C_{eff}\frac{3}{1 + 4.5\left(Et_0/\mu_{init}\right)^{2/3}\dot{\epsilon}^{2/3}} \tag{12}$$

where C_{eff} is effective longitudinal wave speed and given as:

$$C_{eff} = c\left(\frac{2}{a+1}\right)^{1/2}\left(\frac{\sigma_{c,min}}{\mu_{init}}\right)^{1/5} \tag{13}$$

where c is the wave speed, $\sigma_{c,min}$ is the strength of the weakest link in a probability distribution of defects, μ_{init} is the average strength, a is a scaling parameter depending on what type of distribution is chosen (e.g., Wiebull, Gaussian), and t_0 is a characteristic time (s) defined by Zhou et al. [39] as:

$$t_0 = \frac{EG_c}{c\sigma_c^2} \tag{14}$$

To compare with the experimental results, values are taken as: ρ=2,700 kg/m^3, G_c= 70 J/m^2 [43], E= 80 GPa [43], σ_c=240 MPa [44], μ_{init}=σ_c/2 (based on ratios used by Levy and Molinari [12]), and a=0.65 [12]. Strain rate varies. Shown in Figure 12 is the theoretical dominant fragment size plotted against strain rate and the experimental results from the impact tests presented in this paper. The strain rate is estimated as v/h for the drop tests, where v is the impact velocity (m/s) and h is the thickness of the target (m). This simplification results in having only three different strain rates (300 s^{-1}, 400 s^{-1}, and 480 s^{-1}) for the five different impact energies since the impact velocities are the same for two sets of impact energies. The experimental results plotted in Figure 12 are bounded above by the Levy and Molinari model [12] prediction and roughly agree with the Glenn and Chudnovksy equation [31]; with the equation proposed by Grady [15] over-predicting the size by greater than a 100 times. These results experimental confirm the models proposed by the other authors, and for engineering purposes, predict the dominant fragment sizes adequately.

In addition to predicting a dominant fragment size, Grady [15] also explored the theory of brittle fragmentation and noted that brittle materials tend to undergo failure and fragmentation at an elevated elastic strain energy than is predicted using quasi-static methods. He attributed this to nonequilibrium fragmentation (Figure 13). Here, the correlation length λ_c at the onset of fracture determines the initial fracture length that is able to dissipate the strain energy [15]. Fractures at a length of λ_c are not capable of dissipating all of the strain energy stored in the body preceding failure; therefore, cracks proceed to cascade and branch until the strain energy is dissipated at a second, smaller fracture length of λ_e.

Grady [15] suggests that the values for the governing length scales, λ_e and λ_c, are those bounding the region of the Mott plot described by the power-law function (a straight line in a loglog plot):

$$G(r) = Cr^{-D} \tag{15}$$

where r is the fragment size, C is a proportionality constant, D is the fractal dimension, and $G(r)$ is a mapping function. The fractal dimension is a non-negative rational number and was introduced by Mandelbrot [45] to better describe irregular forms that are too complex to be described by Euclidean geometry. Since fractals are able to describe self-similar response at any scale, they have been routinely applied to describing the fragmentation process in all types of materials [46–48]. The fractal dimension of a particle size distribution can be used to estimate the degree of fragmentation during, for example, comminution. Comminution is the process of grinding and crushing a solid body to form smaller fragments. For finer comminution, the fractal dimension is near 2, indicating that the fragmentation occurs primarily on the surface [49]. A value of D closer to 3 suggests the damage is more spatially distributed [49].

The values of λ_e, λ_c and D are all of particular interest here because they yield insight into the dynamic behaviour of the brittle material. A Mott plot representation is a plot of the logarithm of the cumulative fraction of fragments larger than the individual fragment size (i.e., major axis), r, on the independent-axis. The cumulative distribution of the fragment sizes is given by:

$$F(r) = \int_{-\infty}^{r} f(\zeta) d\zeta \tag{16}$$

where $f(\zeta)$ is the probability distribution of data. Plots of the cumulative distributions of the fragments are shown in Mott plot form in Figure 14. The direction of increasing impact energy is highlighted. This is now explored in greater detail.

The values of λ_e, λ_c and D depend on which subset, s, of the total set of fragments, S, is chosen. A least squares optimization algorithm was developed to compute a power-law curve fit in the form of equation (15), thereby determining the values of λ_e, λ_c and D, for all $S - s + 1$ possible subsets of size n, where $100 < s < S$. The "best" subset of size s was selected to maximize the coefficient of determination, R^2, and the range of the bounding length scales, $\Delta\lambda$=λ_c-λ_e, over the respected subset. Shown in Figure 15 is the dependance of R^2 and D on $\Delta\lambda$ for the Red material. Values of $\Delta\lambda$ were determined from Figure 15a at the junction where there R^2 began to decrease significantly. Values of λ_e and λ_c were subsequently obtained from $\Delta\lambda$. The corresponding values of D were then obtained from Figure 15b. In addition, the ratio of:

$$F_G = \frac{\lambda_c}{\lambda_e} \tag{17}$$

was determined. Values of λ_e, λ_c, D and F_G were obtained similarly for the Black and White materials.

Values of λ_e, λ_c, $\Delta\lambda$, D, and F_G for all materials and impact energies are displayed in Table 3. Associated plots are shown in Figure 16. Shown in Figure 16a is the dependence of λ_e on impact energy for all materials. For all cases λ_e is on the order of the peak in the major axis probability density distribution. The value of λ_e decreases for an increase in impact energy. For all cases, the values of λ_e correspond to between 0.55 and 0.65 on the y-axis of the Mott plots in Figure 14. That is to say, between 55 $\%$ and 65 $\%$ of the total number of fragments is larger than λ_e. This is expected since the distributions of fragment sizes in Figure 10 are positively skewed.

Shown in Figure 16b is the dependence of $\Delta\lambda$ on impact energy. The value of $\Delta\lambda$ decreases for an increase in impact energy (Figure 16b). This might be expected since the probability density distribution becomes more narrowband as the impact energy is increased (from Figure 10). Shown in Figure 16c is the dependence of λ_c on λ_e. Here λ_c increases as λ_e increases. Again, this is expected based on the two previous figures.

Shown in Figure 16d is the dependence of the fractal dimension, D, on the impact energy. The results indicate that D increases slightly with an increase in impact energy. This is consistent with previous investigations [49, 50], and is associated with an increase in smaller fragments. In addition, fractal dimensions near 2 suggest the fragmentation processes are primarily surface-related (e.g., comminution). Figure 16e shows that F_G decreases for increasing impact energy for all material types. Again, this might be expected since the value of λ_c decreases more more rapidly than λ_e when the impact energy is increased. Lastly, the relationship between D and F_G is plotted in Figure 16f. The values of D decrease slightly when F_G is increased. More importantly, this plot defines a relationship between two non-dimensional values, where D is a measure of the cascading effect of fragmentation from λ_c to λ_e, and F_G is a measure of the nature of the size distribution (i.e., how narrow-band the distribution is).

5. Scanning Electron Microscopy of the Fracture Surfaces

The dynamic fragmentation of rock is a seemingly stochastic process where cracks initiate, grow, and coalesce along roughened, complicated mineral surfaces to form, as these experimental results and many others indicate, distinct fragments of similar size and shape. Characteristics of these surfaces and the physical mechanisms that drive fragmentation are often overlooked, especially in numerical works, but are explored here. Shown in Figure 17 are scanning electron microscope images of a collection of fragments taken from the 28.0 J impacts for the White and Red materials. The shapes of the fragments are ”blocky”, as might be expected since plagioclase and K-feldspar minerals compose a large percentage of these fragments, and fracturing in these minerals primarily occurs along cleavage planes. There are also many complicated shapes and surface characteristics associated with each mineral type. In addition, the fragments contain ”sharp” corners, which are likely sites of branching of the propagating cracks to form fragments. Crack propagation and branching are now investigated.

5.1. Fracture and Branching

Evidence of crack propagation, especially transgranular cracking, reveals loading history information and the sequence of fracture events as a result of the low-speed impacts. Shown in Figure 18a and b are through-grain cracks found in plagioclase and K-feldspar, respectively. Tensile stresses needed to cause transgranular cracking in these minerals are in the order of 2 GPa [4]. Evidence of even higher localized stresses are found in the form of transgranular cracking in quartz grains (Figures 18c to f). Shown in Figure 18c is the location of the quartz grain containing the crack, which is sandwiched between two plagioclase grains. At a higher magnification (Figure 18d and e), the crack is jagged. The curvilinear, or wave-like, crack propagation in the quartz is attributed to incremental crack extension [51] as a result of non-uniform stress field and mixed-mode fracture [52]. High magnification SEM imagery (Figure 18f) reveals that the crack's surface is relatively smooth. Tensile stresses needed to cause transgranular cracking in quartz are slightly greater than 2 GPa [4, 53], confirming that these stresses are reached during these low-speed impact tests.

Branching is a subsequent stage of fracture following crack propagation. The branching phenomenon has been studied by many researchers, most of whom attempt to seek a necessary condition for branching through comparison of stress states before and after branching [23, 54, 55]. Little appreciation has been given to the physical mechanisms. Shown in Figure 19 are bifurcation points for crack branching on fragment tips. The fragments examined in this figure contain distinct fracture tips and, hence, are assumed to be bifurcation points for cracks. An example of a fragment tip in quartz is shown in Figure 19a. Bifurcation occurs in the granite samples when propagating cracks in other mineral phases encounter quartz because of its higher hardness. An example of a bifurcation point in K-feldspar is shown in Figure 19b. Here a localized region is melted and plastically deformed over much of the tip. The region encompassed by the melt is approximately 38 μm from the fracture tip. Bulk temperatures needed to cause local melting of these minerals are in the order of 830 K [5]. High temperatures are likely associated with grain-on-grain contact produced by global fragment motion. The generation of localized heat during fracture has been noted previously [26, 27]. More evidence of localized melting at bifurcation points is shown in Figure 19c on the fracture tip of a plagioclase fragment. The extent of the plastic deformation region, initiated by melting, is approximately 2.8 μm from the tip. It should be noted that this plagioclase fragment is smaller in size than the K-feldspar fragment presented in Figure 19b, which may account for the smaller plastic tip region. Highlighted in Figure 19c is evidence of a melt connection on the surface of a plagioclase fragment. The circled region is further magnified in Figure 19d. The melt-connection in Figure 19d is approximately 1.3 μm in length, while smaller pieces of melt, roughly 0.3 μm in size, are highlighted on the left. These results confirm the realization of temperatures in the order of 830 K at bifurcation points in K-feldspar and plagioclase fragment tips. Again, these high localized temperatures are a result of grain-on-grain contact brought on by fracture surface motion and recovered heat from exceeding the elastic limit.

5.2. Surface Debris

A dominant feature in all SEM images is the scatter of debris on their surfaces. The nature of the debris is explored further in Figure 20. Debris on the surface of a K-feldspar grain are shown in Figure 20a, while higher magnification images of surface debris on garnet and plagioclase are shown in Figures 20b, c and d, respectively. EDS analysis of the debris on the K-feldspar and garnet surfaces reveals that they are mainly K-feldspar in composition, while plagioclase debris is scattered on the plagioclase surfaces. Similar techniques as used in Figure 7 and gev curve-fits were used to determine the major axis dimension and shape distributions of these fragments . The distributions are shown in Figures 21a (minor axis) and b (shape), with corresponding peak values displayed in Table 4. The value of the smallest fragment size on these surfaces is also displayed in Table 4. At a magnification of 300X (Figure 20a on the K-feldspar surface), the peak in the major axis distributions occurs at 1.36 μm, and the peak in the shape distribution occurs at 1.49. The peak in the distribution decreases to 0.55 μm on the garnet surface (Figure 20b), while the shape value remains similar at 1.51. The peak in major axis distribution on the plagioclase surfaces are slightly less at 0.55 μm and 0.61 μm, respectively. The smallest debris found on the surfaces of the 1,000X-magnification images are approximately 0.30 μm. The physical meaning of these results and highlights from the other experimental results are now discussed further.

6. Discussion

Experimental results indicate that the mass of fragments generated as a result of impact under these loading conditions asymptotically increases as the impact energy is increased. The White material produces the least amount of fragment mass, and the Red material produces the most. This is associated with the larger percentage composition of fracture-prone K-feldspar and plagioclase minerals in the Red material, followed by the Black and then White materials. The asymptotical increase to a finite mass, or volume, of fragments is a result of the use of a steel backing. This enhances crushing/comminution-type loading on the rock, as the other experimental results would indicate. The total number of fragments $\geq$10 μm generated from impact increases substantially as the impact energy is increased. The fragment estimate is as high as 75,000. Estimates of the total number fragments would substantially increase if those $<$10 μm were included.

The peak in the probability distributions of sizes, including minor and major axes and area, decreases and the distribution becomes more narrow-band as the impact energy is increased. Shape results remain consistent over the impact energies studied, with mean values of 1.60 for Red, 1.53 for Black, and 1.48

for White. These results indicate that more dense minerals, or those with higher quartz content, have larger dominant fragment sizes and smaller shapes (smaller aspect ratios) under the impact conditions used here. The size results are bounded above by the Levy and Molinari model [12] prediction, and below by the Glenn and Chudnovksy equation [31], indicating that these models predict fragment size adequately for engineering purposes.

Values of the correlation length scales, fractal dimension, and ratio between λ_c and λ_e were examined in order to gain insight into the dynamic behaviour of the granite materials. The value of λ_e was on the order of the peak in the major axis probability density distribution for all cases. Values of λ_e, λ_c, and F_G decrease for an increase in impact energy. This might be expected since the probability density distribution becomes more narrowband as the impact energy is increased, resulting in a decrease of $\Delta\lambda$ and, by association, λ_e, λ_c, and F_G. The fractal dimension, D, was found to increase as the impact energy was increased. This is consistent with previous investigations [49, 50]. In addition, fractal dimensions near 2 suggest the fragmentation processes involved are primarily surface-related (e.g., comminution). This is consistent with the fragment mass data and the dominant fragment size results.

Physical mechanisms of fragmentation were investigated on fracture surfaces. Through-grain cracks in plagioclase, K-feldspar, and quartz were observed, suggesting that localized tensile stresses in the order of 2 GPa were reached [4, 53]. Bifurcation mechanisms were also examined and the results indicate that bifurcation occurs at quartz grain boundaries due to the relatively higher hardness of this mineral. In K-feldspar and plagioclase, bifurcation regions posses evidence of the realization of localized temperatures in the order of 830 K [5]. High temperatures were created by frictional melting via recovered heat from exceeding the elastic limit and vibration. The region encompassed by the melt was found to vary depending on the size of the fragment containing the fracture tip.

Large amounts of microscopic surface debris are a dominant feature of the fracture surfaces. The nature of the debris was examined at high magnification, which revealed dominant peaks in their distributions at sub-micron values between 0.55 μm and 0.69 μm. Without consideration of the fragment debris, the experimental results presented in Figure 16 indicate that the scale-invariant relationships, as defined by the theory presented by Grady [15], falters when applied to the fragments smaller than λ_e. If the distributions for the microscopic surface debris are extrapolated over the total surface area, as was done to determine the total number of fragments, then it is expected that these microscopic fragments will also represent the peak in the histogram distribution of fragment sizes. This is noteworthy since Grady [15] suggests that the exit length scale, λ_e, can be calculated as:

$$\lambda_e \simeq 3(K_c/\sigma_{hel})^2 \tag{18}$$

where K_c is the fracture toughness of the material (Pa$\sqrt{m}$) and σ_{hel} is its Hugoninot elastic limit (Pa). The Hugoniot elastic limit is the limit of elastic deformation that ceramics can endure before deforming plastically or brittly under dynamic loading. It is defined as:

$$\sigma_{hel} = \frac{1-\nu}{1-2\nu} Y_{yield} \tag{19}$$

where ν is the Poisson ratio and Y_{yield} is the yield stress (Pa). If values of ν=0.11 [5], Y_{yield}=2 GPa [5], and K_c=1 MPa m$^{1/2}$ [5] are used then λ_e is estimated as 0.58 μm

for K-feldspar. This is in the order of the peaks in Table 4, suggesting Grady's [15] estimation of λ_e is plausible.

Our results suggest that the scale-invariant relationship falters when applied to the fragments smaller than $\lambda_e \simeq 0.60\ \mu m$. This is the limit at which coherent, or dominant, fragments are generated from the constitutive materials under these loading conditions. The smallest fragments found on the surfaces were approximately 0.30 μm. This lower limit is referred to as the comminution or grinding limit [5]. Comminution can occur at strain rates as low as 10^{-7} s^{-1} (e.g., low creeping faults to as large as 10^{7} s^{-1} (e.g., seismogenic and impact-related events) [5]. Comminution is controlled by the mechanical properties of the materials being crushed, such as yield and shear strength, fracture toughness, and thermal conductivity [5]. Comminution has been explored by numerous researchers and results indicate that the fragment size limit is sub-micron [56–61]. There have been several explanations for this limit:

1. At a critical size, the smaller particles absorb the compressive and shear forces through cumulative elasticity and relative movement (bulk yield). This limits further fracturing.
2. Below a critical size, cracks cannot initiate or grow under compression, and particles yield plastically instead of fracturing (i.e., the shear strength is grain-size dependent below the critical size).
3. The smaller sizes undergo agglomeration to produce larger clusters, which define an apparent grinding limit [56].
4. Fragments smaller than a critical size are melted as a result of the high strain rates and associated adiabatic conditions [57].

Although not the subject of this paper, results from these experiments indicate comminution plays an important role in the fragmentation and energy dissipation processes; even at very low loading energies. Together, the results presented here indicate that propagating cracks dominate the fragmentation process at mm-size scales, while at micron-grain scales the fragmentation process is dominated by comminution initiated by the kinetics and abrasion between adjacent grain surfaces. This is especially true for the K-feldspar and plagioclase-family minerals, which are much more susceptible to fracture than quartz. Results from Table 4 indicate that comminution processes generate fragments as small as 0.28 μm. Several theories attempt to explain this limit [56–61].

7. Conclusions

Statistical analysis of the fragments generated from the dynamic fragmentation of granite at impact energies of 6 to 28 J indicate larger dominant fragment sizes and smaller shapes (smaller aspect ratios) for the granites with higher quartz content. Scanning electron microscopy revealed evidence of localized tensile stresses and temperatures in the order of at least 2 GPa and 830 K were obtained as a result of the impact. Sub-micron fragments on the fracture surfaces were examined and peaks in the probability distributions were found to be approximately 0.60 μm. This is the limit at which coherent fragments are generated under these loading conditions. The smallest fragments found on the surfaces were approximately 0.30 μm. This is referred to as the comminution limit.

The results presented in this paper offer insight into the catastrophic dynamic fragmentation of rock under low-energy impact and provide useful guidelines for those numerically modelling the fragmentation of rock. Typically, fracture events are modelled from a continuum perspective, and micro-scale aspects of the fragmentation process are not considered due to the computational costs associated with the requirement of having elements less than 100 nm in size in order to fully capture the cracking, comminution and heat generation processes at these fine scales. In conclusion, a more detailed evaluation of fracture mechanisms and their contribution to the evolution of fracture will ultimately lead to more efficient use of brittle materials in engineering applications.

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Table 1: Extrapolating total number of fragments and their estimated mass for Red material at 5.9 J.

Group Type	# Fragments	Mass Estimate (g)	Major Axis (μm)	Shape		
Smallest	2,276	0.08	162	1.45		
Small	1,876	0.08	200	1.45		
Medium	1,375	0.32	501	1.62	Actual	Extrapolated
Large	58	0.40	1,047	1.58	Mass (g)	#
Total	(A) 5,585	(B) 0.88			(C) 1.68	(AxC/B) 10,662

Nomenclature

a	Scaling parameter for the Levy and Molinari [12] prediction of fragment size
$a_{material}$	Material scaling coefficient in power-law function curve-fit
c	Longitudinal wave speed, m/s
C	Proportionality constant
D	Fractal dimension
E	Young's modulus, Pa
$f(\zeta)$	Probability distribution of fragment sizes
$F(r)$	Cumulative distribution of the fragment sizes
F_G	λ_c/λ_e
$G(r)$	Mapping function
G_c	Fracture energy, J/m^2
h	Thickness of target, m
KE	Kinetic energy, J
K_c	Fracture toughness, Pa$\sqrt{m}$
$n_{material}$	Material scaling coefficient in power-law function curve-fit

Table 2: Total fragment number estimates for each material type and impact energy.

Material Type	Impact Energy (J)	# of fragments	Major Axis (μm)	Minor Axis (μm)	Area (μm^2)	Shape	ξ	σ
Red	5.9	10,662	162	145	16,898	1.45	-0.186	0.323
Red	11.0	17,137	155	132	8,511	1.62	-0.177	0.333
Red	15.0	24,162	141	113	5,495	1.59	-0.165	0.298
Red	19.4	35,389	120	100	6,918	1.66	-0.125	0.302
Red	28.0	59,786	115	93	6,166	1.70	-0.130	0.260
Black	5.9	4,199	195	141	18,620	1.54	-0.167	0.268
Black	11.0	16,095	182	102	10,470	1.51	-0.137	0.217
Black	15.0	25,980	151	89	8,913	1.51	-0.129	0.210
Black	19.4	37,520	126	71	10,000	1.55	-0.142	0.209
Black	28.0	74,895	105	65	5,248	1.55	-0.090	0.171
White	5.9	10,537	209	170	16,980	1.47	-0.204	0.325
White	11.0	15,566	204	129	15,850	1.48	-0.150	0.289
White	15.0	28,168	156	105	10,230	1.45	-0.099	0.247
White	19.4	41,006	132	126	9,120	1.51	-0.097	0.230
White	28.0	65,717	117	100	5,012	1.45	-0.078	0.225

Table 3: Values of λ_e, λ_c, $\Delta\lambda$, D, and F_G.

Material Type	Impact Energy (J)	λ_e (μm)	λ_c (μm)	$\Delta\lambda$ (μm)	D	F_G (λ_c/λ_e)
Red	5.9	177	7,521	7,344	1.74	42
Red	11.0	165	7,650	7,485	1.66	46
Red	15.0	168	4,510	4,342	1.92	27
Red	19.4	117	2,751	2,634	1.99	24
Red	28.0	105	2,475	2,370	2.17	24
Black	5.9	210	8,288	8,078	1.66	39
Black	11.0	194	7,425	7,231	1.89	38
Black	15.0	162	4,321	4,159	2.04	27
Black	19.4	124	2,651	2,527	2.00	21
Black	28.0	110	2,575	2,465	2.05	23
White	5.9	222	7,430	7,208	1.69	33
White	11.0	205	7,567	7,362	1.78	37
White	19.4	183	4,640	4,457	1.97	25
White	19.4	180	2,751	2,571	2.28	15
White	28.0	154	2,475	2,321	2.32	16

Table 4: Major axis and shape distributions for debris on the fracture surfaces. The values in brackets are the $\log_{10}$ of the corresponding peak values.

Image	Major Axis (μm)	Shape	Smallest Size (μm)
(a) K-feldspar	1.36 (0.134)	1.49 (0.173)	0.91 (-0.039)
(b) Garnet	0.69 (-0.164)	1.51 (0.180)	0.34 (-0.463)
(c) Plagioclase	0.55 (-0.267)	1.60 (0.205)	0.33 (-0.487)
(d) Plagioclase	0.61 (-0.211)	1.62 (0.211)	0.28 (-0.560)

r	Fragment size, m
R^2	Coefficient of determination
s_{GC}	Average fragment size according to Glenn and Chudnovksy [31], m
s_{Grady}	Average fragment size according to Grady [15], m
s_{Zhou}	Average fragment size according to Zhou [33, 39], m
s_{LM}	Average fragment size according to Levy and Molinari [12], m
s_{GC}	Average fragment size according to Glenn and Chudnovksy [31], m
t_0	Characteristic time defined by Zhou [33, 39], s
v	Impact velocity, m/s
Y_{yield}	Yield stress, Pa
$\alpha_{material}$	Material scaling coefficient in logarithmic function curve-fit
$\beta_{material}$	Material scaling coefficient in logarithmic function curve-fit
$\Delta\lambda$	λ_c-λ_e
$\dot{\epsilon}$	Strain rate, s^{-1}
λ_e	Exit correlation length according to Grady's [15] nonequilibrium fragmentation theory, m
λ_c	Entry correlation length according to Grady's [15] nonequilibrium fragmentation theory, m
μ	Shift parameter in generalized extreme value distribution curve-fit
μ_{init}	Average strength for the Levy and Molinari [12] prediction of fragment size, Pa

ν	Poisson ratio
ρ	Material density, kg/m^3
σ	Scale parameter in generalized extreme value distribution curve-fit
σ_c	Compressive strength of the material before failure, Pa
$\sigma_{c,min}$	Strength of the weakest link in a probability distribution of defects [12], Pa
σ_{hel}	Hugoniot elastic limit, Pa
ξ	Shape parameter in generalized extreme value distribution curve-fit