Small Worlds, Mathematics, and Humanities Computing

by

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Abstract

Primarily the conversation surrounding humanities computing has been mainly focused on defining the relationship between humanities computing and conventional humanities, while the relationship humanities computing has to computers, and by extension mathematics, has been mainly ignored. The subtle effect computers have on humanist research has not been ignored, but the humanities general illiteracy surrounding computers and technology acts as a barrier that prevents a deeper understanding on these effects. This goal of this thesis is to begin a conversation about the ideas, epistemologies, and philosophies surround computers, mathematics, and computation in order to translate these ideas into their humanist counterparts.

This thesis explores mathematical incompleteness, mathematical infinity, and mathematical computation in order to draw parallels between these concepts and similar concepts in the humanities: post-modernism, the romantic sublime and human experience. By drawing these parallels this thesis both provides a general overview of the ideas in mathematics relevant to humanities computing in order to assist digital humanists in correctly translating or interpreting the effects of computers on their own work and a counter argument to the commonly accepted notion that the concepts developed by mathematics are mutually exclusive to those developed in the humanities.

Contents

List of Figures

Chapter 1: Introduction

> I have adopted 'humanities computing' in particular for certain suggestive qualities in the name: a potential still to be taken as an oxymoron, thus raising the question of what the two activities it identifies have to do with each other; the primacy it gives to the 'humanities', preserved as a noun in first position while functioning as an adjective, hence subordinated; and its terse, Anglo-Saxon yoking of Latinate words. I read it first as a challenge to what we think we are doing, then as its name. (McCarty 2)

I was attracted to 'humanities computing' because of the word 'computing'; however, to say that humanities computing is not as I expected it to be is a complete understatement, and even looking back I still do not fully understand what it was that I was expecting. I completed my undergraduate in Mathematics and Physics, and yet I found my interests pulling me towards computers and writing. To the science student, arts credits are sometimes a pleasant distraction, sometimes a university mandated-detour, and always secondary in importance to science credits. As an undergraduate, I was well aware of the division between science and art. There existed two different universes on campus, the science students and the arts students, both of whom had their own culture, their own buildings, their own dress code, and their own views on the world. Rarely, if ever, do these two pockets of reality intersect. My choice to pursue a Master of Arts was a choice of rebellion. As I progressed through my undergraduate career, certain cracks formed in the way I understood the concepts of pure and holistic science. I found the vocal rationalist movement, which was well represented in my class, to be hypocritical in their blanket dismissal of so-called 'irrational' subjects like religion and literature. I had reached a point where I wanted to study something new: to see the world from a different perspective.

I am a scientist and a mathematician; rational thinking is what I am, and it will always be what I am best at. Likewise, I have always been interested in models of the universe: explaining and understanding phenomenon as individual parts of a common whole. As a science student, I had, and to some extend still have, only a limited understanding of what it means to work in the humanities, which is why I chose to pursue a Master of Arts. To me the first step to understanding humanity as different parts of a single whole required me to understand both sides of the science / humanities rift. I like computers, I understand computers, and I have spent the better part of my life studying the philosophy behind them. Here was an opportunity to take that knowledge of computers and use it in a way different from how I was taught, offer that skill to another group who needed it, and perhaps learn something about them in the process. I thought humanities computing represented an opportunity to look at computers from a perspective fundamentally unscientific. Unfortunately, I was only partially correct.

'Humanities computing' and the newer 'digital humanities' (DH) are difficult terms to define partly because they originated out of a change in methodology instead of around a specific topic of study, like numbers in mathematics. Humanities computing claims to originate out of the work of Father Roberto Busa, an Italian priest, who teamed up with IBM in 1949 to create a computational concordance of the works of Thomas Aquinas. In this way, humanities computing began when the first humanities computing project began, and the two are nearly inseparable. Likewise, much of humanities computing exists pragmatically, it is a thing that humanists do. John Unsworth describes humanities computing as such: "So, one of the many things you can do with computers is something that I would call humanities computing, in which the computer is used as a tool for modeling humanities data and our understanding of it" (37). Unfortunately, this definition only opens the door for other more subtle questions. Is humanities computing an academic discipline separate from normal humanities? To what degree does a humanist project need to be digital before it becomes a humanities computing project? Indeed Geoffrey Rockwell wrote a 1999 paper exploring the first question, and the second has spawned an entire genre of opinion papers. However, accepting for a moment that the first question is solved and that humanities computing is indeed separate from conventional humanities, the second question becomes vitally important. Where is the line between the humanities and humanities computing?

"When many of us hear the term digital humanities today, we take the referent to be not the specific subfield that grew out of humanities computing but rather the changes that digital technologies are producing across the many fields of humanist inquiry" (Fitzpatrick 13). As society increasingly relies on digital and computerized technologies non-digital humanists will inevitably encounter computers during their normal routine and the cut and dried interpretation of 'humanities computing' as something a scholar does with a computer becomes much murkier. If a humanist checks her email on a computer, then does that make her a digital humanist? In his aptly titled three minute presentation "Who's In and Who's Out", Stephen Ramsay declares, with glee, that "I think Digital Humanities is about building things" and "if you aren't building you are not engaged in the 'methodologization' of the humanities, which, to me, is the hallmark of the discipline that was already decades old when I came to it" (241). Mark Sample counters in his equally well titled "The Digital Humanities is not about Building, it's about Sharing" where he argues that, "The heart of the digital humanities is not the production of knowledge; it's the reproduction of knowledge" (256). Lincoln Mullen takes the middle ground by claiming that, "Digital humanities is a spectrum… Working as a digital humanist is *not* one side of a binary, the other side of which is working as a traditional scholar" (237). Indeed there seem to be as many definitions of digital humanities as there are digital humanists; however, there are strong central themes, and there are two I would like to bring attention to. The first is the definition of humanities computing as a tool discussed above, and the second is humanities computing as a revelation of thought and sharing, "For the digital humanities, information is not a commodity to be controlled but a social good to be shared and reused" (Spiro 22).

How much of a digital humanities project must be digital is a hotly contested issue, but the underlying tone of the discussion is that the ideas and concepts inside of humanities computing must be absolutely humanist. Lisa Spiro reinforces this point when she points out that: "The values of the digital humanities represent a convergence of several sets of values, including those of the humanities; libraries, museums, and cultural heritage organizations; and the networked culture (19)". The quote is interesting to me not because of what it contains but because of what it leaves out. Conspicuously, the one thing missing from this discussion is the computer itself. This seems strange to me because the computer is the only thing separating humanities computing from the conventional humanities. Let me rephrase the question of degree by putting it thusly: To what degree must a humanist concept be computational before it becomes a concept of humanities computing? This question is uncomfortable because it no longer assumes that concepts in humanities computing are purely humanities concepts, and further it is ambiguous in this field because humanists in general have a poor reputation for understanding 'computational ideas'. "We're illiterate. Besides myself, no one on the CI board can use any of the languages we need to understand to operate with our proliferating digital technologies – not even elementary markup languages" (McGann 53).

In the epigraph Willard McCarty tries to describe the term 'humanities computing' as balancing its humanities and computing counterparts; however, more and more it seems that the emphasis is getting pulled away from computers. Kathleen Fitzpatrick claims that the term "Digital humanities thus grew specifically out of an attempt to make 'humanities computing,' which sounded as though the emphasis lay on the technology, more palatable to humanists in general." Implying that even the term switch from "humanities computing" to "digital humanities" is a move intended to subordinate computation. This becomes especially revealing of the culture as a whole when paired with Katherine Hayles's observation that, "'digital humanities' was meant to signal that the field had emerged from the low-prestige status of a support service into a genuinely intellectual endeavor with its own professional practices, rigorous standards, and exciting theoretical explorations" (43). The intellectual rigor in humanities computing comes from the word 'humanities' and 'computing' is just a tool that helps us get there. In short, computers are a thing we interact with and computing is a thing we do; computers, in the world of humanities computing, are verbs. Unfortunately, the belief that the concepts of humanities computing originate exclusively from the humanities replaces my question of degree with its pragmatic counterpart: Should digital humanist learn to code? This emphasizes the fact that 'computers' in digital humanities are nothing more than a tool.

I wish to ask a related question. What role do computers have in humanities computing? The fact that computers are actively changing our society is extremely hard to dispute, and in at least a few cases the promise of change is what is driving the second definition of humanities computing I pointed out earlier. "The promise of the digital is not in the way it allows us to ask new questions because of digital tools or because of new methodologies made possible by those tools. The promise is in the way the digital reshapes the representation, sharing, and discussion of knowledge" (Sample 255). However, by focusing all of our attention on the relationship between the humanities and humanities computing we lose sight of the equally important relationship between humanities computing and computers. Inserting computers into the humanities is not a neutral act and will change via translation the ideas and concepts that originated out of the humanities; this is not going unnoticed, and in order to resist this pressure to change some choose to push back against the influence of computers. Johanna Drucker is one such scholar:

> Humanists, and the values they embrace and enact, must counter with conceptual tools that demonstrate humanities principles in their operation, execution, and display. The digital humanities can no longer afford to take its tools and methods from disciplines whose fundamental epistemological assumptions are at odds with humanistic method (6).

To other scholars such as Willard McCarty see value in carefully selecting certain principles for adoption into humanities computing. “my initial project, however, will be to deconstruct the unitary notion of computer science so that, with false unities gone, we can ask what aspects of the rich plurality of concern might be relevant to the humanities, and so to humanities computing” (159). Once again, we arrive at a question of degree. To what degree should humanist concepts mix with computational concepts in order to arrive at digital concepts?

In between these two positions is a continuum of possibility that contains various combinations of humanist and computational ideals. From the perspective of degree, I agree with both sides of the argument. Thoughtlessly adopting new methodologies is dangerous because such methodologies insert new and poorly understood ideas into the study as a whole; however, new ideas are always necessary in both the humanities and the sciences in order to sustain continued scholarship. Unfortunately, from this perspective computation is losing because of the definition of humanities computing as dealing solely with humanist concepts. However, this definition is flat out wrong. It is impossible to introduce computers without also introducing the ideas they represent. Johanna Drucker points out that this is exactly what happens when visualizations try to emulate humanist principles: “But the graphical presentation of supposedly self-evident information… conceals these complexities, and the interpretive factors that bring the numeric into being, under a guise of graphical legibility”. Computers interact with computational models. A paper book is not the same as an ebook, and understanding the ebook requires at least a marginal understanding of the object that makes ebooks possible. Unfortunately, translating the print book into an ebook necessarily requires some loss of information as not all aspects of the print book are compatible with its ebook counterpart. The same is true of the philosophies behind both objects. Everything inside of a computer is digital, i.e. constructed out of tiny pieces of indivisible objects, and much of the humanities deal with grey areas that cannot be perfectly translated into such objects. Thus we have arrived back at the problem of degree. Moving to far towards the philosophy of computers risks compromising humanist ideals in favour of computation rigidity; however, sticking to closely to conventional humanism runs the risk of creating a humanities computing indistinguishable from its conventional counterpart.

I do not wish to enter the discussion of degree because silently it is assuming that computational truth and humanist ideals are mutually exclusive. The unwritten assumption here is that every time a project makes space for computational truth it does so by rejecting some humanist ideal. The question of degree is then how much is the humanist willing to give up in order to gain the benefits of computation. I wish to ask a different question which I believe more closely associates with Mark Samples definition of humanities computing as the reproduction of knowledge: Is there a space where both computational truth and humanist ideals can co-exist? This question comes out of my mathematical background. In that field the existence of such spaces between ideas is more important than any question of degree. If we can show that such a space does not exist, then we can know that Drucker is correct; humanist ideals must be protected from computational truth in order to maintain its independence of the sciences; therefore, degree becomes vitally important. However, if such a space does exist then perhaps Willard McCarty is right and we should be incorporating more computational ideas into humanities computing in order to fully explore it. The first step to answering this question is to begin learning about the ideas behind computation.

I will not be entering the discussion of whether or not humanists should learn to code, as a 'programmer' in the context of the current definition of humanities computing is a pragmatic problem and the skill should be distributed according to available work. However, I am about to argue that everyone working in the digital humanities needs to understand at least some of the cultural and philosophical background that makes code possible. Similar to learning a language, there are differing levels of fluency and the ability to construct coherent sentences in programming code is not the same as understanding the cultural background of that code. Not everyone working in the humanities needs to be able to write large books, but everyone needs to be able to read them. The same is true of computer code; knowledge about the internal structure of computer code is necessary to correctly interpret the digital texts that that code produces. Numerical literacy is more than the ability to convert ideas into functional programs. Numerical literacy is about understanding the epistemological assumptions that computers need to make in order to convert data into visualizations: it is about building theoretical models that take advantage of the strengths of computers, and it is about seeing and understanding the whole project as more than just the sum of its separable parts. Numerical literacy is not just about correctly employing scientific methods, but also about interpreting those same methods and understanding why the community adopted them in the first place.

Like it or not, every time a project even touches a computer it interacts with mathematical models. Whenever text is reduced to data, whenever data is visualized, and whenever that visualization is sent via HTML to a user's computer, the computer's internal processor is operating on mathematical objects first and human-readable information second. Neither pleasing colors nor curvy lines will change the fact that visualizations represent numbers before they represent text. Topic modeling is statistics, and any full understanding of topic modeling necessarily requires a basic framework of statistics. All networking and communications platforms employ some form of database structure, which converts human interaction into predefined logical symbols. All video games, even those with no interactive elements, are virtual mathematical worlds constructed using mathematically defined causal relationships. Programs are written in programming languages, and all programming languages are dialects of mathematics. An understanding of mathematics is paramount if computing researchers ever want to view code, output, or data as anything more than just a means to an end.

From my perspective, numbers are more than just data, they are a text in their own right; the same goes for computer code. If humanities computing is a challenge to the humanists to start thinking about their topic in new and interesting ways, then I see no reason why it cannot also be a challenge to scientists to do the same. At least part of this project requires both sides to stop thinking about computers in terms of tools and instead to look at them as topics of research in their own right. Doing so will not only offer everyone a new perspective on these machines that have become so pervasive in modern society, but will also offer those who only need computers for their practical benefit a way to correctly interpret their contribution.

The goal of my thesis is twofold and changes depending on which definition of humanities computing my reader accepts. In both cases my main goal is to translate, as best as I can, the ideas and epistemologies that lie at the foundation of computers in such a way that those with no mathematical background can begin to understand them. To the person whose belief in tool oriented humanities computing is unshakable I offer this translation as a practical tool. If humanist ideals need to be protected from computational truth then the first step in constructing such a barrier is understanding that which is being defended against. I leave the adoption of these ideas and their transformation into the question of degree to someone else. However, buried in this translation is another argument; this time for a question. What exists at the intersection of humanities and computing and why cannot humanities computing occupy this space? By translating these ideas I intend to offer insight into the unexplored intersection of modern mathematics with modern humanism. While this exploration is in no sense complete, my hope is that what I can show will be enough to justify further scholarship. I will do so in four chapters.

In the first chapter, I will begin by attempting to define mathematics and in doing so explain the differences between its pure and applied forms as well as point out how these subjects relate to the humanities. I will define 'proof' and 'objectivity' as they present themselves inside of mathematical discourse. Finally, I will talk about mathematical fundamentalism -- also known as positivism -- why it is separate from mathematics, why humanists are correct in their fear of it, and why it represents a dangerous and self-defeating definition of the subject as a whole.

In the second chapter, I will describe in much more detail the evolving philosophy of mathematics in order to set the stage for the invention of the computer. I will follow a brief history of mathematical thought and will demonstrate how mathematics has transitioned from its ancient to modern forms, and I will describe the crisis that created today's version of mathematics. I will attack the humanist fear of mathematical reductionism by demonstrating that the mathematical project tends to make its largest leaps only when such reductions fail. Likewise, I will demonstrate why the humanist's fear of mathematical worlds as a simplifying monster is incorrect, and why the adoption of mathematical principles is not a rejection of humanist ideals.

In the third chapter, I will discuss the mathematical concept of infinity and how it relates to the humanistic concept of the Sublime, popular especially in the Enlightenment and Romantic periods. I will attack the fundamentalist notion of a universal mathematics by demonstrating that infinity is not an invitation to all knowledge, but instead an important limitation of it. I will talk about mathematical limitations and how mathematicians choose to transcend and to work on top of those limitations. I will talk about mathematical failure and how this failure represents an attempt to understand something indescribable: the Sublime. Finally, I want to completely demonstrate that infinity is definitively less than 'everything'.

Finally, in the fourth chapter I will introduce the computer and describe how the computer is a child of the post-modern mathematics discussed in the previous three chapters and not the static modernistic mathematics that still resides in popular discussion. I will talk about today's mathematical universal models and the assumptions that construct them in order to demonstrate that science still has plenty to learn both about the universe and about itself. Finally, I will end with a discussion of Alan Turing's interpretation of the 'human' through his knowledge in computation, why this interpretation presents an important challenge to humanities computing, and why it remains unanswered.

In the end, my goal is not to convince you, the reader, that computers are the only way forward or that the methodologies of mathematics and the numerical sciences should be adopted wholeheartedly. If the digital humanities is to remain only a field where humanists use computers to further humanist goals then so be it. However, I personally see this as a missed opportunity. The goals and objectives of both humanism and science are not mutually contradictory and they do not need to continue to diverge. In my translation I hope to paint a picture of a new space that humanities computing could possibly occupy. My hope for this thesis is to begin a conversation discussing not what computers have to offer the humanities, but what humanities computing has to offer both the sciences and the humanities equally.

> We are by now well into a phase of civilization when the terrain to be mapped, explored, and annexed is information space, and what's mapped is not continents, regions, or acres but disciplines, ontologies, and concepts. We need representations in order to navigate this world, and those representations need to be computable, because the computer mediates our access to this world, and those representations need to be produced at first-hand, by someone who knows the terrain. If, where the humanists should be represented, we in the humanities scrawl, or allow others to scrawl, 'here be dragons,' then we will have failed. (Unsworth 46)

Chapter 2: Seeing the Mathematical Perspective

My first degree was in mathematics. I am well aware of the typical reaction to that statement. At first, it is one of admiration, 'Wow, you must be smart!' followed by an admission of inadequacy, 'I don't understand it myself', concluded with the assumed but never explicitly stated, 'and I have no reason to ever learn anyway'. It is not as if I cannot relate. I remember almost failing Math 10 in high school. I too wondered in class, but never aloud, how manipulating conic sections could possibly benefit my life. I begrudgingly did my homework and, like a good little boy, worked through the vast majority of my assignments ten minutes before they were due. I absolutely understand the soul-crushing feeling of gazing upon a full page of random equations not having a clue what to do with them; frankly, I still get that feeling when facing down a full page of French philosophy. I have spent days, even weeks, working on large complex problems only to spend an even longer period of time trying to figure out what my solution actually meant, or if it even counted as a solution at all. Yet, even after swearing off this demonic subject shortly after graduating from high school, I still found myself graduating with a major in that very subject six years later.

My introduction to real mathematics came through Micheio Kaku's book, *Hyperspace: A Scientific Odyssey Through Parallel Universes, Time Warps, and the 10th Dimension*. He intended the book to be an introduction to the relatively new physical theory of superstrings, as well as provide a brief history of the ideas that led to its creation. I was in junior high school at the time and thoroughly unfamiliar with all of the many technical details the later portions of the book referenced; however, lack of understanding is not lack of absorption, and many of the core ideas stuck around and formed the foundation of what would eventually become this thesis. In the introduction, Kaku describes his own project as follows:

> This book is about a scientific revolution created by the theory of hyperspace, which states that dimensions exist beyond the commonly accepted four of space and time. Consequently, many physicists are now convinced that a conventional four-dimensional theory is "too small" to describe adequately the forces that describe our universe. In a four-dimensional theory, physicists have to squeeze together the forces of nature in a clumsy, unnatural fashion. Furthermore, this hybrid theory is incorrect. When expressed in dimensions beyond four, however, we have "enough room" to explain the fundamental forces in an elegant, self-contained fashion. (viii ix)

Unification is an important issue in physics because it assumes that all objects are subject to the same set of rules. Outside of physics unification is still important, but its importance varies depending on what is being studied. Each scientist studies different parts of the larger world. One might study astronomy, another biology, and yet another society. In each field, the community of scientists works together to form a framework of study, rules, guidelines, and common facts that allow each group to transmit and interpret data similarly, much like a language. However, these groups create their rules with their own research questions in mind. Sometimes certain groups share rules, but much more frequently, they are entirely independent. Truth discovered in astronomy does not usually contradict truth discovered in biology; however, it does not aid in knowledge creation either. The understanding of the equations that govern planetary motion is generally not useful when trying to understand the digestive system of African monkeys. However, in less than optimal situations academic frameworks can outright contradict each other. Unfortunately, humanities computing has placed itself directly at the center of one such disagreement.

The contradiction appears when we contrast the chosen perspectives of the hard sciences, or the so-called mathematical sciences, and hermeneutics, a subset of the liberal arts and humanities. The hard sciences see the universe as having one singular correct and unchanging perspective from which truth about the universe flows. This perspective transcends the human perspective, but is accessible through rational thinking and experimental demonstration. Truth from this perspective is absolute; correct and incorrect explanations exist, and it is the duty of the scientist to experimentally separate one from the other.

This perspective serves as a loose definition of 'objectivity' because it seeks truth that exists independently from humanity. Contrarily, the hermeneutic tradition views truth as a derivative of a fluid context. There is no correct reading of any text, historical event, or ethical dilemma. Instead, these concepts can be read from several different perspectives, each producing its own interpretation. Truth does not flow from a single objective source, but from the collective human experience. Truth is a collage made up of individual readings that collectively create a world bigger than its individual parts.

This perspective counts as a loose definition of 'subjectivity' because it seeks truth that depends on human experience. This difference between these two perspectives has created a war between fundamentalists on both sides. In the hard sciences terms like 'subjectivity' and 'social construction' are derogatory phrases used to argue that a point is either uninteresting, or useless. Likewise, it has become a cliché to talk about the humanists' dismissal of technology, and the generalized opinion that computers and their singular perspective represent a cold, heartless, and emotionless reality.

Kaku was interested in similar contradictions that exist within the realm of mathematical physics. He references at least two competing theories of the universe: the standard model of quantum mechanics and Einstein's theory of relativity. The standard model, or 'wood' as Kaku referred to it, sees matter as the fundamental element that makes up everything we see and experience, while relativity, or 'marble' as Kaku referred to it, sees geometry as fulfilling the same role. Both theories correctly predict universal phenomenon when limited to the scope that created them: relativity for large objects, standard model for small; however, both theories describe mutually exclusive universes that do not together describe a unified universe. Kaku argued that the reason we as humans see two competing models of physics is because of the limited perspective through which we as humans experience the world. We are physically capable of experiencing only three spatial dimensions, and because of this, we like to impose these same restrictions onto our ideas. Kaku argues that when we take on the objective perspective incorporating ten dimensions we begin to see the universe not as a collection of mutually exclusive theories, but as a single unified world. However, ten dimensions is not something we as humans are physically capable of experiencing. To solve this problem we need to 'see' through the lens of something that is inhuman: mathematics. Kaku makes this argument by following the history of thought behind physics and unification theorems, and uses that discussion to construct what he presents as the answer to these problems: superstrings.

Superstrings are a "twenty-first-century physics that was discovered accidentally in our century" (Kaku 171). To Kaku, superstrings represent a new perspective independent of relativity and the standard model that simultaneously makes both true while also resolving the supposed contradictions that appeared between them. At the time of my initial reading, I did not understand superstrings, the problems they were supposed to solve, or the mathematical perspective that can see them. Likewise, this thesis is not about superstrings or the mathematics behind them. Instead, I offer this explanation of superstrings only to present two extended metaphors that will help in understanding the remainder of my thesis.

The first is that there might exist extra dimensions: places that I personally cannot experience or understand, but which still have a profound impact on the nature of the world around me. The second is that space and understanding are metaphorically linked. The more ideas I look at and the broader the perspective I look at them from, the more likely I am to notice connections between these ideas that I would not have seen while exploring only one. From the perspective of the Digital Humanities, these two principles take on a less cosmic, but still important, form. Firstly the knowledge gained through computerized modeling of text is not generated solely by the text itself, but also the computer, the mathematical models behind computation, and very possibly other factors that nether the sciences nor the humanities have words for. Secondly, a full understanding of any computerized result requires one to look at that result from the broadest perspective possible: understanding each part of the process individually, while also seeing all of them as contributing to a single holistic methodology. Unfortunately, understanding superstrings as the unifying language of physics requires one to understand mathematics as the language of superstrings. I would never return to superstrings because understanding mathematics and computation turned out to be a lot more involved then I first expected. However, while Kaku only intended his argument to unify competing theories in mathematical physics I believe that a similar conceptualization can be useful in bringing together even more diverse and contradictory frameworks such as the rift that separates science and the humanities.

Similar to Kaku, I believe the resolution to this problem is a tenth dimension; I want to look at both definitions 'humanities' and 'computing' from a perspective that will reveal both for what they are. I choose mathematics as this perspective not because I believe mathematics holds the secrets of the universe, as some would argue, but because it rests at the center of this controversy. Humanities computing is the humanities' attempt at incorporating computing into the humanities, while mathematical physics is the sciences' attempt at incorporating math into the sciences. As will become clear, math and computers in this context are nearly the same thing. The first step to solving this controversy is then to discover exactly what mathematics is, and more importantly, what it has to say about perspective.

Pure Mathematics

While preparing for this paper, a colleague of mine working towards his PhD in statistics asked my opinion on a simple, yet profoundly import, question: "Is formal logic true?" The answer to this question is deviously simple in answer, but extraordinarily difficult in explanation. No, formal logic itself cannot be true because it must exist prior to the truth that it creates. Logic defines truth and likewise there is no way of creating logical truth without some prior form of logic. Math, which is heavily based in formal logic, as a solitary unified entity cannot be thought of as being true or false because the study itself defines mathematical truth and therefore must exist before such concepts can even exist. Logic itself cannot justify mathematics, and all of the many philosophies behind the truth of mathematics are philosophies in the purest sense.

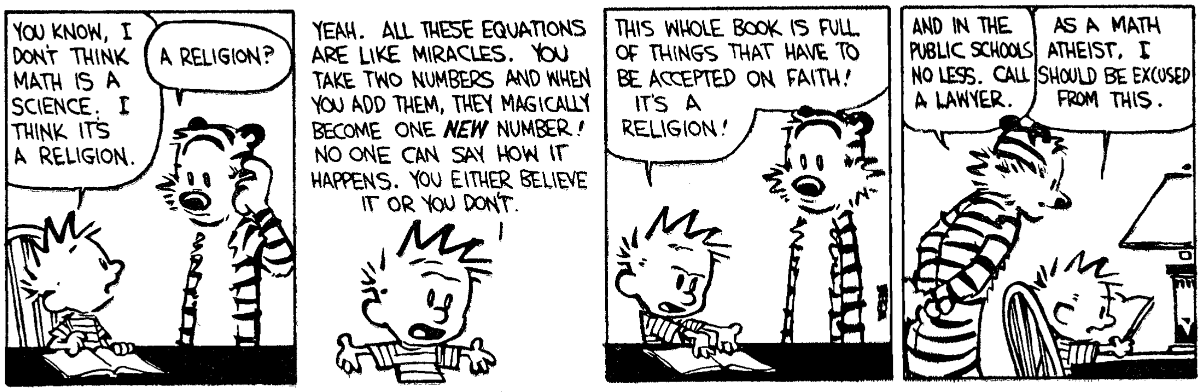
The philosophy behind mathematics does have the same kind of mass appeal that other philosophies generate. Unlike the philosophy of science, the philosophy of math does not feel it needs to spend as much time trying to justify its own existence. Part of this is because 'pure' mathematics is notorious in its unwillingness explain mathematical 'fact', and the other part is because mathematics is mostly interested in solving mathematical problems, and trying to justify why those problems exist is significantly less interesting. In the words of Brian Rotman:

> Where textual critics literize their metalanguage, mathematicians mathematize theirs. And since for mathematicians the principal activity is proving new theorems, what they will ask of any description of their subject is: Can it be the source of new mathematical material? Does it suggest new notational systems, definitions, assertions, proofs? Now it is certainly the case that the accounts offered by Frege [Intuitionism], Brouwer [Platonism], and Hilbert [Formalism] all satisfy this requirement: each put forward a program that engendered new mathematics; each acted in and wrote the play and, in doing so, gave a necessarily truncated and misleading account of mathematics. (2)

The three philosophies that Rotman mentions are the most popular interpretations of three basic perspectives on the philosophy of mathematics, all of which attempt to from a non-mathematical foundation that the rest of mathematics builds off of. Brouwer's mathematical platonism views mathematical objects as facts about the world, and sees mathematics as the science that experimentally discovers these facts. Frege's mathematical intuitionism views mathematical objects as a construction of the human mind. Mathematical objects in this sense are created through the physical act of constructing the objects and performing the proofs. Finally, Hilbert's mathematical formalism sees mathematics as a game. The human creates the rules that mathematical object play by. Mathematics does not exist outside of this game, and the objects themselves should not be seen as anything more than just symbols on a sheet of paper. As long as the rules do not contradict themselves, the formalist is happy. All three philosophies try to assert what mathematicians already know about mathematics: that math is at its core a discussion of mathematical objects. Since all three subjects try to construct mathematics using non-mathematical objects, proper mathematicians rarely need to deal with them. Likewise mathematical philosophy is easily separable from mathematics; it is laughably easy for a mathematics student to make it through an undergraduate carrier without taking a single course in mathematical philosophy (or even mathematical history). Mathematical philosophy can, for all practical purposes, be summed up in the single assumption: ‘mathematics exists’. Such an assumption immediately makes mathematical truth available for those interested in finding it. For my purposes, the interpretation I will be primarily interested in is Hilbert's formalism, which I will discuss in more depth in chapter two; however, the other two philosophies have a role to play as well. As Rotman pointed out above, truth in mathematics exists somewhere inside of all three of them.

Thinking back to another conversation I had with a guest lecturer following his presentation on the assumptions involved in statistical natural language processing, I remember that he described mathematics as a religion. It is an observation he probably borrowed from Bill Watterson's cartoon character Calvin who, while complaining about math homework, makes the same comparison (112).

Figure 1: Calvin and Hobbes



Similarly to how logic defines logical truth, mathematical universes define mathematical truth. Each philosophy creates a different mathematical universe, which defines both mathematical truth and objectivity. This directly mimics how mathematics itself is performed. Any higher level mathematical class typically begins when the teacher dictates a set of rules, or 'axioms', that are to be accepted as irrefutable facts for the remainder of the class. These axioms function as a definition of the topic they express. 'Set axioms' define what a mathematician is allowed to call a 'set', 'group axioms' define what a mathematician is allowed to call a 'group', and 'number axioms' define what a mathematician is allowed to call a 'number'. Mathematicians invent rules, conjuring them out of little more than the mysterious force known as 'mathematical intuition', and then spend the rest of their time arguing about the repercussions of these decisions. Pure mathematics cannot escape from philosophical skepticism and so it does not even try. Unfortunately, this means that mathematics has no foundation. Everything is derived from the layers below it, and the axiomatic layer at the bottom is accepted unconditionally. You either believe it, or you are not participating in mathematical discussion.

In its purest form, mathematical truth is completely divorced from physical truth because the assumed foundation that builds mathematics is not necessarily true in the real world. Math understands that it cannot cage the universe and observe it through a rigorous lens, so it does not try. A pure mathematician has no interest in what the physical universe has to offer because he is more interested in the fantasy worlds that math has created. Applied mathematics does exist, but it neither drives nor makes up the bulk of mathematical research. In the words of G.M. Hardy:

> I will only say that if a chess problem is, in the crude sense, 'useless', then that is equally true of most of the best mathematics; that very little of mathematics is useful practically, and that that little is comparatively dull. The 'seriousness' of a mathematical theorem lies, not in its practical consequences, which are usually negligible, but in the significance of the mathematical ideas which it connects. (16)

Mathematical truth is important only when it links other mathematical truth. In short, math makes itself true.

Central to the mathematical perspectives is the concept of *proof*. The Greeks viewed proof as an argument that definitively resisted doubt; an argument so persuasive that not even the gods could conjure up evidence to refute it. We can think of proof as an object whose existence definitively establishes an indisputable fact: an object whose presence ends, not continues, an argument. Mathematical proof comes in several varieties, all of which are valid in some mathematical systems and not in others depending on the axioms that construct them. Direct proof is the simplest of mathematical proofs and represents a narrative that carries the axioms of a mathematical argument, through a sequence of events, towards its natural conclusion.

Say I wanted to prove that a person, Daphne, is Alice's Great Grandmother. The direct way to prove this relationship is by systematically revealing each of the relationships that bridge them. I begin with an axiomatic basis that contains the following statements: Daphne exists, birth certificates exist, and a birth certificate defines lineage. Then I carry my audience through each of the important events that would lead to Alice's birth: Daphne begat Charlie, Charlie begat Bob, Bob begat Alice. I would verify each claim by presenting a birth certificate that independently verifies the lineage of each successive relationship. My audience might question the validity of any one of the axioms, but in the world where the axioms are all definitively true my conclusion is irrefutable.

My audience has no choice but to assume that my transformations are true because the assertion that they are is a part of the axiomatic basis of my argument. There are, of course, numerous philosophical objections to the above argument, but none of them are relevant in the mathematical sense because mathematicians always assume that they are operating in a world where these objections are false. In essence, a mathematical argument always exists within a 'small world' where the premise of that argument is true. One can still question what relevance that small world has to other small worlds, but the mathematical project itself depends on the assumption that the existence of the world itself cannot be questioned. This is mathematical objectivity, and the only difference between the many philosophical interpretations of mathematics is how they non-mathematically justify the existence of such worlds.

Fiction acts as an easy metaphor for mathematical direct proof. Fiction represents a sequence of transformations that converts the characters and events on the first page of a novel to the characters and events on the last page. All fiction begins with premises: a set of places, people, and circumstances which are designed to be accepted ‘for the sake of argument’ and which are presented to the reader without proof or justification. These premises might be plausible, as is the case with historical fiction, or hypothetical, as is the case with fantasy. These premises are then followed by a description of various events, chosen by the author, which link the presented premise with its natural conclusion. If the fiction is masterfully crafted, then the reader will finish the book feeling that each event in the story led naturally into the event that followed. This creates a causal narrative structure that intuitively feels like it could have written itself.

However, if the narrative is not well constructed, then there might be transformations that do not mesh with the reader’s conceptualization of the constructed world. Say a character acts contrarily to how they have developed, and the author forces them down a path that they would not have normally taken. To the reader, it may feel like the author has intruded into the book and artificially changed it to suit her needs. Depending on the purposes of the book, this intrusion might break its immersive qualities and possibly jeopardize the argument that the text is trying to make. The lasting effect on the reader, accidental or otherwise, would be that they either discard the book as hogwash, or are enticed into looking deeper into the text and exploring the offending chapter possibly leading to the creation of another text.

Mathematical direct proof is nearly identical. It begins with an initial premise that exists only in a manufactured small world where that premise is unconditionally true. This premise is followed by a sequence of events that transforms it into a conclusion that represents the idea that is being proved. Like fiction, the plausibility of the premise is secondary to the construction itself. Mathematical texts can represent plausible ideas, like the natural numbers, the flow of liquids, the simulated transfer of ideas, or bizarre otherworldly objects such as infinite dimensional hyper-planes, irrational numbers, and functional vector spaces, all of which are to be taken 'for the sake of argument'. A masterfully crafted proof feels like it has constructed itself, and that the author's only contribution has been to draw the reader's attention away from unimportant detail. However, if at any point along the proof narrative any of the transformations do not seem to flow naturally from the preceding events, then the argumentative quality of the proof is broken, and the audience's attention turns to the offending lemma. Just as an author might write further books to fill in plot holes or flesh out character development, so too the failure to create a valid mathematical proof tends to generate other proofs that can and have led to the creation of entirely new mathematical fields.

It is at this point that the metaphor begins to break down. Fiction and proof separate when we attempt to define valid transformations. I will accept the actions of a fictional character presented to me only if I can somehow relate to that character; I use the term 'relate' in the loosest possible sense. The concept of human experience is not easily defined, and varies dramatically between perspectives. All aspects of my human experience will contribute towards what I as reader will find convincing. Everything from my age, my culture, my economic status, my education, and my previous exposure to other works of fiction contribute towards how I interpret any text presented to me. Two different readers exposed to the same narrative can have entirely different experiences and therefore experience entirely different arguments. The truth that a single text generates must necessarily represent this multitude of independent readings. In order to have a full understanding of any individual text one must necessarily explore it from a vast assortment of perspectives. However, in the case of mathematical proof, this generalized definition of validity blatantly contradicts the mathematical project.

Alone, the concept of small worlds is insufficient to define the mathematical project because it fails to offer a metric to distinguish between mathematical worlds and non-mathematical worlds. Mathematics is not just the creation of small worlds; it is also the interpretation of them. Mathematics brings with it a number of goals and objectives that must be fulfilled if a small world is to be deemed ‘mathematical’. The most important of these will be discussed in later chapters; however, for now it is only important to understand that these base axioms form the ‘mathematical perspective’.

Early mathematicians viewed the mathematical perspective as an absolute and pure perspective that existed independently from both mathematicians and mathematics as a whole. A proof was true because it aligned with some transcendent rules of rhetoric; however, today modern axiomatic mathematicians treat this perspective as a democratic, and somewhat arbitrary, construction built by the entire community, with varying degrees of ambiguity depending on which mathematical philosophy the mathematician accepts. We interpret certain symbols and concepts in a certain way because we have collectively agreed to do so. These agreements are in some sense axioms in their own right that govern the language through which mathematical ideas are expressed. In this way mathematics itself can be thought of as a small world that itself contains other small worlds; likewise, a small world becomes ‘mathematical’ when it adheres to the rules of the larger mathematical perspective.

Constructed small worlds are infinite, and so are the structures that create them. However, not all of these small worlds are interesting. One of the fundamental tenets of the mathematical perspective is that all mathematical problems have a single definitive mathematical solution. We collectively agreed that all mathematical questions must have a definitive answer. As a consequence, mathematical texts are written in such a way that there can only ever be one definitive interpretation of the ideas it presents. The mathematical perspective reads all mathematical texts and then judges each to be either true or false. Like fiction, the mathematical author expresses ideas through an invented world, and, also like the author of fiction, the author of mathematical texts specifically chooses the world he wants to work in. However, unlike in fiction, the mathematician is not speaking to a varying audience with varying perspective. The only opinion that matters is the opinion of mathematics itself. It is this solitary perspective that interprets the premise of all mathematical works and collectively transforms all mathematical ideas into their respective conclusions. Similar to fiction, works based on a common premise, as is the case with a series of novels, produce complementary structural truth that carries from one work to another. However, unlike fiction, where concepts like human rights might be explored in several unrelated texts, ideas derived from one mathematical text have absolutely no place in any other mathematical world unless a proof that explicitly links those ideas is presented as well. The mathematical perspective's acceptance of a proof mandates that everyone, past and future, working within this perspective accept it as well. However, beyond these fundamental differences, mathematical proof is still a narrative argument. At its core, all mathematical proofs are still an appeal to the intuitions of the reader. If the author fails to capture the interest of mathematics personified, then they have failed to transmit the idea and the proof has failed.

The arbitrary nature of mathematical premises and the fact that all connections between ideas must be explicit does lead to a certain amount of fragmentation in the mathematical project. Consequently, bridges between fragmented ideas tend to be the most important and sought after theorems in mathematics. Remembering Hardy's comment that the seriousness of a mathematical theorem lies not in its usefulness but instead in its ability to link other mathematical ideas, we can see that the highest praise in mathematics goes to those who find connections between the most ideas. Fractured systems do exist, but they generally do not last long as they usually represent unsolved puzzles. Frequently, as is the case of Kaku's string theory, these unifications take the form of a new higher axiomatic premise, which can itself derive the premises of other mathematical worlds. The prototypical example of mathematical unification is Euclidean geometry.

Euclid was a Greek mathematician who wrote the defining text in modern mathematics: *The Elements*. Euclid's work represents the unification of all Greek mathematical knowledge. We do not know whether the mathematical theorems in Euclid's work all originated from Euclid himself, or whether he borrowed them from other mathematicians of his time. However, this distinction is of only minor importance because Euclid's real contribution was not in creating the ideas but in bringing these ideas together. Beginning from the common basis of geometry, Euclid derived the entirety of Greek mathematics. He created a unified perspective for interpreting mathematical premises, which still functions as the prototype of the modern mathematical perspective, as well as a single unified mathematical premise to go along with it. Euclid 's construction was extremely successful and because of this he never had any reason to question the notion that it represented the purest possible representation of space. Euclidean definitions are not intended as new objects in and of themselves, but are instead clean versions of the words they were intended to replace. The linguistic notion of a line is fuzzy and prone to some ambiguity, but Euclid's notion of a line was intended to be exact and without connotative meaning. Euclid also presented the prototypical geometric axioms, which differ subtly in philosophy from their modern counterparts.

Unlike modern axiomatic mathematics, Euclid's axioms were meant to represent ideal forms of physical transformations that mathematicians perform on space. Each of Euclid's axioms represent a tool in the mathematician's tool belt: the straightedge allowed mathematicians to draw lines between two points, the compass allowed mathematicians to draw circles around fixed points. Inside of Euclidean geometry there existed a literal link between real world constructions on space, and the mathematical proof of those constructions. Each axiom represented a philosophical truth about the universe grounded in these physical actions, and the physical interpretation of mathematics was itself the physical act of performing these constructions. Euclid viewed mathematics as a way to extract human error out of human actions and produce a pure mathematical space in its place; an idealized form of the real world. As discussed above, this perspective no longer defines the indisputable core of mathematics. Certain fuzzy qualities of Euclidean mathematics, which even Euclid himself did not like, failed to stand up to the absolute metric that Euclidean geometry was designed to represent. Over time, it became apparent that almost any set of assumptions could be formed into a cohesive perspective, both the physically plausible and the physically absurd, making it difficult to defend the mathematical perspective as being anything other than a collective decision.

Today mathematicians recognize that a subjective lens filters all human interaction with the outside world. Math cannot make any claims about the real world because doing so would contaminate its objective purity with our own humanistic subjective experiences. Likewise, 'indubitability' is a concept that has fallen out of favor. Just because I personally cannot find any reason to doubt a statement does not mean that nobody else ever will. However, this modern perspective was still born out of the old Greek perspective. While few modern pure mathematicians will argue any connection between math and reality, the ethereal mythos surrounding mathematics as a whole still says that math speaks to the real world, and this mythos still strongly influences both the community and the philosophy behind the study.

Mathematics is best understood as a language. Unlike Greek mathematics, modern mathematics only needs to be unquestionable in the sense that the reader understands that the mathematical perspective believes its ideas, but readers themselves can accept or reject any subjective consequences of its ideas at their fancy. In essence, mathematical ideas do not need to be unquestionable; they just need to be transferable. Unfortunately, this does leave math in the awkward position of believing that it does not need to justify its own beliefs. In high school pure mathematical classes, this is indeed the common complaint. It is difficult for students to understand that the point of pure mathematics is that there is no point. Mathematics has no answer to the question "Why do we learn mathematics?", and any teacher faced with this question will be forced into a situation where they will need to either make something up, skirt the issue, or jump straight into the philosophy of mathematics, which the school is not paying them to do.

However, the question still needs to be addressed. Why do we study mathematics? This question is subjective and the answer will change from mathematician to mathematician. I study math because it is about rules. Outside of rules, outside of a framework set up by others, I find it easy to get lost in the preconceptions that build my own perspective. The very act of teaching myself to filter ideas through a perspective that is not my own gives me new insight into my own biases; learning math has taught me to think like somebody else.

I also study mathematics because it is the human race's greatest unified collaborative construction. Each mathematical contribution adds permanent knowledge to a growing pool of research. Mathematics' insistence on a single perspective creates an environment where arguments and discussions eventually end; yet, bridges between old and new ideas are always necessary creating a mathematics that is big enough to accept contributions from both the professional academics and amateur hobbyist indefinitely.

A French mathematician, Pierre de Fermat, presented one of mathematic's greatest puzzles. Born to a wealthy merchant in the late 1620s Fermat enjoyed the freedom to study law, languages, classics, and most importantly mathematics (*Dictionary of Scientific Biography* 4: 566). Among Fermat's most important works are his contributions to number theory, and his proofs concerning various integer equations. However, he did not provide proofs for all of his claims. Written into the margin of his copy of *Arithmetica* Fermat claimed that the equation $a^n + b^n = c^n$ has no integer solutions for a, b, and c for any integer n greater then two; however, Fermat failed to present a proof of that claim in any of his papers. Thousands of mathematicians over three hundred years have tried and failed to rediscover Fermat's claimed proof, and it was not until 1994 when Andrew Wiles successfully found the completion to this proof. The mathematical journal *Annals of Mathematics* honored Wiles by presenting his proof as one of only two papers in their May 1995 edition (the second paper was coauthored by Wiles and included a proof of a minor result that Wiles needed for his construction). Unfortunately, I can offer no quote from the introduction of this important work because no such introduction was ever written. Wiles himself dedicated the first two pages of his paper to a brief overview of the work he built upon, and presented his first theorem on page three. The triumph of his proof lies not in the practical usefulness of the equation, as no engineer has ever needed, or will ever need, to solve Fermat's equation, but in the fact that this proof acts as a bridge between Fermat's riddle and number theory, as well as acting as a bridge between Fermat's mathematics and modern number theory. The importance of Wiles' proof has nothing to do with its technological implications and everything to do with his successful unification of three hundred years of independent mathematical work. This is pure mathematics at its finest.

Applied Mathematics

Truth in mathematical physics is a different beast entirely. The goal of the physics community is to understand the large world around them by submitting intuitive guesses about its structure to a series of physical experiments. In mathematical physics, these guesses take the form of small mathematical worlds that derive experimental predictions. One of the assumptions associated with mathematical physics is that there exists a correct mathematical description of the human environment. Likewise, the theoretical framework surrounding this mathematics is in practice similar to pure mathematics, but stands on an entirely different theoretical foundation. Mathematical physics taints the purity of mathematics by trying to build on the perspective of human experience and not inventing its own. Unlike pure math, which justifies its claim of objectivity by creating a separation between mathematical worlds and the real world, physics justifies its claim to objectivity by defining an objective language that converts repeatable subjective experiences into mathematical objects. For example, every time I look at a green crayon I will experience the color green. Likewise, every time someone else looks at the same green crayon they will also experience the color green. A scientist can point a machine with an electronic eye at the same green crayon and ask the machine to output a mathematical object: 475nm. Because it is safe to assume that the human eye works similarly in almost all humans, they can conclude that all occurrences of the number 475nm corresponds to a prediction that a human observer will subjectively experience green. Unfortunately, this only works in situations where subjective experiences between two individuals are similar. In the case of a literary text, where the subjective interpretation changes between readers, science cannot construct this unified interpretive language without also limiting which interpretations, such as certain related communities, they are predicting.

Mathematical physics chooses its mathematical worlds based solely on how closely they describe physical experience. Those worlds that can accurately predict experiments are valued more highly then those that cannot. Just like in pure mathematics, the most important theorems are those that tie together the most experimental data. Like every science, mathematical physics views itself as a cycle of hypothesis, experiment, and revised hypothesis. The mathematical worlds act as an efficient way to automate hypothesis generation. The process is describable in terms of the ideas presented in Thomas S. Kuhn's *Structure of Scientific Revelations*. The dominant mathematical models represent the paradigm, while normal science works to provide the proofs, predictions, and experiments that make that paradigm useful. Revelations happen whenever there is a fundamental shift in either the axioms that construct the mathematical system, as when Einstein corrected Newtonian physics and updated it with relativistic axioms, or the underlying perspective that interprets the mathematical system, as when quantum mechanics killed the deterministic interpretation of equations in favor of a probabilistic interpretation. "When it was done, even the percentage composition of well-know compounds was different. The data themselves had changed. That is the last of the senses in which we may want to say that after a revolution scientists work in a different world" (Kuhn 135).

The first major paradigm in mathematical physics originated out of Sir Isaac Newton's work on physical motion. Born in 1642, Newton arguably invented calculus and Newtonian physics (arguable in the sense that Gottfried Wilhelm Leibniz also arguably invented calculus). Before Newton, mathematical physics was a tool used by physicists, and did not represent the physical perspective. Early mathematical models were rarely useful outside of the individual problems scientists created these models to address. In the 1500s, two competing models tried to explain planetary motion. The church favored, and enforced, the geocentric model, which claimed that the planets orbited around the earth, while thinkers such as Copernicus favored a heliocentric model that claimed that the earth revolved around the sun. Both models began with a very physical conceptual framework that allowed astronomers to intuitively understand heavenly motion; however, both required cumbersome and complicated corrections in order to accurately predict heavenly motion. Johannes Kepler, who corrected Copernicus's heliocentric model, came up with three laws that described planetary motion. The first is that planets travel in ellipses around the sun, not circles as Copernicus had thought, and the other two are mathematical formulas that describe the speed and location of a planet at any particular point in time.

While important, it would be hard to compare Kepler's work with Newton's mathematical physics. His formulas tell us plenty about the motion of planets, but they say nothing about the motion of objects here on earth. Likewise, the earlier Greek inventor, Archimedes had developed several useful formulas for measuring the flow and displacement of liquids, which are utterly useless in any astronomical application. In both cases math operated as a case-specific tool that, as is the case with all mathematical worlds, gave no insight whatsoever into concepts outside their intended scope.

Newton's work changed that; he completely rewrote the relationship between mathematics and physics. Newton's *Philosophiae Naturalis Principia Mathematica* (not to be confused with Bertrand Russell and Alfred North Whitehead's proposed sequel, *Principia Mathematica,* written in the 1900s) is an important work in the history of science, and the most important work in mathematical science. Newton modeled his text after Euclid's work in geometry. Like Euclid, Newton begins by defining both the axioms of a small world as well as definitions that dictate how that world is to be interpreted. He uses these axioms to prove other statements, which in turn he used to prove even more statements. Like Euclid, who viewed mathematical geometry as an idealized form of physical cartography, Newton's axioms represented an idealization form of motion; Newton's postulates are themselves pure mathematical statements that represent a plausible reality. His first postulate, that all objects in motion stay in motion unless acted on by some other object, is completely inaccessible to all physical experiments; it itself cannot be experimentally refuted or confirmed but can be used to derive other statements which are. Even today, most introductory classes in physics teach the real world by encouraging students to manipulate impossible objects like frictionless surfaces, massless ropes, and ideal springs. The crowning achievement of the *Principia* is Newton's successful derivation of Kepler's heliocentric model for astronomical motion thereby proving for all practical purposes that his model represented the larger physical world, and not just another mathematically small world (*Dictionary of Scientific Biography* 10: 69). Even better, unlike Kepler's model, which could only be applied to celestial bodies, Newton's model could be applied to everything: ballistics on the earth, the motion of a boat through water, and eventually to the physics of human senses like touch, sight, and hearing. Newton's work was far from complete; however, it did provide an extensible base that physicists still use as a premise for their own work.

Applied mathematics represents any attempt to apply mathematical worlds and ideas to real world phenomenon. As pure mathematicians reject the notion of a link between mathematics and reality, we must categorize applied mathematics as different field: related but separate. A computational model is an object that exists under applied mathematics. Computational models are any mathematical world that tries to simulate real world phenomenon: such as Newton's physics. Likewise, a computational model in DH is any mathematical model that tries to represent humanist text: such as network modeling. Truth in applied mathematics exists because of the applied concept of usefulness. Likewise, computational truth in DH is closely linked to a humanist interpretation of the same concept of usefulness. A tool is 'true' in some sense if it is useful to the humanist; likewise, a mathematical principle is true in an applied setting only if its results are useful in achieving the goals of the application. Likewise, applied mathematics also necessarily forces the concept of 'good enough' into the mathematical universe.

Proof in applied mathematics tends to be more relaxed than in pure mathematics. The number π is impossible to represent in decimal notation; however, it can be approximated for practical use. Calculators and computers universally replace π with its decimal approximation (3.14159265359). This is an incorrect representation of π in the pure sense; however, in the applied sense any difference between this value and the real value is small and insignificant. The applied mathematical perspective also operates on a number of assumptions. The most important of these assumptions is that small errors in representation produce small errors in product. The compression rate of concrete is a numerical value and varies slightly inside of any concrete object. The strength of one part of a concrete block is not necessarily the same as the strength of a different part of the same block. From the applied perspective, as long as these variations are small, they will have no impact on the overall strength of the object.

Error also grows in proportion to the scope of the application. The more complex a world is, the greater those small errors propagate. For example, take a container filled with practically identical electrically charged objects. If I measure the electrical power of only one of those objects and multiply by the total number of objects in the bin, the difference between my result and the actual electrical power will be insignificant if the container is small, but might be significant if the container is large. As the scale of the applied application increases, the precision of the applied mathematics must also increase. In this way, pure mathematics still represents an idealized form of applied mathematics; in the case where the applied world is infinitely large, the precision necessary to produce results that are 'good enough' rwill also be infinite. Therefore, applied mathematics is a 'dirty' form of pure mathematics. By adopting computers as a form of humanist research, DH unintentionally adopts these same assumptions. Digital models are not text, and they also are not real world phenomenon; however, as long as the scope of the project is small, the error inserted into the project's results should also remain small.

This is where the generalized computational model enters the picture. The generalized computational model states that wherever there is a practical problem, there also exists a mathematical model that generates a good enough approximation of a practical solution. Physics views the universe as a practical problem. Predicting experimental results is a practical application, and for every experiment, there exists a good enough approximation of a representation of the results. However, the universe is also very large and complex. Small errors in understanding at the subatomic level produce massive errors on a human scale. This is why mathematical physics, as an applied mathematics, begins to look like pure mathematics. Newton's model looks like a pure mathematics because it chooses to adopt the biggest perspective it can: there are pure branches of Newtonian physics. However, these similarities are only in application. Pure mathematics has no concept of 'useful'; its ability to meld with other mathematical knowledge is all that matters. Likewise, applied mathematics has no concept of 'truth' independent of usefulness. A statement is true only if it furthers some objective; the mathematical purity of that object is secondary. The digital humanist's use of mathematical models mirrors that of the applied mathematician; the only thing that matters is the adoption of mathematical methods for the purposes of furthering humanist goals. By extension, a portion of digital humanities becomes an applied mathematics, even if humanists will not admit it.

The universal computational model originates from an observation. Physics likes to see itself as the largest perspective among the sciences. Chemistry is a product of physical interaction, biology is a product of chemical interaction, and society is a product of biological interaction. This hierarchy paints physics as the superset of all sciences. Mathematical physics attempts to build a pure mathematical environment that is useful in predicting experimental outcomes. If such a mathematical world actually exists, and physicists can demonstrate that it is fully computational, then every physical object under physics will also be computational. Chemistry, biology, and all aspects of society become a provable, and highly complex, construction built by this computational model. This is the generalized computation model: the belief that every object and event in the universe is a provable product of a single unified mathematical world -- in short, that the universe is a computer.

Under Newton, the knowledge of a single moment represents a possible axiomatic basis for reconstructing the entire history of the universe, both before and after that single moment. Laplace's demon is a mythical creature described by Pierre-Simon Laplace, a French mathematician, that gained all knowledge of a single moment and extrapolated from it all knowledge about the universe. Today, improvements in information theory, the discovery of chaos, and the advent of quantum mechanics have disarmed Laplace's demon by changing the model from deterministic to probabilistic, and showing that not all algorithms are reversible: implying that information can be lost. However, the general concept that everything in the universe flows mathematically from a single source, the computational model, still persists to this day and lives on in the work of physicists like David Deutsch who talk very seriously about virtual reality and physics simulation. At its core, the belief is that the universe is fundamentally constructible, and all we need to do is figure out the rules of this construction. This idea is central to every claim either that the universe is a computer, or that the language of the universe is mathematics. I will return to this concept in chapter four.

The differences between applied and pure mathematics will continue to evolve throughout this thesis, but for now, it is more important to focus on their similarities. Both applied and pure mathematics maintain a precarious balance between objective and subjective thinking. In both mathematical fields, objectivity acts as a filter for subjective ideas that is separate from individual researchers. The pure mathematicians' claim to objectivity is that their perspective is a constructed perspective; it has been contracted over the course of human history and is independent of any individual mathematician. Likewise, applied mathematicians' claim to objectivity originates from their concept of useful. Applied mathematics tests ideas against goals and objectives. However, pure science has little voice in determining what those objectives are. The ability to predict outcomes can just as easily aid in maximizing food distribution, testing methods for creating social equality, and eradicating diseases, as it can create weapons of mass destruction, enable genocide, and invent new ways of dying. Usefulness is driven by goals, and those goals must still be chosen in advance much the same way that pure mathematics needs to justify its truth before such truth can exist. In both cases, perspective creates truth. Technological advancement is derived from goals, which need to be chosen before these goals are achieved, and mathematical truth is derived from decisions, which need to be made before mathematical truth itself is generated. As much as pure mathematics would like to pretend that it is an arbitrary perspective generated by randomly selected axioms, this conclusion is just not defensible. Mathematicians choose their axioms because they subjectively make sense; we have no other objective explanation. Instead of being the purest form of objectivity, mathematics represents a precarious balance between objectivity and subjectivity. Subjectivity creates the ideas, but objectivity filters them. It is only through this pairing of objective and subjective reasoning that either can offer up any claim of value.

The definition of objectivity that I will be using for the remainder of this thesis is based on both of these claims. In the common vernacular an 'objective opinion' is simply the opinion of someone not related to the project. Likewise, the objectivity in pure mathematics is the opinion of mathematics itself. It does not matter how 'correct' a mathematician believes her proof to be, it is only correct once the mathematical perspective has accepted it. Likewise, a scientific theory only has value once experimental evidence, another objective opinion, declares that it has value. With these in mind, and to prevent my project from getting pulled into the numerous philosophies of objectivity, I will be defining objectivity as the physical act of appealing to another perspective (usurping language is also a distinctly mathematical pastime). Even further, 'thinking objectively' represents a physical act in which one tries to temporarily replace their own subjective thoughts or beliefs with that of the perspective that they are using to support their work.

Fundamentalism

At this point, I would like to say a few words about fundamentalism. I understand that the term 'fundamentalism' is generally reserved for religious fundamentalism; however, I feel this reservation is only useful when one wants to draw a (generally politically motivated) line between religion and other radical ideologies. A fundamentalist is any person, entity, or group of entities, that adheres to a strict code of conduct, moral code, or worldview exclusively. A determining factor of fundamentalists is a reversal of the objectivity / subjectivity continuum referred to in the previous section. As an example, a corporation might adhere to a very strict health and safety regime for the purposes of reducing workplace injuries. The manager's strict adherence to seemingly unimportant rules might annoy certain employees, but if these rules objectively lower workplace injuries, then the adoption of the rules are justified by this fact. The need to reduce workplace injuries in the first place is a subjective desire that deploys an objective set of rules in order to achieve a goal. Likewise, a religious society might impose restrictions on the sexual activities of their members. In much the same way, these restrictions act as an objective filter that restricts subjective experience in order to achieve a definable goal: limiting the spread of diseases, safeguarding a system of inheritance, or indoctrinating a population. (The political claim of a goal and the actual goals themselves are not necessarily the same thing.)

Fundamentalism begins when these purposes are reversed. The company now adopts safety rules because the rules themselves represent some objective truth that must be adhered to. The religious society adopts restrictions on sexual activity simply because it is objectively true that sexual activity needs to be restricted. In both cases the perspective of the rules becomes the subjective perspective of implementer: they believe it because the rules believe it. Other examples: Nature should be preserved because nature is objectively good, women are objectively superior / equal / inferior to men, and, most importantly, objective thinking is objectively better than subjective thinking. Fundamentalism strips objectivity of its context. Without context, objectivity becomes a context itself and through that context justifies its own existence. In this way, by reversing the objective / subjective pairing fundamentalism succeeds only in completely removing the distinction. If what I believe justifies what I think I believe, then I will never have any reason to question myself. Thus, all my thoughts are subjective. This is true of the mathematics I set up early in this paper. Without context, mathematics is nothing more than a fundamentalism, a definition of truth that exists outside of any subjective experience, and those who adhere to the belief that mathematics is the uncontested language of the universe are fundamentalists. From an emotional perspective it might seem like math is better than astrology, just as that same emotional perspective might paint human rights as better than genocide. However, separated from a framework of goals and assumptions, all of these ideologies are structurally identical.

As an example, Evangelical Christianity operates on two fundamental tenets. The first is Christianity which states that the moral corruption of the world is cleansed through the death and resurrection of their savior Jesus Christ, and the second, Evangelism, which states that it is the duty of the Christian to convert others into this doctrine. In this perspective, conversion defines usefulness. The world will become a better place when others convert precisely because the rule implies that it will. Scientific fundamentalism takes the form of positivism, the belief that all knowledge is scientific knowledge. In the case of positivism, usefulness is stripped of any goal oriented structure; usefulness is true simply because it is useful.

David Deutsch argues that the discovery of the scientific method is the most important event in the history of knowledge, and the primary reason humans were able to free themselves of their static societies and began learning at the exponential rate that continues today. "The emergence of science, and more broadly what I am calling the Enlightenment, was the beginning of the end of such static, parochial systems of ideas. It initiated the present era in human history, unique for its sustained, rapid creation of knowledge with ever-increasing reach" (29). Likewise, Evangelical fundamentalists see the birth and death of Jesus as being the single most important event in history. It is the day that the rift between man and God, created by the moral corruption of the world, was repaired through the willful sacrifice of the only perfect human being: "In the beginning was the Word, and the Word was with God, and the Word was God. He was in the beginning with God. All things came into being through Him, and apart from Him nothing came into being that has come into being… And the Word became flesh, and dwelt among us" (New American Standard Bible John 1.1-3,14). The positivist's claim to objectivity is identical to the applied mathematical concept except positivists do not define a framework of usefulness. The positivist thinker values objectivity simply because it is objective; through objective thinking, the world becomes a better place. Science represents objectivity, and the world will become a better place when it becomes more scientific. Evangelical fundamentalism also has a claim to objectivity. As morality is primarily a subjective experience, it is the Evangelical desire to objectively achieve the best subjective experience they can. God wants the best for his people. To that end, he has given us a set of rules that, if followed, will result in objectively happier, content, and peaceful people. God represents objectivity, and the world will become a better place when everyone adopts the views of God.

Deutsch argues that myth is a primitive version of a scientific explanation. The ancient individual looks at the world, and attempts to rationalize what they see based on what they believe. However, these ancient 'bad' scientific explanations are not useful because, unlike Newton's laws, they offer no predictive narrative that can be experimentally tested and expanded upon. “Every other detail of the story, apart from its bare prediction that winter happens once a year, is just as easily variable. So, although the myth was created to explain the season, it is only superficially adapted to that purpose” (20). Such primitive science creates a worldview that is impossible to dispose of because it is untestable. Deutsch argues that the scientific revolution, or the enlightenment, happened because we discovered a new type of explanation; one that is rigid enough to be tested, and disposable enough to be thrown out if found insufficient. Scientific explanations create a dynamic system that favors positive change and improvement. In contrast, the old myth system inhibits change in favor of stagnation. These theories are always capable of explaining events after they happen, but have no predictive value. As both myth and scientific theory are effectively objective theories about the universe, science wins as it outperforms myths at objective and predictive reasoning.

The religious side of the argument views the world not in terms of objective reasoning, but subjective experience. In the Christian world, the career of leaders and speakers does not rest on their ability to make rational and predictive arguments. Instead, they need to be able to relate to, create, and sometimes even control the subjective emotional experience of the listeners. Christians will explain God not as an objective science experiment, but as a personal relationship or experience. Their arguments usually appeal to the emotional wonder of natural experiences and the subjective feeling of greater structure beyond, or in spite of, the selfish hands of man. Religious folk take comfort not in their knowledge of the universe, but in the mystery of the universe. It is the subjective knowledge that they are simply one small part in an ordered system much bigger than they are. Unlike science, academic progress is not seen as the definite goal of the human race. Science tries to achieve subjective happiness by recreating the universe in its own image, and in doing so replaces the divine and infinitely complex system with an inferior and flawed system that we created for ourselves. At best, progress in religion is a lateral shift that changes the relationship humans have with the universe, and at worst, it is a distraction or driving wedge that separates us from God. Science, in essence, is simply a means to create a more structured, but less meaningful, existence; it is a method for replacing God that is ultimately doomed to fail, as God cannot be replaced.

Deutsch's argument against myth hinges on his belief that his perspective is objectively correct. The primary way he makes his argument convincing is by interpreting religious practice from within his own scientific perspective and demonstrating that religion operates as an inferior version of science. In order to accomplish this rhetorical trick Deutsch must convince the reader that the origin of all human endeavors begins with a desire to scientifically understand their surroundings. This assumption is clearly demonstrated when Deutsch chooses the scientific method as the most important point in human history. "Progress that is both rapid enough to be noticed and stable enough to be continue over many generations has been achieved only once in the history of our species" (vii). Likewise, religion falls for the exact same trap when it frames technology as a replacement for God. They argue that if we abandon God the resulting absence creates a subjective hole in our lives that we try to fill with other, sometimes immoral, activities. Science is perpetual only because it can never truly fill that hole in the heart of humanity that is reserved for God and God alone. We peruse science only because we feel empty and have rejected the only thing that can fill that emptiness.

In both cases, each perspective filters the goals of the other through their own lens and argues that the other's failure to adhere to their own rules proves inferiority. Both science and religion operate on different axiomatic structures. Science sees demonstrability, or progress, as the metric that measures knowledge while Christianity sees the marriage of God and man as fulfilling the same role. Christians view the bible as the axiomatic source of God, while science sees experimental demonstrability as the axiomatic source of knowledge. Finally, each defines a different narrative that makes extension of their respective axiomatic bases possible. The scientific method defines a series of actions that need to be performed in order for a new idea to be true; likewise, religion defines which actions constitute a search for God. Given these three ingredients -- single correct perspective, axiomatic belief structure, and a system for convincing narrative -- I have all the ingredients necessary to produce a small world inside of either of these perspectives in the same way I would produce small worlds within the mathematical perspective.

How then are we to choose which of our options are the right options? Unfortunately, the very existence of this question represents one of the key problems associated with fundamentalism. A fundamentalist believes that there can only be one answer. Because the universe can offer only one objective perspective, any deviation from that rule set could imply questions that those rules are incapable of answering. If the positivist were to accept, even for a moment, that the Christian derives subjective value from beliefs that are non-scientific then that would question their own belief that everything of value flows from the scientific method. Likewise, if the Christian fundamentalists were to accept, even for a moment, that science produces value separate from God then that would question their own belief that value flows directly from a relationship with God. The result is ridiculous arguments like the evolution / creation 'debate' which claims to be about facts when in reality it is a fight over objective sources. Any attempt to solve this problem would require introducing a framework of usefulness, which fundamentalists on both sides will never agree on.

Mathematics offers absolutely nothing in the search for the 'correct' perspective. From the mathematical perspective, every valid small world inside of the mathematical perspective holds equal weight. Even the concept of 'valid' is constructed within the mathematical perspective. The fundamentalist mathematician might argue that all worlds that create contradictions should be thrown out, but this is only because one of the assumptions of the mathematical perspective is that contradictions should not exist. A mathematician designs each world they create to express a certain idea, which is true only in and about itself. The existence of a contradiction in a mathematical world only destroys the world in the sense that further mathematical research into it would be uninteresting. More specifically, logically contradictory worlds create logical peculiarities (everything is provable after a contradiction: the principle of explosion), which render much of the mathematician's tool belt useless. The choice to exclude these worlds from the mathematical canon is little more than a logical consequence of the rules we chose to construct the mathematical perspective. It is not that we find these worlds offensive, it is that the mathematical perspective finds them offensive. This separation between my perspective, the perspective of other mathematicians, and the perspective of math defines the foundations of the mathematical project. Likewise, this rejection of an idea by the mathematical perspective is not a permanent judgment on the value of that idea. The mathematical perspective may believe that proof generates truth, but the act of building a proof narrative is still an action performed by humans. It is a tool, given to us by the mathematical perspective, that helps us to discover what mathematics itself believes. While math may have rejected the truth in a contradictory world, this by no means forces the rest of us to accept that that world is without value. On the contrary, that world will live on perpetually in the form of a proof narrative that is tremendously useful in transmitting mathematical ideas to the next generation. In this sense, a contradictory world is less wrong, and more complete. It has exhausted what it is capable of telling us about the mathematical perspective, and therefore has exhausted its research potential from within the mathematical community.

Conclusion

So then, what is the value of mathematics? I believe that this question is closely tied to the value of objective reasoning. Mathematicians deny the subjective value of mathematics, and in a sense the 'philosophy' of mathematics as well, precisely because they recognize that the moment objectivity and subjectivity converge is also the moment that objective reasoning ceases to exist. Objectivity can only exist separate from subjectivity. This separation also implies that objectivity has no value in and of itself. If I were to accept mathematics as the purest form of objectivity, while simultaneously accepting objectivity as the source of all knowledge, the only thing I will have accomplished is to incorporate mathematics into my own personal subjectivity and created a form of mathematical fundamentalism. Objectivity is valuable because it provides a filter through which ideas can be refined: a place where poorly constructed realities can be revealed, and found wanting. Deutsch correctly identifies this as the primary reason why science is powerful. Unlike Deutsch, however, I do not believe that science defines objectivity; I do not even believe that math defines objectivity. Objectivity is an action. Objectivity is about trying to see the world through a perspective that is not my own. To literally push my own thoughts and beliefs aside temporarily and attempt to form new thoughts and beliefs according to the structure of something that is not me. In essence, objectivity is all about questioning one's self.

One of the problems I've had with some members of DH is their unwillingness to question themselves. It is all right and good to use computers to 'further humanist goals', but it is quite another thing to allow computers to challenge the very foundations of humanism. If the goal of humanist scholarship is simply to maintain the epistemic status quo then I feel that adopting a new perspective is a terrible idea. From the perspective of fundamentalist, the introduction of new ideas is inevitably toxic as Johanna Drucker points out: "I cannot overstate the perniciousness of such techniques for the effect of passing [the graphical] construction off as real, and violating the very premises of humanistic inquiry" (para. 23). Toxicity is inevitable; however, I do not see it as a bad thing. Objectivity in the sciences has been and continues to be tremendously useful; this I cannot deny. Kaku's ten dimensions are not visible to the human, but looking at physics from the perspective of ten dimensions, a mathematical entity, reveals tensions, false contradictions, and false dichotomies that cannot be seen through the lens of either quantum mechanics or relativity on their own. Likewise, looking at the human through the lens of mathematics reveals inconsistencies, and structural inequalities that are equally invisible when seen through the lens of our desires. New techniques might be toxic, but that is only because they reveal truth from a fundamentally different perspective. We can either choose to reject this perspective, or we can understand it.

As much as pure mathematicians would love to argue that their rules, their perspectives, are arbitrary, the fact remains that they are not. We subjectively choose these rules. We made a decision to accept some and exclude others. These decisions are important because they are an example of the subjective mathematical intuition filtering the objective mathematical perspective. Mathematical knowledge exists within the play between these two extremes. On one hand, it is useless, on the other, it is useful: one is subjective and the other is objective. These objects cannot exist together at the same time; however, mathematical knowledge still exists somewhere in between.

Chapter 3: A Brief History of Incompleteness

The first time I really began to understand the bizarre nature of mathematics was during a third year undergraduate course on complex numbers. I had been mostly interested in science and applied mathematics so I had yet to fully internalize the arbitrary nature of axiomatic mathematical discourse. Before this course in complex numbers, mathematical topics were generally introduced topically instead of axiomatically. Mathematical object begin with some metaphor that abstracts some element of the natural world: the rest of the course being an exploration of the properties of that mathematical object. Complex numbers can be taught from this perspective, but my professor chose not to. He happened to be a staunch pure mathematician. He never once offered any metaphor to describe what a complex number was, and, worse, seemed unable to provide any feedback on errors beyond 'because it is wrong'. My classmates and I all wanted an answer to the ever-elusive question 'why?' but he refused to give any. Even worse, he seemed unable to even understand why anyone would even desire such a preposterous explanation.

There are several different metaphors I could offer to help explain what a complex numbers is, but I choose to offer none. Complex numbers may have arisen out of necessity, but this professor taught them to me as nothing more than invention. Multiplication of negative numbers has always been a bit of a strange entity in the mathematical world. Take -5 objects, and then multiply them by another -5 objects and magically they become +25; this concept is grade school algebra. The principle is axiomatic; negative numbers when multiplied together produce positive numbers. There may be numerous ways to justify this notion, but the only one that matters is the simple fact that it produces the number system that mathematicians want. Furthermore, this decision introduces some interesting peculiarities. The equation $x^2 = 25$ now has two solutions $x = -5$ and $x = 5$, while the equation $x^2 = -25$ has none. The lack of symmetry in these two equations is precisely the peculiarity that the complex numbers resolve. The equation $x^2 = -25$ has no solutions inside of the universe created by the natural numbers; however, the complex numbers assign it two solutions anyway: $x = i5$ and $x = -i5$. Fittingly, the mathematical object i (j in the engineering world) is a pure invention with no immediate physical metaphor. In order to emphasize the make belief nature of this value Descartes assigned it the name 'imaginary', "Neither the true nor the false roots are always real; sometimes they are imaginary; that is, while we can always conceive of as many roots for each equation as I have already assigned, yet there is not always a definite quantity corresponding to each root so conceived of" (175).

Complex numbers are tremendously useful as a model for electricity, quantum mechanics, and many other physical processes, yet the pure mathematical object that allows these models to exist is distinctly separate from all of these uses. It is a pure invention, and the objects that scientists use it to describe are not. The scientists who chose to use complex numbers to model these processes did so entirely because of their usefulness. Electricity and complex numbers share properties and relationships; both objects interact similarly to other objects around them, and looking at complex numbers and how they interact gives those scientists useful insight into how electrical processes interact.

I bring up this example because I see topic modeling, and all numerical text-mining methods, as being no different. Topics, in the context, are mathematical objects. They are probability distributions that represent the chance of observing particular symbols. Each topic consists of a symbol and the probability of finding that symbol in a text. However, I do consider the choice of terminology here to be rather unfortunate. The terms 'complex numbers' and 'electrical model' are different enough that they imply the correct sense of separation needed to understand both. The study of electricity is not the study of complex numbers, and vice versa; however, understanding complex numbers helps tremendously when learning about electricity. The same is true for topics. 'Topics' in the mathematical sense are like complex numbers; they do not directly represent the topic someone refers to when they talk about a 'topic of conversation' or a 'topic of a chapter'. The study of topics in a book is not the study of topic modeling, and vice versa. However, in theory, since topic models are designed to resemble chapter topics, the study of one should produce useful observations relevant to the study of the other. This represents the pragmatic separation between pure and applied mathematics. They are both different, but the study of one produces observations useful to the study of the other.

To the humanist who prefers to write papers about papers using only quill and parchment, I will happily concede that close reading, a fundamentally non-numeric methodology, and distant reading, a generally numeric methodology, are different. They model the text differently, require different skills from the scholar and, most importantly, produce different results. If numerical methods do not produce useful results or do not resemble the objects that they are trying to model, then any scholar is free to walk away from them, write a paper about why the attempt failed, and learn from that failure. The fear inherent to the skittish humanist about numerical methods should instead be directed at fundamental mathematical science and not mathematics itself. Unfortunately, the success of mathematical models in mathematical physics has produced a culture in which the separation between pure and applied mathematics is becoming meaningless. Since Newton, the scientific models have continuously been refined and reconstructed to the point where any useful difference between the two fields has become negligible. Why differentiate between 'the laws of physics' and 'the equations that describe the laws of physics' when the two are defined identically? Unlike topical studies where the linguistic definition of 'topic' is still important, there is no longer any linguistic definition of physical laws. It has disappeared, and been entirely replaced by the mathematical equations that model them.

This prospect should terrify the humanist because it raises important questions concerning the goals of textual study. If, like physics, mathematical modeling proves to be so effective that there becomes no discernable difference between text and the computerized models that read text, then essentially the humanities will have been replaced by mathematical humanities. If mathematical models can answer all the of the interesting questions better than traditional humanist methodologies then, the popular argument goes, there will no longer be any reason to return to traditional humanist methods beyond a blind fixation on tradition. Yet, I have already mentioned a problem. Tradition itself holds value, and this perspective misses that value completely.

Another question arises: Are there any questions that mathematics cannot answer? Or even more specifically: Where is the separation between 'mathematical' and 'non-mathematical' objects. Why is it that Descartes felt that imaginary numbers were not a 'definite quantity' while today they are? The popular mythology around mathematics states that all of these questions have easy answers, but the reality is that they do not. The actual unexpected answer is to all of these questions is that mathematics does have distinct boundaries, but we are still in the process of figuring out where those boundaries actually are. In order to demonstrate this fact, I need to dive into both the philosophy of mathematics and the history of the project.

Euclidian Geometry

The problem with studying mathematical texts in a humanist setting is that primary texts are generally a worthless source for quotations. Mathematics always begins by inventing a universe, and that universe is separate from the people that created them; therefore, mathematical texts are not in the least bit interested in setting up the framework around these created worlds. The prototypical example of this is Euclid's book *The Elements*. As Eli Moar points out:

> It is instructive to compare *The Elements* to a modern mathematics textbook. You will not find here the usual preface and introduction, a forward to the student and a foreword to the instructor, exercises with answers, appendixes, a bibliography, and an index. Nor will you find words of praise about the merits of the book over its competitors. Right from the first page – indeed, the first sentence – it is down to business. (34)

Conceptually, most mathematical universes are created when a mathematician attempts to simplify a single idea. Euclid created Euclidean geometry when he converted an infinitely large sheet of blank paper into a mathematical world. The rules of Euclid's universe allow a mathematician to freely create objects using idealized versions of tools a real world cartographer would use in order to draw a map. Euclidean geometry is ideal in the sense that it is an attempt to extract human error from these tools. A real world cartographer with a real world straightedge and compass is prone to mistakes, yet no such mistakes can exist on the Euclidean plane. Here, all circles are perfectly circular and all lines perfectly straight. Greek mathematicians viewed Euclidean geometry as the ideal form of the physical world. They recognized that human observation was unreliable. We make mistakes; we perceive things that are not actually there, or our tools are not precise enough to extract the true measure of any particular value. The Greeks were aware of this problem and sought to find ways to separate absolute truth from error-prone conceptual reality. One of their inventions was the axiomatic method.

An axiom is two different, yet very important, concepts bundled into a single object. Firstly, an axiom is an assumption whose truth requires no justification. The Greeks viewed axioms as statements whose truth was self-apparent. They, by their very nature, resisted argument, and were impervious to doubt. Secondly, an axiom is a pure version of some singular concept. A well-built axiom is always phrased in such a way as to reject all connotative meaning. They are designed to contain as little information as possible in order to prevent them from being dragged into unrelated discussions. The Euclidean axioms try to be both of these things. They represent physical actions that the Greeks did not feel needed justification, and each axiom purifies its physical action into a single pure idea. The actions a mathematician is allowed to perform inside of the Euclidean universe are as follows:

- Create a line from any defined point through any other defined point.
- Extend any line indefinitely in either direction.

- Draw a circle with any defined radius centering on any defined point.

Just as the tools a cartographer uses define which actions can be performed on a map, so too the Euclidean axioms define what a mathematician can do on the Euclidean plane. At no point was Euclid required to explain how two points on the plane would be connected, or prove that the circle the third axiom produces is indeed a perfect circle. These truths are justified by their axiom whose validity is self-evident. However, Euclid felt that these specific were intuitively obvious because they represented actions, and actions, unlike arguments, do not intuitively feel like they need explanation. Each axiom represents a process, which is confused only when an unskilled cartographer tries to carry them out.

As the skill of the cartographer increases, so too does the accuracy of their maps. An apprentice cartographer can draw maps at a certain level of precision, a master cartographer can draw maps at an even higher level of precision, and an Olympic cartographer with infinite skill should be able to produce a map with infinite precision. Euclid saw Euclidean geometry as geometry seen from the perspective of the ideal. We, as humans, have no inherent difficulties imagining such an Olympic cartographer, and likewise the Greeks treated geometry as a flawless representation of space.

Figure 2: Constructing an equilateral triangle.

1) Given a line of length one and endpoints A and B.

2) Draw a circle cA with center A and radius l.

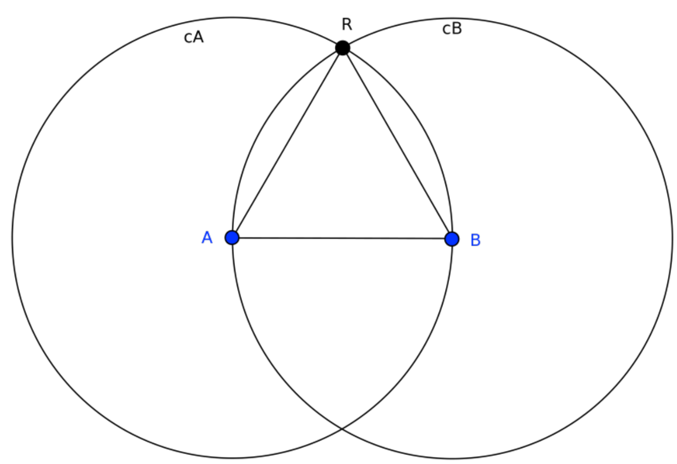
3) Draw a circle cB with center B and radius l.

4) Label the intersection of cA and cB R.

5) Draw a line between R and A.

6) Draw a line between R and B.

7) The triangle ABR is an equilateral triangle with side length l.



Euclidean geometry consists of roughly two different kinds of methods: construction and proofs. Constructions are direct proofs that provide a recipe for constructing a particular object in the Euclidean universe. The very first construction in Euclid's book *The Elements* provides one such recipe for constructing a perfect equilateral triangle, which I demonstrate in figure two. In this way, the Euclidean universe can construct anything that exists within it, and anything that cannot be constructed likewise does not exist. The proof of any object's existence lies in the construction of that object.

The second kind of methodology that Euclid introduced is used to prove statements that do not exist within the mathematical universe, but are true of the universe itself. In the case of the construction above, I would need to argue separately that the triangle I built indeed fits the definition of an equilateral triangle laid out in Euclidean geometry. The assertion that I built an equilateral triangle is not itself an object in the Euclidean universe, and therefore cannot be directly constructed. Likewise, that assertion that a construction is impossible is also not a Euclidean object and also cannot be directly constructed. Non-constructible statements also have mathematical interest and it is useful to determine whether these statements are true. A second proof technique, proof by contradiction, exists for precisely this purpose.

Proof by contradiction, or *redactio ad absurdum*, is a proof technique linked to an important axiom at the heart of the mathematical perspective: the principle of bivalence. The principle of bivalence, also known as the law of the excluded middle, is the axiom in mathematics that assumes that every statement must be either true or false. The middle is excluded in the sense that there can be nothing between these two distinct states. This axiom labels such objects like slightly true or reasonably true as un-mathematical implying that such objects cannot exist within mathematics. Everything that is not definitively false must be definitively true and vice versa. Proof by contradiction creates proofs by taking advantage of this axiom and tries to show that a statement is false by demonstrating that it cannot be true. Once one logical state has been shown to be impossible then bivalence forces the other truth state. For example, if I can prove that I did not write this paper, then clearly someone else must have. The technique itself is fast, easy to execute, and enormously simple to understand, which makes it a low-cost method to quickly verify facts that cannot be directly constructed.

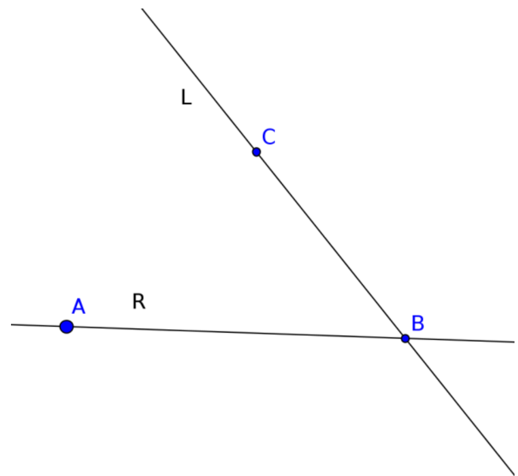
Like any other proof narrative, proof by contradiction begins with a premise that the mathematical perspective has already accepted as true, and then creates a counterfactual world by adding one additional statement to the premise. In nearly all cases, this additional statement is the opposing statement to the one I want to prove. For example, “Assume that World War II didn’t happen.” After the assumed statement, the proof mimics a construction by building a conclusion that flows naturally from the assumed premise. “Therefore, there are no German U-boats on the bottom of the Atlantic Ocean.” Once I reach the conclusion, I will then demonstrate how this conclusion is absurd by demonstrating that it contradicts one or more statements in the generally accepted premise. As an example, I could reveal damning photos of German U-boats I found while exploring the Atlantic Ocean. If the proof is properly constructed, the direct proof that transforms the premise into the conclusion should itself resist doubt, and therefore turn the attention of the audience towards the premise. By definition, everybody in the room should already agree on the premise, so all potential criticism must fall on the additional singled-out statement that the arguer appended to the premise. I began with the assumption that World War II did not happen which would imply that no U-boats were sunk during this fictional war; however, I have proof that there are sunken U-boats on the bottom of the Atlantic Ocean, which means the war must have happened.

This argumentative strategy works because we are naturally blind to the assumptions that construct our ‘common sense’ and we immediately latch onto those statements that the arguer has singled out. Unfortunately, this means that the arguer can lead the audience to question statements that may not contain the contradiction. As an example, the World War II argument mentioned above can easily be reworded to question the validity of photographic evidence. Mathematicians are aware of this problem and deal with it in a rather brutal fashion. A contradiction rules out more than just the assumed statement, it rules out the entire universe that constructed the statement.

Proof by contradiction operates on three separate worlds: the original world that existed prior to the proof attempt, a copy of that world with the additional assumption, and a second copy of the world that includes the converse of the additional assumption. When a contradiction is discovered in the world where the added statement is assumed true then this counts as a proof that this statement cannot be true of the world before the assumption. Since it cannot be true, bivalence forces the statement to be false, giving us reason to accept the third world where the converse is assumed. However, this process only works if the initial world is itself free of contradictions. If both of the constructed worlds contain a contradiction, then bivalence itself has been shown to be false and all three of the worlds need to be discarded. Thus, we have created a situation in which the definition of contradiction creates the proof technique that governs it. Non-contradictory worlds are defined as worlds in which proof by contradiction is a valid proof technique, and likewise proof by contradiction only works in worlds that are non-contradictory. One implies the other. Likewise, anyone who attempts to make a point by constructing a contradiction is necessarily also claiming that their perspective is free of contradictions, because any contradiction in their fundamentals would invalidate their own argument.

In the case of Euclidean geometry, proof by contradiction demonstrates that certain constructions are impossible. Imagine a sheet of paper with three labeled points A, B, and C. The Euclidean axioms imply that I can draw a straight line between any pairing of these three points. Now if I draw a line between two points, A and B, and my line also passes through the third point, C, drawing out the remaining two pairs would produce the same line twice over and our constructed object becomes only a single straight line. However, if line AB does not pass through point C then drawing out all additional lines will produce a triangle. The definition of our axioms make it impossible to produce more than three lines; however, there is no easy way to rule out the possibility of a configuration that produces exactly two lines. Proof by contradiction can demonstrate that such a construction is impossible.

Figure 3: Three points on a surface.



Assume that some setup of three points on the Euclidean plane allows us to draw exactly two lines. I will name the three points A, B, and C, and the two lines R and L. Since R and L are both construable, each of them must pass through at least two points. Without loss of generality I will assume that R is drawn through A and B, while L is drawn through B and C. Our attention turns to the line that is draw through A and C, which I will call x.

Since we can only construct two lines, x must be the same as either R or L. However, if we assume that x = R then A and B are on the same line as A and C which implies that all three points are on the same line. If this were true, then R = L which contradicts our original assumption that R and L are different lines. The same holds true for x = L by symmetry. Therefore, this construction is not mathematically possible. Euclidean constructions populate the Euclidean universe and proofs inform us of truths about that same universe.

One of the consequences of the axiomatic method is *reductionism*. Axiomatic universes necessarily dissolve into their constituent elements. Larger objects are created from combinations and permutations of smaller objects. It is no longer necessary for me to explain how to construct an equilateral triangle because I have already proven that such a construction exists. If at any point I need to equilateral triangle in order to build another object, I can just assume it exists and use the construction given above as justification. Likewise, constructions explain Euclidean objects. There is no meaning to a Euclidean object outside of its recipe for construction; objects gain value through their construction, and nothing more. Yet, deeper meaning did exist to the Greeks even if that meaning was never explicitly stated. The Greeks felt that axiomatic truth was inherently true of the real world as well. Euclidean geometry was not just a study; it was a full, rich, and powerful conceptual representation of space itself.

The Pythagorean cult was a group of Greek mathematicians who believed that the universe was itself a mathematical construction. Integer ratios, a mathematical object, construct musical scales. Pleasant pairings of notes share a definable numeric relationship to one another. Yet, this simplistic relationship explodes into infinite complexity as an artist complicates this simplicity during the construction a masterpiece: "Pythagoras therefore concluded that numerical ratios rule the laws of musical harmony-- and by extension, the entire universe. It was to become an idée fixe with the Pythagoreans, the cornerstone of their world picture" (Maor 19). The Pythagoreans believed that numbers, a concept so mathematically 'pure' that Euclid did not even bother to axiomatize, formed an axiomatic foundation of the universe, and that every object was logically constructed out of this foundation. Part of this belief rested on the common sense notion that the universe is fundamentally measurable; at some level, they believed that there existed an atomic unit that measured the universe.

In mathematics, the 'unit' is a small indivisible quantity that can be multiplied to form any other quantity. Metric distance is defined according the meter, an arbitrary distance that is used as a common point of comparison. Two meters is simply two one-meter units combined together. A half-meter is an integer ratio of a one-meter length. The Pythagoreans believed that given any two arbitrary lengths, there exists some unit that measures both lengths: in effect an axiom that constructed both objects. For example, if I were to hand you two pieces of string, one five centimeters long and another three and a half-centimeters long, we could say that the half-centimeter measures both lengths: the first being exactly ten half-centimeters, and the second being exactly seven. The idea also comes up frequently in our concept of physics. If I were to use a pencil to make two marks on a sheet of paper, it is intuitively easy for me to describe the marks by counting how many carbon atoms make up each mark. We can accept that there may be too many to count, but at some fundamental level we still like to believe that one mark may have exactly two billion graphite atoms, while the other only has only one and a half billion. Either way, the graphite molecules measure the pencil marks. The Pythagoreans' belief that the universe was fundamentally measurable extended to the works of Euclid as well. Anything that is constructible must exist within the real world and therefore must be measurable. Because they built an entire religion out of this simple intuitive truth, it is no surprise that people died when it was discovered that this assumption is demonstratively false.

Imagine a perfect right-angle triangle; also, imagine that the artist of this triangle was mathematically exact when measuring it out. The two sides adjacent to the right angle in this triangle are exactly one unit long. The remaining side, the hypotenuse, has a length that can be determined by using the famous Pythagorean theorem, which certain clay tablets suggest Mesopotamians discovered first (Moar 5-12).

$$1^2 + 1^2 = H^2 => 2 = H^2 => H = \sqrt{2}$$

The hypotenuse is exactly $\sqrt{2}$ units long. Pythagoras did not know anything about $\sqrt{2}$ or the irrational numbers, so the next logical question is to determine what integer ration $\sqrt{2}$ represented. As discussed above every ratio is in the form $\frac{a}{b}$ so equating that to $\sqrt{2}$ we get the following formula

$$\frac{a}{b} = \sqrt{2} => \frac{a^2}{b^2} = 2 => a^2 = 2b^2$$

Since $2b^2$ is an even number a^2 must also be an even number.

The Pythagoreans knew at the time that squaring an even number always produces an even number. Likewise, squaring an odd number always produced an odd number. The only way that a^2 can be even is if a itself is even. Since a is even, we can factor out the 2 resulting in a new variable $a = 2c$. Our equation then becomes,

$$(2c)^2 = 2b^2 => 4c^2 = 2b^2 => 2c^2 = b^2$$

This new equation is similar to the one we had above, and can be manipulated in exactly the same way. Since $2c^2$ is even this must mean that b^2 is even and therefore that b itself is even.

$$b = 2d => 2c^2 = (2d)^2 => 2c^2 = 4d => c^2 = 2d^2$$

We began with this exact same equation. To summarize:

$$\sqrt{2} = \frac{a}{b} = \frac{c}{d} | b = 2d$$

I have found two different values that measure $\sqrt{2}$ with a very important relationship. For every unit b that measures $\sqrt{2}$ there exists another unit d exactly twice as big as b that also measures it. The contradiction in this result is subtle but important. Any value that measures the hypotenuse of this triangle can be replaced with a unit twice as big. Now by the same logic I could also replace this new unit with another unit that is also twice as big. Eventually I will arrive at the completely ludicrous conclusion that I can measure an object using a unit larger than the object itself: I can use a car to measure a molecule. Thus, we have arrived at a contradiction. Unfortunately, the only mathematical resolution to this contradiction is to accept that $\sqrt{2}$ is not an integer ratio, and therefore is not a 'number' in the Pythagorean sense. Likewise, there is no unit that measures all three sides of this constructible triangle.

To understand this proof is to understand that certain important questions suddenly become unanswerable – for example, "How many molecules of graphite are needed to draw all three sides of a unit right triangle?" If Euclidean geometry does indeed represent an idealization of space, then a perfect construction would produce a line that cannot be measured by any, no matter how small, fundamental object.

The common representation of irrational numbers is that they are numbers with an infinite decimal expansion: i.e. π has an infinite number of decimal places. However, this is actually untrue. In reality, irrational numbers simply cannot be represented as decimals. They are definably not numbers under the Pythagorean definition, but they are constructible under the Euclidean definition and thus of space itself. The Pythagoreans were left with an uncomfortable choice: either

Euclidean geometry was wrong and these perfect constructions could not exist, or their own number theory was wrong and numbers were themselves an impossible idealized object.

> According to legend, the Pythagoreans were so shaken by the discovery that $\sqrt{2}$ is irrational that they vowed to keep it a closely guarded secret, perhaps fearing it might have adverse effects on the populace. But one of them, a person by the name of Hippasus, resolved to reveal the discovery to the world. Enraged by this breach of loyalty, his fellows cast him overboard the boat they were sailing, and his body rests to this day at the bottom of the Mediterranean. (Moar 28)

This proof exists as one of the most important proofs in modern mathematics, because it, and its results, acted as both a definition and a refinement of everything that was to come after. While this proof may have challenged the Pythagoreans' belief in a constructible universe, Euclid simply took it as another abstract obstacle that he needed to overcome. As a solution, he decided to use geometry and not number theory as the founding axioms of his mathematical project. The benefit of this is that number theory is fully constructible within geometry, but geometry is not fully constructible within number theory. Numbers can be explained through geometry, but geometry cannot be explained through numbers. Euclid could not provide an explanation as to why the irrationals existed, but he did not have to. They fit within the geometric model; any further explanation would just get in the way of this objective fact. The axiomatic method, designed to transcend human intuition, survived this catastrophic human failure and helped to propel Euclidean geometry, not number theory, to become the underlying framework for all of mathematics. Unlike the Pythagoreans' philosophy, Euclid's geometry has survived thousands of years and has driven the mathematical interest of virtually every mathematician since:

> Certainly no book has had a greater impact on mathematics than the *Elements.* The work has been translated into practically every language and reissued in numerous editions (by

> some accounts, it enjoyed the second largest number of editions after the Bible). Moreover, few works have had a greater number of written commentaries, and commentaries on the commentaries; a modern edition will have perhaps twenty pages of commentary for each page of original text. (Maor 34)

However, this proof also turned mathematics away from object-oriented thinking. The axioms of numbers are based on our concept of objects; we count objects. However, geometry is based on interaction. In this case, interaction won, and the bulk of mathematical research has since focused not on the objects themselves, but on how they interact. The mystery of the irrationals would linger for several thousand years, and only find partial resolution in the discovery of modern quantum mechanics.

Riemannian Geometry

Unfortunately, Euclidean geometry is not perfect. It contains one minor flaw. Another axiom, known as the parallel postulate, proved to be a lasting enigma in the mathematical community:

> If a straight-line falling across two (other) straight-lines makes internal angles on the same side (of itself whose sum is) less than two right angles, then the two (other) straight-lines, being produced to infinity, meet on that side (of the original straight-line) that the (sum of the internal angles) is less than two right-angles (and do not meet on the other side). (Euclid 7)

This is the parallel postulate: Euclid's fifth axiom. This statement tries to assert that there exists within the Euclidean universe an object we intuitively know as a parallel line. The problem Euclid had with this axiom is that it does not read like the other axioms; it reads like a construction, not a fundamental action. Parallel lines feel like an intuitively obvious aspect of geometric space, yet Euclid neither pairs this intuitive fact to some base operation nor proves it using the ones he had already defined. Moreover, Euclid could not throw the concept out because he needed it to prove several important conclusions in his geometry. To a mathematician, the parallel postulate absolutely demands a construction; however, no construction was found despite a thousand years of attempts. German mathematician Georg Friedrich Bernard Riemann settled the problem when he challenged the axiom in the nineteenth century.

When trying to deal with rogue axioms like the parallel postulate, there are always two options: construct it or demonstrate that it is contradictory. Riemann went with the second option. Since nothing could construct the parallel postulate, maybe he could construct a contradiction. He took this idea to its logical extreme by taking not just the parallel postulate, but also all of Euclid's axioms and replaced them with their logical opposites. In the Riemann geometric system, two points defined more than one line, lines cannot be extended infinitely in either direction, and every pair of lines will eventually intersect. I cannot overstate the absurdity of this geometric system, and at the time, mathematicians expected that Riemann would eventually find contradictions in it. Amazingly, Riemann found no contradiction. Even worse, Riemannian geometry proved to be a valid geometric and mathematical system that represents geometry on a curved surface. An airplane traveling from Edmonton to Hong Kong has an infinite number of straight paths to choose from, a runner traversing the globe, kayaking when necessary, will eventually return to the same spot, and if both the runner and the plane choose to circumnavigate the globe, their paths will always cross. Reimann, along with the work of Gauss, Lobachevsky, and Bolyai, proved conclusively that not only was it impossible to prove the parallel postulate from Euclid's other axioms, it could be replaced by just about everything else. Riemannian geometry proved that Euclidean geometry represented only one of an infinite number of possible valid geometries. Riemannian geometry demonstrated conclusively a sentiment that had been slowly growing in the mathematical community: that axioms were not foundational to the universe or even to mathematics itself. They were simply a way of describing small worlds that we ourselves made up.

> This outcome was of the greatest intellectual importance. In the first place, it called attention in the most impressive way to the fact that *proof* can be given of the *impossibility of proving* certain propositions within a given system…. In the second place, the resolution of the parallel axiom question forced the realization that Euclid is not the last word on the subject of geometry. (Nagel & Newman 9)

Riemannian geometry marked the end of an era in mathematics. Early mathematicians never had to question the foundations of mathematics because mathematics was itself axiomatic. It was just something that existed objectively to the universe. Nobody needed to question its completeness, correctness, or its consistency simply because each of these facts was self-evident and did not require proof. Questioning the consistency of the Euclidean world was just as absurd as questioning the existence of writing in a novel. However, Riemannian geometry turned this all on its head. Euclidean geometry was now one of many. Instead of it being the axiomatic set that defined the core of mathematical knowledge, it was a historically relevant, but fundamentally unimportant, possible universe in an infinite sea of possible universes. Mathematics was no longer a concrete study, but instead a language that transformed intuitive objects into mathematical objects.

Mathematical Consistency

Unfortunately, Riemannian geometry also symbolized a split in the mathematical community itself. Some embraced the arbitrary nature of axiomatic sets as a benefit to mathematics. It was a way of finally separating the human from the math. If an axiomatic set was only special because a human assigned it privilege, then by accepting and studying this sea of arbitrary axiomatic worlds we remove the human intuition of correct vs. incorrect systems and enter into a world of true mathematical purity. However, there were others, symbolized by Newton, who still believed that the universe was still somehow fundamentally mathematical, and the failure of one axiomatic set only set the stage for another more powerful set waiting to be discovered. However, remembering Pythagoras, both camps were quick to agree that human intuition is unreliable, and in order to move forward, and to maintain the purity of mathematics for all time, it was necessary to rebuild mathematics from the ground up.

A mathematical world is consistent if it is incapable of proving both sides of a contradiction. The iconic inconsistent system is the axiomatic set containing the single element “this statement is false”. The statement is a problem because as an axiom its truth implies its falsity; the first constructed object in that world is a proof that the axiom is itself false. Proof in any mathematical sense requires a firm separation between true and false statements, and therefore any mathematical system necessarily needs to be consistent. The consistency of Euclidean geometry had until this point been assumed; however, moving forward this assumption could no longer stand. It was no longer ‘good enough’ to assume the consistency of mathematics itself: “Since the Euclidean axioms were generally supposed to be true statements about space (or objects in space), no mathematicians prior to the nineteenth century ever considered the question whether a pair of contradictory theorems might some day be deduced from the axioms” (Nagel & Newman 13). Any new foundational axiomatic set needed to come with a proof of its own consistency. This need for an explicit proof originates from the fact that paradoxes can and have been constructed in even the most intuitively consistent system.

Unlike other mathematical systems discussed thus far, set theory has no definitive formulation. There are most certainly popular and unpopular formulations, but due to a philosophical dispute none of them holds the same weight that Euclidean geometry once had. One such formulation comes from Gottlob Frege. Frege was a supporter of a Platonic view of mathematics and thus saw mathematical objects as representing real world objects. Similarly Frege’s set theory tries to assert that physical element by defining sets using only one axiom, “Given any property P, there exists a (unique) set A consisting of those and only those things that have property P" (Smullyan & Fitting 11). There are no restrictions on what property P could be, and thus this formulation assumes that anything that we as humans can identify as common between objects can used to form a group of said objects. The problem with this set axiom is that it creates a paradox first constructed by Bertrand Russell in 1901.

Under Frege's axiom, we can define a set that contains all blue objects in a specific room, all apples on the planet, or even all words that begin with the letter Q. Sets can also contain other sets. Say I wanted to define a set that contains both me, and all the objects that are a part of my person. All of the clothes I'm wearing, the bags I carry, and the objects inside of those bags will be considered elements of my extended person. If I wanted to study the interaction of several people inside of a public location, like a subway system, I would need to define a new set that contained all the people I wished to study. However, people in my study are themselves a set that contains all of the objects on their person. Therefore, my study would contain sets that themselves contain other sets. This is where the problems start cropping up.

Imagine a set of all sets: a set that contains all valid sets under my singular definition. Because this set is itself a set, it must necessarily contain itself. Noticing that the set of all sets contains itself, we can now split every possible set into two separate groups: those sets that contain themselves and those sets that do not contain themselves. Classical groups like 'all the apples in a barrel' will not contain themselves because 'all the apples in a barrel' is not itself an apple. However, abstract sets like 'the set that contains all sets' count as a set that does contain itself. With the exception of the groups themselves, it is easy to sort every possible set into one of the two categories. Categorizing the groups themselves into their own categories is much more difficult. The 'set containing all sets that do not contain themselves' proves to be problematic. If we assume that this set does not contain itself then we are forced to sort it into its own category, implying that it does indeed contain itself. Likewise, if we assume that it does contain itself then we must sort it into the other category forcing it to not contain itself. Both are contradictions. Therefore, the entire axiomatic system is itself contradictory, and just like the statement, 'this statement is false,' this single intuitive statement that has been used to create all of set theory is itself contradictory.

In order to get around this self-contradictory notion of sets it became apparent that sets needed to be constructed from base elements no differently than Euclidean objects are constructed from base processes. The Zermelo-Franko (ZF) set axioms are one of several attempted axiomatic systems for set theory. They are the most cited, and therefore most successful, axiomatic set system. Like Euclid, the ZF axioms define a number of processes that build sets out of other defined sets. This axiomatic set avoids the contradictory nature of its more intuitive brother by being incapable of constructing a set that contains all sets. However, because the ZF axioms do not represent some intuitive notion of sets, it is difficult to defend them as being the final definition of mathematical sets.

One example of a set theory that deviates from ZF is Zermelo-Franko-Choice (ZFC) set theory. The axiom of choice is the assumption that for any set there exists a choice function for every object in that set. A choice function is any method for separating an entity from its set and representing it individually. The axiom of choice is the single most controversial axiom in set theory, if not all of mathematics, for good reason. The axiom of choice mirrors the parallel postulate in the sense that it intuitively feels like a statement about sets, instead of an action that constructs them. As well, valid set theories can be constructed without the inclusion of the axiom of choice. That argument about the inclusion of the axiom of choice has necessitated a separation of the ZF axioms into two separate universes: ZF set theory with choice excluded, and ZFC set theory which includes choice. Of course, a small rift like this is easy to exploit, and hundreds of possible alternatives and replacements have been suggested.

Mathematical Completeness

This brings me to the second desired property required of the new mathematical foundation: completeness. In order to choose which set theoretic framework is the correct set theoretic framework we need to find the axiomatic system that offers a complete description of mathematical sets. Completeness merely suggests that the axiomatic system is powerful enough to prove the existence of every true statement within that universe. A complete mathematical system should be able to find a definitive solution to every mathematical problem, while at the same time ignoring every non-mathematical problem. Likewise, completeness can be thought of as a definition of mathematical objects; if we can find an axiomatic set that constructs all proofs that mathematicians find interesting, then we can use that axiomatic set as a definition of mathematical objects. Completeness is the assumption that allows us to claim that a mathematical model can perfectly represent that which it is modeling: a complete set theory, then, should be able to solve every set theoretic problem, a complete physics should be able to solve every physical problem, and a complete topic modeling should be able to solve every topic problem.

Likewise, one of the most important set theoretic problems came to us from a German mathematician by the name of Georg Cantor. Cantor, born 1845, is the father of modern set theory and dictated many of the fundamental tenets that must be true before an axiomatic set can claim to be a set theory. Among Cantor's greater achievements is a constructed proof that infinity comes in several flavors. More importantly, he proved that the infinite set containing all of the rational numbers is necessarily smaller then the infinite set containing all of the irrational numbers; an observation that mirrors Euclid's discovery that geometry can construct numbers but not the other way around. I will discuss the details of this proof in the next chapter, but what is important now is a question that originated from this proof.

While important, Cantor's discovery presented a puzzle to set theorists. If the set of all irrationals is bigger than the set of all rational numbers, does there exist any set with quantities between these two infinities? Cantor hypothesized that such a quantity could not exist, but was unable to provide a proof of that conjecture. The problem became known as the continuum hypothesis (Smullyan & Fitting 9). In 1940 Kurt Gödel provided a conclusive, but unsatisfactory, answer to this problem.

Kurt Gödel's contributions to mathematics cannot be overstated, "The character and achievement of Gödel, the most important logician since Aristotle, bear comparison with those of the most eminent mathematicians" (*Dictionary of Science Biography* 17: 348). Born in 1906 and raised in Vienna, Gödel was interested in the more bizarre questions that sat at the root of mathematics: mathematical discourse itself. One of Godel's accomplishments is a demonstration that the ZF axioms were not powerful enough to disprove the continuum hypothesis. Paul Cohen finished the proof in 1963 by proving that neither assumption, true or false, would result in a contradiction, therefore proving that the continuum hypothesis is completely independent of the ZF axioms. Just s in the case of the parallel postulate and the axiom of choice, picking a solution for the continuum hypothesis would produce two different, but completely valid, set theoretic worlds. Thus Cantor's Continuum Hypothesis became a hole in mathematics itself, a space between these two infinities that modern mathematics currently has no way of probing.

Dealing with independent quantities is a legitimate problem. Logically we can treat them in a number of ways. If we, like Euclid and the parallel postulate, tried to accept the ZFC axioms as the axiomatic foundation of mathematics, we could easily throw out the continuum hypothesis as being non-mathematical and simply just move on. If the abyss is independent then it is clearly outside the world we are interested in studying. Yet failure to find a solution does not itself rule out the possibility that another, more powerful, axiomatic set can construct it. Contradiction and independence are related, but opposite problems. Contradictions imply that both sides of a binary can be demonstrated. Likewise, independence implies that neither side of a binary can be demonstrated. If there is to be a new foundation to all mathematical study, then it must contain neither of these entities; bivalence must be absolute, and these entities present exceptions to that rule which need to be addressed. Guessing random axiomatic systems is an inefficient way of solving this problem, and it is now necessary to prove that such a logical system even exists.

The ‘Hilbert program’ is a mathematical framework developed by David Hilbert, a proponent of a ‘formalist’ philosophy of mathematics who had been inspired by Alfred North Whitehead and Bertrand Russell’s attempt to rebuild all current mathematical knowledge from a single axiomatic foundation. The Hilbert program, in its most basic formulations, is an attempt to contain all of mathematics, including its philosophy, inside a single logical structure. Hilbert formally proposed a weaker version of his program among an influential list of 23 important unsolved mathematical problems. “When we are engaged in investigating the foundations of a science, we must set up a system of axioms which contains an exact and complete description of the relations subsisting between the elementary ideas of that science” (447). Once again, Gödel answered this challenge with a proof that rocked the very foundations of mathematics itself. Gödel’s incompleteness theorem is a proof that no axiomatic system can both be complete and consistent. The details of his proof are complicated, and reproducing it here in full would be entirely impossible. However, I do wish to convey the spirit of his proof as present by Ernest Nagel and James R. Newman.

Principia Mathematica (PM) is the axiomatic system designed by Alfred North Whitehead and Bertrand Russell for use in their monumental three-volume book by the same name (not to be confused with Newton’s *Principia Mathematica*). The project was never completed, due in part to the difficulty of the project, the outbreak of World War II, and Gödel’s result, which proved that the main goal of the project was logically impossible. Gödel originally constructed his proof within the framework of the PM axiomatic set; however, the mathematical community has since accepted Gödel’s result as applying to all deductive formal systems. Gödel’s proof both recognizes and takes advantage of the separation between objective mathematical systems and subjective mathematical language, as discussed in Chapter One. He took this separation one step further by modeling language about mathematics as a proper mathematical system. Gödel recognized that every meta-mathematical statement is also a sequence of symbols that can be viewed as proper mathematical objects. The interpretation of these symbols remains within the framework of meta-mathematics but the symbols themselves were objects inside the PM mathematical world. For example the statement 'x proves y' is also a string containing ten symbols. Likewise, all three of PM statements, English language texts, and large numbers can all be thought of as longs sequences of symbols which themselves need to be interpreted. Gödel created a system that translates logical statements in PM into a unique large number that represented that statement, and by doing so showed that every logical statement has a constructible pair inside of number theory. In short, he created two objective systems, mathematical deductive logic, and mathematicians talking about mathematical deductive logic, and then showed how to translate one into the other.

The genius behind Gödel's proof is his strict separation between objects and language about objects while at the same time recognizing that language about objects can itself be separated into objects and language about objects: in order to argue that x is true, I need to produce a long series of symbols that prove x to be true. Gödel numbers allow us to reinterpret PM statements as statements about number theory; every PM proof is a number theoretic construction. Gödel continued the construction by using two previously known valid PM functions. Dem(x, y) is true if the statement with Gödel number x is a proof of the statement with Gödel number y. Sub(x, y, z) returns the Gödel number of the statement that is constructed by finding the statement with Gödel number x and replacing the symbol represented by Gödel number y with the statement represented by Gödel number z. Using the laws of PM, and this mapping from statements into numbers, he constructed an object in PM that looks like this:

$$sub(n, 17, n) = \ g(\sim(\exists x) \mid Dem(x, \mathrm{sub}(\mathrm{n}, 17, \mathrm{n})))$$

This equation loosely translates to the English language sentence, "The proof that this statement is true is not constructible." I recognize that most humanists will not have followed how Gödel created this object; however, the important point is that it is a valid and constructible member of the PM axiomatic set. This object exists in all valid formal systems. If we assume that PM can either prove or disprove this statement, we arrive at a contradiction. Therefore, we must conclude that either this statement is insoluble inside of PM or else that PM is inconsistent. Through a bizarre twist of intuitive logic, both of these contradictions imply that the above statement is true.

Gödel's proof is itself a proof that the statement is true. However, Gödel's work acts on the PM axioms but does not act within them. The proof itself exists on the linguistic level and shows that there is at least one true statement within PM that PM itself cannot prove. In essence PM is incomplete and, more importantly, all possible constructive axiomatic systems are also incomplete. The very existence of this kind of argument completely eradicates the possibility of collapsing mathematical language into a self-contained mathematical world. Yet, it also serves as a demonstration of mathematics' inability to filter its own ideas. The complete spectrum of perspectives that Gödel used in his proof is dizzying; however, he still proved a statement. Demonstrating conclusively that no matter how 'pure' pure mathematics is, Gödel showed that our entire concept of proof is still mathematical ideas filtered through the human perspective. The community has accepted his proof and it is now a part of mathematical canon. Along with completely derailing Hilbert's project, Gödel's proof has created some lasting chaos in the mathematical community and has raised important questions that we still don't have answers for. What truly exists at the foundation of mathematics?

Conclusion

There exists another important statement for both rationality and mathematics, but it is not a structural axiom. Occam's Razor states that, given any set of competing hypothesis, the simpler explanation is the correct one. In mathematics the statements represents the subjective quality of mathematical beauty. A mathematical proof is beautiful when it is small, concise, and easy to understand. Mathematical beauty also has a practical side; it is easier to spot mistakes in smaller objects. In 1976 when Kenneth Appel and Wolfgang Haken presented their computer assisted proof of the 'four colors problem' (the mathematical hypothesis that any two dimensional boundary map can be colored using only four colors), the Toronto crowd was stunned -- not only because a solution had been found to an old mathematical problem, but also because of how extraordinarily ugly it was.

> Mathematician after mathematician expressed uneasiness with a proof in which a computer played a major role. They were bothered by the fact that more than 1000 hours of computer time had been expended in checking some 100,000 cases and often suggested (hoped?) that there might be an error buried in the hundreds of pages of computer printout. (Wilson 216)

The proof that Appel and Haken had supplied was large, complicated, and impossible to verify by hand at the time; however, it was still a valid proof. Hilbert's project was also large, unwieldy, and completely inaccessible to anything but the most devout logician; it was also wrong. Unfortunately, the razor itself has no way to distinguish between these two concepts because the details of the both are complex. However, even though it does not drive proof, the concept of mathematical beauty still drives mathematical research. Even though the four colors problem has been 'solved' there is still serious research going into the pursuit of a simpler, more 'intuitive', proof. From a formalist perspective, the concept of beauty is 'non-mathematical' because it does not represent any distinct rules that are independent of the subjective mathematician; however, any description of mathematics is still incomplete without some description about why mathematicians prefer beautiful concepts. Likewise, all philosophies of math are incomplete for similar reasons. They all focus on a single portion of mathematics and make the claim that that portion constructs everything else. Intuitionism sees math as a pure mind game, yet cannot explain the success of mathematics as a means of explaining universal phenomenon. Likewise, Platonism jumps to the other extreme by claiming that math is an objective phenomenon and ignores the communal, subjective aspects of mathematics. Mathematical beauty is about understandability, and understandability is intimately connected with both the human mind and the human environment.

Any defined form of demonstration is then a small world. In practice small worlds are powerful entities capable of realizing truth much faster and more efficiently than any method of non-formal demonstration. However, small worlds are also demonstrably incomplete. A small world or any system of demonstration is fundamentally limited in the kinds of truth it can produce: humanist methods produce humanist knowledge and mathematical methods produce mathematical knowledge. Those truths that a small world is capable of demonstrating can be extracted quickly and efficiently, giving the entire system a favorable aura: especially to results-based funding sources. However, the main complaint I hear from the humanities and religion about mathematical and demonstration-based sciences, is that it makes assumptions about the world while ignoring a much more subtle truth. Mathematics itself both supports and justifies this concern. Small worlds deny that which they cannot explain, and as Gödel pointed out, there are will always be important facts about a small world that cannot be explained by them.

What does all of this have to do with DH? Depending on which definition of humanities computing you prefer, the answer could be very little or a lot. Computers are a personification of mathematical worlds. Whenever you translate text into a form that computers can read, you are transforming it into a mathematical object. Like Gödel's numbers, text exists on two different levels: a long sequence of symbols, and the ideas represented in that text. Gödel's proof is relevant because it demonstrates what humanists already know: that text is more than the sum of the symbols that constructs it. Likewise, Gödel's proof also demonstrates that mathematics as well is more than the sum of the symbols that construct it. Computers and all mathematical worlds operate on the symbols themselves, not the ideas that those symbols represent. Therefore, infographics, visualizations, computer analysis, computer interpretation, and all other forms of computer models of text are incomplete. The fear that numerical representations misrepresent text is justified.

However, this incompleteness cannot be eradicated by changing the representation, or denying the link to mathematics altogether; it is intrinsic to all defined systems of representation. However, to jump to the other extreme and state that these representations have no value also implies that classical humanist methods are themselves complete. Is this true? I cannot answer that, but I can say that the fact that DH exists in the first place implies that computers offer something that conventional humanist methodology cannot.

To conclude I will once again return to the question proposed in the first chapter. Why is mathematics useful? Mathematical systems were designed to have no connection to the outside world, and exist entirely independently of the human mathematician. However, this description of mathematics is wrong. We can also think of mathematics as an objective entity, a being that sees the world in a certain way, and through mathematical methods we humans learn more and more about how that objective entity thinks: this interpretation is also wrong. The truth lies somewhere in between. Mathematics should be useless, because it has no clear way of demonstrating what value it has. Those who demand full rational justification should have abandoned mathematics long ago. Yet, the success of Newtonian physics and other successful mathematical models have continued the legend that somehow mathematics is useful. In fact, even I find myself incapable of denying it. Formality is incomplete, but this says nothing about the human mind. Even though Gödel's statement is formally undecidable within PM, he still managed to prove it. How? Why can we come up with new valid methods of proof when it is has been demonstrated that nothing formal can generate them?

This is why I strongly question fundamentalism or structuralism of any sort. The single worst perception about mathematics is that it is somehow pure, somehow grounded on a firm, logical, and completely rational basis. Mathematical systems actually make less sense the deeper you dig into them. Yet, like many academics who have struggled with this same question before me, I have no choice but to conclude that at some level it is working anyway. For the vast majority of mathematicians, mathematical philosophy is unnecessary because mathematics justifies itself. Unlike science, which loves to talk about why science is important, mathematics cares only about mathematical worlds and very little else. We do not need to know where it came from or how it creates meaning. We only need to know how it operates and how to participate, and both of these problems are significantly less complicated than self-justification and meaning creation[1]. Even more importantly, every failure in mathematics also signals an expansion of mathematical thought. The failure of Euclid marked the beginning of new axiomatic systems previously unheard of. The failure of completeness marked the beginning of the mathematics of the mind and the incorporation of topics like artificial intelligence, decision making, and complexity that math was previously incapable of approaching. Every failure becomes a triumph as we slowly become aware of a bigger, more complicated, and more interesting world.

My interest in mathematics is that it is a gateway between the three main branches of thought. It is a science used constructively to control and model the universe around us, it is an interpretation of the limits of human intuition and imagination, and it is a religion that constructs undemonstrative worlds. My personal definition of the axiom is only obtainable by combining both the ancient and modern definitions. Unlike the Greeks, I do not believe that axioms contain any level of inherent unarguable truth, because what is unarguable today will not remain unarguable tomorrow. However, I also do not accept that mathematics is universally neutral. As inexplicable as it is, there is a link between mathematical universes and the real universe that they try to represent. There is a structure to the universe, even if it is only accessible by regularly tearing down the walls we keep building for ourselves. The answer must lie somewhere between these two quantities -- somewhere between the small worlds that rationality creates, and the large world that regularly defies them.

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[1] Although, Rotman talks extensively about a modern semiotic / linguist interpretation of mathematics that is worth exploring.

Chapter 4: The Mathematics of Limitations

> Dedekind said, with respect to the concept of set, that he imagined a set as a closed sack that contains completely determinate things – but things which one does not see, and of which one knows nothing except that they exist and are determinate. Somewhat later, Cantor gave his own conception of a set. He drew his colossal figure upright, made a magnificent gesture with his raised arm, and said, with an indeterminate gaze: 'A set I imagine as an abyss'. (*From Kant to Hilbert* 836)

Zeno, an ancient Greek philosopher, proposed a series of paradoxes that he intended as proof that our concept of motion was an illusion. Imagine a runner standing at the starting point of a race. Once the race begins, but before it finishes, the runner must cross the midway point of the race. This task requires a finite amount of effort distributed over a finite period of time. Once the runner crosses the first midway point he is again tasked with completing half the remaining distance before he can finish the race; a task which again requires a finite amount of effort distributed over a finite period of time. Every subsequent half-length of the race marks the beginning of a new half-length, which eventually generates an infinite number of half-races that he needs to complete before he can finish. Zeno questioned how it was possible for the runner to complete the race when there were clearly an infinite number of obstacles standing between the runner and the finish line. Since no man can overcome infinity, Zeno concluded that the man should be incapable of completing the race.

Likewise, it is possible to manipulate Gödel's argument from the previous chapter in order to make a similar claim about the universe. Static axiomatic systems are incomplete and unable to answer every question one might ask about a given system. Like the runner's infinite series of obstacles, mathematical science also faces a seemingly infinite number of mathematical systems, or 'paradigms', as Thomas S. Kuhn would label them. Each paradigm shift represents a change in both the structure and the perspective of the mathematical model expanding the ideas that that model can represent. In the case of physics, each of these paradigms follows a fairly linear sequence of mathematical models derived from 'first principles': Newtonian physics, Einstein's relativity, Quantum mechanics, and string theory (this, of course, is a massive oversimplification). Each model, according to this loose interpretation of Gödel, must be incomplete, and will always fail as a complete model of the universe.

As an argument against motion, the argument and its derivatives are easily invalidated. Legend has it that a fellow Greek philosopher, Diagnoses, rebutted Zeno's argument by simply standing up and moving freely around the room. Likewise, the scientific response to any challenge about the motion of scientific, or mathematical, advancement is identical. From the perspective of the engineer, the problem of incompleteness is a simple matter of decimal places. Actual practical problems never require true mathematical precision. Numerical models only ever need to be 'good enough' to accomplish the task. An architect will never need to know the mathematically precise weight that a certain foundation can handle; it is only necessary to know, within safe tolerances, that the foundation is strong enough to hold the building on top of it. Advancements in mathematical models produce better predictions about the world, which in turn produce higher towers. A single mathematical model might be able to design with confidence a three-story building, while another thirty, and yet another three hundred. At no point will the architect need to produce an infinitely tall building, which results in the general uselessness of infinite precision. In the past humans could not build fly, but today we can. In the past humans could not live in space, but today we are. Today humans cannot reliably cure cancer, but in the future, we will.

Gödel cripples normal science working on mathematical models because its goal is to fill out missing details in the current paradigm; however, there will always be details missing. Nevertheless, proponents of scientific advancement will quickly point out that science is not about the dogmatic acceptance of a single theory; instead, science represents a furnace that boils out the impurities in a constant flow of ideas. David Deutsch goes so far as to label all scientific paradigms as misconceptions: "I have often thought that the nature of science would be better understood if we called theories 'misconceptions' from the outset, instead of only after we have discovered their successors. Thus we could say that Einstein's Misconception of Gravity was an improvement on Newton's Misconception" (446). Deutsch claims that it is not the ideas that science produces that justify its importance to humanity, but instead the process itself, a framework through which bad ideas can be tested and ultimately discarded: a statement with which I wholeheartedly agree. The scientific process is independent from both the ideas it generates as well as any individual paradigm the community happens to accept. The ideas that science produces may be individually incomplete, but the claim is that science as a methodology is not. As each paradigm is tested and replaced, during one of Kuhn's scientific revelations, our knowledge improves and brings us closer to a truer representation of the universe. Every time the runner finishes a half-length, he transforms something previously impossible into something now possible. Like Diogenes, if faced with any argument that the runner is not moving, the scientist need only point upwards at the passing international space station as definitive proof that science is indeed in motion. Yet, some would also argue that the comparison is wrong.

Zeno's argument is a logical fallacy, and the runner will indeed finish the race. Deutsch argues that science faces no such barrier and can expand infinitely, which is a concept he built into the title of his book *The Beginning of Infinity*. From his perspective, infinity is not a limitation but a challenge. There is nothing standing between the scientist and infinity, nor is there a reason for the scientist to stop running. Given enough time, science will eventually solve every problem that dares stand in its way. The fact that the scientist will never reach the end of his race I, at worst, an inconsequential limitation of decimal points, and, at best, definitive proof that science provides a permanent benefit to humanity. Thus, science is always at the 'beginning of infinity', as Deutsch phrases it, and constantly traveling towards, but never arriving at, some ideal scientific utopia.

<u>Sublime</u>

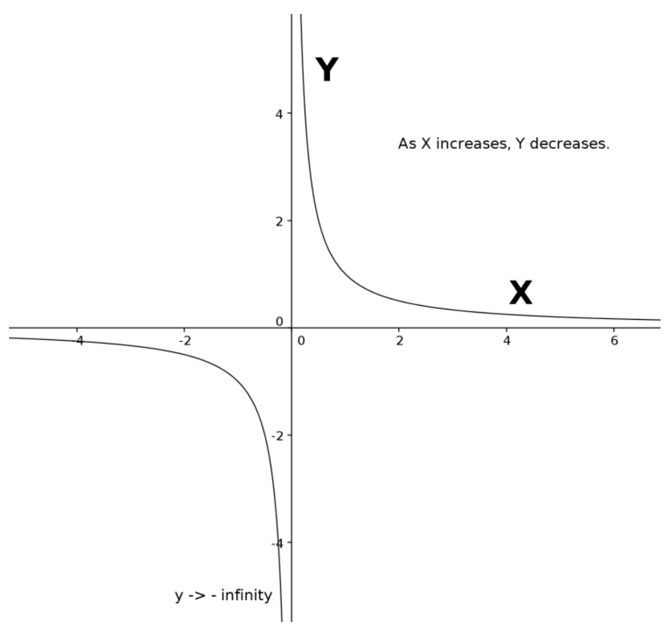
Infinity in the humanities is a different concept entirely, mostly because it does not contain an explicit definition of motion. Philip Shaw offers the following partial definition of the humanities' analogue of infinity: the Sublime. "Whenever experience slips out of conventional understanding," writes Shaw, "whenever the power of an object or event is such that words fail and points of comparison disappear, then we resort to the feeling of the sublime. As such, the sublime marks the limits of reason and expression together with a sense of what might lie beyond these limits" (2). Unlike scientific infinity, this definition of 'sublime' bundles together both a concept of infinity and a concept of limitations. The sublime exists beyond what we can perceive and imagine, and any attempt to represent it is doomed to fail; however, knowledge is gained through that failure. Motion in the humanities is also much more difficult to define, as the humanities generally do not construct material objects, like the International Space Station, that can be proudly displayed as material proof of progress. Problems of antiquity in the humanities are still problems of today, and absolutely nothing has been, or ever will be, solved conclusively. Motion in the humanities is not a runner, but instead a large stack of books. Each scholar's contribution to humanist discourse represents a new entry into an expanding corpus, but all the observer sees is a mountain of books. From a distance, this mountain of knowledge is easy to spot, but it is extremely difficult to make any guesses as to where, if anywhere, it is going. Unfortunately, this makes it easy to claim that it is not going anywhere. And from a certain perspective, that is true. The humanities seek the sublime, but since the sublime cannot be demonstrated explicitly, the humanities must seek failure in order to progress. Diogenes cannot look at a pile of books and claim instinctively that it is moving. The motion in this example is the attempt to understand the books, not the physically manifested mountain.

Motion in DH is an important question because a large portion of DH creates objects in a very literal sense: archives and libraries produce databases, text mining produces numerical values, and computer visualizations produce computer code. However, it is important for DH scholars to ask themselves how they define motion. Is DH like science in the sense that it can point at those things that it produces as justification that it is moving, or is the concept too materialistic to be the real value of DH? I feel that the answer to this will dramatically vary depending who is responding. As a programmer and a mathematician, I will happily accept that the material form of my work justifies my own motion and will gladly produce material objects as proof that I have accomplished something. To at least a small extent, I feel that digital designers, performers, and artists would agree with me. However, I know of plenty who would scoff at the idea that visualizations are themselves the reason we build visualizations, that the existence of an archive justifies making archives, or that the existence of the International Space Station justifies the science that put it there. As usual, both sides represent a fundamentalist extreme. Any real justification for work in any field would practically make use of some mixture of both perspectives. Likewise, DH as a mixture of science and humanities forces itself into a perspective somewhere between infinity and the sublime. However, as a mathematician, I dislike invoking either term so casually because mathematical infinity is a concept filled with subtlety and any simplification will inadvertently lead to misconceptions and logical fallacies.

Limits

Introductory calculus is taught at the high school level, and a minimum of first-year calculus is an almost universal requirement for science students at any level. As well, calculus is mostly driven by applied work and is therefore not generally taught axiomatically. The success of calculus is well deserved. Invented alongside Newtonian physics, calculus is the language that joined together math and physics. Calculus is the language of continuous relationships, a relationship that mimics many properties of nature. It not only provides the equations that represent continuous relationship, but also provides the means to calculate them, simultaneously giving science a way to model physical relationships as well as predict the outcomes of experiments involving them. Unlike number theory, which begins with numbers, calculus primarily operates on functions, which are relationships between numbers. Calculus is similar to geometry in that it does not operate on any assumptions regarding the measurability or atomic nature of the elements it manipulates. A function is a generalized mathematical object that represents the relationship between one object and another. For example, the relationship between a child and an adult is a 'growth' function. The function of 'dawn' transforms night into daytime, and its inverse 'dusk' transforms day back into night. 'Time' is a function that transforms the future into the past. Likewise, there is also a function connecting the number of sun tanners in one country with the number of workplace safety violations in another, even if it not causal as well as relatively uninteresting. (The study of statistics can be thought of as an attempt to extract hidden meaning from these types of relationships.) Calculus is not interested in the objects themselves, but instead the properties of relationships that these objects create. Functions, not the objects they represent, are the main focus of calculus, making it the perfect fit for a process-centered study like physics.

Figure 4: Graphing a function.



Take the function $f(x) = \frac{1}{x}$ as an example. This function operates on real numbers, which means it takes as input any real number and transforms it into another real number. (See figure four.) The function f operates on x and transforms it into y. As we increase the input value x, the resulting y decreases. If x = 10, then y = 0.1. If x = 100 then y = 0.01 and so on. There is also a commonality between this function and Zeno's runner. Every time x increases, y is forced closer to an unreachable finish line. For every y value even slightly above zero, such as 0.0000000001, we can calculate the x value that will create it: 10000000000. However, at no point will any x value produce a y that is equal to zero. The formula $y = \frac{1}{0}$ has no solution because it is not constructible within this function. More importantly, zero is exactly the first such value that is not constructible.

Zero marks the *limit* of this function, it acts as a barrier, and everything larger than zero is constructible. Likewise, we can use negative x values to produce any value less than zero. Zero, the only real number not constructible within the function, acts as a *singularity* within this function. Like Zeno's finish line, zero is a value that interacts with the function but is invisible to it. The word 'singularity' in this context is a point of ambiguity, a value that offers no insight whatsoever into the nature of its context. Zero is a kind of tear or rip in the function as it represents a different object depending from which direction we approach it. Approaching from the right causes the function to explode to infinity, but approaching from the left causes it to explode towards negative infinity. Several possible interpretations exist. I can think of the singularity as a process that teleports the value from negative to positive infinity crossing every intermediary point in a single instant, or I could think of the singularity as a line that wraps around the universe and crosses some ideal point in order to transition from positive to negative infinity. I can speculate all I want because each interpretation exists independently of the object itself. The singularity verifies none of the interpretations, and it is precisely this fact that makes it a singularity. I could not even invent a value to represent this function at zero because the true value at this point is simultaneously somewhere, nowhere, and everywhere at once. To express this idea from within the language of calculus we say that zero is the limit of $y = \frac{1}{x}$ as x approaches *infinity*. Mathematicians use limits to express the fact that the function is traveling towards zero. Some functions actually arrive at their limits, but $f(x) = \frac{1}{x}$ does not. This function's limit is forever beyond its reach.

Zero may not be constructible from within certain functions, but it certainly is constructible from within larger mathematical systems such as the Cartesian plane. In this larger system, there is nothing infinite or unreachable about zero. The same goes for Zeno's runner. During his argument, Zeno creates a function that equates the number of half-races the runner has completed to the entire length of the race; the full length of the race represents the limit of the accumulation of half-races. Just like $\frac{1}{x}$, we can calculate exactly how many half-lengths are necessary to finish any distance less than the full length of the race. However, the end of the race itself is a singularity and exists beyond Zeno's constructed perspective. Assuming it takes the runner half an hour to finish the first leg of the race and that he maintains a constant velocity, we can surmise that it will take him roughly one quarter of an hour to finish the next leg of the race, an eighth of an hour to complete the next, and so on. Just as the limit of the accumulation of half-lengths of the race approaches the full length of the race, so to the limit of accumulations of half durations approaches one hour. From within Zeno's function the one-hour mark is also a singularity, the function cannot actually reach an hour, and therefore cannot see what the runner will do beyond this period. Yet, from a larger perspective nothing singular happens at the one-hour mark. It comes just as the runner finishes the race. The unmentioned corollary to Zeno's argument is that if the runner cannot finish the race, then the universe itself is incapable of crossing the one-hour mark. More specifically, if motion does not exist then neither does time. As a description of the relationship between location and time, i.e. motion, it fails miserably at accurately describing either of those quantities. Is there a deeper philosophical question associated with Zeno's paradox? Yes, but only in the sense that the paradox itself is blind to the concept of motion and therefore incapable of telling us anything meaningful about it.

'Infinity' is an umbrella term that represents an extremely large number of mathematical objects, entities, and processes. Unfortunately, human intuition tends to think of infinity in terms of 'everything'. We like to view infinity as a perspective that sees everything and a container that holds everything. The fallacy here is that infinity itself is a concept built by humans, and it cannot represent an idea we ourselves cannot understand. From the mathematical perspective, infinity does not represent the existence of knowledge, but instead symbolizes its absence. Zeno abused infinity by using it to hide the runner's final destination; once hidden he asks us to accept that the finish line does not exist. Inside his constructed perspective, the limitations of the race suddenly vanish; because of this, he asks us to accept that the race will continue forever without end. He denies the existence of motion based solely on his own inability to understand motion. Likewise, all uses of the word 'infinity' carry with them the same degree of ambiguity. I could easily rework the argument to say that the runner is now immortal and can run forever. With this newfound freedom, I argue that the runner, free from the shackles of mortality, is free to run towards and eventually to reach every location everywhere, nothing beyond his reach. However, such an argument is still flawed. Even now, the poor fool is incapable of flight and therefore trapped on the planet earth. He may be convinced that he is now free of his limitations, but the simpleton will never be able to reach orbit, and as long as he believes he can access everything everywhere, he has no reason to ever try. So too by arguing that science represents some beginning of infinity, Deutsch is also obscuring the possible limitations of science's reach. Asking us to believe that science can solve everything and anything simply because it goes on forever is no different then arguing that an immortal runner can reach every location everywhere. The key difference between Gödel's theorem and Deutsch's infinity is that Deutsch defines science as a process and not an object. We know the limitations of static systems, but we need more information before we can translate that observation to processes.

Calculus was not available to the Greeks when Zeno first presented his paradoxes. Aristotle was already aware of the problems associated with mixing mathematical objects from two different systems. The Pythagoreans, through their demonstration of irrational numbers, had already shown that properties of magnitudes were not necessarily compatible with properties of numbers. Zeno's argument relied heavily on the assumption that splitting a line in half infinitely and counting to infinity represented the same object. Just as a person cannot count to infinity, jump over an infinite number of hurdles, or run for an infinite number of seconds, so too humanity shouldn't be able to run an infinite number of lengths. Yet, in all but the last of those cases the infinite entities are inherently measurable. We can count numbers, hurdles, seconds, and measured lengths, but as the Pythagoreans proved in the previous chapter, not all lengths are measurable and therefore countable. Intuitively the only infinities that humanity cannot overcome are those that can be measured, yet lengths aren't always measurable and experience tells us that it is indeed possible to perform an infinite number of immeasurable tasks. "Hence Zeno's argument," said Aristotle,

> makes a false assumption in asserting that it is impossible for a thing to pass over or severally to come in contact with infinite things in a finite time. For there are two senses in which length and time and generally anything continuous are called 'infinite': they are called so either in respect of divisibility or in respect of their extremities. So while a thing in a finite time cannot come in contact with things quantitatively infinite, it can come in contact with things infinite in respect of divisibility. (Aristotle 6:2)

At the time Aristotle didn't have the language or the mathematics necessary to discovery why these infinities were different, but he did recognize a bad model for what it was: a bad model.

Aristotle's lasting contribution to infinity, however, is not his explanation of Zeno's error, but in his denial of the concept of infinity altogether. Every line can be split, so the process of splitting a line only ends when the mathematician decides to stop splitting the line. Zeno's argument, however, requires us to imagine a line that has already been split in half an infinite number of times. In a separate work, *Metaphysics*, Aristotle discussed the difference between 'potential infinity', a process that does not ever need to stop, and 'actual infinity', the value such a process arrives at once it has completed. Aristotle, through Zeno, recognized that artificially created infinite entities were not comparable to other more valid mathematical objects; likewise, there was also no known way to produce such a comparison. The very concept of infinity acted like a singularity from within the study of mathematics itself. Anything produced using it was ambiguous because infinity masked the absence of knowledge. Worse, Aristotle did not even believe that such a concept was capable of existing inside the human mind. Unlike geometry, infinity was a concept too big to fit inside the human head, and any attempt to represent it would be incomplete. Any attempt to talk about an infinite object constituted an attempt to squeeze a transcendent idea into a human-created container and was doomed to failure. Likewise, Aristotle placed a ban on the concept of infinity. We can talk about infinite processes and arbitrary large, or small, finite objects that those processes create, but any reference to a complete infinity is itself a mistake and has no place within logic and mathematics. When we say that the limit of $y = \frac{1}{x}$ is 0, we are only saying that the term zero represents the behavior of the function as x explodes; we are not saying that zero is an actual value represented by the function.

Limits are not unlike how an astronomer would study the sun's corona. Unable to point a telescope directly at the sun, the astronomer would use a strategically placed black dot on the lens of the telescope that blocks all of the direct sunlight from the sun's core allowing the telescope to take in only the light from the corona. Mathematicians designed limits to block out the infinite value itself, while revealing the behavior of functions very close to it. In the particular case of $y = \frac{1}{x}$ we got lucky; the limit of that function converges on a mathematically unambiguous value. In any other context zero is a viable quantity, so it makes sense that we would mistake the limit of this function for an actual mathematical quantity as well. However, we still must recognize that the limit of this function and the value zero are two different mathematical entities that just so happen to share many of the same attributes. The limit of x as y approaches zero is much different. In this case, x diverges to infinity. Infinity, according to Aristotle, is not a valid quantity. So apart from informing us that the sequence never stops getting bigger, a potentially infinite process, we can infer nothing more about its behavior at this limit.

The Problem of Irrationality

If Aristotle's rule were still held today, my story would end abruptly here; thankfully, it does not. Mathematics still had to wrestle with the existence of another bizarre, yet important, mathematical object: π. Notoriously, π is an irrational number and likewise its decimal expansion contains an infinite number of numbers. However, if we deny that an infinite number of decimal points can exist, we are left with a weird inability to accurately represent a value that the community has already agreed is valid.

Anyone even remotely familiar with mathematics will eventually face down the identity 0.999... = 1. This curious mathematical fact generates a lot of challenges by students, and tends to be a banned topic in popular mathematical forums (specifically the xkcd.com mathematical forum) due to its powerful ability to spawn long, heated, and pointless arguments. Intuitively 0.999… just feels smaller than 1. Any student would be hard-pressed to find any value between 0.999… and 1, but that only means that it is the closest value less than 1. Yet, the way we have defined decimal expansion makes this intuition completely wrong. The problem lies in the convergent properties of decimal expansion. The number π is not representable by decimal expansion, but every measured correction brings us closer to the true value of π. Likewise, we can write out a potentially infinite series with a limit at π. The limit of this sequence and π are still not the same entity, but as long as there is no ambiguity, there is no reason why we cannot use the limit to represent the object. However, it is no good for our definition of value to be so broken. There is no symmetry in defining 1 as being 1, and π as being the limit of its decimal expansion. So in order to standardize the system, the mathematical community agreed to define all numbers as being the limit of their decimal expansion. Under this definition, 0.999... is a potentially infinite series where every correction brings it closer to its limit: 1. Likewise, the infinite series 0.5000000… converges on 0.5 (the trailing zeros are left out as a matter of convenience). Decimal expansions themselves represent other values, and do not exist as values themselves.[2]

However, if we forget this definition for a moment and think of infinite decimal expansions as a potentially infinite process then it is easy enough to see that 0.999… could be less than 1; at some point, the mathematician writing out the value will give up and produce a final nine. The process might go on forever, but at all times during the process of writing it out there will always be an enigmatic final nine that guarantees that the final constructed value is definitively less than 1. There is nothing actually wrong with this reasoning as it can be used to produce a completely valid mathematical universe; however, this system has no way of representing irrational, and is therefore not how modern 'real' numbers were defined.

Two terms are important when discussing mathematical infinity: 'convergence' and 'divergence'. Functions converge when their limits represent a mathematical value that the community accepts. Functions diverge when their limit represents an ambiguous value. For example, the sequence 1 + -1 + 1 + -1… diverges as it is forever trapped jumping between one and zero never arriving at either. Likewise, the function 1/x also diverges for x, as it explodes towards the ambiguous value ∞.

Those in mathematics like to think of limits as a very concrete concept, and from a meta-mathematical perspective, they are. However, when looking at them from the perspective of the functions themselves, limits take on very fuzzy qualities that mimic the sublime. Philip Shaw describes the sublime as "The moment when the ability to apprehend, to know, and to express a thought or sensation is defeated. Yet through this very defeat, the mind gets a feeling for that which lies beyond thought and language" (3). It is hard to view a mathematical function from a subjective perspective; however, if functions could talk, if they could describe their own experiences, then

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[2] If all of this is getting confusing, do not feel bad. The entire field of analytical calculus taken at the honors level is effectively just normal calculus dancing precariously around this and other technicalities. It is not important to understand the argument, just to understand that it is there.

zero would seem to be a very sublime entity to the function $f(x) = \frac{1}{x}$. The function knows that the limit is there, it can sense the limit, interact with the limit, but it is incapable of describing the limit. Physical singularities present a similar problem. A black hole is an object whose gravitational pull is so powerful that it overpowers even the forces that prevent two objects from occupying the same space. A black hole crushes matter so infinitesimally small that it ceases to exhibit any of the characteristic properties that it had before entering the black hole. Just like the function, the current model of black holes has no way of experiencing, measuring, or even modeling the center of a black hole: it is a singularity, a point that is structurally beyond the reach of this current model. The only things we can know about it are those things that are very close to it, but the singularity itself is forever beyond our reach. We cannot describe it, yet we try anyway.[3]

From the perspective of mathematical proof, attempting to describe something that cannot be described is a fool's errand. Mathematicians can only learn through construction what can be constructed; however, humanists gain knowledge about the sublime whenever their constructions fail. Aristotle's ban on infinity is a ban from the perspective of mathematical construction. It is a recognition that the knowledge of infinity is beyond the scope of the mathematical project. Math can only create knowledge about things that it can construct, and Aristotle did not believe that infinity could be constructed. To the experimental physicist a mathematical model is only as good as the predictions it can offer. However, usefulness is of no interest to the pure mathematicians. Even though set theory has no definitive structural framework, we still have knowledge about sets. Likewise, the founder of set theory, George Cantor, did not need to know if a formal definition of infinity could exist, he just needed to try.

Set Theory

[3] Although, at the time of writing this, I just discovered that Stephen Hawking had, quite literally, just rewritten the book on black holes; therefore, everything I just said is probably wrong.

In 1874, German mathematician George Cantor wrote the paper that marked the end of Aristotle's ban on complete infinity and marked the beginning of mathematical 'set theory'. As we have seen, set theory is a mathematical framework for treating collections of objects, or sets, as objects themselves. Set theory has since exploded in both importance and utility. The axioms of set theory are fundamental in the sense that they can be used to rebuild number theory, graph theory, and nearly every other mathematical field making it the natural successor to Euclid's geometry, if such a successor can even exist. Among Cantor's large list of accomplishments was his recognition that the natural numbers served two important mathematical purposes: the first being a symbol of quantity: the 'cardinal numbers'. When describing a collection of apples, we might say that the collection contains six apples. In this case, six represents the cardinality of the set of apples. Likewise, a fleet of ten ships would represent a set with cardinality ten. The second role that Cantor described that numbers represent is order: the 'ordinal numbers'. The set of all runners in a marathon might have cardinality one hundred, but each runner is also assigned a rank that represents their completion order. The first runner to cross the finish line will be assigned one, the second two, and so on.

Set theory defines two different sets of numbers: one that represents quantities, the cardinal numbers, and one that represents order, the ordinal numbers. The cardinal numbers represent the size of a set, while the ordinal numbers represent their order or ranking of a particular element within that set. For the finite natural numbers, and their representation within set theory, order and quantity parallel each other perfectly. The set that represents 'seven' both contains seven elements as well as represents the seventh element inside the set of all natural numbers. Buried within this definition is a problem. In order for seven to be the seventh element of the set of natural numbers, one must first accept that there is a set of natural numbers. Clearly, the natural numbers are a potentially infinite quantity. The only way to group all of the natural numbers into a single set is by first completing this potentiality, and then drawing a line around it.

At first, the community was wary about Cantor's definition. At best, naming infinity was simply a useless absurdity; however, some critics, like Leopold Kronecker, worried that trying to contain infinity within the mathematical universe would introduce contradictions and ambiguities that could prove cancerous to the study as a whole. Much of the criticism Cantor received bordered on abusive and certainly contributed to his depressive fits and eventual death.

> It was Cantor's hope to obtain a better-endowed, more prestigious professorship in Berlin, but in Berlin the almost omnipotent Kronecker blocked the way. Completely disagreeing with Cantor's view on "transfinite numbers," he thwarted Cantor's every attempt to improve his standing through an appointment to the capital…. He died in 1918 in Halle University's psychiatric clinic. Schoenfliess was of the opinion that his health was adversely affected by his exhausting efforts to solve various problems, particularly the continuum problem, and by the rejection of his pioneering work by other eminent mathematicians. (*Dictionary of Science Biography* 3: 52)

Recognizing the validity of the set of all natural numbers opened the floodgates for a vast assortment of new and interesting questions, but the problem of ambiguity still remained. Intuitively the set of all natural numbers should be bigger than the set of all even numbers; however, we can't count all the elements in each of these sets so we have no provable way of verifying this hypothesis. Cantor solved this problem by capturing the concept of counting and specifying it to the point of absurdity: with interesting results. When we count a collection of objects, we are assigning each object in that set to a natural number. Given a barrel of apples, I can count them by reaching into the barrel, grabbing one of the apples, and assigning it a number. The first apple I pull out is number one, the second is number two, and so on until I run out of apples in the barrel. Whatever number gets assigned to the last apple I pull out of the barrel represents the cardinality of the set because I have shown that my barrel of apples contains the same number of elements as the mathematical object assigned to the last apple. Likewise, we can compare the cardinality of two arbitrary sets, say a barrel of apples and another barrel of oranges, by pulling one object out of each barrel and setting them next to each other. Whichever barrel runs out of elements first has the smaller cardinality. Cantor extended this pairing principle to infinite sets by saying that two sets have the same cardinality if there exists a process by which we can pair exactly one element in the first set with exactly one element in the second.

Imagine comparing the set of all even numbers to the set of all natural numbers. The natural hypothesis is that the natural numbers contain twice as many elements as the even numbers because they represent the natural numbers with every second element removed. However, Cantor's pairing principle says something different. Reaching into both barrels, we might pull out the number one from the natural numbers and the number two from the even numbers and assign them as a pair. Continuing on: two and four might be the next pair, three and six the next, four and eight, five and ten, and so on. I will represent this process using the function $even = natural * 2$. This function has no singularities. Every natural number, no matter how high, has a pair in the evens, and vice versa. Because there are no elements that will remain unpaired once we finish counting, we have no choice but to accept that the natural numbers have the same cardinality as the even numbers. The standard metaphor when discussing this concept was first presented by David Hilbert as the Grand Hotel Paradox.[4]

Imagine a hotel consisting of one hallway with an infinite number of sequentially numbered hotel rooms. At the entrance of the hotel, a single employee manages the hotel via a loudspeaker that carries her voice to every room in the building. Every room is currently occupied. A potential patron enters the building and asks the employee for a room for the night. In a conventional hotel, the employee turns the new client away. However, in Infinity Hotel she can create an empty room. In this hotel the manager turns on the microphone and orders everybody in the building to pack up

[4] The earliest published reference is generally attributed to Gamow who himself attributes the metaphor to, "the unpublished, and even never written, but widely circulating volume: 'The Complete Collection of Hilbert Stories' by R. Courant" (17).

their room, and reoccupy the room sequentially next down the hallway. Whoever is in Room 1 moves to Room 2, whoever is in Room 2 moves to Room 3, and whoever is in Room 10,000,000,000 moves to Room 10,000,000,001. In a normal hotel whoever is in the last room would return to the front desk and complain that there is no room for them to occupy; however, due to the potentiality of Infinity Hotel, no matter where a person is in the building there is always a 'next' room for them to enter. Once the move is complete, room number one is now empty and ready for the new patron to occupy. Mathematically, this strange ability to create empty space out of nowhere stems from the fact that infinity plus one is still infinity.

Yet, the identity goes further. If a busload of patrons arrives, we can still make room for all of them by performing the same task multiple times. Likewise, even if an infinite number of patrons arrive we can still make room; however, instead of telling everyone to move one room up, we tell them to move to the room exactly double their room number: a result of the fact that infinity times two is still infinity. In Infinity Hotel, there is no largest room value and therefore, no matter how many rooms we tell them to move up, nobody will ever have to double back and occupy the first room.

Let us look at a slightly different example that Deutsch brings up: Infinity Hotel garbage disposal (172-174). Let us say it is garbage day and the manager wishes to remove all of the garbage from Infinity Hotel. The garbage collector's task in this case is very similar to Zeno's interpretation of the runner. He would need to collect the garbage from an infinite number of hotel rooms, and thus complete an infinite number of tasks. Deutsch argues that the garbage man could complete this task rather simply. He gets on the manager's microphone and orders the person in Room 1 to move all of his trash to Room 2; they have half an hour to complete this task. Then he orders the occupant of Room 2 to move all their garbage to Room 3; they have fifteen minutes to complete this task. Likewise, each room number would have half the time the previous room had in order to move their garbage plus the garbage of everyone before them. Barring the herculean effort this would take to complete for some of the later rooms, all of the garbage will have been moved after one hour. The infinite sequence of garbage swapping will be complete in this hour. All of the garbage has left the building, but where has it gone? Deutsch argues that the garbage has disappeared into a singularity and is in fact 'nowhere'. Indeed, from the perspective of the inhabitants this is exactly so, it has just vanished, disappeared, ceased to exist. Similar to how a black hole eats matter, all of the garbage has disappeared into an enigmatic and unreachable final room. The inhabitants cannot see where it has gone, and I doubt that they will care. I assume that anybody who grew up in residential suburbia can relate to this feeling. Every Tuesday we put our garbage out on the curb, and the next day it is magically gone.

Imagine for a moment that the garbage was not being passed from room to room by the occupants and was instead being collected by the garbage collector himself. The manager simply tasked every room's occupant to leave their garbage in front of their room's door and the garbage collector himself collected it all. Like the occupants, the garbage collector has half an hour to pick up all the garbage in front of the first room, a quarter of an hour for the second, and so on for each consecutive room. Just like in the first example this garbage collector will have collected all of the garbage in the hotel after an hour has passed, yet unlike the first the garbage has not disappeared into a singularity as we know exactly where it has gone; the garbage collector is carrying all of it. The final step in garbage disposal happens after the first hour has elapsed. Once collected, all the collector needs to do is walk outside the building and toss everything into the garbage dumpster.

The difference between these two disposal systems is subtle, yet important. In both examples the hotel itself does not offer a description of how the garbage made it to the dumpster outside the hotel. In the second example, I had to reveal an additional step to the process. It will take the collector one hour plus whatever time he needs to cart it all to the dumpster. The first example is no different. The original algorithm is sufficient to convince all of the hotel guests that the garbage has vanished from the hotel, but unlike the second, it offers no insight into where the garbage has gone. Like the second example, we need another step, one external to the hotel that adequately explains the disappearance in the first: ordinal infinity.

As mentioned previously, the natural numbers serve two distinctly different roles: order and quantity. Infinity Hotel makes use of them both. Cardinal infinity represents the hotel itself: a box that contains an infinite number of objects -- in this case, hotel rooms. However, there is also an ordinal infinity at play here. Every room is labeled sequentially with a natural number. Since the hotel itself represents a complete infinity we could theoretically label the last room, the completion of the hotel, Room ∞. Just like Zeno's finish line, Room ∞ does not actually represent a room in our specific hotel, but that does not mean it does not exist. Room number Infinity is the ordinal limit of the endless progression of room numbers. It is the first room bigger than all of the rooms in the hotel, and since this place did not receive a command to send garbage to the next room, as it is not a natural number, all of the garbage ended up there and is waiting for the garbage collector. Ordinal infinity represents the next value after an infinite process has completed. It is the first length between Zeno's finish line and wherever the runner collects his award. Anyone programmed to only be able to see hotel rooms will not be able to see it, because it is not a hotel room. It is sublime. Anyone stuck inside the hotel would be utterly incapable of seeing it; however, it still plays a vital role in the functioning of the hotel. In mathematics, whenever we talk about a boxed infinity we are always talking about an infinite process plus a singular element, the 'closure', that acts as the last element of the infinite process: how we choose to frame infinity can hide this element. In both the cases of Zeno's runner and Deutsch's garbage disposal, the chosen perspective obscures the final element.

It is important to observe that cardinal infinity, the hotel itself, is not the same as ordinal infinity, the garbage dumpster. If we add one new room to cardinal infinity, we still have cardinal infinity. If we multiply cardinal infinity by two, or even itself, we still have cardinal infinity. However, ordinal infinity, which set theory has named w, is different. Once all the garbage has made its way to the dumpster there is no reason why a garbage truck, labeled $w + 1$, couldn't pick up the garbage and drop it off at a landfill, $w + 2$. Potential infinity pretends to be an end. It is difficult to convince ourselves to move beyond a barrier, if we believe that that barrier does not exist. However, in this context, infinity is a cloud. It obscures knowledge and tries to convince us that those things that lie beyond are beyond our reach. We could, if we wanted to, create another Infinity Hotel behind the first one with room numbers labeled $w + 1$, $w + 2$, $w + 3$, and so one. Since this itself represents a potentially infinite process, we can complete that too, to create $2w$. Yet, even now there is no reason why we cannot continue: $2w$, $3w$, $4w$, w^2, w^w, $w^{w^{w^w}}$ and so on until we reach the ultimate limit e_0: after which comes $e_0 + 1$, $e_0 + w + 1$ and on and on for potentially as long as we want. Yet, at no point will any of these ordinal numbers represent cardinality bigger than Infinity Hotel. We have transcended infinity, and learned something about it. Yet, I have found myself trapped inside yet another infinity.

An important concept in the study of infinite cardinals is the concept of self-similarity (a concept explored in much more detail in mathematical 'chaos theory'). That is an object that can be identical to a subset of that same element. A part equals the whole. In fact, this observation defines infinity. An object is infinite if and only if it is self-similar. Hotel Infinity is infinite because multiple infinities can fit inside of it. We can take a subset of the rooms in that building and create the entire building. The bizarre nature of cardinal infinity is that it represents everything we think we know about the concept of everything. To claim that we know everything is to claim that we have achieved a set of knowledge that is self-similar to the set of all knowledge. Cardinal infinity seems to fit that bill perfectly; if we try to add stuff to it, we only get more of what we started with. In fact, any attempt at expansion results in nothing at all. When we say the universe is infinite, we are really saying that it is infinite in the cardinal sense. Future explorers who find new worlds outside of those that we already know about will not disprove our current notion that the universe is infinite. Exploring infinity makes what we know bigger without invalidating our assertion about its size. Infinity is not a claim that the universe is too big to grasp; it is a claim that the universe is just more of the same. No matter how high we count in the natural numbers, we never arrive at a point where we suddenly count something new. Infinity in this context is achievable. Counting is potentially infinite, yet it is still always moving towards a definable object. Likewise, we know that everything just slightly less than cardinal infinity will eventually be constructed within a potentially infinite process. If I believe that scientific knowledge is always increasing, and that the universe is infinite, then I become blind to the last element, the last thing that science is capable of teaching me. If I suddenly claim that there is no last object, then it does not take much of a leap to conclude that scientific knowledge will eventually become all knowledge.

The humanities also confront infinity. However, unlike science, which seems to converge on some level of absolute truth, the humanities diverge. They create knowledge, without any clear indication of where that knowledge is going. Unfortunately, without this metric it becomes too easy to argue the superiority of science over the humanities. At best, the humanities also diverge towards the same infinity that science converges on and both represent two paths towards the same goal. However, unlike convergent functions, not all divergent functions go anywhere. Remember that the function $1 - 1 + 1 - 1 \ldots$ is potentially infinite. It diverges, but it never gets anywhere. The worst-case scenario for the humanities is that the limit of its pursuit is less than infinity. Given the choice between a function that we know converges on infinity versus a function that might converge on infinity, it seems obvious to pick the one that we know goes somewhere. It seems that the rationalists are right and we will eventually evolve away from everything essentially non-scientific. However, this argument is flawed because it assumes that infinity equals everything, and this of course is the fundamental problem with infinity. It always hides what it cannot reveal.

True Infinity

Just defining ordinal and cardinal infinity on the natural numbers is important, but it is not nearly important enough to warrant Cantor the prestige he has now. The mark of all great mathematicians is not their ability to define boundaries, but to surpass them. The mathematical community's decision to define real numbers as the limit of arbitrary sequences of natural numbers has its own peculiarities. Just like Zeno's runner who has to complete an infinite number of lengths between the start and the finish line of his race, so too there are an infinite number of irrational numbers between any two real numbers. There are an infinite number of points on the line between 1 and 2, just as there are an infinite number of points on the line between 0.000000001 and 0.000000002. One of the repercussions of this potentially infinite process is that it really dissolves the common sense notion of order that drives everything I talked about in Hilbert's grand hotel. If there are an infinite number of rooms between any two rooms, then how could we possibly define what the next room is? Likewise, garbage collection inside of an infinity hotel defined on the rational numbers would look a lot different than the infinity hotel I defined on the natural numbers. Guests at Irrationally Infinite Hotel cannot be ordered to move their garbage to the next room because they have no way of knowing what the next room is. No matter which room they guessed to be the next room, they would still find an infinite (i.e. self-similar) number of rooms between them and their guess. We might still be able to collect some garbage by telling everyone to move their garbage to the next natural number up, i.e., their room number + 1, but even after artificially completing that infinity we would still be left with an infinite number of garbage bins between the ordinals w and $w + 1$. The self-similarity of this solution means that it is identical to getting everyone to throw their garbage out a window into a garbage bin specially designed for their room and their room alone. However, these observations only recognize that the two hotels are dissimilar; we still do not have any ways of demonstrating which one is bigger. This is the same observation that Aristotle made, but he too had no way of comparing them. Cantor also recognized this dissimilarity, but this time he had a system of comparison: the pairing principal first published in 1892.

Imagine a process very similar to counting. All of the rooms in Infinity Hotel sit in one barrel on my left, and all the rooms in Irrational Infinity Hotel sit in a second barrel on my right. One at a time, I pull one object out of each barrel and pair them together. Since both barrels are infinite, I need to artificially end the process, which itself produces an infinitely long list. On this list there are two columns, a number representing a room in Infinity Hotel, and a second symbol, right next to it, representing a single room in Irrationally Infinite Hotel. If both sets contain the same number of elements, then every room in one hotel should be paired with a room in the other hotel, and every room in both hotels is on the list. I will begin the proof by assuming that I have created such a list and that every room in both hotels is on it. In order to contradict this assumption I will construct an element that is not on the list using Cantor's diagonal argument.

I will start by looking at the real number that is paired with Room 1. Since this number is an irrational number, it is represented by a unique infinite decimal expansion. I will then look at only the first digit of this decimal expansion and write down any digit that is not equal to it. So, if the first digit of element one was 5, then I will write down any other digit, say 4.[5] Repeating this process, I look at the second digit of element 2 and once again write down any other digit different from the one I find. My process of looking up digits and writing down any other digit creates a potentially infinite process. Artificially ending this infinite process produces an infinitely long sequence of numbers, and thus an irrational number. However, since this irrational number differs by at least one digit from every element on my list, I must conclude that my constructed irrational number is also not on the list. This contradicts my initial assumption that all rooms in both hotels are on the list. Thus, each hotel has a different cardinality. In addition, since Hotel Uncountable Infinity is the only hotel with elements not on the list, I will also conclude that it contains more rooms than

[5] I also need to avoid writing down digits 0 and 9 in order to avoid constructs similar to 0.999… = 1.

Infinity Hotel. Therefore, it is possible to overfill Infinity Hotel; I just need to get everyone from Hotel Uncountable Infinity to arrive at once.

The proof given above represents an early accomplishment of set theory, but also a very important problem with it. Set theory uses term C to represent the infinite cardinal numbers where C_1 is the cardinality of the natural numbers and C_2 is the cardinality of the irrational numbers. Likewise, C_2 has also been shown to represent the cardinality of many other mathematical objects: such as the cardinality of all the points along Zeno's race and the power set of the natural numbers (the set of all combinations of natural numbers). The value C_2 is important enough to warrant its own name: the continuum. Unfortunately, ordinal numbers are not so lucky. The existence of w_2, the irrationally infinite ordinal, is much more controversial. Unlike the infinite cardinals, which can be constructed, the infinite ordinals beyond w cannot be constructed inside of set theory. Ordinal numbers are unable to construct an ordinal analogue of the continuum, which poses two really difficult questions: How do we apply functions, like garbage disposal, across the continuum (what limit does it represent?) and what, if anything, lies between C and C_2? The answer to the first question lies within the 'axiom of choice', which is one of the most controversial axioms in all of mathematics. The axiom of choice simply assumes that functions can magically operate on the continuum and all other sets, which allows us to construct objects like w_2. Interestingly, without the axiom of choice we cannot even assign a name to all of the rooms in hotel continuum. Some work has been done on the second question, which is the continuum hypothesis, but those results can be summed up with the statement, 'Math doesn't have a clue.'

So, now that we have arrived at the continuum can we finally conclude that we have reached everything? Once again the answer is no. Cantor also proved that the cardinality of the power set of A is necessarily bigger than the cardinality of A itself ($P(A) > A$). Taking the power set of the continuum produces a new hotel that is even bigger than an irrationally infinite hotel. Just as the continuum has rules different from countable infinity, so too this new even larger monstrosity would be equally incomparable to the continuum. Yet even then taking the power set of this new monstrosity creates a set even larger still. Using the power set function we can create a potentially infinite process that creates an infinity of infinities ($C_1, C_2, C_3, \ldots$). Unfortunately, mathematics currently has no method for taking the limit of this process, and therefore we have no way of describing anything bigger. Does this mean we have finally arrived at everything? By now any such claim should be met with extreme skepticism.

The beauty of mathematics is that we do not actually need to construct 'everything' in order to figure out its properties. We just need to assign it a name and see what happens. We can think of everything as a set: a set that contains all sets. However, this only brings me back to Russell's paradox, a theorem I discussed in the previous chapter. The set of all sets is contradictory and cannot exist within mathematics. We can describe infinity, but only by trapping ourselves in other infinities. 'Everything', however, is beyond our reach. 'Everything' is sublime.

Conclusion

The problem with trying to define infinite epistemologies, like Deutsch's infinity, is that it will make a silent claim of similarity; everything it discovers will resemble that which it has already discovered. Deutsch accepts the conclusion that knowledge will increase forever, but the first sentence on the first page of the introduction traps him in the conclusion that all of this new knowledge will still resemble modern science: "Progress that is both rapid enough to be noticed and stable enough to continue over many generations has been achieved only once in the history of our species. It began at approximately the time of the scientific revolution, and is still under way" (1). Unfortunately, this puts science in the same uncomfortable situation that the humanities are facing right now: what if science / humanities needs to change?

Static perspectives prevent us from seeing our own barriers, and even while everyone I have talked to in the humanities agrees with this statement (and in fact use it as a reason to avoid mathematics), it amazes me how quickly the belief that we have achieved a dynamic perspective decays into a static perspective. How is it that the humanities, a discourse that refuses to define its own terms simply because such a definition would reject the multiplicity of the study, is afraid of another way of thinking? No matter how many times the humanities claim to be non-restrictive, they still represent a divergent infinite process. Science is no different. Science may always be increasing, but that does not mean it will ever produce knowledge that is unscientific. The limit of scientific knowledge may well be all scientific knowledge, but there is absolutely no way of justifying the claim that all scientific knowledge equals all knowledge. In fact, science's link to mathematics guarantees its own incompleteness. It is not just that each scientific paradigm is incomplete; any forward-moving system is equally and identically incomplete. I offer the same argument for the humanities. Humanist scholarship may not accept mathematical methodologies as their own, but by rejecting them, the humanities limit themselves to those ideas that humanist methodologies can demonstrate.

I may be reaching significantly outside the scope of Gödel's original proof, but the truth that it hints at says that not just algebra is limited, but demonstration itself is limited. Even processes like science are themselves systems of proof that generate worlds. They may be dynamic, constantly changing words, but they are still worlds. Deutsch's argument that science is somehow better than religion, or any other dogmatic subject, because it is dynamic is blind to its own static nature. The claim of separation between science and dogma needs a static symbol that can be used to identify scientific knowledge. Deutsch uses the example of the Spartans as an unchanging brittle and static society that was doomed to failure, but I see the scientific method as being no different. It is a static set of rules that defines a system of justification. I cannot disagree that at the present moment it is incredibly useful, and it will continue to be useful for a long time. However, I will not agree that it is without limitations and can continue on 'forever'. To claim that we have arrived at the 'correct' system of knowledge generation is identical to the Spartan claim that they had arrived at the 'perfect' civilization. To deny the limitations of science, or any other perspective for that matter, is logically unjustifiable. It is also fundamentalism. Moreover, everything I just said applies to classical humanism as well. The moment they deny the usefulness of a method simply because they do not understand it is the same moment they trap themselves within their own perspective.

The power in objective thinking is that it allows us to transcend the barriers that subjective perspectives force on us; however, objectivity merely changes barriers without removing them. In some sense, this is why the humanities are bigger. Humanists create limitations for the sole purpose of breaking them. They do not cling to a single methodology or a single way of doing things, and most importantly, they recognize that not only have our ideas changed over time, but so have their methods. Truth, in the humanist sense, is not an achievable, tangible object but is instead sublime. It is truth gained through the failure to contain truth. This is remarkably similar to the mathematical concept of truth by failure, and indeed bringing them together might create ideas that reside outside of the 'infinity' that each generates alone.

Even though I know the limits of math, it gives me a language to attempt to understand the world. Every time that attempt fails, I find myself implicitly understanding more about what I am studying. Ironically, this is not that far from Deutsch's interpretation of science. Science too is a language that attempts to understand the universe, and through its failure, he understands more about the world around him. Both sides are true. Both sides are false.

Chapter 5: Virtual Reality

The failure of the Hilbert model of mathematics introduced a large degree of chaos into the mathematical world. The roots of mathematical thought were no longer obvious, and rebuilding mathematics on a known foundation was equally futile. Unfortunately, nobody understands what post-Hilbert mathematics is supposed to look like or even if we need to such a distinction. Solomon Feferman writes:

> One reason the working mathematician can ignore the question of need of foundational axioms… is that the mathematics of the 99% group I indicated earlier can easily be formalized in ZFC and, in fact, in much weaker systems…. The continuum itself, or equivalently the power set of the natural numbers, is *not* a definite mathematical object. Rather it's a conception we have of the totality of "arbitrary" subsets of the set of natural numbers, a conception that is clear enough for us to ascribe many evident properties to that supposed object… but which cannot be sharpened in any way to determine or fix that object itself…. I think we are left to regard the question: Does mathematics need new axioms?, as primarily a philosophical one. And if you agree with me on that, then we have the discouraging conclusion that we can expect as many answers to the question as there are varieties of the philosophy of mathematics. (403 - 407)

The continuum itself poses a strange barrier in mathematics. Inside of the continuum, everything we want to understand about mathematics feels achievable. Inside of this bubble we can usefully operate on infinity, and most other mathematical quantities; however, once we try to expand beyond the continuum the whole the concept of mathematical precision begins to break down. We can see and describe a few of the objects outside, but meaningful understanding is sparse and only serves to remind us of that which we do not know. Of course, there is no pressing need for mathematics to expand beyond the continuum as the vast majority of useful mathematical work operates within this barrier. Mathematics in its current form is 'good enough' to continue as a useful subject for another hundred, if not thousand, years. However, 'good enough' is a distinctly impure mathematical concept and one that should not rest easy with any pure mathematician.

Now that we have discovered the bubble within which classical mathematics resides, we can begin to look objectively at mathematics itself. Computers are very much a product of this bubble. Computers represent the space that this old mathematics operated inside, and in this post-Hilbert mathematical world, we can study computers themselves as proper mathematical objects. Instead of agonizing over the incompleteness of mathematics, the next step in the mathematical program is to learn how to discern what exists within this bubble and what does not; this is where computer science enters the picture.

As a part of Hilbert's objectives, he proposed another question important to this new (post-modern?) mathematics. The 'Entscheidungsproblem' asked the mathematical community to find an algorithm to determine whether a question, answerable only by yes or no, was decidable within mathematics. This question lies at the root of all research into computer science, a branch of applied mathematics, and is foundational to the theoretical framework that makes the modern computer possible. Computer science is the study of mathematical algorithms, and their primary goal is to transform useful, or fictional, objectives into computer algorithms. They seek to discover the difference between computable tasks, non-computable tasks, and the various shades of grey in between: heuristics.

Unfortunately, the mathematical framework surrounding computers is not well known, and their reception by the non-mathematical community has not been that of a community accepting the computer's inherent incompleteness, but rather the exact opposite. For example, Joseph Weizenbaum created a computer program called ELIZA between 1964 and 1966 that tried to model human conversation. Weizenbaum, like many other computer scientists, was interested in whether a computer could be programmed to understand and speak human language. As a test run, ELIZA ran a script called DOCTOR that tried to imitate a Rogerian psychotherapist; Weizenbaum choose this profession simply because it was easy to model. All ELIZA had to do was absorb whatever the human said to it, and return it in the form of a question. The reaction to Weizenbaum's work surprised even him.

> I was started to see how quickly and how very deeply people conversing with DOCTOR became emotionally involved with the computer and how unequivocally they anthropomorphized it. Once my secretary, who had watched me work on the program for many months and therefore surely knew it to be merely a computer program, started conversing with it. After only a few interchanges with it, she asked me to leave the room. (370)

Langdon Winner, in his 1986 article *Mythinformation,* describes the so-called 'computer revolution':

> What one often finds emphasized, however, is a vision of drastically altered social and political conditions, a future upheld as both desirable and, in all likelihood, inevitable.... As these technologies become less and less expensive and more and more convenient, all the people of the world, not just the wealthy, will be able to use the wonderful services that information machines make available. Gradually, existing differences between rich and poor, advantaged and disadvantaged will begin to evaporate. Widespread access to computers will produce a society more democratic, egalitarian, and richly diverse than any previously known. Because "knowledge is power," because electronic information will spread knowledge into every corner of world society, political influence will be much more widely shared. With personal computer serving as the great equalizer, rule by centralized authority and social class dominance will gradually fade away. The marvelous promise of a "global village" will be fulfilled in a worldwide burst of human creativity. (590)

In a very short period, computers have entered our society's common definition of 'normal'. The computer enables communication, mediates business transactions, offers directions, provides entertainment, and manages schedules. Yet, very few of us know how to interact with the computer beyond the gated community of structured paths that companies like Apple and Microsoft have built for the market. Unfortunately, this creates a world in which mathematics filters every facet of human experience, while we collectively pretend that it is not there. The illusion is that mathematics somehow represents the natural or correct filter. By submitting ourselves to this filter, all of the problems of the world will somehow find their resolution. The natural extension of this philosophy is of course the universal computational model: the belief that the universe is itself a singular solitary computer.

This position is rightfully contested in humanist discourse, as has already been demonstrated in previous chapters. However, it is all too easy for the argument against the universal computational model to dissolve into an argument against the computer. Humanists like Drucker might be wary around computers because they silently insert decisions and assumptions into humanist discussion, but I would argue that these insertions are precisely the point of computational inquiry. The entire purpose of computers is to provide a medium for testing such assumptions. Computers exist as an embodiment of these decisions, and to argue that the act of making such decisions taints the humanities as a whole, is also to argue that computers have no place in humanist discussion.

Unfortunately, popular culture has created an incorrect mythos that computers are real, that computers generate reality, and that that reality is definitively mathematical. Taken from this perspective it is no wonder that a study like the humanities, which has for the most part moved away from rigid structuralism, resists it. However, this perspective is unfair as it fails to recognize that computers have forced mathematics to face a similar problem. Computers themselves are axiomatic, in exactly the same way that the mathematical perspective is axiomatic; however, just like in mathematics there exists a necessary layer on top of computers that cannot be incorporated into the computer itself. If Hilbert's project had succeeded then things would look very different, but it did not. Computers were born out of the failure of mathematical modernism, and the computational model inherits this legacy. The modern computer is a machine born out of crisis both mathematical, the failure of the Hilbert model, and social, World War II. Likewise, the philosophy and science behind computers should not be of a narrative of completeness but instead incompleteness; it is a machine born out of the need to rediscover everything we thought we knew about mathematics.

Classical Computation

Classically the computational model began with Newtonian physics. That model defines an object as a bundle of observable quantities. As a simplistic example, a single chess piece has assigned to it a location, a color, and a rank. Collectively these attributes make up the current state of the piece. The possible states of a particular chess piece represent elements in a set. There are 65 possible locations (64 squares plus one location representing 'off the board'), two possible colors, and six possible ranks giving rise to 780 possible states any particular chess piece can occupy. If we were to think of the entire board as one single object then we can likewise enumerate all the possible states of the board and would find that number to be exponentially larger. In general, the state of an individual token, or the board as a whole, will never change unless first acted on by some other entity. When a pawn takes a queen the queen's state is changed; likewise, when a king castles, the corresponding rook is displaced to another location. The 'laws of physics' of chess are the rules that dictate how the players can move the pieces. The chess rules forbid a single token to change color or a pawn to move backwards. Here unfortunately is where the metaphor between Newtonian physics and chess breaks down. In chess, an external free agent chooses the transformations that happen to the board. In Newtonian physics, the universe itself makes all of the decisions.

In billiards, one single shot can put every ball on the table into motion. The state of any particular ball will only change when it interacts with either another ball or the edge of the table. However, we can still think of any given state of a billiard ball as being one of an infinite set of allowable states. Every time two balls interact, each undergoes a transformation that changes its state to another permissible state. These new states might then interact with other states changing the system once again. Like chess, the rules of these interactions are static and unambiguous. When a queen takes a rook, there is no question about where the rook is transferred; likewise, when a ball hits another ball there is exactly one definitive state that each ball will be transformed into.

In Newtonian physics, a real world object, like a baseball, has a number of different attributes: location, velocity, mass, density, and so on. Each of these collectively forms the state of the baseball and various equations dictate how that baseball can transform. I can pick up the baseball and apply a force to that ball which will change its state. Time then acts on the object until it finally hits a wall and its state changes again. Likewise, just like the chessboard, we can generalize all of the objects into one single object. The universe hands my senses a single snapshot of itself, then time acts on that snapshot changing its state to another allowable configuration. What I see looks very similar to an old-fashioned movie reel. I see one state; time transforms it to another state, then another, and another. The classical laws of physics are the rules that dictate how time transforms one universal state into another. If we know the laws of physics, then we can determine every state that the universe will occupy and, in the case of full determinism, Laplace's demon -- the idea that all knowledge in the universe can be extracted from a single moment in time -- becomes a reality.

The classical model of physics works by approximating what we see around us; however, it does rely on one faulty assumption. The classical model demands that there be no separation between the state of an object and the measurements we make about it: i.e., that an object and how I perceive that object are the same. When I measure the location of a chess piece, I am also measuring the state of that same chess piece. If I find it to be on a certain square I can say unambiguously that the state of that chess piece is represented by the square I found it on. The quantum computational model, on the other hand, begins with a radically different assumption, one that fundamentally separates the states of a system from the measurements we make about it. The classical model is deterministic, but the quantum model is fundamentally statistical.

One possible way to define statistics is by characterizing it as a mathematical attempt to link unknown, and possibly ideal, objects with their real and practical approximations. Consequently, statistics works with two related, but different, collections of data. The 'population' represents the true, mathematically pure object or collection of objects that the statistician is studying. In the case of a survey, the population always refers to the results of the survey inside of the theoretical world where every member of the target audience had filled it out truthfully. In the case of the roll of a die, which is also statistical, the population refers to the complete set of results that that die produces: past, present, and future. In all but the most trivial cases, it is practically impossible to know the true population of any statistical problem. Getting every human on the planet to fill out a survey is only slightly less impossible than knowing every result a die has and will ever produce. For most scenarios, the population distribution is an ideal unknowable truth that we are trying to approximate.

The second collection of data is called the 'sample' and in statistics refers to a real collection of observations that an observer makes about a population. If I roll a die six times I will produce a sample containing six observations; likewise when I survey a dozen people I produce a sample of a dozen survey results. Every time I measure a population, I am effectively reaching into a giant bag of possible results, selecting observations at random, and adding it to my sample. In the case of a die, every measurement of a die roll represents one outcome from an incredibly large bag containing every die roll that has ever and will ever happen. Likewise, asking one person a question will result in an answer selected from every answer every human would ever give. Mathematical statistics studies the relationship between sample and population, and asks what relationship exists between the population and sample.

A fundamental assumption important to both applied mathematics and statistics is the 'rule of large numbers'. The rule of large numbers states that the larger a sample is, the more closely it resembles the population. If I were trying to determine if the population of a country supports their current leader, I would begin by asking a sample of the population a few key questions. This small sample would represent the opinions of a small selection of the population. A sample of one person would represent only one opinion, while a sample of two would represent a slightly larger group of opinions. The rule of large numbers is an observation that as I ask my question to a greater and greater number of people, the opinion represented by my dataset is increasingly representative to the opinion of the population of the country. Once I have asked everybody my question, my sample becomes the population. Every text an author writes represents a sample of every text she has every written and will ever write. The rule of large numbers allows us to assume that a sample will resemble a population, and that a single work will have similarities to a large corpus of works by the same author. Thus, structural analysis of this text could reveal authorship where authorship is unknown, because this unknown text might resemble another text of unknown origin. Topic modeling takes this philosophy one step further by assuming that an idea expressed in a paper represents a sample of every time that idea is ever represented and uses this information to try to extract the context of a specific sequence of letters.

The rule of large numbers also employs a second important element: the choice function. Whenever an observation is made, someone or something needs to choose an element from the population to add to the sample. If I were to interview only people who favored the current leader of a country, the opinion of my sample would not represent the opinion of the population. Likewise, choosing ten words sequentially from a text will probably better represent the idea that that sentence is trying to express than will the author's work as a whole. How we choose samples affects how well the samples represent the population. In this way the ‘algorithm’ that we use to choose samples is an important, but easy to hide, aspect of any statistical study. The best choice of sample is one that equally represents each sub-population according to its proportion. A survey needs to sample both sides of a political debate in order to discern the true ‘will of the people,’ just as a topic model needs access to both a writer’s fictional and non-fictional works in order to discern an overall style. A biased choice function does not invalidate the rule of large number. Even in the case of extreme bias a journalist will run out of left-leaning candidates to interview forcing them to interview right-leaning candidates. In this case the rule of large numbers still holds; however, a much larger sample size is needed in order to accurately describe the true ‘will of the people’. Likewise, any bad choice function biases information and exponentially increases the necessary sample size needed to simulate a population. In order to prevent this slowdown the choice function that statisticians use is a mathematical construct that represents our intuitive notion of unbiased choice: randomness.

Statistics is important for classical mechanics only because measurements of an object’s state are imperfect. Physical tools are never precise enough to accurately represent a physical object. Whenever I measure the length of an object, I will inadvertently find that its true length is between two of the ticks on my ruler forcing me to guess its exact size. In a similar fashion, the usage of a single word in a text might not align perfectly with how a dictionary defines that word. Similar to Euclidean geometry, classical mechanics views this introduction of randomness as a direct result of the imperfection of the observer and not because of the mathematical model itself. Classical mechanics always assumes that there is a true length of the object, which acts as the mean of the population of imperfect measurements made about it. Every time I make such a guess, I am inserting a small element of randomness into my data that I will need to sort out statistically. The analogue in textual studies would be if I were to assume that a text had an ideal true interpretation that was difficult to extract because the physical act of interpretation requires small amounts of guesswork that subtly sabotage every attempt. Naturally, this mirrors our common sense notion of the world. If you and I were to look at a statue or read a book, both of us would naturally expect that the other person is seeing the same object, reading the same book, and experiencing the same sensual experiences. This is of course not true for modern hermeneutics, and it is equally not true for modern quantum mechanics. In a quantum mechanical system, randomness is a core feature of the model itself and not a byproduct of human imperfection. That difference is best represented through the mysterious properties of light.

Quantum Computation

The history of light is a complicated topic, to which I cannot do proper justice in the time I have allotted; however, unlike previously examples, which have progressed through notable leaps, the science of light developed through gradual shifts and the modern theory cannot be solely attributed to a single reasonably sized group of notable scientists. Approximately twenty years after Newton published his *Principia* he published another book titled *Opticks.* In it, Newton detailed his 'corpuscular' theory of light where he modeled light after trillions of tiny particles. Unfortunately, this book, while influential, never gained the following his previous work had because it never offered any real advantage over its direct competition the 'wave theory of light' championed by rival physicist Christian Huygens from Holland (Beech 42).

In physics, a wave represents the transmission of energy through some medium. If I were to drop a rock into a pool of water, the action of the rock hitting the water would momentarily push some of the water below the surface of the pool. The water hit by the rock responds by pushing or pulling the water around it, which in turn does the same. The result is a ring of concentric waves that propagates around the area where the rock hit the water; the energy moves through the water, but the water itself stays where it is. If I were to drop two rocks into the same pond at relatively close proximity, both rocks would form their own independent systems of rings. If both sets of waves simultaneously pull up on a portion of the water the ripple would reach a greater height than if it had been acted on by only one ring: this is called constructive interference. Likewise, if one wave pulls up on a section and another pushes down then the section itself will not move and the waves pass by as if nothing had happened: this is called destructive interference.

The wave interpretation of light models light in exactly the same way as energy is transferred in water. A light source acts as a metaphorical rock which perturbs the cosmic 'aether' causing a ring of light-waves that propagate out from the source in exactly the same way that waves propagate around a rock in water. The theory's success is best demonstrated through British physicist Thomas Young's 1803 experiment, which is still popular today as an introductory demonstration of the wave nature of light. In the Young's double slit experiment a single beam of light (modern experiments usually use a laser) is split into two separate point sources each representing a metaphorical rock dropped into water. Under the corpuscular theory of light, the students should see two independent blobs of light on the screen, each corresponding to one of the one of the holes the light could pass through; however, what any observer will actually see is a series of alternating bright and dark spots representing the interference patter associated with the wave theory of light. The bright spots represent points where the two split beams interfere constructively; the dark spots represent points where the two beams interfere destructively.

Unfortunately, as time went on, holes began appearing in the wave theory of light. In 1889 Albert Michelson and Edward Morley tried to experimentally demonstrate the aether directly. Unfortunately, this experiment failed, lending strong evidence to the growing notion that light, unlike water waves, did not make use of a medium. Unfortunately, without a medium, the theory wave theory of light needed to accept that light must transmit itself, resulting in a reintroduction of some elements of the corpuscular theory: a problem that only found some conclusion in the modern introduction of quantum mechanics. Combining the wave theory and the corpuscular theory produces a bizarre compromise that can still technically be interpreted classically, but needs to dance precariously around its limitations. This interpretation views the laser as an object that produces trillions of 'light packages', or photons, that travel towards the two slits of the experiment. Half of these photons travel through the first slit and the other half travels through the second slit. The students still see an interference pattern because the two groups of photons still interfere with each other. However, they actually produce an interference sample instead of an interference population. In the event that there are an infinite number of photons, the interference pattern would be identical to the pure wave theory of light. However, because there are trillions of photons hitting the screen, the rule of large numbers states that what we see looks almost exactly like the ideal light wave.

Unfortunately, classical mechanics has no way to answer definitively the following question: What happens when we send only one photon through Young's experiment? Common sense tells us that one of two results is possible. The first possibility is that the photon behaves like a particle and chooses to travel through one of the slits. In this situation the photon would have nothing to interfere with and would not generate an interference pattern. The other common sense option is that the particle would act like a wave and split itself in two as it interacted with the slits. Each photon would travel through different slits and interfere with each other resulting in an extremely dim, but complete, interference pattern. The actual result of the single photon resides somewhere in between and is difficult to describe due to its blatant disregard for common sense. When I send a single photon through the apparatus only a single photon will appear on the screen behind the slits; however, it will not appear in the spot it should have appeared in if it had not interfered with anything. Instead, it will appear inside one of the bright spots of the interference pattern. If, ten years, later I send another solitary photon through the apparatus, another solitary speck will appear on the screen. This time, however, it will probably be located inside a different bright spot in the same interference pattern. Over time, as the photons build up, the specks on the screen will begin to resemble the interference pattern. The result of this experiment is then identical to the case where I send trillions of photons through the slits only I now have no intuitive way to explain what the photon is interfering with.

David Deutsch tries to describe this effect in *the fabric of reality* by having his reader imagine that “each tangible photon has an accompanying retinue of shadow photons, and that when a photon passes through the other one of our four slits, some shadow photons pass through the other three slits” (*Fabric of Reality* 44). Half of these virtual photons travel through the top slit, the other half travel through the bottom slit. The entire group of photons interferes with each other creating the entire interference pattern, but only the real photon is visible when it hits the screen. The entourage of virtual photons represents the population, and the universe randomly selects one to appear in our universe.

Mathematical quantum mechanics interprets the same experiment in a slightly different way. Instead of a photon, the mathematician deals with ‘probability waves’ that represent a photon. The probability wave interacts with both slits just as a pond interacts with two rocks. However, once the wave hits the screen, the probability wave is collapsed into a single random sample, which we visually experience. This interference pattern then represents the population of possible locations where the electron could be measured. When we look at the screen, we are seeing only one sample from this possible population. Likewise, when you and I look at the same statue what we are actually seeing is two different samples of the same population. The statue produces a probability distribution and as each photon of light hits our eye, it represents only one random sample in that population. However, even a brief glance at a statue will produce trillions of photon impressions on my eye. Since large samples tend to look like actual distributions, I can still infer with ‘good enough’ accuracy that what I see represents the actual probability distribution. When you and I look at the same statue we will both see different samples; however, these samples will be so similar that neither of us will ever be able to tell the difference. Under the Many Worlds Interpretation of quantum mechanics, supported by David Deutsch and several others, the 'multiverse' is a set of all universes where each universe represents one possible observation. As individuals we are born into a random member of these universes, and likewise, observe one of the outcomes at random.

Unlike classical mechanics, which treats states and measurements as the same thing, quantum mechanics needs to abstract both state and measurement as related but entirely independent objects. The state in quantum mechanics is the probability wave itself, while the measurement is only ever a sample of that distribution. As a second difference, the sub-states of the classical model do not interact with each other. The mathematical entity Rook + Pawn has no meaning in a classical system, but does in a quantum setting. The interference pattern we see on the screen in Young's experiment is the state represented by the superposition of the world where the electron passes through the top slit and the world where the electron passes through the second. However, the classical model still represents a solid theoretical framework if we recognize these differences and speak only about the states of a system and not its measurable attributes.

The computational model views the universe as a series of transforming states. The universe presents each state to us as a single object that simultaneously representing all of the smaller objects inside of it. In chess, the state of the board includes the state of all the pieces on the board as well as the player to move. Likewise, the state of a quantum system includes the individual states, or 'probabilities', of all the particles within that system. The computer presents us a state, which it chooses through an algorithm. The computer then selects the next state and presents it to us through a process no different than the interactions of a billiard table. Just like an old film projection system, our senses experience only one state of the universe at a time. They just update quickly enough that our senses interpret them as motion.

Under the computational model, there is no 'becoming'. One state does not become another state; there is no philosophical moment where we experience both at the same time. Instead, every time the computer updates the state of the system the computer deletes the old state and replaces it with the new one. A chessboard does not become a new state as a player takes a move; instead, it discards the old position and inserts a new one in its place. Quantum mechanics generates a model only marginally different. In it, every pair of states has a continuum of other states between them: similar to the difference between countable infinity and the continuum. Whenever an algorithm updates a state, it is also choosing a state from a population of states. In the case where this population is infinite, a choice is only possible if the infinity can be ordered. The natural numbers can be ordered, and therefore we can easily define algorithms that can choose elements from them. However, operations on the continuum cannot be ordered unless we first assume that such an ordering is possible. The only reason choice exists inside of the continuum is because mathematicians choose to assume the 'axiom of choice'. In quantum mechanics, every update requires the algorithm to choose a state, and every measurement requires another algorithm to choose an observation. The transition from structure to observation requires the universe to make two choices, and our best math has absolutely no way of modeling such a decision outside of an assumption.

A computer is any machine that makes choices. Viewing the universe as a state chooser solves a philosophical problem in physics. It is one thing to say that the universe is governed by equations, but it is quite another thing to materialize these equations, throw them into the air, and just hope that they fly away. Mathematical formulas only indicate directions on how a choice is to be made; they do not act out the choices that they make. In order for a formula to be realized, a computer must act on the equation and extract its decision.

Mathematical Computation

Computation is the mathematical branch dedicated to realizing equations; it is also something of a misfit among mathematical pursuits because it is surprisingly difficult to abstract computation away from physical processes. The equation $1 + 1 = 2$ includes three distinct parts. To the left of the equals sign we have the statement $1 + 1$. This statement implies a set of objects and a process to compute the decision. To the right of the equation we have the result 2. This is the conclusion of the equation. Each equation can be thought of as an incomplete proof. The statement $1 + 1$ is the premise, while the statement 2 is the result. The third part is then the proof narrative itself, the actual physical act of performing the equation. Pure mathematicians do not like to think about transformations, as long as they know that a proof narrative exists, the actual realization of that narrative is unnecessary; however, this is untrue for applied mathematics.

When a bank needs to know how much interest to pay to a client, the pure mathematical notion that 'the answer exists' is absurd because they require an actual number. The equation $1 + 1$ is easy enough to realize as even the most basic mathematical students have the answer memorized and require no additional work. However, most people have not memorized complicated equations like $9{,}752{,}341 + 1{,}528{,}945$. In order to realize these types of equations something needs to perform the proof narrative (11,281,286).

All realizable mathematical functions operate in much the same way. Anyone trained to perform an algorithm should be able to realize them. Computation talks about algorithms by abstracting each task as a sequence of symbols creating a valid mathematical universe, but the process itself always requires a physical contribution, consumption of energy, or the expenditure of resources. I first internalized this fact during a class on differential equations. My teacher wrote an equation on the board and a student immediately asked him how he realized that statement. The professor answered by informing the class that he employed the 'method of staring'. That is he wrote all of the symbols onto the board, took two steps away from it, and stared at them until the correct answer magically came to him. He then noted that the corresponding proof was easy to produce and went on with the lecture. Like it or not, the process of realizing equations cannot be wholly separated from the human brain.

Computation is, and will always be, a physical process. An equation can exist independently of the mathematician, but an unsolved equation needs a computer to make it whole. The original definition of a 'computer' is simply a term given to one who computes. A computer in these terms is a clerk working with a pen and parchment trying to realize a mathematical system. The task might involve anything: dividing a family's inheritance, working out a complicated tax system, or calculating the wages of laborers. Before the invention of the electronic computer, all of these equations were realized by hand. The initial theory of algebra, the language of algorithms, came from the Persian mathematician Muhammad ibn Musa al-Khwarizmi through his book *al'jebra*. Similar to Euclid's work, *al'jebra* is an exhaustive text that combines all methods for solving linear equations known at the time. As an example, consider the equation 4 + x = 14. We can solve for the variable x by subtracting 4 from both sides of the equation $4 + x - 4 = 14 - 4$ then realizing the addition and subtraction statements to find our answer; $x = 10$.

An algorithm, a term also derived from al-Khwarizmi's work, is then a set of rules by which a result can be found. If a student is instructed to add together two numbers, say four and three, they have two choices: either they have memorized the result or they need to employ an algorithm to find it. Early abacuses are a result of this process. Given the task of calculating 4 + 3 an early computer would start by sliding all of the beads of their abacus to one side. The computer then counts out four beads, sliding each to the right. Then the computer counts out another three beads and slides all of those to the right. After this process, he only needs to count all of the beads on the right of the abacus to find out what the result is. At no point in this process is the computer required to make a decision. In fact, any and every computer following the same algorithm should come to the exact same result. In this way, the decision the computer makes is wholly separable from the hardware that the decision was made on. Al-Khwarizmi algebra worked wonderfully as a tool for specifically solving linear equations, but as mathematics branched further away from number theory and tackled other problems involving other mathematical entities, mathematics began looking for a more general definition of computation.

In 1936, Alan Turing published the paper, "On Computable Numbers, with an Application to the Entscheidungsproblem." In it, he invented what we now refer to as the 'Turing Machine'. The Turing Machine is a theoretical construct that Turing hoped would be a suitable generalization of the actions of a computer, and that could serve as a standardized definition for computation. At the time Turing was competing with like-minded Alonzo Church who had independently produced 'Lambda calculus' which Church also hoped would fill a similar role. Like the human computer, the Turing machine is a physical object, yet is still general enough that it is not restricted to a single specific hardware. I could build a Turing Machine out of electricity, clockwork parts, gas powered machines, toilet paper roles, and even rocks on a beach (http://xkcd.com/505).

The Turing machine is comprised of two primary physical elements: the head, and a long, potentially infinite, paper tape. The Turing machine also comes programmed with a dictionary of rules that govern how it should operate. The machine starts in some initial state then proceeds according to these rules. The head reads the symbol on the sheet of paper directly below it, and then looks that symbol up in its dictionary. It then executes a set of commands from a limited vocabulary that the dictionary provides. Allowable commands include erasing the symbol the head is currently on, writing a new symbol, moving one space to the right, moving one space to the left, or any combination of these three actions. Additionally the machine can be programmed with multiple dictionaries, or states, and each action set can contain orders for the machine to look up the next command in a different dictionary.

As an example, we could create a Turing machine that can find the inverse of any random sequence of zeros and ones. Given the sequence 000110110101, I would start the head of the Turing machine on the first zero. Then assign the Turing machine the following dictionary.

0: E, W1, R (Erase the symbol, write a 1, move to the right)

1: E, W0, R (Erase the symbol, write a 0, move to the right.)

_: Halt (Halt when head reaches a space with no symbol.)

The Turing machine would start by reading the first zero and follow the appropriate commands. It would replace that symbol with a 1 and move to the next symbol, executing what if found there. One by one, the machine would switch each symbol, finishing only when the sequence ended. This machine is an abstraction of what a real human computer would do when computing values. If a human were tasked with adding 76 and 5 the human would look at the six, remember it, look at the 6, write down a 1, carry the extra one, then look at the new one and the old seven, write down an eight, and the computation is done.

Like all good mathematics, the true genius behind Turing's machine is not in its ability to abstract algorithms, but in the way it separated the physical from the real. Turing recognized that his machine was still just a machine and therefore static. The dictionaries of any real Turing machine would need to have some physical static presence. The symbols might be levers or switches that the head triggers that cause a cascading set of predetermined mechanisms. This is clockwork, and these kinds of machines had been discovered long ago. The true genius behind Turing's work is his recognition that certain static machines, hard-programmed to act on a specific set of clockwork instructions, can be tricked into performing the clockworks of another machine.

In the same paper, Turing demonstrated this programmability by mimicking the path taken by Gödel. Just as Gödel showed that every logical statement about mathematical systems could be expressed as an object inside of mathematics, Turing showed that the states of a Turing machine could be expressed as a 'descriptive number', the analogue of 'Gödel number', a number unique to that Turing machine. Since a descriptive number is itself a number, and in turn a long, possibly infinite, sequence of symbols, it can act as the input of yet another Turing machine. This new Turing machine, a machine that can read the states of other machines, reads this input and uses it to construct the dictionary of the machine the number represents. In this way, it can mimic every possible Turing machine, and therefore every possible computer. Turing concluded by showing that such a machine actually exists, and in doing so showed how to program a static machine to perform the tasks of other static machines: a process that is no longer static. It was later shown that Turing's machine was computationally equivalent to Church's lambda calculus: i.e., that every algorithm expressible within Church's calculus was also computable on a Turing machine and vice versa. This led to the proposition of the Church-Turing thesis: anything computable is computable on a Turing machine. The hardware found in all modern computers varies between manufacturers, architects, and firmware; however, the idea of computational equivalence allows us to treat them all as the same construct. The term 'computer' in the sense of hardware is synonymous with the concept of Turing equivalence. Any machine whose program can be run on a Turing Machine is a computer.

The basic theoretical framework of the modern computer is nearly identical to that proposed by Turing. The biggest difference is that modern computers only need two symbols: 0 and 1. Each symbol represents a power setting: on or off. There are several computational architectures on the market; the most common currently is the x86 architecture; however, the increased popularity of smartphones means that the ARM architecture is rapidly catching up. Each computer operates on a long string of zeroes and ones. The processor divides these strings into blocks, and interprets each block as a call to one of its many, possibly hundreds, of hard-coded instructions. Just like the Turing machine, each instruction represents a basic task the computer is wired to perform: insert a bit into memory, change a bit from 0 to 1, etc. The machine itself is a static system. It can only perform those operations that it is created with.

Programs themselves are also Turing machines. The string of zeros and ones represent the descriptive number of the Turing machine represented in the programing language that the hard coded instructions mimic. Assembly is a programming language that directly mimics machine code; however, it represents each instruction as a series of human readable characters. The C programming language, for example, takes this one step further by abstracting common series of instructions as individual functions. Python, written in C, takes this abstraction even further by assigning symbols to entire objects. Computers represent a hierarchy of Turing machines. Each machine interprets its own symbols into a descriptive number that the machine below understands. In this way, a user clicking on a symbol on their desktop is itself a symbol that is translated into a command that is in turn translated into machine code. The processor operates on it and sends it back to the user interface, which translates it back into the colorful patterns that the user recognizes.

The possibilities of functional computing are potentially infinite. All computable programs are ordered sequences of numbers. Since there are an infinite number of such sequences, there are also an infinite number of tasks a computer can perform. However, this infinity is definitively countable. Computers have limitations, one of which is their complete and utter inability to work with non-sequential data. This is less of a problem than one would think. Modern computers have perfected the concept of sequence and symbols to the point where the average user does not even realize that they are working with ordered symbols. I can use my mouse to click on an icon on my desktop, which opens a program. I can type characters into a keyboard and sounds come out of the speakers. Many common users do not even abstract the computer beyond the input and output devices they interact with. Modern computers can work on other kinds of sequences as well: chronological ordering, spatial ordering, and everything in between are all perfectly fine systems of organization. Likewise, the modern cliché that computers 'should just work' has designed a generation of machines that intentionally hides the layers of translation that mediate the transition between cause and effect on a computer screen. When I move the mouse to the right, the curser on the screen moves to the right. I neither interact with, nor participate in anything that happens in the interim; the 'laws of physics' of the computer mouse are completely invisible.

The same is true for the universal computer in physics. Our interactions with the universe are like a user working on a computer. When I throw a ball, I do not need to think about how that ball moves, only that when I throw it in one direction it moves in that direction. Just as the computer user has no access to the processes that facilitate the transition between the press of a button and the computer printing a character onto the screen, so too I have no access to the computational events that facilitate cause and effect. The equations that govern the movement of particles represents a computer, which, when activated, translates symbols into the universe's machine code through translation software that is unimportant to the mathematical model. Worse, these translations are not directly observable. They are true statements about the universe that we have no access to; however, this is not a problem. As long as our mathematical language is computationally equivalent to the universe's language, no important information is lost. The question has transformed again: Is modern mathematics computationally equivalent to the universe?

Every computer necessarily works with two distinct layers, both of which operate oppositely to what our common intuition says about them. On the physical layer, there are the things that we sense. Objects I touch, measurements I read, the burning sensation I feel when I put my hand in flame, the feeling of support when I sit in a chair, all of these are measurable outcomes of resolved equations. The universe hands our brain input in the form of a state, then our brain computes a response. Everything physical is really just the process of differing computers realizing equations and handing them to other computers. Sense from a computational perspective is nothing more than this transfer of data.

On the second layer there exists the computer, which uses physical processes to realize abstract entities. The algorithms that govern this computer are its 'laws of physics'. The base processes of this computer are the axioms of the entire universe. These axioms themselves are sublime truths in the sense that we have absolutely no way of either directly observing these processes or proving them using logic that they themselves have not already constructed. What we get is a reversal of expectations. What we see around us is actually a virtual world generated by these physical ideal truths inside the mathematical construction. In the computational model, we exist within a constructed virtual reality, generated by an ideal physics beyond what we can demonstrate. To borrow from popular culture, the computational model is quite literally *the Matrix*: a world that hides its own physicality inside the illusion of the physical.

The humanist analogue to this concept is that of language itself. Language is a construct; it has evolved alongside humanity. Whenever we try to express an idea, it is always through some form of language. Ideas and concepts are in some sense virtual. Language and custom operate as the computer, and that computer builds everything we say and do. Perspective can be just as much of a cage as the computational Matrix that hides the real world from us. In order to escape from his virtual world Neo needed help from the outside world. He needed to interact with something that existed beyond his perspective. The same is true for language. Language can change because it is constantly bumping into events that defy the conventions it sets for itself. When I speak with someone else, I make mistakes, I miscommunicate, and I offer explanations that do not exactly represent what I am trying to explain. These imperfect interactions offer language something the computational model does not have: a way to interact with entities beyond itself. Unfortunately, no such change is possible in an ideal computational model. In the rationalist's world, our virtual reality cannot interact with any other entity precisely because the rationalist denies the existence of anything it could interact with. The foundations of the computational model are precise and cannot change, because they construct the concept of change itself. If the Hilbert model of mathematics had succeeded then the argument would end here; however, it failed. Such a world cannot exist, and the computational perspective does interact with the outside world -- every time it fails to model it.

<u>Human Computation</u>

The computational model is itself nothing more than common sensical intuitive deductions that dance precariously around hard mathematical truth. The 'good enough' principle allows us to bypass Gödel, by allowing us to assume that we are only after practical truths. The 'good enough' principle allows us to still be excited about the potentiality of scientific progress because anything beyond that limit must be impractical and useless. Finally, the 'good enough' principle offers the illusion of everything because it denies the practicality of those objects beyond itself. The perception that the computational model is a clean, emotionless object with impossibly straight edges is itself a construction with absolutely no basis in reality. The model itself is the exact opposite of this; a piecewise collage of related ideas that only begin to resemble humanity when we see it at an enormous distance. The only reason we prefer it right now is because it is tremendously useful. The computational model is to the physical sciences what penicillin was to medicine: a miracle drug that has proven itself repeatedly, and has pushed humanity towards an explosive technological era that offers with it the hope of unlimited potentiality. However, if the humanist is to tame the computer, 'good enough' is not good enough, and that might be its greatest strength.

While prototyping a system for visualizing the Orlando project, I was reminded by Susan Brown that what humanists are really interested in is not wide trends or overarching generalities, but instead those moments where the model breaks. Humanists do not want to hide the fuzziness, they want to see it, stretch it, and reveal all of its hidden subtleties. Unfortunately, I feel that the humanist conversation has turned more towards distorting visualizations in order to force them to represent this fuzziness. While interesting, this concept misses the primary means through which mathematics, and therefore computers, communicates: failure. Proof by contradiction is itself the attempt to gain knowledge through the failure of an idea. Likewise, visualization can never be a means to communicate subjective complexity because it will always bring with it the bias of the mathematical model that constructed it. If the humanities are to use the computer to its full potential, then they need to see visualizations, and perhaps data itself, not as a construction of their ideas but as a proof by contradiction. In computer science, this is exactly how computation is teaching us about humanity. When a computer scientist claims that the human is a computer, they are really claiming that they are going to model the human as a computer and if the model fails then they have learned something new, but if it succeeds then that is all the more incentive to keep going. These biases form the beating heart of the computational model and cannot be easily removed. Likewise, if the humanist is to accept the computer then they need to at least understand the computational idea of proof by contradiction. It may not represent a canonical 'humanist method' but portions of it does represent the point in mathematics where models break down, and by focusing on these points humanist goals are achievable.

Alan Turing codified the computational equivalent of 'good enough' in a second paper titled 'Computing Machinery and Intelligence'. Turing makes the argument that a discrete state computer, that is, a computer that transitions from state to state given sufficiently large complexity, can mimic a human being to the point where no one would be able to tell the difference. The claim is that such a program would be 'good enough' to be human. The argument he uses to make this claim involves the experimental equivalent of the 'good enough' principle: the Turing Test. The Turing Test involves two humans and one computer. One of the humans is the examiner and only gets a keyboard and a computer display. The remaining human and the computer are the test subjects. The examiner is tasked with typing questions into her computer terminal and based on the two responses determine which one is human. The other human also has a keyboard and display and needs to type back answers to the questions that the examiner sends. Likewise, the computer also receives as input the question in the form of a string of characters and functionally returns a new string of characters as its answer. Turing argued that if the computer were sufficiently programmed it could fool the examiner into thinking that it is human. If the examiner cannot figure out which one of the subjects is human, with higher certainty than a random guess, then for all intents and purposes, Turing argues, that the computer can think. After all, the only reason we believe each other to be thinking entities is because we see each other act in such a way that convinces us that the other is a thinking entity. We make decisions, and so do computers.

The Turing Test works by reducing written language to a long sequence of symbols. Turing's hypothesis is that if we, as humans, can transform sequences of symbols into other sequences of symbols then we are essentially computers. He sees no reason why the computer cannot do likewise. From the perspective of the examiner, any response would either erode or reinforce her belief that the object she is talking to is human. All the computer needs to do to prove its humanity is transform each sequence of letters into other convincing sequences. The key is that the output needs to be a convincing output, not a correct output. Computers have a reputation for performing perfect arithmetic, but Turing's imitation machine would need to recognize this and insert mathematical errors as well as slow down its own replies in order to compensate. The complexity of such a task is enormous, and even Turing recognized this difficulty. Before the computer could accurately recognize and convert written speech it would need to be able to accurately recognize as well as interpret the entire human condition: sarcasm, emotions, metaphor, and personality. Corrections would need to be made for all human imperfections, mistaken algebra, friend-of-friend stories, misquotations, imperfect grammar, emotional attachment to certain subjects, etc. However, Turing recognizes that computers are a potentially infinite quantity. We can always add more processors that perform more and faster complex calculations, and everything I mentioned so far, from sarcasm to inside jokes, can be thought of as state variables. He saw absolutely no reason why we could not program a computer to perform the function "Fine thank you very much." = F("How are you today?"). At the very least, Turing challenged his readers to answer the question "Why not?" What part of human speech is not simply just converting symbols that represent ideas into other symbols that also represent ideas? The computational model may rest on an ample supply of unchallenged assumptions, but so does our understanding of ourselves. The Turing Test is a challenge for anyone interested to take these assumptions seriously and come up with a legitimate reason why humans are not computers. Where, if at all, does this model fail?

The classic response to Turing's challenge comes in the form of John Searle's 'Chinese room argument'. Searle envisions a man trapped inside a room connected to the outside world only through a small slit in the wall just big enough to slide paper through. Inside this room contains only a desk, paper, writing tools, and one extremely large dictionary containing enough rules to adequately transform sequences of Chinese characters into other sequences of Chinese characters. The man himself cannot speak or comprehend any form of Chinese. The man's only task inside this small room is to compute responses to any input he receives. Whenever such an input arrives, he merely follows the steps dictated to him through the machine. He checks the first character and looks it up in the dictionary, then performs whatever tasks the dictionary asks him to do. Eventually when his tasks completes, he will have produced a second sheet of paper filled with Chinese characters. He then slides that sheet of paper out the same slot from which he received the input. Since the dictionary is sufficiently complicated, the woman outside the box finds the response to her query satisfactory and walks away satisfied. Searle argues that since the ideas expressed in those characters are never revealed to the man inside the box, the box itself cannot be thought of as a thinking entity. So too a computer, which acts in the same way, cannot be thought of as a thinking entity.

I argue that the Chinese room argument fails as an answer to Turing's challenge because it fails to understand the computational model. More specifically, Searle's argument is correct, but only from an axiomatic basis different from the basis of Turing's argument. Searle tries to appeal to understanding as a subjective concept: qualia. Imagine a scientist born blind who studies to become an expert on vision. He might have complete knowledge of the mechanics of vision, might be able to accurately simulate vision on a computer, and might even be able to build a robot that acts on visual stimulus, however, when he cures his own blindness the first moment of sight will produce in him a subjective experience that no amount of mechanical training could have prepared him for. In his 1980 article Searle argues two points: firstly that "I have inputs and outputs that are indistinguishable from those of a native Chinese speaker, and I can have any formal program you like, but I still understand nothing," and secondly that the computer does not 'explain' computer understanding because, "we can see that the computer and its program do not provide sufficient conditions of understanding since the computer and the program are functioning, and there is no understanding" (418). The argument rightfully points out that mathematics as a purely formal system not only fails to account for this level of understanding, but also outright denies it. However, the Hilbert model failed and this interpretation was dead before Searle wrote this paper.

The mechanics behind attention and the differences between the conscious and the unconscious brain are not well understood. Let us return to the Matrix example and consider the scene where Morpheus teaches Neo to fight. Unlike conventional martial arts, Neo did not learn to fight through conventional teaching techniques. Instead, Morpheus simply attached him to a computer and downloaded a fighting program. When Neo opens his eyes after the procedure he immediately utters, "I know Kung Fu." This occurrence is similar Searle's: "let the individual internalize all of the elements of the system. He memorizes the rules in the ledger and the data banks of the Chinese symbols, and he does all the calculations in his head…. All the same, he understands nothing of the Chinese" (419). When Neo learned Kung Fu, did he also learn to understand Kung Fu? Is he just following some preprogrammed set of instructions? Are his arms flailing around madly without the help of his conscious brain, or did he also gain the conscious mind of a martial arts expert? These questions are not addressed in the movie, and indeed the computational model of the brain does not have an answer to them yet. The question is open, and nothing has been decided one way or the other. However, there is also no point of failure. Nobody has found anything definitive that would prove that subjective experience is itself not information that can be downloaded into the brain, and there is no reason to suspect that the human brain has access to some fundamental source that no other physical object has access to. However, to Searle this is not an open question. In fact, the problem is so firmly solved from his perspective that he frequently questions why he even needs to address it: “Actually I feel somewhat embarrassed to give even this answer to the systems theory because the theory seems to me so unplausible to start with” (419).

The difference is, of course, axiomatic: “The study of the mind starts with such facts as that humans have beliefs, while thermostats, telephones, and adding machines don’t. If you get a theory that denies this point you have produced a counter-example to the theory and the theory is false” (Searle 420). Searle began with the assumption that humans have something that machines do not, and any statement that denies this is immediately false. In Searle’s world this fact is axiomatic, it cannot be questioned. Likewise, all of his arguments are an appeal to this axiom. However, the simple fact that he is arguing axiomatically does not make him wrong as Turing is also arguing axiomatically: Turing assumes that all decision-making machines are computers. It does mean, though, that the two arguments are independent of each other as they both operate inside different and mutually contradictory universes. Likewise, further discussion about the ‘correctness’ of Searle’s argument is likely to descend into a fundamentalist squabble similar to the ‘evolution’ discussion brought up in Chapter One. Therein lies the problem, neither argument can be ‘wrong’. Turing invented an axiomatic system in order to frame a hypothesis, and this axiomatic system is at odds with a different axiomatic system that Searle is clearly not willing to give up.

Searle has not pointed out a failure in the computational model, only that the sciences and the humanities understand knowledge differently. To the computer scientist, the knowledge in the computer is stored in the dictionary not the man. The man interpreting the dictionary may not have access to the ideas because he does not understand Chinese, but mathematically he is acting as little more than an extra physical appendage to the whole contraption: the ideas are real, the physical processes are illusions. The dictionary is equally a part of the computer as a whole, but represents the abstract rules that the computer uses. Searle’s argument hinges on his belief that the human is the only conscious part of the machine, consciousness in this sense being the understanding that links each symbol with the idea that it symbolizes. Searle also believes that the dictionary only contains a collection of 'formal' symbols and therefore cannot possibly also represent the subjective half of the equation. However, this cannot be true as mathematics is not purely 'formal' and its symbols cannot be definitively separated from their meaning. The mathematician intuitively understands that mathematical systems exist as more than just a formal system, and that multiplicity is encoded in the mathematical text itself. There is no reason why a dictionary cannot contain the information necessary to produce a subjective experience within its computer because it contains the entirety of mathematics and, even though subjectivity is poorly understood in a mathematical context, must also contain the intuitive 'subjective' portion of mathematics beyond formal symbols. From this perspective the Chinese room could contain Searle's 'understanding' as it is encoded in the book itself; however, Searle will never accept this premise because his belief that only humans can 'understand' is axiomatic: i.e., he would argue that mathematics only contains 'subjective' qualities when it is in interaction with a human. To question this belief is to question his notion of truth itself. As discussed in the first chapter, the mathematical perspective does not require a human to make it true, and a computer does not require a human to make it understand. Searle's argument is a demonstration of the failure of communication between science and the humanities, but it is not a failure of the computational model itself because the computational model is a hypothesis and so is Searle's.

Turing's paper concludes with a series of counter-arguments to his proposed experiment. Of these arguments, Turing brings up 'The Mathematical Argument', which is his way of facing Gödel's idea of incompleteness mentioned in Chapter Two. He recognizes that there are inherent limitations found in that particular theorem, as well as many others that followed. However, this did not pose a problem: "The short answer to this argument is that although it is established that there are limitations to the powers of any particular machine, it has only been stated, without any sort of proof, that no such limitations apply to the human intellect" (56). Math is limited, but so are humans. It is utterly naïve to assume that there are no inherent barriers limiting what we as humans can do. Biological processes govern our brains and those processes are only capable of so much. The question was never whether computers are limited, because the limitations of mathematics are known. Turing understood the limitations inherent to the mathematical model, but chose to use it anyway. Instead, the more interesting question is how these limitations compare to the human intellect. Which one is more limited: the computer or the human?

The admission that computers are limited unfortunately opens the floodgates to numerous poorly worded objections. Anticipating this, Turing immediately followed this comment with quick responses to the closely related 'argument from consciousness', and 'argument from various disabilities.' These arguments can be generalized as the set of arguments of the form 'computers cannot do X', where X is some subjective event thought to be unique to the human experience. Various examples include falling in love, feeling an emotion produced by a sonnet, caring about someone, getting angry, appreciating art, enjoying strawberries and cream, etc. His response to this argument is an appeal to the uncertainty surrounding the above-mentioned limitations. Computers are a potentially infinite entity, and just because computers cannot do these things now is not an argument that they will never be able to do them.

> I believe they [these arguments] are mostly founded on the principle of scientific induction. A man has seen thousands of machines in his lifetime. From what he sees of them he draws a number of general conclusions. They are ugly, each is designed for a very limited purpose, when required for a minutely different purpose they are useless, the variety of behavior of any one of them is very small, etc., etc. Naturally he concludes that these are necessary properties of machines in general. (57)

The only reason we cannot conceive of a computer doing X is because we have not yet built a computer that can do X. This very closely mimics the potentiality argument of science mentioned in the previous chapter. As the memory and processing power of these digital computers increase so too will their ability to perform new algorithms. What is one day impossible for the computer will eventually become possible given sufficient expansion of its computational power; however, Turing's project is not nearly as ambitious as the physics project. He does not intend to build a universe, merely a human. Remembering that the entire thesis of his paper rests around building an experiment to test a hypothesis, Turing's appeal to infinity is not intended as a conclusive argument, only a reminder that at the time the paper was written there was, and remains, no direct concrete method for comparing the two: Why not? The answer he leaves to his readers.

Chapter 6: Conclusion

> One currently influential philosophical movement goes under various names such as postmodernism, deconstructionism, and structuralism, depending on historical details that are unimportant here. It claims that because all ideas, including scientific theories, are conjectural and impossible to justify, they are essentially arbitrary: they are no more than stories, known in this context as 'narratives'. Mixing extreme cultural relativism with other forms of anti-realism, it regards objective truth and falsity, as well as reality and knowledge of reality, as mere conventional forms of words that stand for an idea's being endorsed by a designated group of people such as an elite or consensus, or by a fashion or other arbitrary authority. And it regards science and the Enlightenment as no more than one such fashion, and the objective knowledge claimed by science as an arrogant cultural conceit. (Deutsch 314)

In Chapter One, I tackled the challenge of explaining mathematical objectivity. Objectivity is an important concept in both mathematics and science. However, the mathematical claim of objectivity requires a distinct separation between the mathematician and the mathematical perspective. The same is true in the sciences. Science is useful because it provides a method for challenging subjective beliefs and intuitive connections. However, objectivity becomes fundamentalism when this separation is lost. Fundamentalism exists wherever objectivity is accepted as a replacement and justification for subjectivity. However, the separation between the human and the mathematical is itself subjectively believed, and this belief forms the mathematical perspective. Objectivity is the physical attempt to understand one's own belief structure through the eyes of someone else. In mathematics, this someone else represents the mathematical perspective itself. This goes against everything the rationalist will say about mathematics. The popular perception believes that what they believe about the universe is dictated by how those beliefs are justified. In mathematics the opposite is true. Belief in mathematics requires no justification, and always precedes mathematical truth. Justification is important, but only in the sense that it builds secondary truth on top of an unjustifiable structure of beliefs. The rationalist would argue that something that cannot be justified, cannot be demonstrated, should not be believed. Against this argument, there is no rebuttal; however, if this premise were to be accepted then pure mathematics would argue that mathematics itself should not be believed. It exists as an entity independent of our world, devoid of usefulness, and truth. What is true of a mathematical world is not true for the real world, and vice versa. Pure mathematicians are fundamentalists in the sense that they strictly adhere to their rules, precisely because they are rules; this is what makes mathematics possible. Yet, these rules are also what separate math from everything else, and give it the aura of objectivity that mathematics is known for. Mathematics is separate because it is constructed separately; it exists only because it believes it exists.

In Chapter Two, I talked about the inherent limitations of 'formal' mathematics. Whenever mathematics declares what it believes, it simultaneously defines what it can and cannot explain and represent. All axiomatic systems are limited. Every time something is reduced to a well-defined system of representation, there will always be things that that system excludes. Whenever someone defines knowledge around strict methodological boundaries, they are also separating themselves from that which their methods cannot demonstrate. Information is always lost. This is true for every mathematical model. This fact directly affects humanities computing as it forces the humanist to accept that all digital representations will fail to perfectly represent that which they want to represent. This is why humanists like Johanna Drucker dislike mathematics, because these limitations restrict the level of complexity that mathematical models represent; however, this is also why these same people fail to understand computers. From the perspective of mathematicians, the only thing scarier than bias that they are aware of is bias that they are not. Mathematicians prefer to work in environments where the limitations and biases are explicitly presented prior to the discussion and computers present such an environment.

In Chapter Three, I challenged the dogma surrounding infinity. It has become profoundly cliché to make the claim that the universe is infinite. In rationalist science the word 'infinity' has become an invitation to knowledge: a word that represents the limitless power of rational thinking. This definition of infinity acts as a justification to the argument that certain specific mathematics is immune to the limitations of normal abstract science. However, mathematical infinity is an illusion. It does not exist in the abundance of information, but in the absence of it. In physics the black hole exists as a singularity because we cannot currently model what goes on at its center; however, the potentiality of science claims that eventually we will discover a scientific model that can. Unfortunately, this claim could be false because it silently assumes that such a model will still be recognizably scientific and self-similar in the same way that mathematical infinity is self-similar. Each of Cantor's infinities represents a definable pattern where each element acts according to a set of rules that defines its infinity. Each infinity contains copies of itself, but these copies say nothing more about the original than the original says about itself. Infinity in this context is not a claim of multiplicity but instead a claim of similarity. Everything inside of the infinity will be similar to everything else inside that same infinity: the elements themselves change, but the rules do not. Likewise, the elements of science change, but the claim of infinite science prevents science itself from changing and forces it to remain subject to its own incompleteness.

Finally, in Chapter Four, I talked about computers and the computational model. Computers represent the legitimate offspring of mathematics crisis and the culmination of the last thousand years of mathematical thought. Computers may create formal systems, but they act on them as an objective observer acting on a mathematically small world. In every possible sense, computers are the personification of modernistic mathematics. Computer programs themselves are proofs that transform every result they spit into truth. Yet, modern computer science is not about algorithms, but instead heuristics. Computer science understands that computers are inherently limited machines built on top of a rickety framework of assumptions. Yet, instead of working to replace the framework, the goal of computer science is to push this framework to its utter extreme in order to learn more about the limitations of the computer and compare those limitations to the world around them. Unlike conventional wisdom that claims that knowledge is gained when error is reduced, computer science learns precisely because the limitations of computers are already known. Whenever things go wrong, and they frequently do, this self-knowledge provides an outlet to explain precisely why things have gone wrong. Instead of worrying about the inherent limitations of the computer, computer science works to build on top of it: to use this knowledge of mathematical limitation to study the limits of other poorly understood systems.

To conclude I wish to repeat Turing's question: what is the difference between a computer and a human. I ask this because it precedes a question I believe to be important to humanities computing: what is the difference between humanities computing and computation? It might seem that these two fields are entirely unrelated entities just as the computer and the human might seem like entirely unrelated entities. However, the similarities between postmodernism and incompleteness, the sublime and infinity, and the human and the computer should not be discarded so readily. Unfortunately, for every similarity I point out someone else will produce two dissimilarities. This is why I cannot enter the conversation of degree in humanities computing. As long as someone wants to find a distinction between two objects, they will find something, and likewise as long as the majority of people working in humanities computing demand that it is only interested in humanist problems then that too will also be true. However, humanists are not the only group of academics guilty of failing to understand their opposition, as David Deutsch so marvellously demonstrates in the epigraph. He too is looking for something to separate his 'correct' philosophy from that which he deems to be 'incorrect'. As long as the perception of separation exists, then separation exists.

Thinking back on the Mark Sample quote in the introduction, "The heart of the digital humanities is not the production of knowledge; it's the reproduction of knowledge" (256). I cannot help but think that the space that humanities computing occupies can be used for more than just creating and analysing digital text. What if humanities computing become more than just humanists using computers? What if humanities computing dedicated itself to the translation of ideas between these two divided fields? What if humanities computing gave up the question of degree in favour of the 'reproduction' and translation of knowledge between the humanities and computers in both directions? I cannot conclusively demonstrate the space that this new humanities computing will occupy, but in my attempt to translate the ideas of mathematics into the realm of the humanities I hope that I have demonstrated that such a realm can exist. Further, I do not believe this direction deviates that far from what humanities computing is already doing. Transforming humanist works into digital works is an important first step in such a project, but moving ideas in the other direction is an equally important second step.

In the end, the point of this essay was to ask these questions. If this is not the direction others wish to take, then humanities computing can continue on its current course. From my perspective, how humanities computing wishes to define itself is axiomatic. Each decision it makes will dictate what knowledge it generates, and every potential 'humanities computing' will produce different and important knowledge. If the field wishes to maintain its current course as a distinctly humanist endeavour then so be it. However, I believe that there is a space big enough to contain both fields and I also believe that occupying that space does not require the sacrifice we think it will require.

I would like to end on a quote from a man influential in both English literature and Christian philosophy. I would name C.S. Lewis as one of the strongest influences on both my Christian faith and my philosophical leanings. At the end of his book *The Last Battle* Queen Lucy and the rest of the inhabitants of Narnia leave the old Narnia as it is about to be destroyed and entire a new real

Narnia that the old one was copied from. From there, they effortlessly climb a mountain and, looking out over the vast Narnia below them, discover at the top a garden that itself contained another Narnia:

> Lucy looked hard at the garden and saw that it was not really a garden but a whole world, with its own rivers and woods and sea and mountains. But they were not strange: she knew them all.
>
> "I see," she said. "This is still Narnia, and more real and more beautiful than the Narnia down below, just as *it* was more real and more beautiful than the Narnia outside the stable door! I see … world within world, Narnia within Narnia…."
>
> "Yes," said Mr. Tumnus, "like an onion: except that as you continue to go in and in, each circle is larger than the last." (207)

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