Creation of a water market in the Athabasca oil sands region: will vertical integration create incentives for entry deterrence?

by

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Abstract

The implications of creating a water market in the Athabasca region of Northern Alberta are examined with the objective of creating a system which encourages efficient water usage by oil sands mining producers. An analysis is performed considering entry to the oil industry when there is no constraint on available water supply versus a situation where the available water supply is constrained. Vertically integrated incumbent oil firms can strategically increase their capacity investment in the downstream oil market to exercise market power in the upstream water market, resulting in entry deterrence when there is a constraint on the water supply. In the absence of a constraint on the water supply, we show that the market will be no more efficient than the current water allocation system.

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I. Introduction

The oil sands in Northern Alberta, Canada are among the largest proven oil reserves in the world. These deposits are unconventional oil, rather than traditional crude oil. The extraction of these unconventional oil reserves via mining is far more water intensive than traditional crude oil. In fact, the sole source of water for oil sands mining is fresh water, used at an average rate of 2.5 barrels of fresh water per barrel of oil produced. Total oil sands production used over 1.2 billion barrels of water in 2013 to produce 0.7 billion barrels of oil (Alberta Environment and Sustainable Resource Development, 2014). According to the Pembina Institute, the total oil sands water usage in 2011 would be equivalent to the annual residential water use of 1.7 million Canadians. Aside from a one-time application fee for a water licence and any related water infrastructure cost specific to the operation, water is effectively a free commodity for use in the oil production process. This would suggest that there is potential for overuse of an essentially free input to the production process. This thesis examines the benefits and consequences of setting up a water market in the Athabasca region.

The following analysis will focus on oil sands mining operations which are massive in scale. According to the Government of Alberta, there were 114 active oil sands projects in 2013, with 6 of these projects being mining operations. Imperial Oil's Kearl mine is not yet considered fully operational, but is one example of the massive scale of this type of project. The Kearl project has a $12.9 billion dollar price tag and its expected capacity is 40 million barrels per year (Lewis, 2014); the project has a water allocation equivalent to over 600 million barrels of water per year (Alberta Environment and Sustainable Resource Development, 2014). These oil sands mining projects are large scale operations which yield a massive capital cost and are challenged with selling their oil at market prices. Given the current mining technology and the scale of these projects, we see that water is an integral component to the production of oil: the resulting production function faced by the oil producers is increasing in both capital investment and in water. This suggests that water and capital are complementary inputs to the production process.

We conjecture that in such a scenario, and with the introduction of a water market, capital investment may be used to drive up the resultant water price in order to deter entry.

Early literature in strategic entry deterrence by Dixit (1979) and Spence (1977) found that investment in capacity by a monopolist can be used to deter entry as it creates a credible threat to the entrant's future profits. Gilbert and Vives (1986) extended the analysis to multiple, non-cooperating, incumbent firms to find that it can still be profitable for these firms to invest in capacity to deter entry, provided the entrant faces a fixed cost, or entry cost. To operate a mining project, the oil producers must invest in large amounts of capital, and these large-scale capital projects require a correspondingly large water allocation to accommodate the high volume of production. The distinction is that this scenario, unlike the classic models of entry deterrence, includes oil firms who operate as price takers in the oil market and thus have no ability to directly impact entrants' profits in this market. The model proposed in this paper is constructed to examine the impact of the creation of an upstream water market which causes these firms to become vertically integrated.

Vertical integration has long been a topic of economic debate as to whether integration creates efficiencies or whether it restricts competition. Kuhn and Vives (1999) looked at the welfare impact of vertical integration of an upstream monopolist when the downstream industry is imperfectly competitive. They show that vertical integration actually increases output downstream, due to cost efficiencies from eliminating double marginalization; however they neglect to consider any potential strategic effects on the market. Salinger (1988) found that an upstream monopolist can limit access to the upstream input, if vertical integration results in market foreclosure; this will cause a resultant price increase in the downstream market. De Fontenay and Gans (2005), however, show that there is a greater incentive for vertical integration under upstream competition than under upstream monopoly, thus competition enhances the potential for strategic vertical integration. Normann (2011), found empirical evidence that supports the hypothesis that markets containing vertically integrated firms are less competitive and when the vertically integrated firm is present, prices will generally be higher than in nonintegrated industries. Our model examines whether the creation of a vertically integrated set of incumbent oil firms will in fact be more efficient, regarding water usage and cost minimization or whether the firms will use the fact that capital investment and water usage are complements to deter entry.

To our knowledge this type of analysis has not yet been applied to the oil and gas sector. The current literature on vertical integration focuses on the exercise of market power in the downstream market which can be identified as an increase in the downstream price. Our paper is unique in that, as far as we can find, it is the first paper to suggest that the vertically integrated firms may exercise market power in the upstream market, not the downstream market. The oil firms can overinvest in capital (hereafter stated as the capacity) downstream to strategically manipulate the upstream market due to the complementarily of capacity and water. We will show that the companies can credibly increase their capacity, thereby increasing their water demand to the point where they have used up their entire water allocation, resulting in an increase in the water price upstream.

Advocates for water policy reform in Alberta have promoted the creation of a water market to stimulate efficient use of water and to reduce waste. Percy (2005) advocated a market based approach to water allocation as a potential solution to scarcity. However, adoption of a water market in Northern Alberta may result in a market that does not function effectively; the fact that there are few players in the region raises questions about the potential for a viable market in terms of facilitating water exchanges and transfers (Adamowicz, Percy, & Weber, 2010).

The absence of an efficient water market currently exists in southern Alberta, where transfers of allocations between water licensees have been allowed for over a decade. These transfers may take one of two forms: temporary transfers do not require any official approval by the government, while permanent transfers of water rights are approved by the regulator. The price which changes hands for either type of transaction, however, is not determined by the market; the price is privately agreed to between the buyer and seller. Although market proponents may think that allowing water transfers will facilitate more efficient use of water, the 'market' has produced a different result: since the allowance of transfers, very few transfers have actually taken place. Despite publically searching for a purchasable water allocation, the town of Okotoks, Alberta had to undergo water rationing, over a period population growth, for several years. The municipality was finally able to acquire an additional water allocation in 2013. (Patterson, 2013). A possible explanation for the lack of activity in this market is that about 75% of the water licenses in the South Saskatchewan River Basin are for irrigation or agriculture purposes. The vast majority of these licenses are controlled by associations called irrigation districts – these districts have rules whereby the district has to approve the transfer of a water license held by a member. Market power of the irrigation districts is a likely reason why few water market transactions have taken place to date in Southern Alberta. The market power seen in southern Alberta is not unlike the potential market power the oil producers would hold in the north under a water market.

The remainder of this thesis will be organized as follows: Section II will provide a background on the oil sands operations, the production and current water licensing, and usage in the region. Section III will define the model and describe the results for the case where there is no entry in the market, which results in an efficient market outcome where price equals marginal cost; it will then go on to discuss the case where there may be entry in the market. The findings here are that, in the absence of a binding constraint on water supply, accommodation will be optimal whereas if the water supply constraint is binding, it may be optimal for firms to deter entry. Section IV will provide a brief numerical example. Section V is a discussion of the findings and section VI will conclude the paper.

I. Background

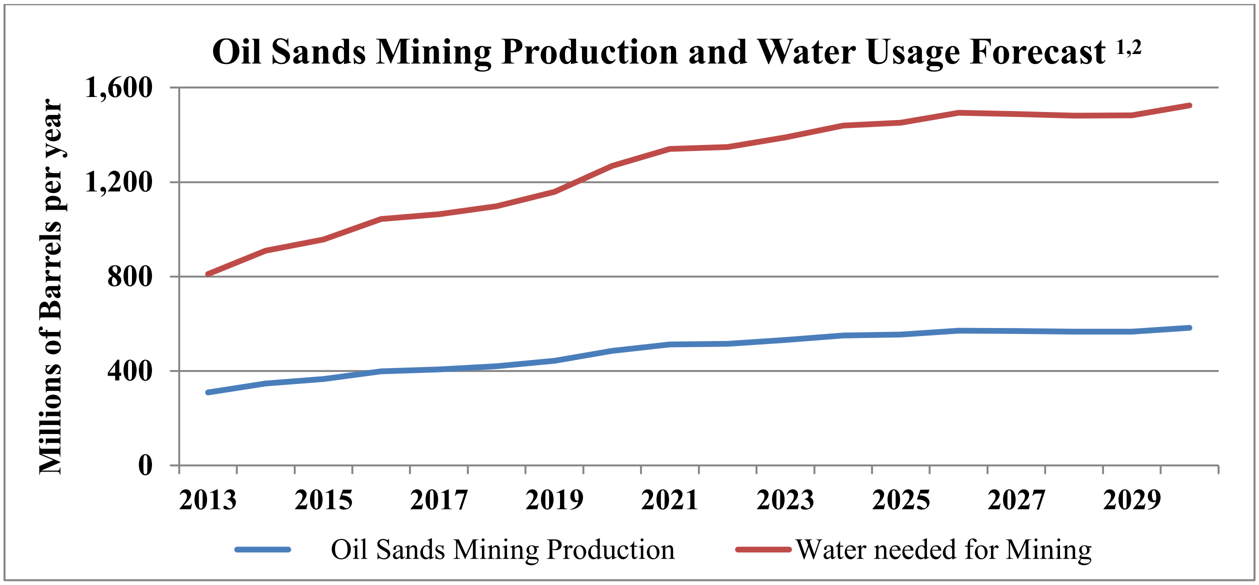
The majority of oil sands activities (and all oil sands mining activity) takes place in the Athabasca watershed, located in northern Alberta, which is home to the Athabasca River. Flows from the Athabasca River originate in the Rocky Mountains and the river flows east through the northern part of the province, eventually emptying into the Peace-Athabasca delta in the North

Eastern Alberta. As of 2010, about seventy-five percent of the water allocations from the Athabasca River Basin were for industrial purposes, the vast majority of which are used for the purposes of bitumen mining near Fort McMurray, Alberta (Alberta Environment, 2010). Other remaining water allocations for commercial and municipal purposes are comparatively small and are dispersed along the length of the Athabasca River.

The Athabasca River is fed by the Columbia Glacier and as such it experiences significant fluctuations in seasonal flows. The flows in the spring and summer, when the snow melts and runs off, are typically five to six times greater than the winter flows when the river is frozen over (Environment Canada, 2014). Schindler and Donahue (2006) caution that climate warming trends show potential for significantly decreased river flows in the Prairie Provinces in the future. Although the current river flows are adequate to meet the needs of the water users in the area, potential growth in population and industry activity, along with decreased flows indicates that water constraint is a strong possibility in the future.

There are currently two different technologies being employed in the recovery of oil, or more specifically bitumen, from the oil sands: mining and in-situ technology. In 2013, mining activities accounted for nearly fifty-five percent of all oil production from the oil sands. Mining activities, however, consumed almost ninety percent of the total fresh water used in the extraction and upgrading of bitumen from the oil sands, while in-situ recovery projects used just over ten percent (Alberta Environment and Sustainable Resource Development, 2014). Although the area and absolute quantity of oil sands reserves are very large, much of the known reserves are not fully recoverable with the current available technology. Mineable reserves account for less than 10% of the total reserves but are estimated to make up over one-fifth of the recoverable total (Griffiths, Taylor, & Woynillowicz, 2006). Mining activities have been the predominant method of oil sands extraction for the past five decades. Although production through in-situ technology is growing rapidly, the Canadian Association of Petroleum Producers is forecasting the annual volume of mining production to nearly double over the next 15 years.

Chart 1:



1. Production mining forecast taken from CAPP's June 2014 release
2. Average water usage per barrel from GOA oil sands information portal, 2013 data

The above graph demonstrates a dramatically increasing demand for water in mining operations over the next 15 years, and the primary source of this water will be from the Athabasca River. Although the water allocations currently outstanding to the existing oil producers will be sufficient to satisfy this projected demand, there are still potential constraints. This projection is based on currently planned operations and their expected future output; entry to the industry could place further demands on the water supply. Also it is possible that drought or climate change may cause shocks to the future expected water supply in the area. Forward looking firms will realize potential water scarcity could threaten their future profitability and may find it in their best interest to act strategically.

The current legislation does not allow for in-situ operations to get licenses for water withdrawals from any major rivers – they are restricted to using water mainly from groundwater (aquifers) and some from surface water/runoff. As per Alberta Energy Regulator's Directive 81 (2012), in-situ operators are subject to stringent guidelines regarding water recycling and usage rates that are mandatory for their operations; no analogous legislation exists regarding mandatory recycling rates for mining operations.

Oil sands mines face no mandatory recycling rate or regulation on the efficiency of their operations regarding the volume of water used per barrel of oil produced. Mining operations used a total of 164 million m^3 of water in 2013, most of which is from the Athabasca River (Alberta Environment and Sustainable Resource Development, 2014). There are currently five firms with approved or operating oil sands mining operations, as of November 2014 (Alberta Environment and Sustainable Resource Development, 2014) and these five firms hold close to 75% of the industrial water allocations from the Athabasca. As previously mentioned, the mining industry is characterized by entry and exit of firms: Imperial Oil is one of the five firms which have recently entered that market, following Canadian Natural Resources in 2008 and Shell in 2002. Both Suncor and Syncrude are established market leaders and have been operating their oil sands mines since 1967 and 1978, respectively. In contrast to this, there are around 30 different companies with operational in-situ projects, and many more companies with proposed and approved in-situ projects, indicating that these operations have a different scale and less market power than the mining operations. Given that in-situ production is far less water intensive, we have focused our model on water usage from mining operations only.

The northern part of Alberta is sparsely populated and the main industrial activity in the area is oil and gas. In earlier years, the industry's contribution to the economy was welcomed, but a recent period of rapid growth in production and the number of projects has many criticizing the environmental footprint of these industrial activities. There is a large First Nation population in the northern part of the province who historically survived by hunting and fishing, and uses the Athabasca River as their main source of transportation. Increasing pollution levels in the Athabasca River are correlated to increasing water usage by the industry (Jasechko, Gibson, Birks, & Yi, 2012). This is of concern, especially to the First Nations who use the river as a food source and for their livelihood. For the purpose of this paper, we chose to focus only on the strategic impact of water usage. However creation of a mechanism to incorporate the cost of pollution to the environment and to society into the water price is one area for future research.

For the model, we section the water users in the Athabasca watershed region into three groups: the local community, the First Nations community, and the oil producers. Each group is assumed to be grandfathered into the market with an initial endowment equal to the amount of water allocated to them under the current licensing system. Under the market these three groups are assumed to hold the total regional water supply which they are then able to sell to each other or to new demanders of water. The First Nations communities in Canada are governed by slightly different regulations, which are federal and not provincial, and as such they do not actually hold any water licenses. However the Constitution Act of 1982 does protect Aboriginal rights to Aboriginal title, which includes traditional and new economic uses of water (Walkem, 2007). Therefore, although these rights are not explicitly licensed, our model recognizes that the First Nations in the Fort McMurray, Fort Chipewyan and surrounding areas do have a claim to water rights. Under the water market model, the First Nations are assumed to be grandfathered into the market with an initial endowment equal to their current water utilization.

II. Model

Consider an economy with consumers and suppliers of water, operating in an upstream water market, including oil producing firms. Vertically integrated oil firms then use the water as an input to produce oil in the downstream market. The following model consists of four stages: Stages 1 and 2 are based on a Stackelberg leader-follower model, where the incumbent oil producers make their choice of capacity in stage 1 and a potential entrant makes his choice of capacity (and entry) in stage 2. In stage 3, water suppliers choose the quantities of water to supply the market, based on a Cournot framework. In stage 4 we derive the water demands for the consumers, to find the producer demand function, which gives the water price as a function of the quantity demanded.

Case 1: The static model without entry

We begin by deriving the water demand of consumers, which is stage 4. There are n_1 final consumers from the local community and n_2 final consumers from the First Nation, who consume water and a basket of consumer goods. These final consumers choose their water demand a_i to maximize the objective function: $v_i = x_i + a_i\left(b - \frac{a_i}{2}\right) - pa_i$ where the second term denotes the benefits derived from water consumption and x_i is the net benefit from consumption of all other consumer goods. Note that p is the water price and b is a constant, which represents the consumer's maximal willingness to pay for water.

There are also intermediate consumers of water who use the water as an input to some production process. There are n_2 intermediate First Nation consumers who use water for transportation, fishing, etc. and J intermediate consumers who use the water as an input to the oil production process. The first nations choose a^f to maximize their production function, given by $a^f\left(\gamma - \frac{a^f}{2}\right) - pa^f$, where γ is a positive constant. Each oil producer chooses $a^o$to maximize its profit function, given by $e(a^o; k_j) - pa^o - k_j r - F$, where $e(a^o; k_j)$ denotes the firm's energy production function, k_j is the amount of capital (i.e., capacity), r is the price per unit of capacity and F is a fixed cost. We assume that $e(a^o; k_j) = a^o\left(s - \frac{a^o}{2}\right) + k_j(z - k_j) + a^o k_j$, where s and z are positive constants which represent the maximal marginal products of water and capacity, respectively. We also assume that the price of energy is equal to 1.

Each individual in any group will choose to consume water to maximize his or her respective objective function, leading to choices that equate marginal benefits from consumption of water to the marginal costs of doing so. Assuming interior solutions, the marginal conditions are formally expressed as follows:

(1.1)	Final Consumers:	$b - a_i - p = 0$
(1.2)	First Nation (intermediate):	$\gamma - a^f - p = 0$
(1.3)	Oil Companies:	$s + k_j - a^o - p = 0$

We assume that $p < \min\{b, \gamma, s\}$. Solving first order conditions (1.1) to (1.3) yields the water demand function for each of the groups, which are linear and decreasing in price. Equation (1.3) reveals that capacity k_j is a strategic complement to water. This implies that the water demand for the oil producers is stated as an increasing linear function of capacity as well as being a function of price.

Let $Q = Q^o + Q^f + Q^l$ denote the total quantity of water supplied in the market, where Q^o, Q^f, and Q^l are the total quantities supplied by the oil industry, the first nation and the local community, respectively. Adding up the total water demands for all consumers from 1.1 to 1.3, we obtain the market clearing condition for the water market: $Q = bn_R + n_2\gamma + Js + K_I - p(n_R + n_2 + J)$. Solving for the water price, we have:

$$(2) \qquad p(Q, K_I) = \frac{n_R b + n_2\gamma + Js + K_I - Q}{N},$$

that is, the inverse water market demand function, where $N = n_R + n_2 + J$ for $n_R = n_1 + n_2$ and $K_I = \sum_1^J k_j$.

Stage 3: The water endowments (quotas) are Y^o for the oil companies, Y^f for the First Nation and Y^l for the local community. The total available water supply is given by $Y = \mathrm{Y}^l + \mathrm{Y}^f + \mathrm{Y}^o$. We assume that the water endowment for each group is greater than the sum of their marginal willingness to pay:

$$\mathrm{Y}^l > n_1 b, \qquad \mathrm{Y}^f > n_2(\gamma + b), \qquad \mathrm{Y}^o > Js$$

As we will demonstrate below, these conditions ensure that the water quotas do not bind in a static model where there is no entry in the oil and gas industry. This is a modeling strategy. It allows us to consider a case in which the water quota constraint for the oil producer binds when there is potential entry. We will show that a binding water quota constraint for the oil industry is a necessary condition for incumbents to have incentives to strategically deter entry.

a. Oil producers choose $q_j^o \in (0, y^o)$ to maximize Π_j, where $Q^o = \sum_1^J q_j^o$ and $Y^o = \sum_1^J y^o$

$$(3.1) \qquad \Pi_j = (p - c)q_j^o + a^o\left(s - \frac{a^o}{2}\right) + k_j(z - k_j) + a^o k_j - pa^o - k_j r - F$$

We assume that $z > r$.

b. The local government choose $Q^l \in (0, Y^l)$ to maximize:

(3.2) $$\Pi^l = (p-c)Q^l + J\Pi_j + \frac{n_1}{2}(b-p)^2$$

c. The First Nations government chooses $Q^f \in (0, Y^f)$ to maximize:

(3.3) $$\Pi^f = (p-c)Q^f + \frac{n_2}{2}(\gamma - p)^2 + \frac{n_2}{2}(b-p)^2$$

The profits for each group (equations (3.1) to (3.3) above) include the profits earned from the sale of water: when price equals the marginal cost of water, there will be zero contribution to profits from the sale of water. However, when the price of water is greater than the marginal cost, each group will find it profitable to sell water as they gain a profit of $p - c$ for each unit of water sold. The remaining terms in the profit functions denote the revenues/benefits from using water, less the cost incurred, measured in terms of consumer surplus foregone.

Assuming interior solutions, the optimization problems yield the following first order conditions:

(4.1) $$(p-c) + \left[q_j^o - a^o(p, k_j)\right]\frac{dp}{dQ} = 0$$

(4.2) $$(p-c) + [Q^l - n_1 a_i(p)]\frac{dp}{dQ} = 0$$

(4.3) $$(p-c) + [Q^f - n_2 a^f(p) - n_2 a_i(p)]\frac{dp}{dQ} = 0$$

Solving for the groups' total water supplies, we obtain the water supply, given as follows:

(5.1) $$Q^o = Js + K_I - JNc + (N-1)Jp$$

(5.2) $$Q^f = n_2(\gamma + b) - \mathrm{N}c + \mathrm{p}(N - 2n_2)$$

(5.3) $$Q^l = n_1 b - Nc + p(N - n_1)$$

Adding up (5.1) to (5.3) yields

(6) $$Q = n_R b + n_2\gamma + Js + K_I - Nc(J+2) + pN(J+1)$$

Using the producer demand function (equation 2) derived in stage 4, this can be solved for price to get $p^* = c$. This indicates that the water market is efficient and competitive, as none of the users has the desire to buy or sell more water than is required to meet their own needs since we have obtained a price equal to marginal cost in equilibrium. The complementarity of water and capacity here will have no impact on the downstream market; although K_I appears in the supply function (6) and in the producer demand function (2), the two cancel out in equilibrium, resulting in a water price that is not dependent on the choice of capacity. Thus, in the absence of entry into the market, vertical integration will have no impact on the equilibrium outcome in the downstream market as oil producers will use water at the same cost as they do under the current system since $p^* = c$.

Entry in the oil and gas industry

The model of the previous section assumes that there is no entry in the oil and gas industry. We now consider a situation where there is potential entry into the oil and gas industry. The only difference with respect to the previous model is that the entrant's water demand must be included in the derivation of the producer demand function as he will need water to enter the oil market. The entrant will be denoted by the subscript $J+1$ and the water demand for the entrant will have the same form as for the incumbents: $a^o\left(p; k_{J+1}\right) = s + k_{J+1} - p$.

Case 2: The case with slack water quotas

Consider the case where there is an entrant and the water quotas are not binding (as we assumed with the choice of quantities above). Here we obtain the following inverse demand function:

$$p = \frac{n_R b + n_2 \gamma + (J+1)s + K_I + k_{J+1} - Q}{N+1} \tag{7}$$

Suppliers choose their quantities to supply as above, only now they take into account the demand of the entrant when choosing a quantity to supply. The outcome of stage 3 is that the entry drives up the water price so that $p > c$, resulting in the following function for the water price:

$$p = \left[\frac{s + k_{J+1} + (J+2)(N+1)c}{JN + 2N + J + 3}\right]$$

Due to the complementarity of water and capacity, the price is dependent on the entrant's choice of capacity and will increase with scale of entry. The price is again not dependent on the incumbent producer's choice of capacity, indicating that choice of capacity is not strategic for the incumbents.

In stage 2, the entrant will choose capacity $k_{J+1}$to maximize Π_{J+1}:

$$\Pi_{J+1}(p,k_{J+1}) = \frac{a^o\left(p,k_{J+1}\right)^2}{2} + k_{J+1}\left(z-r-k_{J+1}\right) - F$$

(8.1) First order conditions: $s - k_{J+1} - p + z - r + \left(p - s - k_{J+1}\right)\frac{dp}{dK} = 0$

In stage 1, the incumbents will choose capacity k_j to maximize Π_j:

$$\Pi_j(p,q_j^o,k_j) = (p-c)q_j^o + \frac{a^o\left(p,k_j\right)^2}{2} + k_j\left(z-r-k_j\right) - F$$

(8.2) First Order condition: $s - k_j - p + z - r + (q_j^o - s - k_j + p)\frac{dp}{dK} = 0$

In fact, the choice of capacity k_j by the incumbents is the same as in the case where there is no entry into the industry. When the water allocations given to the industry are not binding, the incumbent firms have no incentive to deter entry to the market. This result is surprising as it indicates that, in the absence of a binding constraint on the water supply, the incumbents are actually better off by allowing entry and earning additional profits from the sale of water.

The cases when the oil industry's water quota binds

Now we consider the case where there is a binding constraint on the water supply. We look at the case $Q^o = Y^o$, where the oil producers wish to supply more water in the market than they have available. The incumbent oil firms are able to deter entry to the industry, provided that their water constraint is binding ($Q^o = Y^o$). As we will discuss in the section v, this creates the necessary condition that will make the water price dependent on the incumbent producers' choice of k_j. For simplicity, we will just examine the least restrictive case where the only suppliers facing a supply constraint are the oil firms, however all cases have been calculated in the appendix.

Let $\mathrm{Q} = Y^o + Q^l + Q^f$, so that only the oil firms face a binding constraint on the water supply. There are two possible outcomes: accommodation and deterrence. Although it is possible for the incumbents to deter entry when the water supply is limited by water availability, we will examine the payoffs between the cases where an entrant is allowed into the market versus when they are deterred.

Case 3: Accommodation

Accommodation is analyzed first. The same first order conditions are used to derive the water demands in stage 4. As before, water demand can be stated as a function of price, only here, it becomes a function of Y^o, not Q^o:

$$p = \frac{n_R b + n_2 \gamma + (J+1)s + K_I + k_{J+1} - Y^o - Q^l - Q^f}{N+1} \tag{9}$$

Since the water quota Y^o here is fixed, the price function is now dependent on the capacity choices of the oil producers. This can be seen after stage 3 when we can simplify the price to:

$$p = \frac{(J+1)s + K_I + k_{J+1} - Y^o + 2(\mathrm{N}+1)\mathrm{c}}{2N + J + 3} \tag{10}$$

We saw in the prior cases that the choice of water quantity was dependent on the incumbent's capacity, but the capacity reduced out of the price function when we solved for the quantities, resulting in a price and was not dependent on the capacity choice. When the water demand exceeds the supply, the choice of price becomes strategic, as the function for price is increasing in capacity. Thus the incumbent producers can select a higher K_I to drive up the water price.

As before, when there was no constraint on the water supply, the entrant will choose capacity to maximize profits Π_{J+1}, subject to this revised producer demand function (10) and the first order conditions (8.1). The incumbents choose capacity to maximize Π_j , according to the same first order conditions as before (8.2), subject to the above price function (10) to obtain:

$$K_I{}^{Accom} = \frac{(2N + J + 3)[(2N + J + 2)Js + (2N + J + 3)J(z - r)]}{\big((2N + J + 3)(2N + J + 4) + (J + 1)(2N + J + 2)\big)}$$

$$+ \frac{\big((2N + J + 3)(2N + J + 4) + (2N + J + 2)\big)Y^o}{\big((2N + J + 3)(2N + J + 4) + (J + 1)(2N + J + 2)\big)(2N + J + 4)}$$

$$- \frac{(2N + J + 2)J[(J + 1)s - Y^o + 2(\mathrm{N} + 1)\mathrm{c}]}{\big((2N + J + 3)(2N + J + 4) + (J + 1)(2N + J + 2)\big)}$$

Water price:

$$p^{Accom} = \frac{(2N + J + 4)[(J + 1)s - Y^O + 2(\mathrm{N} + 1)\mathrm{c}]}{\big((2N + J + 3)(2N + J + 4) + (J + 1)(2N + J + 2)\big)}$$

$$+ \frac{(J + 1)[(2N + J + 2)s + (2N + J + 3)(z - r)]}{\big((2N + J + 3)(2N + J + 4) + (J + 1)(2N + J + 2)\big)}$$

$$+ \frac{Y^o}{\big((2N + J + 3)(2N + J + 4) + (J + 1)(2N + J + 2)\big)}$$

Case 4: Entry deterrence

Entry deterrence will occur when the entrant cannot make positive profits in equilibrium: the entrant will not enter the market when the incumbents choose a level of capacity K_I, such that $\Pi_{J+1} = 0$. Once again, we have the same producer demand function (9) as in the accommodation case above where $Y^o = Q^o$. Subject to the producer demand function (9), the incumbents choose a higher level of capacity than they would in the case of entry, subject to:

$$\Pi_{J+1} = \frac{a^o\big(p, k_{J+1}\big)^2}{2} + k_{J+1}\big(z - r - k_{J+1}\big) - F = 0$$

Since the entrant's profits are zero, he will not find it profitable to enter the market and we find that the incumbent's deterrence capacity is:

$$K_I^{Det} = Y^o + 2(\mathrm{N} + 1)(\mathrm{s} - \mathrm{c}) + \frac{(2N + J + 2)(z - r)}{2}$$

$$- \frac{((z - r)^2 + 4\beta)^{\frac{1}{2}}(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{2(2N + J + 2)}$$

Where:

$$\beta = \frac{2(2N + J + 2)^2 F - (z - r)^2 (2N + J + 3)^2}{2(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}$$

The water price becomes:

$$p^{Det} = s + \frac{z - r}{2} - \frac{((z - r)^2 + 4\beta)^{\frac{1}{2}} (4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{2(2N + J + 2)^2}$$

We find that the deterrence capacity here is higher than the accommodation capacity for the incumbent firms. The equilibrium price is lower in the case where deterrence is optimal, since the entrant will demand no water in equilibrium when we have deterrence. Due to the fact that the capacity choice of the incumbents is higher under deterrence than accommodation, we can see that the price would in fact be higher than the accommodation case should the entrant decide to enter the market when the incumbents choose this capacity level.

We see that deterrence will only occur when there is less slack in the constraint on the water supply: the scarcer water is for the incumbents, the more likely they are to deter entry. In addition to this, entry deterrence requires a sufficiently large fixed cost, in the absence of a large enough fixed cost it becomes too expensive for the incumbents to invest in a capacity level high enough to deter entry.

III. Example

To illustrate the outcomes from the model when there is a constraint on the water supply, we look at the following example. This example assumes that $J = 6, n_1 = 200\ and\ n_2 = 60$. It also assumes that $c = 2, b = 8, \gamma = 12, s = 38, z = 840\ and\ r = 590$. We examine the effect on capacity choice and the profits of the incumbent producers under two variations of fixed costs and two variations of water constraint Y^o. The table below lists the water price, fixed cost, water constraint, and the profit of an incumbent firm under each of 3 cases.

Table 1:

Variable	Value Under Accommodation			Value Under Deterrence		
	Case A	Case B	Case C	Case A	Case B	Case C
Price	\$3.81	\$3.81	\$3.10	\$3.25	\$6.84	\$3.25
F	41,125	40,000	41,125	41,125	40,000	41,125
Y^o	1,260	1,260	1,650	1,260	1,260	1,650
K_I	1,704	1,707	1,712	1,758	3,657	2,110
Profit	\$221	\$1,346	\$370	\$248	-\$52,893	-\$1,876

Case A is an example of the conditions required to make entry deterrence optimal. The profits are greater when entry is deterred for this level of water quota and fixed cost. We can see that a change to either the water constraint or the fixed cost will affect the optimality of entry deterrence: entry deterrence will not be optimal in case B or in case C. Case B looks at a lower fixed cost to the industry; when the fixed cost is too low, we can see that it becomes too costly for the incumbents to deter entry, resulting in negative profits, therefore accommodation will be optimal. In case C, we consider a higher water quota than case A, while leaving the fixed cost the same. If the water quota is not sufficiently small, it will also be too costly for the incumbents to invest in the entry deterring capacity and they will prefer to accommodate entry into the market.

IV. Discussion

The objective of this thesis has been to examine whether the creation of a market for water in Northern Alberta would create efficiencies, or whether it would be a source of inefficiency for the oil mining industry. We look at four main outcomes which would be possible if oil producers were vertically integrated and operating in the water market. Case 1 considers a ‘status quo’ where there is no entry into the industry. The market is operating efficiently in the absence of entry, which is demonstrated by price equal to marginal cost. Such a price is analogous to the situation we have under the current legislation: in the current system, users do not pay for water and will use it according to that marginal cost required to fetch the water. Although an efficient market, case 1 provides no incentive to reduce consumption of water or to conserve, thus it is just as efficient as the current water licensing system.

Case 2 considers accommodated entry into the oil market, when the water quotas of the incumbent producers are not binding. Entry into the industry brings the price of water up beyond the marginal cost, allowing the water suppliers to earn a positive profit from selling the water. They are thus better off in the situation where there is entry than in the case where there is no entry into the market. However, as the incumbent oil producers' profits are increasing with water sales, we see an increase in their market power which comes from the mark-up of charging a water price greater than the marginal cost. This mark-up suggests that the market is not competitive and puts a potential entrant at a disadvantage compared to the incumbent producers. The water price with entry is increasing with the capacity of the entrant, which is intuitive: due to the complementarity of water and capacity, we would expect a larger scale of entry to require a greater amount of water, thus driving up the price even higher. The incumbents' investment in capacity in this case is the same as in case 1: their choice of capacity is not strategic here since the water price is not dependent on the capacity level of the incumbents.

Case 3 considers entry accommodation when the water quotas of the incumbent oil producers are binding. The limitation placed on the water suppliers regarding the quantity available to supply the market drives up the price of water, beyond the price we saw in case 2. The imposition of the constraint on the supply of water by the incumbents, results in a producer demand function, and hence a water price, that is dependent on the capacity choice of the incumbent producers. Since these firms are constrained in their water availability, the capacity choice they make becomes strategic. Given the current technology, a greater capacity will require more water for each producer, driving up the water price in equilibrium. Although entry is not deterred in this case, the higher water price and the strategic link between the price and capacity do provide motivation for incumbent producers to act strategically. In fact, we see that a reduction in the water quota to a quantity lower than the amount the incumbent producers had been using when unconstrained will effectively drive up the water price while reducing the profits and capacity levels of the incumbents when a new entrant enters the market.

Case 4 considers entry deterrence when the water quotas of the incumbent oil producers are binding. Incumbent producers can exercise their market power by choosing a level of capacity that is so high that the entrant will not find it profitable to enter the industry. The capacity choice in this case is higher than all of the other cases, indicating that the incumbents have chosen their capacity level, not due to efficiencies in the returns on capacity, but they have chosen a high capacity in order to drive up the resultant water price and prevent entry into the market. It will be optimal to deter entry when the profits in the case of entry are lower than the profits will be in the case of entry deterrence. Deterrence is typically optimal only when the entrant faces a high enough fixed cost. If the fixed cost is too low, it will be optimal to accommodate entry. In this model, we can show that it is both necessary and sufficient for the incumbents to have a binding constraint on their water endowment, in order for the choice of capacity to become strategic. If the choice of capacity is strategic, they are able to choose a higher capacity levels such that they can exercise market power and deter entry. The water endowments may be binding or non-binding for the local community and the First Nation as these do not impact the interaction between the choice of capacity and the water price. This is why we have chosen to examine the least restrictive case here, where the only binding constraint is $Q^o = Y^o$ (for other cases, please see appendix).

Proposition 1: A binding water quota for the oil producers $Q^o = Y^o$ is both necessary and sufficient for the capacity to be a strategic variable in the water market.

Proof:

In the case of entry, we have producer demand function:

$$p = \frac{n_R b + n_2 \gamma + (J+1)s + K_I + k_{J+1} - Q^l - Q^f - Q^o}{N+1}$$

We can see that the price is increasing in the total capacity of the incumbents K_I. However, when the water constraint is non-binding, the incumbents will choose their water supply to maximize profits, and will select the quantity according to:

$$Q^o = Js + K_I - J(\mathrm{N}+1)c + NJp$$

Since this contains the term K_I, as does the price function, the expression will reduce to obtain:

$$p = \frac{n_R b + n_2\gamma + s + J(\mathrm{N}+1)c + k_{J+1} - Q^l - Q^f}{(N + NJ + 1)}$$

This is no longer a function of K_I, indicating that the choice of capacity is not strategic. Note that neither Q^l or Q^f is a function of Q^oor K_I, as such they will have no effect on whether or not the choice of capacity is strategic. When we have $Q^o = Y^o$ as a fixed value, the oil firms do not choose Q^o according to the formula above, since they sell their entire allocation. Therefore the capacity term will not cancel out of the price function and the choice of capacity will depend on K_I and will thus be strategic.

Deterrence causes additional inefficiencies due to the fact that the incumbent producers actually incur an additional cost to deter entry. In the other equilibriums, where entry is accommodated, the optimal choices of both capacity and water demand are chosen at the level where the marginal cost to use an additional unit is equal to the marginal benefit from using another unit of the input. When the incumbents choose their capacity level to deter entry, they choose a level that is not on their reaction function. That is, they choose a level of capacity where the marginal cost is equal to the marginal benefit plus some additional premium σ.

<u>Proposition 2</u>: Under entry deterrence, the choice of capacity is greater than the efficient capacity such that $MB + \sigma = MC$

Proof: $$MB + \sigma = MC \leftrightarrow$$

$$s + k_j - p + z - r + \sigma = 2k_j \leftrightarrow$$

$$s + z - r - k_j - p < 0$$

$$s + z - r - Y^o - 2(\mathrm{N}+1)(\mathrm{s}-\mathrm{c}) - \frac{(2N+J+2)(z-r)}{2}$$

$$+ \frac{(2N+J+2)((z-r)^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)^2}$$

$$- s - \frac{z-r}{2} + \frac{((z-r)^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)^2}$$

$$< 0$$

$$\left[\frac{4F-(z-r)^2}{(4N^2+4NJ+16N+J^2+8J+14)}\right]^{1/2}$$

$$< \frac{2(2N+J+2)[Y^o+2(\mathrm{N}+1)(\mathrm{s}-\mathrm{c})]}{(2N+J+3)(4N^2+4NJ+16N+J^2+8J+14)}$$

$$+ \frac{(2N+J+2)(2N+J+1)(z-r)}{(2N+J+3)(4N^2+4NJ+16N+J^2+8J+14)}$$

$$\frac{4F}{(z-r)^2} - 1$$

$$< \frac{(4N^2+4NJ+16N+J^2+8J+14)}{(z-r)^2}\left(\frac{2(2N+J+2)[Y^o+2(\mathrm{N}+1)(\mathrm{s}-\mathrm{c})]}{(2N+J+3)(4N^2+4NJ+16N+J^2+8J+14)}\right.$$

$$\left.+ \frac{(2N+J+2)(2N+J+1)(z-r)}{(2N+J+3)(4N^2+4NJ+16N+J^2+8J+14)}\right)^2$$

Given the restriction that we have a sufficiently small fixed cost F in the industry (in proportion to $(z-r)$) to ensure that the firms will earn positive profits in equilibrium, we can see that the marginal cost here in fact does exceed the benefits from the entry deterrence level of capacity.

Taken together, these four cases demonstrate that the water market will create inefficiencies, as opposed to incentives to use water more efficiently. The magnitude of the inefficiencies will be predicated on whether or not the water quotas allocated to the incumbent oil firms are sufficiently large. When there is a lot of slack in the water constraint so that the firms have an excess of water to supply, a slight mark-up in the price is evident. However, as the water quotas become very restrictive as compared to the current water utilization of the industry, we see an increasing incentive for a strategic choice of capacity in order to deter entry and lessen competition for scarce inputs. Although this case is not of concern given the current water supply and utilization for the industry, it becomes increasingly likely in the future due to the potential for supply shocks and entry into the industry. The emphasis here is that the water quotas are an important factor in determining the outcome and should water scarcity become an issue in the region, there will likely be more discussion around alternative water policies such as a water market to incent conservation. We have shown that, given the current industry structure, creation of a water market under conditions of scarcity will likely have the opposite effect; firms will choose a higher capacity than they would in the absence of a market, thereby using a greater amount of water than would be considered efficient.

V. Conclusion

This thesis has shown that the industry outcome will be highly dependent on the initial water quotas and on the amount of slackness between the available water and the amount of water quantity the oil producers would like to supply in the market. Similar to the previous literature on entry deterrence we show that an increase in investment in capacity by the incumbent producers will result in the potential for deterrence of an entrant. However, the likelihood of deterrence relies heavily on the ability of the incumbents to strategically manipulate the water price via their capacity choice; this will be dependent on the existence of binding water quotas for the industry. We have shown the unique result that the producers can manipulate the upstream market price for water via their capacity choice to deter entry downstream.

Thus a change from the current water allocation policy to the creation of a water market will likely not have any beneficial effects on water usage in the oil sands region. Since entry to the oil market is likely, the market will include an inefficient mark-up on the price even when water quotas are slack. This creates an inefficient outcome as the incumbent oil producers can increase their profits at the expense of the entrant, who is faced with a higher water price. On the other hand, if there is a shock which reduces the available water supply, or if there is a lot of entry into the industry, it is probable that the water quotas will become binding. We have seen that the industry is characterized by high fixed costs so it is likely that entry deterrence will be the result, should the industry's water quota's become binding. It is important to note that the water quotas may become binding in the future due to events exogenous to the water market itself. Changes to the scale of the mining operations, water demands for reclamation of tailings ponds, emergence of new industries and the effects of climate change will all impact future water availability. This suggests the potential for the oil industry to become less and less competitive under a vertically integrated structure, an outcome which is not beneficial to the other water market participants or to society as a whole, since deterrence will result in more water usage for oil production, above the point water would be used in an efficient equilibrium.

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VI. Appendix

Table of Contents

Section 1.0: Entry is desirable

Water Demand:

i. Local Community: n_1 consumers
ii. First Nations: n_2 consumers
iii. Oil Producers: J producers

Final Water Consumers: $n_1 + n_2 = n_R$

Water Suppliers:

- Local Community is endowed with Y^l units of water
- First Nations are endowed with Y^f units of water
- Oil Producers are endowed with Y^o units of water

Let $Y = Y^l + Y^f + Y^o$ be fixed.
In this scenario, the quantities sold will be less than the endowments Y_i

Stage 3: Find water demand
Utility of a representative final consumer i:
$x_i + a_i\left(b - \frac{a_i}{2}\right)$
Budget Constraints:
$w_i = x_i + p^a a_i \qquad \forall\, i = 1, \dots, n_R$

Where:
x_i = Utility derived from the basket of consumer goods
a_i = Water demand (final)
a^o = Water usage (intermediate, oil producers)
a^f = Water usage (intermediate, First Nation)
p = Price of water
w_i = Income or wealth

Utility Maximization for final water consumer i:

$\max_{\{a_i\}}\left(x_i + a_i\left(b - \frac{a_i}{2}\right) - pa_i\right)$

Then $\quad 0 = -p + b - a_i$

$\quad \Longrightarrow a_i(p; b) = b - p$

Income Maximization for intermediate First Nations production

$\max_{\{a^f\}}\left(a^f\left(\gamma - \frac{a^f}{2}\right) - pa^f\right)$

Then $\quad 0 = -p + \gamma - a^f$

$\quad \Longrightarrow a^f(p; \gamma) = \gamma - p$

Profit maximization for oil producers (in the oil market):

$$\Pi_s^e = e(a^o; k_j) - pa^o - k_j r = a^o\left(s - \frac{a^o}{2}\right) + k_j(z - k_j) + a^o k_j - pa^o - k_j r - F$$

Where
k_j = Capacity or capital cost for producer j
z and s are constants

r= price of capital

F= fixed cost

$$\max_{\{a^o\}}(\Pi_s^e) = \max_{\{a^o\}}\left[a^o\left(s - \frac{a^o}{2}\right) + k_j(z - k_j) + a^o k_j - pa^o - k_j r - F\right]$$

Then $\quad 0 = s + k_j - a^o - p$

$\Longrightarrow a^o(p; k_j) = s + k_j - p$

Let:

$$K_I = \sum_1^J k_j$$

Supply of Water

For final consumers: $n_R \cdot a_i(p; b)$

For intermediate First Nation: $n_2 \cdot a^f(p; \gamma)$

For incumbent oil producers: $J \cdot a^o(p; k_j)$

Market clearing condition:

$$Q = n_R \cdot a_i(p; b) + n_2 \cdot a^f(p; \gamma) + J \cdot a^o(p; k_j)$$

$$Q = n_R(b - p) + n_2(\gamma - p) + K_I + J(s - p)$$

$$Q = bn_R + n_2\gamma + Js + K_I - p(n_R + n_2 + J)$$

Let $N = n_R + n_2 + J$

Producer demand function

$$p(Q, K_I) = \frac{n_R b + n_2\gamma + Js + K_I - Q}{N}$$

Where $\frac{dp}{dQ} = -\frac{1}{N}$

<u>Stage 2</u>: All groups choose water supply simultaneously

a. Oil producers will choose $q_j^o \in (0, y^o)$ to maximize Π_j, where

$$Q^o = \sum_{s=1}^{J} q_j^o \ and \ Y^o = \sum_{1}^{J} y^o$$

$$\max_{\{q_j^o\}} \left((p-c)q_j^o + a^o\left(s - \frac{a^o}{2}\right) + k_j(z-k_j) + a^o k_j - pa^o - k_j r - F \right)$$

$$\max_{\{q_j^o\}} \left((p-c)q_j^o + (\frac{1}{2})(s+k_j-p)^2 + k_j(z-k_j) - k_j r - F \right)$$

$$\Rightarrow \left(\frac{dp}{dQ} q_j^o + (p-c) - (s+k_j-p)\frac{dp}{dQ}\right) = 0$$

$$q_j^o = s + k_j - Nc + (N-1)p$$

Since $Q^o = Jq_j^o$

$$Q^o = Jq_j^o = Js + K_I - J\mathrm{N}c + (N-1)Jp$$

b. Local Community

The local community has indirect utility function:

$$v_i(\cdot) = x_i \ + \frac{(b-p)^2}{2}$$

Where

$$x_i = \frac{(p-c)Q^l + J\Pi_j}{n_1}$$

The local government will choose $Q^l \in \ (0, Y^l)$ to maximize:

$$\Pi^l = (p-c)Q^l + J\Pi_j + \frac{n_1}{2}(b-p)^2$$

First Order conditions:

$$\frac{dp}{dQ}Q^l + (\mathrm{p}-\mathrm{c}) - n_1(b-p)\frac{dp}{dQ} = 0$$

$$\frac{-Q^l}{N} + \mathrm{p} - \mathrm{c} + \frac{n_1}{N}(b-p) = 0$$

$$Q^l = n_1 b - Nc + p(N - n_1)$$

c. First Nation

First Nations will maximize profits:

$$\Pi^f = \Pi_s^{fa} + \Pi_s^g$$

$$\Pi^f = (p-c)Q^f + {}^1\!/_2\,(\gamma - p)^2$$

Need to add the consumer surplus to the First Nations producer surplus.

A resident i in the First Nations will maximize their utility as follows:

$\max_{\{a_i\}}(w_i - pa_i + a_i(b - \frac{a_i}{2}))$ which yields: $a_i(p;b) = b - p$

Substitute the demand functions into the objective function to derive the indirect utility function for a representative resident í.

$$v_i(w_i, p, b) = w_i - p(b-p) + (b-p)\left(b - \frac{(b-p)}{2}\right)$$
$$v_i(\cdot) = w_i + (b-p)\left(b - p - \frac{(b-p)}{2}\right)$$
$$v_i(\cdot) = w_i + \frac{(b-p)^2}{2}$$

Where $\quad w_i = \Pi^f / n_2$

The First Nations government chooses $Q^f \in (0, Y^f)$ to maximize:

$$\Pi^f = (p-c)Q^f + \frac{n_2}{2}(\gamma - p)^2 + \frac{n_2}{2}(b-p)^2$$

Assuming an interior solution, the first order condition is:

$$\frac{dp}{dQ}Q^f + (\mathrm{p} - \mathrm{c}) - n_2(\gamma - p)\frac{dp}{dQ} - n_2(b-p)\frac{dp}{dQ} = 0$$
$$\frac{-Q^f}{N} + (\mathrm{p} - \mathrm{c}) + \frac{n_2}{N}(\gamma - p) + \frac{n_2}{N}(b-p) = 0$$
$$Q^f = n_2(\gamma + b) - \mathrm{Nc} + \mathrm{p}(N - 2n_2)$$

So, we have:

$Q = Q^o + Q^f + Q^l$ where

$$Q^o = Js + K_I - JNc + (N-1)Jp$$
$$Q^f = n_2(\gamma + b) - \mathrm{Nc} + \mathrm{p}(N - 2n_2)$$
$$Q^l = n_1 b - Nc + p(N - n_1)$$

Find the quantities of water:

$$Q = Js + K_I - JNc + (N-1)Jp + n_2(\gamma + b) - \mathrm{Nc} + \mathrm{p}(N - 2n_2) + n_1 b - Nc + p(N - n_1)$$

$$Q = n_R b + n_2\gamma + Js + K_I - Nc(J+2) + \mathrm{p}(2N + NJ - J - n_R - n_2)$$
$$Q = n_R b + n_2\gamma + Js + K_I - Nc(J+2) + \mathrm{pN}(J+1)$$
$$Q = n_R b + n_2\gamma + Js + K_I - Nc(J+2) + \mathrm{N}(J+1)\left[\frac{n_R b + n_2\gamma + Js + K_I - Q}{N}\right]$$
$$Q(2N + NJ) = (n_R b + n_2\gamma + Js + K_I)(2N + NJ) - N^2 c(J+2)$$
$$Q = n_R b + n_2\gamma + Js + K_I - c\left(\frac{N^2(J+2)}{N(J+2)}\right)$$

$$Q = n_R b + n_2\gamma + Js + K_I - Nc$$

$$p = \frac{n_R b + n_2\gamma + Js + K_I - [n_R b + n_2\gamma + Js + K_I - Nc]}{N}$$

$$p^* = c$$

Stage 1: Oil producers choose capacity k_j

$$\max_{\{k_j\}} \left(q_j^o (p - c) + \left(\frac{1}{2}\right)\left(s + k_j - p\right)^2 + k_j\left(z - k_j - r\right) - F \right)$$

$$q_j^o \frac{dp}{dK} + s + k_j - s\frac{dp}{dK} - k_j\frac{dp}{dK} - p + p\frac{dp}{dK} + z - 2k_j - r = 0$$

$$\frac{1}{N}\left(q_j^o - s - k_j + p\right) + s - k_j - p + z - r = 0$$

$$\left(q_j^o - s - k_j + p\right) + N(s - k_j - p + z - r) = 0$$

$$k_j = \frac{N(z - r) + (N - 1)s - (N - 1)p + q_j^o}{N + 1}$$

$$k_j = \frac{N(z - r) + (N - 1)s - (N - 1)p + \ s + k_j - Nc + (N - 1)p}{N + 1}$$

$$Nk_j = N(z - r) + Ns - Nc$$

$$k_j{}^* = (z - r) + s - c$$

Results:

$$K_I{}^* = J(z - r) + Js - Jc$$

$$Q^* = n_R b + n_2\gamma + 2Js + J(z - r) - (J + N)c$$

$$p^* = c$$

$$Q^{o*} = 2Js + J(z - r) - 2Jc$$

$$Q^{f*} = n_2(\gamma + b) - 2n_2\mathrm{c}$$

$$Q^{l*} = n_1 b - n_1 c$$

$$\Pi^{f*} = \frac{n_2}{2}[(\gamma - c)^2 + ((b - c)^2]$$

$$\Pi^{l*} = J\Pi_j^* + \frac{n_1}{2}(b - c)^2$$

$$\Pi_j^* = [2s + (z - r) - 2c](p^* - c) + \left(\frac{1}{2}\right)\left(s + k_j - p\right)^2 + k_j\left(z - k_j - r\right) - F$$

$$\Pi_j^* = \left(\frac{1}{2}\right)(2s + (z-r) - 2c)^2 - \left((z-r) + s - c\right)^2 + (z-r)((z-r) + s - c) - F$$

$$\Pi_j^* = \left(\frac{1}{2}\right)(s-c)^2 + \left(\frac{1}{2}\right)\left(s - c + (z-r)\right)^2 - F$$

$$\Pi_j^* = (s-c)^2 + (s-c)(z-r) + \left(\frac{1}{2}\right)(z-r)^2 - F$$

Section 2.1: Entry Accommodation

Water Demand:

i. Local Community: n_1 consumers
ii. First Nations: n_2 consumers
iii. Oil Producers: J producers
iv. New Entrant (oil market): J+1th producer

Water Suppliers:

- Local Community is endowed with Y^l units of water
- First Nations are endowed with Y^f units of water
- Oil Producers are endowed with Y^o units of water

Let $Y = Y^l + Y^f + Y^o$ be fixed.

Stage 4: Find water demands

Supply of Water (derived in section 1)

For final consumers: $n_R \cdot a_i(p; b)$

For intermediate First Nation: $n_2 \cdot a^f(p; \gamma)$

For incumbent oil producers: $J \cdot a^o\left(p; k_j\right)$

For entrant: $a^o\left(p; k_{J+1}\right)$

Market clearing condition:

$$Q = n_R \cdot a_i(p; b) + n_2 \cdot a^f(p; \gamma) + J \cdot a^o\left(p; k_j\right) + a^o\left(p; k_{J+1}\right)$$

$$Q = n_R(b-p) + n_2(\gamma - p) + K_I + J(s-p) + s + k_{J+1} - p$$

$$Q = bn_R + n_2\gamma + (J+1)s + K_I + k_{J+1} - p(n_R + n_2 + J + 1)$$

Let $N = n_R + n_2 + J$

Producer demand function

$$P\left(Q, K_I, k_{J+1}\right) = \frac{n_R b + n_2\gamma + (J+1)s + K_I + k_{J+1} - Q}{N+1}$$

Where $\frac{dp}{dQ} = -\frac{1}{N+1}$

Stage 3: All groups choose water supply simultaneously

a. Oil producers:

$$\max_{\{q_j^o\}} \left((p-c)q_j^o + (\frac{1}{2})\left(s+k_j-p\right)^2 + k_j\left(z-k_j\right) - k_j r - F \right)$$

$$\Rightarrow \left(\frac{dp}{dQ}q_j^o + (p-c) - \left(s+k_j-p\right)\frac{dp}{dQ}\right) = 0$$

$$q_j^o = s + k_j - (N+1)c + Np$$

Since $Q^o = Jq_j^o$

$$Q^o = Jq_j^o = Js + K_I - J(N+1)c + NJp$$

b. Local Community

The local government will choose $Q^l \in (0, Y_1)$ to maximize:

$$\Pi^l = (\mathrm{p}-c)Q^l + J\Pi^f + \frac{n_1}{2}(b-p)^2$$

First Order conditions:

$$\frac{dp}{dQ}Q^l + (\mathrm{p}-\mathrm{c}) - n_1(b-p)\frac{dp}{dQ} = 0$$

$$\frac{-Q^l}{N+1} + \mathrm{p} - \mathrm{c} + \frac{n_1}{N+1}(b-p) = 0$$

$$Q^l = n_1 b - (N+1)c + p(N+1-n_1)$$

c. First Nation

The First Nations government chooses $Q^f \in (0, Y_2)$ to maximize:

$$\Pi^f = (\mathrm{p}-\mathrm{c})Q^f + \frac{n_2}{2}(\gamma-p)^2 + \frac{n_2}{2}(b-p)^2$$

Assuming an interior solution, the first order condition is:

$$\frac{dp}{dQ}Q^f + (\mathrm{p}-\mathrm{c}) - n_2(\gamma-p)\frac{dp}{dQ} - n_2(b-p)\frac{dp}{dQ} = 0$$

$$\frac{-Q^f}{N+1} + (\mathrm{p}-\mathrm{c}) + \frac{n_2}{N+1}(\gamma-p) + \frac{n_2}{N+1}(b-p) = 0$$

$$Q^f = \mathrm{p}(N+1-n_2-n_2) - (\mathrm{N}+1)\mathrm{c} + n_2\gamma + n_2 b$$

$$Q^f = n_2(\gamma+b) - (\mathrm{N}+1)\mathrm{c} + \mathrm{p}(N+1-2n_2)$$

So, we have:

$Q = Q^o + Q^f + Q^l$ where

$Q^o = Js + K_I - J(N+1)c + NJp$

$Q^f = n_2(\gamma + b) - (\mathrm{N}+1)\mathrm{c} + \mathrm{p}(N + 1 - 2n_2)$

$Q^l = n_1 b - (N+1)c + p(N + 1 - n_1)$

Find the quantities of water:

$$Q = Js + K_I - J(N+1)c + NJp + n_2(\gamma + b) - (\mathrm{N}+1)\mathrm{c} + \mathrm{p}(N + 1 - 2n_2) + n_2(\gamma + b) - (\mathrm{N}+1)\mathrm{c} + \mathrm{p}(N + 1 - 2n_2)$$

$$Q = Js + n_2\gamma + n_R b + K_I - (J+2)(N+1)c + \mathrm{p}(JN + N + J + 2)$$

$$Q = Js + n_2\gamma + n_R b + K_I - (J+2)(N+1)c + (JN + N + J + 2)\left[\frac{n_R b + n_2\gamma + (J+1)s + K_I + k_{J+1} - Q}{N+1}\right]$$

$$Q(JN + 2N + J + 3) = (N+1)[Js + n_2\gamma + n_R b + K_I] - (J+2)(N+1)^2 c + (JN + N + J + 2)\left[n_R b + n_2\gamma + (J+1)s + K_I + k_{J+1}\right]$$

$$Q = Js + n_2\gamma + n_R b + K_I + \frac{(JN + N + J + 2)(s + k_{J+1}) - (J+2)(N+1)^2 c}{JN + 2N + J + 3}$$

Then the price becomes:

$$p = \left(\frac{1}{N+1}\right)\left[s + k_{J+1} + \frac{(J+2)(N+1)^2 c - (JN + N + J + 2)(s + k_{J+1})}{JN + 2N + J + 3}\right]$$

$$p = \left[\frac{s + k_{J+1} + (J+2)(N+1)c}{JN + 2N + J + 3}\right]$$

<u>Stage 2</u>: Entrant J+1 chooses capacity k_{J+1}

$$\max_{\{k_{J+1}\}}\left(\left(\frac{1}{2}\right)\left(s + k_j - p\right)^2 + k_{J+1}\left(z - k_{J+1} - r\right) - F\right)$$

$$s - k_{J+1} - p + z - r + \left(p - s - k_{J+1}\right)\frac{dp}{dK} = 0$$

$$(N+1)(s-k_{J+1}-p+z-r)+p-s-k_{J+1}=0$$

$$k_{J+1}=\frac{(N+1)(z-r)+Ns-Np}{N+2}$$

$$k_{J+1}(N+2)=(N+1)(z-r)+Ns-N\left(\frac{s+k_{J+1}+(J+2)(N+1)c}{JN+2N+J+3}\right)$$

$$\begin{aligned}k_{J+1}&(N+2)(JN+2N+J+3)\\&=(JN+2N+J+3)(N+1)(z-r)+(JN+2N+J+3)Ns\\&-N\big(s+k_{J+1}+(J+2)(N+1)c\big)\end{aligned}$$

$$\begin{aligned}k_{J+1}&[(N+2)(JN+2N+J+3)+N]\\&=(JN+2N+J+3)(N+1)(z-r)+(JN+2N+J+2)Ns\\&-N(J+2)(N+1)c\end{aligned}$$

$$k_{J+1}^{**}=\frac{(JN+2N+J+3)(N+1)(z-r)+(JN+2N+J+2)N(s-c)}{(N+2)(JN+2N+J+3)+N}$$

Interpretation: since $P\big(Q,K_I,k_{J+1}\big)$ becomes a function of k_{J+1}only once we solve for Q, this means that the incumbent's choice of capacity does not depend on the entrant and they have no incentive to deter entry under this scenario.

<u>Stage 1</u>: Incumbents choose k_j to maximize profits

$$\max_{\{k_j\}}\left(q_j^o(p-c)+\left(\frac{1}{2}\right)\big(s+k_j-p\big)^2+k_j\big(z-k_j-r\big)-F\right)$$

$$q_j^o\frac{dp}{dK}+s+k_j-s\frac{dp}{dK}-k_j\frac{dp}{dK}-p+p\frac{dp}{dK}+z-2k_j-r=0$$

$$k_j=\frac{(N+1)(z-r)+Ns-Np+q_j^o}{N+2}$$

$$K_I=\frac{J(N+1)(z-r)+NJs-NJp+Q^o}{N+2}$$

$$K_I^{**}=J(z-r)+Js-Jc$$

$$k_j^{**}=(z-r)+s-c$$

Results:

$$k_{J+1}^{**} = \frac{(JN+2N+J+3)(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} + \frac{N(JN+2N+J+2)[s-c]}{(N+2)(JN+2N+J+3)+N}$$

$$(JN+2N+J+3)p = s + (J+2)(N+1)c + \frac{(JN+2N+J+3)(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} + \frac{N(JN+2N+J+2)[s-c]}{(N+2)(JN+2N+J+3)+N}$$

$$p^{**} = \frac{s+(J+2)(N+1)c}{JN+2N+J+3} + \frac{(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} + \frac{N(JN+2N+J+2)[s-c]}{(JN+2N+J+3)[(N+2)(JN+2N+J+3)+N]}$$

$$K_I^{**} = J(z-r) + Js - Jc$$

$$k_j^{**} = (z-r) + s - c$$

$$a_i{}^{**} = (b - p^{**})$$

$$a^{f**} = (\gamma - p^{**})$$

$$a^{o**} = \left(s + k_j^{***} - p^{**}\right)$$

$$a^{o**} = s + k_j^{***} - \frac{s+(J+2)(N+1)c}{JN+2N+J+3} - \frac{(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} - \frac{N(JN+2N+J+2)[s-c]}{(JN+2N+J+3)[(N+2)(JN+2N+J+3)+N]}$$

$$Q^{**} = 2Js + n_2\gamma + n_R b + J(z-r) - Jc + \frac{(JN+N+J+2)(s+k_{J+1}) - (J+2)(N+1)^2 c}{JN+2N+J+3}$$

$$q_j^{o**} = 2s + (z-r) - 2c + N(p^{**} - c)$$

$$Q^{o**} = J[2s - (N+2)c + (z-r) + Np^{**}]$$

$$Q^{o**} = J2s - J(N+2)c + J(z-r) + \frac{NJs + NJ(J+2)(N+1)c}{JN+2N+J+3} + \frac{NJ(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} + \frac{N^2 J(JN+2N+J+2)[s-c]}{(JN+2N+J+3)[(N+2)(JN+2N+J+3)+N]}$$

$$Q^{f**} = n_2(\gamma + b) - (\mathrm{N}+1)\mathrm{c} + p^{**}(N+1-2n_2)$$

$$\Pi^{f**} = (p^{**} - c)Q^f + \frac{n_2}{2}[(\gamma - p^{**})^2 + (b - p^{**})^2]$$

$$\Pi^{f^{**}} = (p^{**} - c)[n_2(\gamma + b) - (\mathrm{N} + 1)\mathrm{c} + p^{**}(J + 1 + n_1)] + \frac{n_2}{2}[(\gamma - p^{**})^2 + (b - p^{**})^2]$$

$$Q^{l^{**}} = n_1 b - (N + 1)c + p^{**}(N + 1 - n_1)$$

$$\Pi^{l^{**}} = (p^{**} - c)Q^{l^{**}} + J\Pi_j^{**} + \frac{n_1}{2}(b - p^{**})^2$$

$$\Pi^{l^{**}} = (p^{**} - c)[n_1 b - (N + 1)c + p^{**}(N + 1 - n_1)] + J\Pi_j^{**} + \frac{n_1}{2}(b - p^{**})^2$$

$$\Pi_j^{**} = q_j^{o^{**}}(p^{**} - c) + \left(\frac{1}{2}\right)\left(s + k_j^{**} - p^{**}\right)^2 + k_j^{**}\left(z - k_j^{**} - r\right) - F$$

$$\Pi_j^{**} = (p^{**} - c)(2s + (z - r) - (N + 2)c + Np^{**}) + \left(\frac{1}{2}\right)\left(s + k_j^{**} - p^{**}\right)^2 + k_j^{**}\left(z - k_j^{**} - r\right) - F$$

$$\Pi_j^{**} = (p^{**} - c)((z - r) + 2(s - c) + N(p^{**} - c)) + \left(\frac{1}{2}\right)\left(s + k_j^{**} - p^{**}\right)^2 + k_j^{**}\left(z - k_j^{**} - r\right) - F$$

$$\Pi_j^{**} = (p^{**} - c)\big((z - r) + 2(s - c)\big) + N(p^{**} - c)^2 + \left(\frac{1}{2}\right)(s + (z - r) - c + s - p^{**})^2 - ((z - r) + s - c)(s - c) - F$$

$$\Pi_j^{**} = (p^{**} - c)\big((z - r) + 2(s - c)\big) + N(p^{**} - c)^2 + \left(\frac{1}{2}\right)(s + z - r - c)^2 + (s + (z - r) - c)(s - p^{**}) + \left(\frac{1}{2}\right)(s - p^{**})^2 - \big((z - r) + s - c\big)(s - c) - F$$

$$\Pi_j^{**} = (s - c)(z - r + s - c) + p^{**}(s - c) + N({p^{**}}^2 - 2cp^{**} + c^2) + \left(\frac{1}{2}\right)(s + z - r - c)^2 + \left(\frac{1}{2}\right)({p^{**}}^2 - 2sp^{**} + s^2) - ((z - r) + s)(s - c) - F$$

$$\Pi_j^{**} = p^{**}\left[\left(N + \tfrac{1}{2}\right)p^{**} - c(2N + 1)\right] + \left(\frac{1}{2}\right)(s - c + z - r)^2 + \tfrac{1}{2}s^2 + Nc^2 - F$$

$$\Pi_{J+1}^{**} = \left(\frac{1}{2}\right)\left(s + k_{J+1}^{**} - p^{**}\right)^2 + k_{J+1}^{**}\left(z - k_{J+1}^{**} - r\right) - F$$

$$k_{J+1}^{**} = \frac{(N + 1)(z - r) + Ns - Np^{**}}{N + 2}$$

$$\Pi_{J+1}^{**} = \left(\frac{1}{2}\right)\left(\frac{(N+2)(s-p^{**})+(N+1)(z-r)+N(s-p^{**})}{N+2}\right)^2 - \left(\frac{(N+1)(z-r)+Ns-Np^{**}}{N+2}\right)^2 + \frac{(z-r)[(N+1)(z-r)+N(s-p^{**})]}{N+2} - F$$

$$\Pi_{J+1}^{**} = \left(\frac{1}{2}\right)\left(\frac{2(N+1)(s-p^{**})+(N+1)(z-r)}{(N+2)}\right)^2 - \left(\frac{(N+1)(z-r)+N(s-p^{**})}{N+2}\right)^2 + \frac{(z-r)[(N+1)(z-r)+N(s-p^{**})]}{N+2} - F$$

Results: interpretation and comparison

$$k_j^{**} = (z-r)+s-c$$

Here k_j^* is the same as in Section 1. The capacity chosen by the incumbents will be the same when it is optimal to accommodate entry as it is in the case where entry is not desirable.

$$k_{J+1}^{**} = \frac{(JN+2N+J+3)(N+1)(z-r)+(JN+2N+J+2)Ns-N(J+2)(N+1)c}{(N+2)(JN+2N+J+3)+N}$$

$$k_{J+1}^{**} = \frac{(JN+2N+J+3)(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} + \frac{N(JN+2N+J+2)[s-c]}{(N+2)(JN+2N+J+3)+N}$$

Since, by assumption, (z-r) >0 and s > c, it follows that $k_j^* > k_{J+1}^*$since

$$\frac{(JN+2N+J+3)(N+1)}{(N+2)(JN+2N+J+3)+N} < 1$$

And

$$\frac{N(JN+2N+J+2)}{(N+2)(JN+2N+J+3)+N} < 1$$

$$p^{**} = \left[\frac{s+k_{J+1}^{**}+(J+2)(N+1)c}{JN+2N+J+3}\right]$$

$$p^{**} = \frac{s+(J+2)(N+1)c}{JN+2N+J+3} + \frac{(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} + \frac{N(JN+2N+J+2)[s-c]}{(JN+2N+J+3)[(N+2)(JN+2N+J+3)+N]}$$

Price: Given the modelling assumptions (s > 0, c > 0, z > r), the price here is strictly positive and we have p > c.

Comparison of profits Π_{J+1}^{**} and Π_j^{**}:

$$\Pi_j^{**} = (p^{**} - c)((z - r) + 2(s - c) + N(p^{**} - c)) + \left(\frac{1}{2}\right)\left(s + k_j^{**} - p^{**}\right)^2 + k_j^{**}\left(z - k_j^{**} - r\right) - F$$

$$\Pi_{J+1}^{**} = \left(\frac{1}{2}\right)\left(s + k_{J+1}^{**} - p^{**}\right)^2 + k_{J+1}^{**}\left(z - k_{J+1}^{**} - r\right) - F$$

Only Producer j gets profits from the sale of water. By assumption, p > c, s > c and (z – r) > 0 so the portion of profits Π_j^{**} from the sale of water are positive. (Or in the case of section 1, p=c yields zero profits from the sale of water).

Consider the addition to profits from the use of water in oil production, which yields the terms: $\left(\frac{1}{2}\right)\left(s + k_j^{**} - p^{**}\right)^2$ and $\left(\frac{1}{2}\right)\left(s + k_{J+1}^{**} - p^{**}\right)^2$. If we assume the incumbent earns greater profits from this term then:

$$\left(s + k_j^{**} - p^{**}\right)^2 > \left(s + k_{J+1}^{**} - p^{**}\right)^2$$

$$\left(s + k_j^{**}\right)^2 - sp^{**} - k_j^{**}p^{**} + p^{**2} > \left(s + k_{J+1}^{**}\right)^2 - sp^{**} - k_{J+1}^{**}p^{**} + p^{**2}$$

$$\left(s + k_j^{**}\right)^2 - k_j^{**}p^{**} > \left(s + k_{J+1}^{**}\right)^2 - k_{J+1}^{**}p^{**}$$

$$\left(s + k_j^{**}\right)^2 - \left(s + k_{J+1}^{**}\right)^2 > p^{**}(k_j^{**} - k_{J+1}^{**})$$

$$2s\left(k_j^{**} - k_{J+1}^{**}\right) + ({k_j^{**}}^2 - {k_{J+1}^{**}}^2) > p^{**}(k_j^{**} - k_{J+1}^{**})$$

$$2s\left(k_j^{**} - k_{J+1}^{**}\right) + \left(k_j^{**} - k_{J+1}^{**}\right)\left(k_j^{**} + k_{J+1}^{**}\right) > p^{**}(k_j^{**} - k_{J+1}^{**})$$

$$2s + \left(k_j^{**} + k_{J+1}^{**}\right) > p^{**}$$

By examination of the price function, we can see that the above is true in equilibrium.

Now consider the portion of the profit function concerning the profits with respect to the choice of capacity, given by the terms: $k_j^{**}\left(z - k_j^{**} - r\right)$ and $k_{J+1}^{**}\left(z - k_{J+1}^{**} - r\right)$. If we assume the incumbent earns greater profits from this term then:

$$k_j^{**}(z - r) - {k_j^{**}}^2 > k_{J+1}^{**}(z - r) - {k_{J+1}^{**}}^2$$

$$(k_j^{**} - k_{J+1}^{**})(z - r) > {k_j^{**}}^2 - {k_{J+1}^{**}}^2$$

$$(k_j^{**} - k_{J+1}^{**})(z - r) > \left(k_j^{**} - k_{J+1}^{**}\right)\left(k_j^{**} + k_{J+1}^{**}\right)$$

$$(z - r) > \left(k_j^{**} + k_{J+1}^{**}\right)$$

The above statement will not be true unless k_{J+1}^{**} is negative. Therefore the entrant earns a greater profit from this term than the incumbent. Then we know that, with respect to profits:

$$\Pi_j^{**} > \Pi_{J+1}^{**} \Leftrightarrow$$

$$(p^{**}-c)[(z-r)+2(s-c)+N(p^{**}-c)]+2s+\left(k_j^{**}+k_{J+1}^{**}\right)-p^{**}$$
$$> \left(k_j^{**}+k_{J+1}^{**}\right)-(z-r)$$

$$\Leftrightarrow$$

$$p^{**}[(z-r)+2(s-c)]-c(z-r)-2c(s-c)+N(p^{**}-c)^2 > p^{**}-2s-(z-r)$$

$$(p^{**}+1)(z-r)+2s(p^{**}+1)+N({p^{**}}^2-2p^{**}c+c^2)$$
$$> p^{**}+c[2p^{**}+(z-r)+2(s-c)]$$

$$(p^{**}+1)[(z-r)+2s]+N\left({p^{**}}^2+c^2\right) > p^{**}+2c(N+1)p^{**}+c[(z-r)+2s]-2c^2$$

$$(p^{**}-c+1)[(z-r)+2s]+N(p^{**}-c)^2 > p^{**}+2cp^{**}-2c^2$$

$$(p^{**}-c+1)[(z-r)+2s]+N(p^{**}-c)^2 > p^{**}+2c[p^{**}-c+1]-2c$$

$$(p^{**}-c+1)[(z-r)+2s-2c]+N(p^{**}-c)^2 > p^{**}-2c$$

$$(p^{**}-c+1)\left[k_j^{**}+s-c\right]+N(p^{**}-c)^2 > p^{**}-2c$$

$$(p^{**}-c)\left[k_j^{**}+s-c\right]+k_j^{**}+s-c+N(p^{**}-c)^2 > (p^{**}-c)-c$$

$$(p^{**}-c)\left[k_j^{**}-1+s-c\right]+k_j^{**}+s+N(p^{**}-c)^2 > 0$$

This inequality will hold for all inputs based on our prior assumptions, so $\Pi_j^{**} > \Pi_{J+1}^{**}$.
Note that both the quantity or water supplied by oil producers $q_j^{o\,**}$, and the price of water are higher in case 2.1 than in case 1, indicating that there will be higher profits for the incumbents in the case where entry is accommodated than in the case where entry is not desirable. [Especially since the incumbent's choice of k_j^{**} is the same in both cases.]

In this case, since the entrant will enter the market if they are earning positive profits in equilibrium, only then will entry be optimal, if the following holds:

$$\left(\frac{1}{2}\right)\left(s+k_{J+1}^{**}-p^{**}\right)^2+k_{J+1}^{**}\left(z-k_{J+1}^{**}-r\right) > F$$

$$\left(\frac{1}{2}\right)\left(\frac{(J+2)(N+1)(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N}\right.$$
$$\left.+\frac{(J+2)(N+1)\{2(J+2)(N+1)+2(JN+2N+J+3)\}[s-c]}{(JN+2N+J+3)[(N+2)(JN+2N+J+3)+N]}\right)^2$$
$$+\left(\frac{(JN+2N+J+3)(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N}+\frac{N(JN+2N+J+2)[s-c]}{(N+2)(JN+2N+J+3)+N}\right)$$
$$*\left(\frac{(J+3)(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N}-\frac{N(JN+2N+J+2)[s-c]}{(N+2)(JN+2N+J+3)+N}\right) > F$$

Section 2.2: Entry Deterrence is Optimal (& constrained water supply)

<u>Section 2.2.i:</u> Supply is limited by availability ($\boldsymbol{Y = Q}$)

Consider the case where Q=Y such that the water sales are constrained by the water supply. As before, water demand can be stated as a function of price:

$$p\left(Q, K_I, k_{J+1}\right) = \frac{n_R b + n_2 \gamma + (J+1)s + K_I + k_{J+1} - Q}{N+1}$$

The entrant will choose a non-negative k_{J+1} to maximize:

$\Pi_{J+1} = (\frac{1}{2})\left(s + k_{J+1} - p\right)^2 + k_{J+1}\left(\alpha - k_{J+1}\right) - F$, where: $\alpha = (z - r) > 0$

In the case where deterrence is optimal, we know that the entrant will not enter the market and can therefore not have positive profits, so we assume $\Pi_{J+1} = 0$.

Differentiating with respect to k_{J+1} and assuming and interior solution, we have:

$$\frac{d\Pi_{J+1}}{dk_{J+1}} = \left(s + k_{J+1} - p\right)\left(1 - \frac{1}{N+1}\right) + \left(\alpha - 2k_{J+1}\right) = 0$$

These first order conditions imply that

$$s + k_{J+1} - p = \frac{(N+1)(2k_{J+1} - \alpha)}{N}$$

Hence

$$\Pi_{J+1} = (\frac{1}{2})\frac{(N+1)^2(2k_{J+1} - \alpha)^2}{N^2} + k_{J+1}\left(\alpha - k_{J+1}\right) - F$$

$$\Pi_{J+1} = \frac{(N+1)^2(2k_{J+1} - \alpha)^2 + 2N^2[k_{J+1}\left(\alpha - k_{J+1}\right) - F]}{2N^2}$$

Since $\Pi_{J+1} = 0$ we have

$$(N+1)^2(2k_{J+1} - \alpha)^2 + 2N^2\left[k_{J+1}\left(\alpha - k_{J+1}\right) - F\right] = 0$$

$$(N^2 + 2N + 1)\left(4k_{J+1}^2 - 4k_{J+1}\alpha + \alpha^2\right) + 2N^2 k_{J+1}\alpha - 2N^2 k_{J+1}^2 - 2N^2 F = 0$$

$$k_{J+1}^2(2N^2 + 8N + 4) - \alpha k_{J+1}(2N^2 + 8N + 4) + \alpha^2(N+1)^2 - 2N^2 F = 0$$

$$(2N^2 + 8N + 4)\left[k_{J+1}^2 - \alpha k_{J+1}\right] - [2N^2 F - \alpha^2(N+1)^2] = 0$$

Let:

$$\beta = \frac{2N^2F - \alpha^2(N+1)^2}{2N^2 + 8N + 4}$$

And assume

$$\left(\frac{F}{\alpha^2}\right) > \max\left[\frac{(N+1)^2}{2N^2}, \left(\frac{1}{4}\right)\right]$$

Then the above equation reduces to

$$k_{J+1}^2 - \alpha k_{J+1} - \beta = 0$$

So $k_{J+1}^{***} = (\alpha + (\alpha^2 + 4\beta)^{\frac{1}{2}})/2$

From the first order conditions we know that

$$k_{J+1}^{***} = \frac{(N+1)[Ns + (N+1)\alpha] - N(n_R b + n_2\gamma + (J+1)s + K_I - Y)}{N^2 + 4N + 2}$$

Combining to solve for K_I^{***} :

$$\left(\alpha + (\alpha^2 + 4\beta)^{\frac{1}{2}}\right) = \frac{2(N+1)[Ns + (N+1)\alpha] - 2N(n_R b + n_2\gamma + (J+1)s + K_I - Y)}{N^2 + 4N + 2}$$

$$\left(\alpha + (\alpha^2 + 4\beta)^{\frac{1}{2}}\right)(N^2 + 4N + 2) \\ = 2N(N + 1 - J - 1)s + 2(N+1)^2\alpha - 2N(n_R b + n_2\gamma + K_I - Y)$$

$$2NK_I = 2N(N - J)s + (2N^2 + 4N + 2 - N^2 - 4N - 2)\alpha - 2N(n_R b + n_2\gamma - Y) \\ - (\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2)$$

$$K_I^{***} = (N - J)s - (n_R b + n_2\gamma - Y) + \frac{N^2\alpha - (\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2)}{2N}$$

Note that $K_I^{***} > 0$ for sufficiently large s and, consistent with other modelling assumptions, we should have $Y > n_R b + n_2\gamma$.

Results (Section 2.2.i):

$$p = \frac{n_R b + n_2\gamma + Js + K_I - Y}{N}$$

$$p^{***} = s + \frac{N^2\alpha - (\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2)}{2N^2}$$

$Q^{***} = Y$

$Q^{o^{***}} = \mathrm{Y}^{o}$

$q_j^{o^{***}} = \mathrm{y}^{o}$

$Q^{f^{***}} = \mathrm{Y}^{f}$

$Q^{l^{***}} = \mathrm{Y}^{l}$

$$a_i{}^{***} = (b - p^{***}) = b - \ s + \frac{(\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2) - N^2\alpha}{2N^2}$$

$$a^{f^{***}} = (\gamma - p^{***}) = \gamma - s + \frac{(\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2) - N^2\alpha}{2N^2}$$

$$a^{o^{***}} = \left(s + k_j^{***} - p^{***}\right) = k_j^{***} + \frac{(\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2) - N^2\alpha}{2N^2}$$

$$\Pi^{f^{***}} = (p^{***} - c)\mathrm{Y}^{f} + \frac{n_2}{2}[(\gamma - p^{***})^2 + (b - p^{***})^2]$$

$$\Pi^{l^{***}} = (p^{***} - c)\mathrm{Y}^{l} + J\Pi_j^{***} + \frac{n_1}{2}(b - p^{***})^2$$

$$\Pi_j^{***} = \mathrm{y}^{o}(p^{***} - c) + \left(\frac{1}{2}\right)\left(s + k_j^{***} - p^{***}\right)^2 + k_j^{***}\left(z - k_j^{***} - r\right) - F$$

$k_{J+1}^{***} = 0$

$$K_I^{***} = (N - J)s - (n_R b + n_2\gamma - Y) + \frac{N^2\alpha - (\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2)}{2N}$$

Compare p^{***} to $p^{**}$from case 2.1:

$$p^{**} = \frac{n_R b + n_2\gamma + (J + 1)s + K_I^{**} + k_{J+1}^{**} - Q^{**}}{N + 1}$$

$$p^{***} = \frac{n_R b + n_2\gamma + Js - Y + K_I^{***}}{N}$$

$p^{***} > p^{**}$ implies that

$$\frac{(N + 1)[n_R b + n_2\gamma + Js - Y + K_I^{***}]}{N(N + 1)} - \frac{N\left[n_R b + n_2\gamma + (J + 1)s + K_I^{**} + k_{J+1}^{**} - Q^{**}\right]}{N(N + 1)} > 0$$

$$n_R b + n_2\gamma + Js + (N + 1)[K_I^{***} - Y] - N\left[s + K_I^{**} + k_{J+1}^{**} - Q^{**}\right] > 0$$

We can see that

$$p^{***} > p^{**} \leftrightarrow n_R b + n_2\gamma + Js + (N+1)[K_I^{***} - Y] > N\left[s + K_I^{**} + k_{J+1}^{**} - Q^{**}\right]$$

$$K_I^{**} = J(z - r + s - c)$$

$$k_{J+1}^{**} = \frac{(JN + 2N + J + 3)(N+1)(z - r)}{(N+2)(JN + 2N + J + 3) + N} + \frac{N(JN + 2N + J + 2)[s - c]}{(N+2)(JN + 2N + J + 3) + N}$$

$$Q^{**} = Js + n_2\gamma + n_R b + K_I + \frac{(JN + N + J + 2)(s + k_{J+1}) - (J+2)(N+1)^2 c}{JN + 2N + J + 3}$$

$$[K_I^{**} - Q^{**}] = \frac{(J+2)(N+1)^2 c - (NJ + N + J + 2)\left(s + k_{J+1}^{**}\right)}{JN + 2N + J + 3} - n_R b - n_2\gamma - Js$$

$$\begin{aligned} N[s + K_I^{**} - Q^{**}] \\ &= \frac{N(J+2)(N+1)^2 c - N(NJ + N + J + 2)\left(s + k_{J+1}^{**}\right)}{JN + 2N + J + 3} \\ &- N[n_R b + n_2\gamma + (J-1)]s \end{aligned}$$

$$K_I^{***} - Y = (N - J)s - n_R b - n_2\gamma + \frac{N^2\alpha - (\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2)}{2N}$$

$$\begin{aligned} n_R b + n_2\gamma + Js + (N+1)[K_I^{***} - Y] \\ &= (N^2 - NJ + N)s - N(n_R b + n_2\gamma) \\ &+ \frac{(N+1)\left[N^2\alpha - (\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2)\right]}{2N} \end{aligned}$$

$$\begin{aligned} n_R b + n_2\gamma + Js + (N+1)[K_I^{***} - Y] - N[s + K_I^{**} - Q^{**}] \\ &= N(N - J + 1)s - N(n_R b + n_2\gamma) \\ &+ \frac{(N+1)\left[N^2\alpha - (\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2)\right]}{2N} \\ &+ \frac{N(NJ + N + J + 2)\left(s + k_{J+1}^{**}\right) - N(J+2)(N+1)^2 c}{JN + 2N + J + 3} \\ &+ N[n_R b + n_2\gamma + (J-1)]s \end{aligned}$$

$$\begin{aligned} n_R b + n_2\gamma + Js + (N+1)[K_I^{***} - Y] - N[s + K_I^{**} - Q^{**}] \\ &= N^2 s + \frac{(N+1)N(z - r)}{2} - \frac{(N+1)(\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2)}{2N} \\ &+ \frac{N(NJ + N + J + 2)\left(s + k_{J+1}^{**}\right)}{JN + 2N + J + 3} - \frac{N(J+2)(N+1)^2 c}{JN + 2N + J + 3} \end{aligned}$$

$$n_R b + n_2\gamma + Js + (N+1)[K_I^{***} - Y] - N[s + K_I^{**} - Q^{**}]$$
$$= N^2 s + \frac{(N+1)N(z-r)}{2} - \frac{(N+1)(\alpha^2+4\beta)^{\frac{1}{2}}(N^2+4N+2)}{2N} + \frac{N(NJ+N+J+2)s}{JN+2N+J+3} + \frac{N(NJ+N+J+2)k_{J+1}^{**}}{JN+2N+J+3} - \frac{N(J+2)(N+1)^2 c}{JN+2N+J+3}$$

$$\Longrightarrow\ p^{***} > p^{**} \leftrightarrow \{n_R b + n_2\gamma + Js + (N+1)[K_I^{***} - Y] - N[s + K_I^{**} - Q^{**}]\} > Nk_{J+1}^{**}$$

This implies that $p^{***} > p^{**}$ if and only if:

$$N^2 s + \frac{(N+1)N(z-r)}{2} - \frac{(N+1)(\alpha^2+4\beta)^{\frac{1}{2}}(N^2+4N+2)}{2N} + \frac{N(NJ+N+J+2)s}{JN+2N+J+3} - \frac{N(J+2)(N+1)^2 c}{JN+2N+J+3} > \frac{N(NJ+N+J+2)k_{J+1}^{**}}{JN+2N+J+3}$$

$$\frac{(N+1)(z-r)}{2} - \frac{(N+1)(\alpha^2+4\beta)^{\frac{1}{2}}(N^2+4N+2)}{2N^2} + \frac{(J+2)(N+1)^2(s-c)}{JN+2N+J+3} > \frac{(NJ+N+J+2)k_{J+1}^{**}}{JN+2N+J+3}$$

Note that:

$$(z-r)^2 + 4\beta = (z-r)^2 + \frac{4N^2F - 2(z-r)^2(N+1)^2}{N^2+4N+2}$$

$$(z-r)^2 + 4\beta = \frac{(z-r)^2[N^2+4N+2-2N^2-4N-2] + 4N^2F}{N^2+4N+2}$$

$$(z-r)^2 + 4\beta = \frac{N^2(4F-(z-r)^2)}{N^2+4N+2} > 0$$

Provided F is sufficiently large to ensure: $2\sqrt{F} > (z-r)$ or:

$$F > \frac{(z-r)^2}{4}$$

Then $p^{***} > p^{**}$ if and only if:

$$\frac{(N+1)(z-r)}{2}-\frac{(N+1)(N^2+4N+2)}{2N^2}\left[\frac{N^2(4F-(z-r)^2)}{N^2+4N+2}\right]^{\frac{1}{2}}+\frac{(J+2)(N+1)^2(s-c)}{JN+2N+J+3} > \frac{(NJ+N+J+2)k_{J+1}^{**}}{JN+2N+J+3}$$

$$\frac{(N+1)(z-r)}{2}-\frac{(N+1)(N^2+4N+2)}{2N^2}\left[\frac{N^2(4F-(z-r)^2)}{N^2+4N+2}\right]^{\frac{1}{2}}+\frac{(J+2)(N+1)^2(s-c)}{JN+2N+J+3} - \frac{(NJ+N+J+2)k_{J+1}^{**}}{JN+2N+J+3} > 0$$

From the last term in the above expression:

$$\frac{(NJ+N+J+2)k_{J+1}^{**}}{JN+2N+J+3} = \frac{(JN+N+J+2)(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} + \frac{N(JN+2N+J+2)^2[s-c]}{(JN+2N+J+3)[(N+2)(JN+2N+J+3)+N]}$$

Subtracting from the prior expression, we obtain:

$$\frac{(N+1)(z-r)}{2}-\frac{(N+1)(N^2+4N+2)}{2N^2}\left[\frac{N^2(4F-(z-r)^2)}{N^2+4N+2}\right]^{\frac{1}{2}}+\frac{(J+2)(N+1)^2(s-c)}{JN+2N+J+3} - \frac{(JN+N+J+2)(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} - \frac{N(JN+2N+J+2)^2[s-c]}{(JN+2N+J+3)[(N+2)(JN+2N+J+3)+N]} > 0$$

$$\frac{(J+2)(N+1)(N^2J+2N^2+2NJ+6N+2J+6)[s-c]}{(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]} + \frac{(N^2J+2N^2+NJ+6N+2)(z-r)}{2[(N+2)(JN+2N+J+3)+N]} > \frac{(N^2+4N+2)}{2N^2}\left[\frac{N^2(4F-(z-r)^2)}{N^2+4N+2}\right]^{\frac{1}{2}}$$

$$\frac{2N^2(J+2)(N+1)(N^2J+2N^2+2NJ+6N+2J+6)[s-c]}{(N^2+4N+2)(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]} + \frac{N^2(N^2J+2N^2+NJ+6N+2)(z-r)}{(N^2+4N+2)[(N+2)(JN+2N+J+3)+N]} > \left[\frac{N^2(4F-(z-r)^2)}{N^2+4N+2}\right]^{\frac{1}{2}}$$

$$\left[\frac{2N(J+2)(N+1)(N^2J+2N^2+2NJ+6N+2J+6)[s-c]}{(N^2+4N+2)(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]} + \frac{N(N^2J+2N^2+NJ+6N+2)(z-r)}{(N^2+4N+2)[(N+2)(JN+2N+J+3)+N]}\right]^2 > \frac{(4F-(z-r)^2)}{N^2+4N+2}$$

Equation 1:

$$\frac{(N^2+4N+2)}{4}\left[\frac{2N(J+2)(N+1)(N^2J+2N^2+2NJ+6N+2J+6)[s-c]}{(N^2+4N+2)(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]} + \frac{N(N^2J+2N^2+NJ+6N+2)(z-r)}{(N^2+4N+2)[(N+2)(JN+2N+J+3)+N]}\right]^2 + \frac{(z-r)^2}{4} > F$$

From prior analysis above (in order to have a positive root), we have that the following must hold: (Equation 2)

$$F > \frac{(z-r)^2}{4}$$

Given these items along with our modelling assumptions that s –c > 0 and z – r > 0, the above equality will hold, provided F is bounded above by equation 1 and below by equation 2. It will then follow that $p^{***} > p^{**}$.

Compare equation1 above to the value of F required to allow for entry in the accommodation case (2.1), where we had:

$$\left(\frac{1}{2}\right)\left(\frac{(J+2)(N+1)(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} + \frac{(J+2)(N+1)\{2(J+2)(N+1)+2(JN+2N+J+3)\}[s-c]}{(JN+2N+J+3)[(N+2)(JN+2N+J+3)+N]}\right)^2 + \left(\frac{(JN+2N+J+3)(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} + \frac{N(JN+2N+J+2)[s-c]}{(N+2)(JN+2N+J+3)+N}\right) * \left(\frac{(J+3)(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} - \frac{N(JN+2N+J+2)[s-c]}{(N+2)(JN+2N+J+3)+N}\right) > F$$

$$\frac{(N+1)^2[(J+2)^2(N+1)^2+2(JN+2N+J+3)(J+3)](z-r)^2}{2[N^2J+2N^2+3NJ+8N+2J+6]^2}$$
$$+\left(\frac{2(J+2)^3(N+1)^4[s-c](z-r)}{(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]^2}\right)$$
$$-\frac{(J+2)^2(N+1)^2\{N^2-2(N+1)\}[s-c](z-r)}{[N^2J+2N^2+3NJ+8N+2J+6]^2}$$
$$-\left(\frac{(J+2)^2(N+1)^2\langle\{N(JN+2N+J+3)\}^2-\{2(J+2)(N+1)+2(JN+2N+J+3)\}^2\rangle[s-c]^2}{(JN+2N+J+3)^2[N^2J+2N^2+3NJ+8N+2J+6]^2}\right)$$
$$> F$$

We can see that the third and fourth terms of the above expression are negative and that the first two positive terms less the other terms will yield a smaller upper bound for F than the bound imposed by equation 1 above; therefore we can see that the constraint from case 2.1 which is necessary for entry will ensure a lower value of F, ensuring that we have $p^{***} > p^{**}$.

Compare K_I^{***} with K_I^{**}

$$K_I^{***} = (N-J)s - (n_R b + n_2\gamma - Y) + \frac{N^2(z-r) - ((z-r)^2+4\beta)^{\frac{1}{2}}(N^2+4N+2)}{2N}$$

$$K_I^{**} = J(z-r) + Js - Jc$$

If $K_I^{***} > K_I^{**}$ we have:

$$K_I^{***} - K_I^{**} > 0$$

$$K_I^{***} - K_I^{**} = (N-J)s - (n_R b + n_2\gamma - Y) + \frac{N^2(z-r) - ((z-r)^2+4\beta)^{\frac{1}{2}}(N^2+4N+2)}{2N} - J(z-r) - Js + Jc$$

$$K_I^{***} - K_I^{**} = Y - (n_R b + n_2\gamma) + (N-2J)s + \frac{(N-2J)(z-r)}{2} - \frac{((z-r)^2+4\beta)^{\frac{1}{2}}(N^2+4N+2)}{2N} + Jc$$

In the above expression, for $K_I^{***} - K_I^{**} > 0$ we need:

$Y > n_R b + n_2\gamma$ which is satisfied by the model specification.

$$(N-2J)s + \frac{(N-2J)(z-r)}{2} > 0 \iff N > 2J \iff n_R + n_2 > J$$

This inequality will hold only when the number of incumbent oil producers is sufficiently low and the number of non-oil producers in the market is sufficiently high, by comparison.

Intuitively, this will be the case when there is little competition in the oil market, indicating that there may be some barrier to entry, such as high capital or fixed costs.

We also need to ensure that the positive numbers on the LHS below are sufficiently large and the expression on the RHS is sufficiently small to ensure:

$$Y - (n_R b + n_2\gamma) + (N-2J)s + \frac{(N-2J)(z-r)}{2} + Jc > \frac{((z-r)^2 + 4\beta)^{\frac{1}{2}}(N^2+4N+2)}{2N}$$

Also (from above) we know $2\sqrt{F} > (z-r)$ and

$$\frac{((z-r)^2 + 4\beta)^{\frac{1}{2}}(N^2+4N+2)}{2N} = \frac{(N^2+4N+2)}{2N}\left[\frac{(4F-(z-r)^2)}{N^2+4N+2}\right]^{1/2}$$

Then the RHS above must be sufficiently small (meaning F must be sufficiently small), while still satisfying$2\sqrt{F} > (z-r)$.

Comparing Profits:

$$\Pi_j^{***} = \mathrm{y}^o(p^{***}-c) + \left(\frac{1}{2}\right)\left(s + k_j^{***} - p^{***}\right)^2 + k_j^{***}\left(z - k_j^{***} - r\right) - F$$

$$\Pi_j^{**} = (p^{**}-c)q^{o^{**}} + \left(\frac{1}{2}\right)\left(s + k_j^{**} - p^{**}\right)^2 + k_j^{**}\left(z - k_j^{**} - r\right) - F$$

Since we have shown above that $p^{***} > p^{**}$ and by definition $\mathrm{y}^o > q^{o^{**}}$, we have:

$$\mathrm{y}^o(p^{***}-c) > (p^{**}-c)q^{o^{**}}$$

In regards to the second term of the profit function and the modelling assumption that s is sufficiently large to ensure s > p:

$$\left(s + k_j^{***} - p^{***}\right)^2 > \left(s + k_j^{**} - p^{**}\right)^2 \iff k_j^{***} - p^{***} > k_j^{**} - p^{**}$$
$$k_j^{***} - k_j^{**} > p^{***} - p^{**}$$
$$K_I^{***} - K_I^{**} > J(p^{***} - p^{**})$$
$$(N+1)(K_I^{***} - K_I^{**}) > J\left(K_I^{***} - Y - K_I^{**} - k_{J+1}^{**} + Q^{**}\right)$$
$$(N-J+1)[K_I^{***} - K_I^{**}] > J\left(Q^{**} - Y - k_{J+1}^{**}\right)$$
$$(N-J+1)[K_I^{***} - K_I^{**}] + Jk_{J+1}^{**} > J(Q^{**} - Y)$$

Given the fact that we showed $K_I^{***} > K_I^{**}$ we know that the LHS of the above inequality is greater than zero. Also, by specification of the model $Y > Q^{**}$ so the RHS above is strictly negative. This implies that the contribution to profits Π_j^{***} from this term will be greater than the contribution to profits Π_j^{**} from this term.

If we assume the third term of the profit function (the profits from the choice of capital) is larger for Π_j^{***} than for Π_j^{**} we have:

$$k_j^{***}\left(z - k_j^{***} - r\right) > k_j^{**}\left(z - k_j^{**} - r\right)$$

$$k_j^{***}(z-r) - k_j^{***2} > k_j^{**}(z-r) - k_j^{**2}$$

$$(z-r)(k_j^{***} - k_j^{**}) > k_j^{***2} - k_j^{**2}$$

$$(z-r)(k_j^{***} - k_j^{**}) > k_j^{***2} - k_j^{**2}$$

$$(z-r)(k_j^{***} - k_j^{**}) > (k_j^{***} - k_j^{**})(k_j^{***} + k_j^{**})$$

$$(z-r) > (k_j^{***} + k_j^{**})$$

$$(z-r) > \left(\frac{(N-J)s - (n_R b + n_2\gamma - Y)}{J} + \frac{N^2\alpha - (\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2)}{2JN}\right) + (z-r) + s - c$$

The above in equality is obviously not true, which indicates that this term will be larger for Π_j^{**} than for Π_j^{***}, which will give:

$$0 < \left(\frac{(N-J)s - (n_R b + n_2\gamma - Y)}{J} + \frac{N^2\alpha - (\alpha^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2)}{2JN}\right) + s - c$$

$$\Leftrightarrow$$

$$0 < 2N^2 s - 2N(n_R b + n_2\gamma - Y) + N^2(z-r) - ((z-r)^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2) - 2NJc$$

$$((z-r)^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2) < 2N(Ns - Jc) - 2N(n_R b + n_2\gamma - Y) + N^2(z-r)$$

Since Ns > Jc, z > r and $n_R b + n_2\gamma < Y$, the RHS of the above is positive. As per the prior assumption that $2\sqrt{F} > (z-r)$, the LHS will also be positive. Therefore the contribution of this term to the profits Π_j^{***} will only be greater in this case than in 2.1, provided the value of F is such that these inequalities are satisfied.

Then we have that $\Pi_j^{***} > \Pi_j^{**}$ $\Leftrightarrow$

$$\begin{aligned} p^{***}y^o - p^{**}q^{o**} - c(y^o - q^{o**}) + (N-J+1)[K_I^{***} - K_I^{**}] + Jk_{J+1}^{**} - J(Q^{**} - Y) \\ > 2N^2 s - 2N(n_R b + n_2\gamma) + 2NY + N^2(z-r) \\ - N(N^2 + 4N + 2)\left[\frac{(4F - (z-r)^2)}{N^2 + 4N + 2}\right]^{1/2} - 2NJc \end{aligned}$$

$$\begin{aligned} p^{***}y^o - p^{**}q^{o**} - c(y^o - q^{o**}) + K_I^{***} - K_I^{**} - J(p^{***} - p^{**}) \\ > 2N^2 s - 2N(n_R b + n_2\gamma - Y) + N^2(z-r) \\ - N(N^2 + 4N + 2)\left[\frac{(4F - (z-r)^2)}{N^2 + 4N + 2}\right]^{1/2} - 2NJc \end{aligned}$$

In this case, we also know that the following inequalities must hold:

$$s + k_j - (N+1)c + Np \geq y^o$$
$$n_1 b - (N+1)c + p(N+1-n_1) \geq Y^l$$
$$n_2(\gamma + b) - (N+1)c + p(N+1-2n_2) \geq Y^f$$

From the first order conditions used to derive the water demand, these can also be written as:

$$s + k_j - p + (N+1)(p-c) \geq y^o$$
$$n_1(b-p) + (N+1)(p-c) \geq Y^l$$
$$n_2(\gamma + b - 2p) + (N+1)(p-c) \geq Y^f$$
$$or$$
$$a_j^o + (N+1)(p-c) \geq y^o$$
$$n_1 a_i + (N+1)(p-c) \geq Y^l$$
$$n_2(a_i + a^l) + (N+1)(p-c) \geq Y^f$$

In this case, each group is assumed to want to supply more water than their initial endowment, so that the total quantity supplied ends up being equal to their total industry supply Y. We can write these constraints as a function of the individual water demands to obtain some intuition behind what is happening here. These can be rewritten as:

$$p \geq c + \frac{y^o - a_j^o}{N+1}$$
$$p \geq c + \frac{Y^l - n_1 a_i}{N+1}$$
$$p \geq c + \frac{Y^f - n_2(a_i + a^l)}{N+1}$$

This gives us:

$$p \geq c + \frac{max\left(y^o - a_j^o, Y^l - n_1 a_i, Y^f - n_2(a_i + a^l)\right)}{N+1}$$

We can see that, in this case, the price will be influenced by the group with the largest producer surplus.

For the oil producers, they derive utility (and, ultimately, profits) from using water in the production process, which translates into their individual water demand a_j^o. They also derive profits from the sale of water in the market. The term $(N+1)(p-c)$ from the inequality $a_j^o + (N+1)(p-c) \geq y^o$ represents the additional water quantity that the oil producers wish to supply in the market to earn additional profits. You can see that if p = c, as in section 1, the oil producers would not care to supply any water above their water demand a_j^o. In the case we are examining here where $a_j^o + (N+1)(p-c) \geq y^o$ the oil producers wish to supply a greater quantity of water than their initial endowment, but are constrained by the supply. This would likely be the case when the price is high and they stand to earn a large profit by doing so.

Similarly, both the local community and the First Nation constraints are written as functions of their individual water demands. They too would like to supply more water than they need to satisfy their own needs, the amount $(N+1)(p-c)$ is representative of the additional water they would like to supply to earn additional profits from the sale of water.

Section 2.2.ii: $\boldsymbol{Q^o = Y^o, Q^l < Y^l, Q^f < Y^f}$

Consider the variation to 2.2.i above where $q_j^o = y^o$ or $Q^o = Y^o$ for the oil producers, but the First Nation and the local community have more water than they choose to supply. [Only the oil producers are constrained].

$$p = \frac{n_R b + n_2 \gamma + (J+1)s + K_I + k_{J+1} - (Y^o + Q^l + Q^f)}{N+1}$$

Here, as in case 2.1, the first nations and local community will chose their water to supply to maximize their profits to obtain:

$$Q^f = n_2(\gamma + b) - (\mathrm{N}+1)\mathrm{c} + \mathrm{p}(N + 1 - 2n_2)$$

$$Q^l = n_1 b - (N+1)c + p(N+1-n_1)$$

Then the price becomes:

$$p = \frac{(J+1)s + K_I + k_{J+1} - Y^o + 2(\mathrm{N}+1)\mathrm{c}}{2N + J + 3}$$

The entrant will choose a non-negative k_{J+1} to maximize:

$$\Pi_{J+1} = (\tfrac{1}{2})\left(s + k_{J+1} - p\right)^2 + k_{J+1}\left(\alpha - k_{J+1}\right) - F$$, where: $\alpha = (z - r) > 0$

In the case where deterrence is optimal, we know that the entrant will not enter the market and can therefore not have positive profits, so we assume $\Pi_{J+1} = 0$.

Differentiating with respect to k_{J+1} and assuming and interior solution, we have:

$$\frac{d\Pi_{J+1}}{dk_{J+1}} = \left(s + k_{J+1} - p\right)\left(1 - \frac{1}{2N + J + 3}\right) + \left(\alpha - 2k_{J+1}\right) = 0$$

These first order conditions imply that

$$s + k_{J+1} - p = \frac{(2N + J + 3)(2k_{J+1} - \alpha)}{2N + J + 2}$$

Hence

$$\Pi_{J+1} = (\frac{1}{2})\frac{(2N+J+3)^2(2k_{J+1}-\alpha)^2}{(2N+J+2)^2} + k_{J+1}\left(\alpha - k_{J+1}\right) - F$$

$$\Pi_{J+1} = \frac{(2N+J+3)^2(2k_{J+1}-\alpha)^2 + 2(2N+J+2)^2[k_{J+1}\left(\alpha - k_{J+1}\right) - F]}{2(2N+J+2)^2}$$

Since $\Pi_{J+1} = 0$ we have

$$(2N+J+3)^2(2k_{J+1}-\alpha)^2 + 2(2N+J+2)^2[k_{J+1}\left(\alpha - k_{J+1}\right) - F] = 0$$

$$(2N+J+3)^2\left(4k_{J+1}^2 - 4k_{J+1}\alpha + \alpha^2\right) + 2(2N+J+2)^2[k_{J+1}\alpha - k_{J+1}^2 - F] = 0$$

$$k_{J+1}^2 2(4N^2+4NJ+16N+J^2+8J+14) - \alpha k_{J+1}2(4N^2+4NJ+16N+J^2+8J+14) + \alpha^2(2N+J+3)^2 - 2(2N+J+2)^2F = 0$$

$$2(4N^2+4NJ+16N+J^2+8J+14)\left[k_{J+1}^2 - \alpha k_{J+1}\right] + \alpha^2(2N+J+3)^2 - 2(2N+J+2)^2F = 0$$

Let:

$$\beta = \frac{2(2N+J+2)^2F - \alpha^2(2N+J+3)^2}{2(4N^2+4NJ+16N+J^2+8J+14)}$$

$$\left(\frac{F}{\alpha^2}\right) > \left[\frac{(2N+J+3)^2}{2(2N+J+2)^2}\right]$$

Then the above equation reduces to

$$k_{J+1}^2 - \alpha k_{J+1} - \beta = 0$$

So $k_{J+1}^{***,ii} = (\alpha + (\alpha^2+4\beta)^{\frac{1}{2}})/2$

Where

$$(\alpha^2+4\beta)^{\frac{1}{2}} = \left[\frac{4(2N+J+2)^2F - (z-r)^2(4N^2+4NJ+8N+J^2+4J+4)}{(4N^2+4NJ+16N+J^2+8J+14)}\right]^{1/2}$$

$$(\alpha^2+4\beta)^{\frac{1}{2}} = (2N+J+2)\left[\frac{4F-(z-r)^2}{(4N^2+4NJ+16N+J^2+8J+14)}\right]^{1/2}$$

From the first order conditions we know that

$$k_{J+1} = p - s + \frac{(2N + J + 3)(2k_{J+1} - \alpha)}{2N + J + 2}$$

$$-(4N^2 + 4NJ + 16N + J^2 + 8J + 14)k_{J+1}^{***,ii}$$
$$= (2N + J + 2)[K_I - Y^o - 2(\mathrm{N} + 1)(\mathrm{s} - \mathrm{c})] - (2N + J + 3)^2\alpha$$

$$k_{J+1}^{***,ii} = \frac{(2N + J + 3)^2\alpha - (2N + J + 2)[K_I - Y^o - 2(\mathrm{N} + 1)(\mathrm{s} - \mathrm{c})]}{(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}$$

Combining to solve for $K_I^{***,ii}$:

$$\alpha + (\alpha^2 + 4\beta)^{\frac{1}{2}} = \frac{2(2N + J + 3)^2\alpha - 2(2N + J + 2)[K_I - Y^o - 2(\mathrm{N} + 1)(\mathrm{s} - \mathrm{c})]}{(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}$$

$$\left(\alpha + (\alpha^2 + 4\beta)^{\frac{1}{2}}\right)(4N^2 + 4NJ + 16N + J^2 + 8J + 14)$$
$$= 2(2N + J + 3)^2\alpha - 2(2N + J + 2)[K_I - Y^o - 2(\mathrm{N} + 1)(\mathrm{s} - \mathrm{c})]$$

$$2(2N + J + 2)K_I$$
$$= 2(2N + J + 3)^2\alpha + 2(2N + J + 2)[Y^o + 2(\mathrm{N} + 1)(\mathrm{s} - \mathrm{c})]$$
$$- \left(\alpha + (\alpha^2 + 4\beta)^{\frac{1}{2}}\right)(4N^2 + 4NJ + 16N + J^2 + 8J + 14)$$

$$K_I^{***,ii} = Y^o + 2(\mathrm{N} + 1)(\mathrm{s} - \mathrm{c}) + \frac{(2N + J + 2)\alpha}{2}$$
$$- \frac{(\alpha^2 + 4\beta)^{\frac{1}{2}}(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{2(2N + J + 2)}$$

Results (Section 2.2.ii):

When entry is deterred, the price will become:

$$p = \frac{Js + K_I - Y^o + 2(N + 1)c}{(2N + 2 + J)}$$

$$\beta = \frac{2(2N + J + 2)^2F - \alpha^2(2N + J + 3)^2}{2(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}$$

$$p^{***,ii} = s + \frac{\alpha}{2} - \frac{(\alpha^2 + 4\beta)^{\frac{1}{2}}(4N^2 + 4NJ + 16N + J^2 + 8J + 14)}{2(2N + J + 2)^2}$$

$$k_{J+1}^{***,ii} = 0$$

$$K_I^{***,ii} = Y^o + 2(\mathrm{N}+1)(\mathrm{s}-\mathrm{c}) + \frac{(2N+J+2)\alpha}{2} - \frac{(\alpha^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)}$$

$$Q^{f^{***,ii}} = n_2(\gamma+b) - (\mathrm{N}+1)\mathrm{c} + p^{***,ii}(N+1-2n_2)$$

$$Q^{l^{***,ii}} = n_1 b - (N+1)c + p^{***,ii}(N+1-n_1)$$

$$Q^{o^{***,ii}} = Y^o$$

$$\Pi_{J+1}{}^{***,ii} = 0$$

$$\Pi^{f^{***,ii}} = Q^{f^{***,ii}}(p^{***,ii}-c) + \frac{n_2}{2}[(\gamma-c)^2 + ((b-c)^2]$$

$$\Pi^{l^{***,ii}} = Q^{l^{***,ii}}(p^{***,ii}-c) + J\Pi_j^{***,ii} + \frac{n_1}{2}(b-c)^2$$

$$\Pi_j^{***,ii} = y^o\left(p^{***,ii}-c\right) + \left(\frac{1}{2}\right)\left(s + k_j^{***,ii} - p^{***,ii}\right)^2 + k_j^{***,ii}\left(z - k_j^{***,ii} - r\right) - F$$

Comparison with accommodation case and analysis:

$p^{***,ii} > p^{**}$ if and only if:

$$s + \frac{z-r}{2} - \frac{((z-r)^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)^2} > \frac{s+(J+2)(N+1)c}{JN+2N+J+3} + \frac{(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} + \frac{N(J+2)(N+1)[s-c]}{(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]}$$

$$s + \frac{z-r}{2} - \frac{((z-r)^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)^2} > \frac{(N+1)(z-r)}{[N^2J+2N^2+3NJ+8N+2J+6]} + \frac{2(N+1)s}{[N^2J+2N^2+3NJ+8N+2J+6]} + \frac{(J+2)(N+1)[N^2J+2N^2+3NJ+7N+2J+6]c}{(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]}$$

$$\frac{(N^2J+2N^2+3NJ+6N+2J+4)s}{[N^2J+2N^2+3NJ+8N+2J+6]} - \frac{(J+2)(N+1)[N^2J+2N^2+3NJ+7N+2J+6]c}{(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]} + \frac{(N^2J+2N^2+3NJ+6N+2J+4)(z-r)}{2[N^2J+2N^2+3NJ+8N+2J+6]} > \frac{((z-r)^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)^2}$$

$$\left[\frac{(2N+J+2)^2}{(4N^2+4NJ+16N+J^2+8J+14)}\right]\left\{\frac{(N^2J+2N^2+3NJ+6N+2J+4)s}{[N^2J+2N^2+3NJ+8N+2J+6]} - \frac{(J+2)(N+1)[N^2J+2N^2+3NJ+7N+2J+6]c}{(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]} + \frac{(N^2J+2N^2+3NJ+6N+2J+4)(z-r)}{2[N^2J+2N^2+3NJ+8N+2J+6]}\right\}^2 + \frac{(z-r)^2}{4} > F$$

This will be true, based on the finding from case 2.1 that the fixed costs are sufficiently small that the incumbents and the entrant can earn positive profits in equilibrium.

$K_I^{***,ii} > K_I^{**} \leftrightarrow K_I^{***,ii} - K_I^{**} > 0$

$$K_I^{***,ii} - K_I^{**} = Y^o + 2(\mathrm{N}+1)(\mathrm{s}-\mathrm{c}) + \frac{(2N+J+2)(z-r)}{2} - \frac{(\alpha^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)} - J[(z-r)+s-c]$$

$$K_I^{***,ii} - K_I^{**} = Y^o + (2\mathrm{N}+2-\mathrm{J})(\mathrm{s}-\mathrm{c}) + \frac{(2N+2-J)(z-r)}{2} - \frac{(\alpha^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)}$$

Since N>J, we know the first three terms will be strictly positive, so $K_I^{***,ii} - K_I^{**} > 0$ if and only if:

$$Y^o + (2\mathrm{N}+2-\mathrm{J})(\mathrm{s}-\mathrm{c}) + \frac{(2N+2-J)(z-r)}{2} > \frac{(\alpha^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)}$$

This will be true based on the previously discussed upper bounds placed on F.

<u>Section 2.2.iii:</u> $\boldsymbol{Q^o = Y^o, Q^l < Y^l, Q^f = Y^f}$

Consider the variation to 2.2.i and 2.2.ii above where $q_j^o = y^o$ or $Q^o = Y^o$for the oil producers, $Q^f = Y^f$ for the First Nation, but the local community has more water than they choose to supply ($Q^l < Y^l$).

$$p = \frac{n_R b + n_2\gamma + (J+1)s + K_I + k_{J+1} - (Y^o + Y^f + Q^l)}{N+1}$$

Here, as in case 2.1, the local community will chose their water to supply to maximize their profits to obtain:

$Q^l = n_1 b - (N+1)c + p(N+1-n_1)$

Then the price becomes:

$$p = \frac{n_2(b+\gamma) + (N+1)c + (J+1)s + K_I + k_{J+1} - (Y^o + Y^f)}{2N+2-n_1}$$

The entrant will choose a non-negative k_{J+1} to maximize:

$\Pi_{J+1} = (\frac{1}{2})\left(s + k_{J+1} - p\right)^2 + k_{J+1}\left(\alpha - k_{J+1}\right) - F$, where: $\alpha = (z - r) > 0$

In the case where deterrence is optimal, we know that the entrant will not enter the market and can therefore not have positive profits, so we assume $\Pi_{J+1} = 0$.

Differentiating with respect to k_{J+1} and assuming and interior solution, we have:

$$\frac{d\Pi_{J+1}}{dk_{J+1}} = \left(s + k_{J+1} - p\right)\left(1 - \frac{1}{2N+2-n_1}\right) + \left(\alpha - 2k_{J+1}\right) = 0$$

These first order conditions imply that

$$s + k_{J+1} - p = \frac{(2N+2-n_1)(2k_{J+1} - \alpha)}{2N+1-n_1}$$

Hence

$$\Pi_{J+1} = (\frac{1}{2})\frac{(2N+2-n_1)^2(2k_{J+1}-\alpha)^2}{(2N+1-n_1)^2} + k_{J+1}\left(\alpha - k_{J+1}\right) - F$$

$$\Pi_{J+1} = \frac{(2N+2-n_1)^2(2k_{J+1}-\alpha)^2 + 2(2N+1-n_1)^2[k_{J+1}\left(\alpha - k_{J+1}\right) - F]}{2(2N+1-n_1)^2}$$

Since $\Pi_{J+1} = 0$ we have

$$(2N+2-n_1)^2(2k_{J+1}-\alpha)^2 + 2(2N+1-n_1)^2[k_{J+1}\left(\alpha - k_{J+1}\right) - F] = 0$$

$$(2N+2-n_1)^2\left(4k_{J+1}^2 - 4k_{J+1}\alpha + \alpha^2\right) + 2(2N+1-n_1)^2[k_{J+1}\alpha - k_{J+1}^2 - F] = 0$$

$$\begin{aligned} k_{J+1}^2&(8N^2 + 24N - 8n_1N - 12n_1 + 2{n_1}^2 + 14) \\ &- \alpha k_{J+1}(8N^2 + 24N - 8n_1N - 12n_1 + 2{n_1}^2 + 14) + \alpha^2(2N+2-n_1)^2 \\ &- 2(2N+1-n_1)^2 F = 0 \end{aligned}$$

Let:

$$\beta = \frac{2(2N+1-n_1)^2 F - \alpha^2(2N+2-n_1)^2}{(8N^2+24N-8n_1N-12n_1+2n_1{}^2+14)}$$

$$\left(\frac{F}{\alpha^2}\right) > \left[\frac{(2N+2-n_1)^2}{2(2N+1-n_1)^2}\right]$$

Then the above equation reduces to

$$k_{J+1}^2 - \alpha k_{J+1} - \beta = 0$$

So $k_{J+1}^{***,ii} = (\alpha + (\alpha^2+4\beta)^{\frac{1}{2}})/2$

Where

$$(\alpha^2+4\beta)^{\frac{1}{2}} = \left[(z-r)^2 + \frac{8(2N+1-n_1)^2 F - 4(z-r)^2(2N+2-n_1)^2}{(8N^2+24N-8n_1N-12n_1+2n_1{}^2+14)}\right]^{\frac{1}{2}}$$

$$(\alpha^2+4\beta)^{\frac{1}{2}} = \left[\frac{(2N+1-n_1)^2[4F-(z-r)^2]}{(4N^2+12N-4n_1N-6n_1+n_1{}^2+7)}\right]^{1/2}$$

From the first order conditions we know that

$$k_{J+1} = p - s + \frac{(2N+2-n_1)(2k_{J+1}-\alpha)}{2N+1-n_1}$$

$$k_{J+1}^{***,iii} = \frac{(2N+2-n_1)^2\alpha}{(2N+1-n_1)^2} - \frac{[n_2(b+\gamma)+(N+1)c+(J-2N-1+n_1)s+K_I-(Y^o+Y^f)]}{(2N+1-n_1)}$$

Combining to solve for $K_I^{***,iii}$:

$$(2N+1-n_1)^2\left(\alpha+(\alpha^2+4\beta)^{\frac{1}{2}}\right) = (2N+2-n_1)^2\alpha - (2N+1-n_1)[n_2(b+\gamma)+(N+1)c+(J-2N-1+n_1)s+K_I - (Y^o+Y^f)]$$

$$(2N+1-n_1)K_I$$
$$= (2N+2-n_1)^2\alpha - (2N+1-n_1)^2\left(\alpha + (\alpha^2+4\beta)^{\frac{1}{2}}\right)$$
$$- (2N+1-n_1)[n_2(b+\gamma) + (N+1)c + (J-2N-1+n_1)s - (Y^o+Y^f)]$$

$$K_I^{***,iii} = Y^o + Y^f - n_2(b+\gamma) - (N+1)c - (J-2N-1+n_1)s + \frac{(4N+3-2n_1)\alpha}{(2N+1-n_1)}$$
$$- (2N+1-n_1)(\alpha^2+4\beta)^{\frac{1}{2}}$$

Results (Section 2.2.iii):

When entry is deterred, the price will become:

$$p = \frac{n_2(b+\gamma) + Js + (N+1)c + K_I - (Y^o+Y^f)}{(2N+1-n_1)}$$

$$p^{***,iii} = s + \frac{(4N+3-2n_1)\alpha}{(2N+1-n_1)^2} - (\alpha^2+4\beta)^{\frac{1}{2}}$$

$$k_{J+1}^{***,iii} = 0$$

$$K_I^{***,iii} = Y^o + Y^f - n_2(b+\gamma) - (N+1)c - (J-2N-1+n_1)s + \frac{(4N+3-2n_1)\alpha}{(2N+1-n_1)}$$
$$- (2N+1-n_1)(\alpha^2+4\beta)^{\frac{1}{2}}$$

$$Q^{f^{***,iii}} = Y^f$$
$$Q^{l^{***,iii}} = n_1 b - (N+1)c + p^{***,iii}(N+1-n_1)$$
$$Q^{o^{***,iii}} = Y^o$$
$$\Pi_{J+1}{}^{***,iii} = 0$$
$$\Pi^{f^{***,iii}} = Y^f(p^{***,iii} - c) + \frac{n_2}{2}[(\gamma-c)^2 + ((b-c)^2]$$
$$\Pi^{l^{***,iii}} = Q^{l^{***,iii}}(p^{***,iii} - c) + J\Pi_j^{***,iii} + \frac{n_1}{2}(b-c)^2$$
$$\Pi_j^{***,iii} = y^o\left(p^{***,iii} - c\right) + \left(\frac{1}{2}\right)\left(s + k_j^{***,iii} - p^{***,iii}\right)^2 + k_j^{***,iii}\left(z - k_j^{***,iii} - r\right) - F$$

Comparison with accommodation case and analysis:
$p^{***,iii} > p^{**}$ if and only if $p^{***,iii} - p^{**} > 0$:

$$s+\frac{(4N+3-2n_1)(z-r)}{(2N+1-n_1)^2}-((z-r)^2+4\beta)^{\frac{1}{2}}-\frac{s+(J+2)(N+1)c}{JN+2N+J+3}$$
$$-\frac{(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N}$$
$$-\frac{N(J+2)(N+1)[s-c]}{(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]}>0$$

$$\frac{(J+2)(N+1)[N^2J+2N^2+3NJ+7N+2J+6][s-c]}{(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]}$$
$$+\frac{\{(4N+3-2n_1)[N^2J+2N^2+3NJ+8N+2J+6]-(2N+1-n_1)^2(N+1)\}(z-r)}{(2N+1-n_1)^2[N^2J+2N^2+3NJ+8N+2J+6]}$$
$$-((z-r)^2+4\beta)^{\frac{1}{2}}>0$$

Then $p^{***,iii}-p^{**}>0$ if and only if:

$$\left(\frac{(4N^2+12N-4n_1N-6n_1+{n_1}^2+7)}{4(2N+1-n_1)^2}\right)\left\{\frac{(J+2)(N+1)[N^2J+2N^2+3NJ+7N+2J+6][s-c]}{(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]}\right.$$
$$\left.+\frac{\{(4N+3-2n_1)[N^2J+2N^2+3NJ+8N+2J+6]-(2N+1-n_1)^2(N+1)\}(z-r)}{(2N+1-n_1)^2[N^2J+2N^2+3NJ+8N+2J+6]}\right\}^2$$
$$+\frac{(z-r)^2}{4}>F$$

This condition will again be satisfied by the assumptions made in prior cases that F must already be sufficiently small to ensure that the entrant and incumbents can make positive profits.

$$K_I^{***,iii}>K_I^{**}\ \leftrightarrow K_I^{***,iii}-K_I^{**}>0$$

$$K_I^{***,iii}-K_I^{**}=Y^o+Y^f-n_2(b+\gamma)-(N+1)c-(J-2N-1+n_1)s+\frac{(4N+3-2n_1)\alpha}{(2N+1-n_1)}$$
$$-(2N+1-n_1)(\alpha^2+4\beta)^{\frac{1}{2}}-J[(z-r)+s-c]>0$$

$$K_I^{***,iii}-K_I^{**}=Y^o+Y^f-n_2(b+\gamma)-(N+1-J)c+(2N-2J+1-n_1)s$$
$$+\frac{(4N+3-2n_1-2NJ-J+Jn_1)(z-r)}{(2N+1-n_1)}-(2N+1-n_1)((z-r)^2+4\beta)^{\frac{1}{2}}$$

$K_I^{***,iii}-K_I^{**}>0$ if

$$\frac{Y^o+Y^f-n_2(b+\gamma)-(N+1-J)c+(2N-2J+1-n_1)s}{(2N+1-n_1)}$$
$$+\frac{(4N+3-2n_1-2NJ-J+Jn_1)(z-r)}{(2N+1-n_1)^2}>((z-r)^2+4\beta)^{\frac{1}{2}}$$

By the assumptions that $Y^f > n_2(b+\gamma),\ s > c,\ z > r$ we know that the RHS above is positive and based on the prior discussion, this inequality will hold true based on the restrictions placed on F.

<u>Section 2.2.iv:</u> $\boldsymbol{Q^o = Y^o, Q^l = Y^l, Q^f < Y^f}$

Consider the variation to 2.2.i and 2.2.ii above where $q_j^o = y^o$ or $Q^o = Y^o$for the oil producers, $Q^l = Y^l$ for the local community, but the First Nation has more water than they choose to supply ($Q^f < Y^f$).

$$p = \frac{n_R b + n_2\gamma + (J+1)s + K_I + k_{J+1} - (Y^o + Y^l + Q^f)}{N+1}$$

Here, as in case 2.1, the first nation will chose their water to supply to maximize their profits to obtain:

$$Q^f = n_2(\gamma + b) - (N+1)c + p(N+1-2n_2)$$

Then the price becomes:

$$p = \frac{n_1 b + (J+1)s + (N+1)c + K_I + k_{J+1} - (Y^o + Y^l)}{(2N+2-2n_2)}$$

The entrant will choose a non-negative k_{J+1} to maximize:

$$\Pi_{J+1} = (\tfrac{1}{2})\big(s + k_{J+1} - p\big)^2 + k_{J+1}\big(\alpha - k_{J+1}\big) - F$$, where: $\alpha = (z-r) > 0$

In the case where deterrence is optimal, we know that the entrant will not enter the market and can therefore not have positive profits, so we assume $\Pi_{J+1} = 0$.

Differentiating with respect to k_{J+1} and assuming and interior solution, we have:

$$\frac{d\Pi_{J+1}}{dk_{J+1}} = \big(s + k_{J+1} - p\big)\left(1 - \frac{1}{2N+2-2n_2}\right) + \big(\alpha - 2k_{J+1}\big) = 0$$

These first order conditions imply that

$$s + k_{J+1} - p = \frac{(2N+2-2n_2)(2k_{J+1} - \alpha)}{2N+1-2n_2}$$

Hence

$$\Pi_{J+1} = (\frac{1}{2})\frac{(2N+2-2n_2)^2(2k_{J+1}-\alpha)^2}{(2N+1-2n_2)^2} + k_{J+1}\big(\alpha - k_{J+1}\big) - F$$

$$\Pi_{J+1} = \frac{(2N+2-2n_2)^2(2k_{J+1}-\alpha)^2 + 2(2N+1-2n_2)^2[k_{J+1}(\alpha - k_{J+1}) - F]}{2(2N+1-2n_2)^2}$$

Since $\Pi_{J+1} = 0$ we have

$$(2N+2-2n_2)^2(2k_{J+1}-\alpha)^2 + 2(2N+1-2n_2)^2\left[k_{J+1}(\alpha - k_{J+1}) - F\right] = 0$$

$$(4N^2 + 8N - 8n_2N - 8n_2 + 4{n_2}^2 + 4)\left(4k_{J+1}^2 - 4k_{J+1}\alpha + \alpha^2\right) \\ + (8N^2 + 8N - 16n_2N - 8n_2 + 8{n_2}^2 + 2)[k_{J+1}\alpha - k_{J+1}^2 - F] = 0$$

$$k_{J+1}^2(8N^2 + 24N - 16n_2N - 24n_2 + 8{n_2}^2 + 14) \\ - \alpha k_{J+1}(8N^2 + 24N - 16n_2N - 24n_2 + 8{n_2}^2 + 14) + \alpha^2(2N+2-2n_2)^2 \\ - 2(2N+1-2n_2)^2F = 0$$

Let:

$$\beta = \frac{2(2N+1-2n_2)^2F - \alpha^2(2N+2-2n_2)^2}{2(4N^2 + 12N - 8n_2N - 12n_2 + 4{n_2}^2 + 7)}$$

$$\left(\frac{F}{\alpha^2}\right) > \left[\frac{(2N+2-2n_2)^2}{2(2N+1-2n_2)^2}\right]$$

Then the above equation reduces to

$$k_{J+1}^2 - \alpha k_{J+1} - \beta = 0$$

So $k_{J+1}^{***,iv} = (\alpha + (\alpha^2 + 4\beta)^{\frac{1}{2}})/2$

Where

$$(\alpha^2 + 4\beta)^{\frac{1}{2}} = \left[(z-r)^2 + \frac{+2(2N+1-2n_2)^2F - \alpha^2(2N+2-2n_2)^2}{(4N^2 + 12N - 8n_2N - 12n_2 + 4{n_2}^2 + 7)}\right]^{1/2}$$

$$(\alpha^2 + 4\beta)^{\frac{1}{2}} = \left[\frac{4(2N+1-2n_2)^2F - (z-r)^2(4N^2 + 4N - 8n_2N - 4n_2 + 4{n_2}^2 + 3)}{(4N^2 + 12N - 8n_2N - 12n_2 + 4{n_2}^2 + 7)}\right]^{1/2}$$

From the first order conditions we know that

$$(2N+3-2n_2)k_{J+1} = (2N+1-2n_2)s - (2N+2-2n_2)\alpha - (2N+1-2n_2)p$$

$$(4N^2 + 8N - 8n_2N - 8n_2 + 4{n_2}^2 + 5)k_{J+1} \\ = (2N+2-2n_2)(2N+1-2n_2)s - (2N+2-2n_2)^2\alpha - (2N+1 \\ - 2n_2)[n_1b + (J+1)s + (N+1)c + K_I - (Y^o + Y^l)]$$

$$(4N^2+8N-8n_2N-8n_2+4{n_2}^2+5)k_{J+1} = (2N+1-2n_2)(2N+1-2n_2-J)s-(2N+2-2n_2)^2\alpha-(2N+1-2n_2)[n_1b+(N+1)c+K_I-(Y^o+Y^l)]$$

$$k_{J+1}^{***,iv} = \frac{(2N+1-2n_2)(2N+1-2n_2-J)s-(2N+2-2n_2)^2\alpha}{(4N^2+8N-8n_2N-8n_2+4{n_2}^2+5)} - \frac{(2N+1-2n_2)[n_1b+(N+1)c+K_I-(Y^o+Y^l)]}{(4N^2+8N-8n_2N-8n_2+4{n_2}^2+5)}$$

Combining to solve for $K_I^{***,iv}$:

$$\alpha+(\alpha^2+4\beta)^{\frac{1}{2}} = \frac{(2N+1-2n_2)(2N+1-2n_2-J)s-(2N+2-2n_2)^2\alpha}{(4N^2+8N-8n_2N-8n_2+4{n_2}^2+5)} - \frac{(2N+1-2n_2)[n_1b+(N+1)c+K_I-(Y^o+Y^l)]}{(4N^2+8N-8n_2N-8n_2+4{n_2}^2+5)}$$

$$K_I^{***,iv} = Y^o+Y^l-n_1b-(N+1)c+(2N+1-2n_2-J)s - \frac{(8N^2+16N-16n_2N-16n_2+8{n_2}^2+9)(z-r)}{(2N+1-2n_2)} - \frac{(4N^2+8N-8n_2N-8n_2+4{n_2}^2+5)(\alpha^2+4\beta)^{\frac{1}{2}}}{(2N+1-2n_2)}$$

Results (Section 2.2.iv):

When entry is deterred, the price will become:

$$p = \frac{n_1b+Js+(N+1)c+K_I-(Y^o+Y^l)}{2N+1-2n_2}$$

$$p^{***,iv} = s - \frac{(8N^2+16N-16n_2N-16n_2+8{n_2}^2+9)(z-r)}{(2N+1-2n_2)^2} - \frac{(4N^2+8N-8n_2N-8n_2+4{n_2}^2+5)(\alpha^2+4\beta)^{\frac{1}{2}}}{(2N+1-2n_2)^2}$$

$$k_{J+1}^{***,iv} = 0$$

$$K_I^{***,iv} = Y^o + Y^l - n_1 b - (N+1)c + (2N+1-2n_2-J)s - \frac{(8N^2+16N-16n_2N-16n_2+8{n_2}^2+9)(z-r)}{(2N+1-2n_2)} - \frac{(4N^2+8N-8n_2N-8n_2+4{n_2}^2+5)(\alpha^2+4\beta)^{\frac{1}{2}}}{(2N+1-2n_2)}$$

$${Q^f}^{***,iv} = n_2(\gamma+b) - (\mathrm{N}+1)\mathrm{c} + p^{***,iv}(N+1-2n_2)$$

$${Q^f}^{***,iv} = n_2(\gamma+b) - (\mathrm{N}+1)\mathrm{c} + (N+1-2n_2)s - \frac{(8N^2+16N-16n_2N-16n_2+8{n_2}^2+9)(z-r)}{(N+1-2n_2)} - \frac{(4N^2+8N-8n_2N-8n_2+4{n_2}^2+5)(\alpha^2+4\beta)^{\frac{1}{2}}}{(N+1-2n_2)}$$

$${Q^l}^{***,iv} = Y^l$$

$${Q^o}^{***,iv} = Y^o$$

$${\Pi_{J+1}}^{***,iv} = 0$$

$${\Pi^f}^{***,iv} = {Q^f}^{***,iv}(p^{***,iv} - c) + \frac{n_2}{2}[(\gamma-c)^2 + ((b-c)^2]$$

$${\Pi^l}^{***,iv} = Y^l(p^{***,iv} - c) + J\Pi_j^{***,iv} + \frac{n_1}{2}(b-c)^2$$

$$\Pi_j^{***,iv} = y^o\left(p^{***,iv} - c\right) + \left(\frac{1}{2}\right)\left(s + k_j^{***,iv} - p^{***,iv}\right)^2 + k_j^{***,iv}\left(z - k_j^{***,iv} - r\right) - F$$

Section 2.2.v: $\boldsymbol{Q^o < Y^o, Q^l = Y^l, Q^f = Y^f}$

In the case where the oil producer's available water supply is not constrained by the initial endowment Y^o, there will be no strategic choice of capacity to deter entry, as in this case the water price does not depend on the incumbents choice of capacity. Here, as in case 2.1 where we saw entry accommodation, we can see that the K_I term in the price function is dependent only on the incumbents choice of water supply, not on the actions of any potential entrant.

$$p = \frac{n_R b + n_2\gamma + (J+1)s + K_I + k_{J+1} - (Q^o + Y^f + Y^l)}{N+1}$$

Where the incumbents will choose the following quantity of water to maximize profits:

$$Q^o = Jq_j^o = Js + K_I - J(N+1)c + NJp$$

As such, the price will be determined by

$$p = \frac{n_R b + n_2\gamma + s + k_{J+1} + J(N+1)c - (Y^f + Y^l)}{N + NJ + 1}$$

Here, we will not see entry deterrence since the incumbent's choice of capacity will have no effect on the price. All parties will maximize their own profits which will not have dependencies on the actions of the others.

Section 2.3: Accommodation is Optimal (& constrained water supply)

<u>Section 2.3.i:</u> Supply is limited by availability ($\boldsymbol{Y = Q}$)

Consider the case where Q=Y such that the water sales are constrained by the water supply. The same first order conditions as before apply to the water demands. As before, water demand can be stated as a function of price:

$$p(Y, K_I, k_{J+1}) = \frac{n_R b + n_2\gamma + (J+1)s + K_I + k_{J+1} - Y}{N+1}$$

In this case, the supply is determined by the available water endowments, given by:

$Y = \mathrm{Y}^l + \mathrm{Y}^f + \mathrm{Y}^o$

<u>Stage 2:</u> Choice of capacity by the entrant:

Entrant chooses capacity $k_{J+1}$to maximize Π_{J+1}:

$$\Pi_{J+1}(p, k_{J+1}) = \frac{a^o\left(p, k_{J+1}\right)^2}{2} + k_{J+1}\left(z - r - k_{J+1}\right) - F$$

First Order condition: $s - k_{J+1} - p + z - r + \left(p - s - k_{J+1}\right)\frac{1}{N+1} = 0$

$$Ns - (N+2)k_{J+1} - Np + (N+1)(z-r) = 0$$

$$k_{J+1} = \frac{(z-r)(N+1)^2 + N(N-J)s - N(n_R b + n_2\gamma) - NK_I + NY}{(N^2 + 4N + 2)}$$

The price then becomes:

$$\begin{aligned}(N^2 + 4N + 2)(N+1)p \\ &= (N+1)(N+2)(n_R b + n_2\gamma + K_I - Y) + (N+1)(NJ + 2J + 2N + 2)s \\ &+ (z-r)(N+1)^2\end{aligned}$$

$$p = \frac{(N+2)(n_R b + n_2\gamma + K_I - Y) + (NJ + 2J + 2N + 2)s + (z-r)(N+1)}{(N^2 + 4N + 2)}$$

<u>Stage 1:</u> Choice of capacity by the incumbents:

Incumbent oil producers choose capacity k_j to maximize Π_j:

$$\Pi_j(p, q_j^o, k_j) = (p-c)q_j^o + \frac{a^o\left(p, k_j\right)^2}{2} + k_j\left(z - r - k_j\right) - F$$

First Order condition: $s - k_j - p + z - r + (y^o - s - k_j + p)\frac{1}{N+1} = 0$

$$Ns - (N+2)k_j - Np + (N+1)(z-r) + y^o = 0$$

$$k_j = \frac{Ns - Np + (N+1)(z-r) + y^o}{N+2}$$

$$K_I = \frac{JNs - JNp + J(N+1)(z-r) + Y^o}{N+2}$$

Results:

$$\widehat{K_I}^{***,i} = \frac{NJ(N^2 + 2N - NJ - 2J)s + Y^o}{(N^2 + 4N + 2 + NJ)(N+2)} + \frac{J(N+1)^2(z-r) - NJ(n_R b + n_2\gamma - Y)}{(N^2 + 4N + 2 + NJ)}$$

$$\hat{p}^{***,i} = \frac{(N+2)(N^2 + 4N + 2)(n_R b + n_2\gamma - Y) + Y^o}{(N^2 + 4N + 2 + NJ)(N^2 + 4N + 2)} + \frac{[(N^2 + 4N + 2 + NJ)(NJ + 2J + 2N + 2) - NJ(N^2 + 2N - NJ - 2J)]s}{(N^2 + 4N + 2 + NJ)(N^2 + 4N + 2)} + + \frac{(N^2J + N^2 + 4NJ + 4N + 4)(N+1)(z-r)}{(N^2 + 4N + 2 + NJ)(N^2 + 4N + 2)}$$

$$\hat{Q}^{***,i} = \mathrm{Y}^o + \mathrm{Y}^f + \mathrm{Y}^l$$

$$\widehat{k_{J+1}}^{***,i} = \frac{(N+1)^2(z-r)}{(N^2 + 4N + 2 + NJ)} + \frac{N(N-J)s}{(N^2 + 4N + 2)} - \frac{N(n_R b + n_2\gamma - Y)}{(N^2 + 4N + 2 + NJ)} - \frac{N^2J(N^2 + 2N - NJ - 2J)s + NY^o}{(N^2 + 4N + 2)(N^2 + 4N + 2 + NJ)(N+2)}$$

Compare to deterrence case:

$$p^{***} = s + \frac{N^2(z-r) - ((z-r)^2 + 4\beta)^{\frac{1}{2}}(N^2 + 4N + 2)}{2N^2}$$

Where

$$\frac{2N^2F - \alpha^2(N+1)^2}{2N^2 + 8N + 4}$$

$$\hat{p}^{***,i} = \frac{(N+2)(N^2 + 4N + 2)(n_R b + n_2\gamma - Y) + Y^o}{(N^2 + 4N + 2 + NJ)(N^2 + 4N + 2)} + \frac{[(N^2 + 4N + 2 + NJ)(NJ + 2J + 2N + 2) - NJ(N^2 + 2N - NJ - 2J)]s}{(N^2 + 4N + 2 + NJ)(N^2 + 4N + 2)} + \frac{(N^2J + N^2 + 4NJ + 4N + 4)(N+1)(z-r)}{(N^2 + 4N + 2 + NJ)(N^2 + 4N + 2)}$$

We can see that the price in the case of deterrence is dependent on the fixed cost of water. However, in the accommodation case, the price will be dependent on the initial water allocation Y^o. If the initial water allocation is very large, the price will be low, whereas if the initial allocation is not large, the price will be higher. Intuitively, we see that, when the allocation is smaller and the price is higher, there will be greater incentive for the oil producers to deter entry.

Compare capacity:

$$K_I^{***} = (N-J)s - (n_R b + n_2\gamma - Y) + \frac{N^2\alpha - (\alpha^2+4\beta)^{\frac{1}{2}}(N^2+4N+2)}{2N}$$

$$\widehat{K_I}^{***,i} = \frac{NJ(N^2+2N-NJ-2J)s + Y^o}{(N^2+4N+2+NJ)(N+2)} + \frac{J(N+1)^2(z-r) - NJ(n_R b + n_2\gamma - Y)}{(N^2+4N+2+NJ)}$$

We can see by simple inspection (due to the size of the denominators) that the accommodation capacity $\widehat{K_I}^{***,i}$ will be lower than the choice of capacity in the case of deterrence.

<u>Section 2.3.ii:</u> $(\boldsymbol{Q^o = Y^o, Q^l < Y^l, Q^f < Y^f})$

Consider the case where the water constraint is binding for the oil producers, $Q^o = Y^o$, but not for the First Nation of for the local community. The same first order conditions as before apply to the water demands. As before, water demand can be stated as a function of price:

$$p = \frac{(J+1)s + K_I + k_{J+1} - Y^o + 2(\mathrm{N}+1)\mathrm{c}}{2N+J+3}$$

In this case, the supply is determined by $Q = Q^f + Q^l + \mathrm{Y}^o$

$$Q^f = n_2(\gamma + b) - (\mathrm{N}+1)\mathrm{c} + p(N+1-2n_2)$$
$$Q^l = n_1 b - (N+1)c + p(N+1-n_1)$$
$$Q^o = Y^o$$

<u>Stage 2:</u> Choice of capacity by the entrant:

Entrant chooses capacity $k_{J+1}$to maximize Π_{J+1}:

$$\Pi_{J+1}(p, k_{J+1}) = \frac{a^o\left(p, k_{J+1}\right)^2}{2} + k_{J+1}\left(z - r - k_{J+1}\right) - F$$

First Order condition: $s - k_{J+1} - p + z - r + \left(p - s - k_{J+1}\right)\frac{1}{2N+J+3} = 0$

$$(2N+J+4)k_{J+1} = (2N+J+2)s + (2N+J+3)(z-r) - (2N+J+2)p$$

$$k_{J+1} = \frac{(2N+J+2)(2N+J+3)s + (2N+J+3)(2N+J+3)(z-r)}{\big((2N+J+3)(2N+J+4) + (2N+J+2)\big)} - \frac{(2N+J+2)[(J+1)s + K_I - Y^o + 2(\mathrm{N}+1)\mathrm{c}]}{\big((2N+J+3)(2N+J+4) + (2N+J+2)\big)}$$

The price then becomes:

$$p = \frac{(2N+J+4)[(J+1)s + K_I - Y^o + 2(\mathrm{N}+1)\mathrm{c}]}{\big((2N+J+3)(2N+J+4) + (2N+J+2)\big)} + \frac{(2N+J+2)s + (2N+J+3)(z-r)}{\big((2N+J+3)(2N+J+4) + (2N+J+2)\big)}$$

<u>Stage 1:</u> Choice of capacity by the incumbents:

Incumbent oil producers choose capacity k_j to maximize Π_j:

First Order condition: $s - k_j - p + z - r + (y^o - s - k_j + p)\frac{1}{2N+J+3} = 0$

$$(2N+J+4)k_j = (2N+J+2)s + (2N+J+3)(z-r) + y^o - (2N+J+2)p$$

$$\begin{aligned}\big((2N+J+3)(2N+J+4) + (J+1)(2N+J+2)\big)(2N+J+4)K_I \\ &= \big((2N+J+3)(2N+J+4)\big)(2N+J+2)Js \\ &+ \big((2N+J+3)(2N+J+4)\big)(2N+J+3)J(z-r) \\ &+ \big((2N+J+3)(2N+J+4) + (2N+J+2)\big)Y^o \\ &- (2N+J+2)(2N+J+4)J[(J+1)s - Y^o + 2(\mathrm{N}+1)\mathrm{c}]\end{aligned}$$

$$K_I = \frac{(2N+J+3)[(2N+J+2)Js + (2N+J+3)J(z-r)]}{\big((2N+J+3)(2N+J+4) + (J+1)(2N+J+2)\big)} + \frac{\big((2N+J+3)(2N+J+4) + (2N+J+2)\big)Y^o}{\big((2N+J+3)(2N+J+4) + (J+1)(2N+J+2)\big)(2N+J+4)} - \frac{(2N+J+2)J[(J+1)s - Y^o + 2(\mathrm{N}+1)\mathrm{c}]}{\big((2N+J+3)(2N+J+4) + (J+1)(2N+J+2)\big)}$$

Results:

$$\widehat{K_I}^{***,ii} = \frac{(2N+J+3)[(2N+J+2)Js+(2N+J+3)J(z-r)]}{\big((2N+J+3)(2N+J+4)+(J+1)(2N+J+2)\big)} + \frac{\big((2N+J+3)(2N+J+4)+(2N+J+2)\big)Y^o}{\big((2N+J+3)(2N+J+4)+(J+1)(2N+J+2)\big)(2N+J+4)} - \frac{(2N+J+2)J[(J+1)s-Y^o+2(\mathrm{N}+1)\mathrm{c}]}{\big((2N+J+3)(2N+J+4)+(J+1)(2N+J+2)\big)}$$

$$\hat{p}^{***,ii} = \frac{(2N+J+4)[(J+1)s-Y^o+2(\mathrm{N}+1)\mathrm{c}]}{\big((2N+J+3)(2N+J+4)+(J+1)(2N+J+2)\big)} + \frac{(J+1)[(2N+J+2)s+(2N+J+3)(z-r)]}{\big((2N+J+3)(2N+J+4)+(J+1)(2N+J+2)\big)} + \frac{Y^o}{\big((2N+J+3)(2N+J+4)+(J+1)(2N+J+2)\big)}$$

<u>Section 2.3.iii:</u> $(\boldsymbol{Q^o = Y^o, Q^l < Y^l, Q^f = Y^f})$

Consider the case where the water constraint is binding for the oil producers and the first nation, $Q^o = Y^o$ *and* $Q^f = Y^f$, but not for the local community. The same first order conditions as before apply to the water demands. As before, water demand can be stated as a function of price:

$$p = \frac{n_R b + n_2\gamma + (J+1)s + K_I + k_{J+1} - (Y^o + Y^f + Q^l)}{N+1}$$

Here, the local community will chose their water to supply to maximize their profits to obtain:

$$Q^l = n_1 b - (N+1)c + p(N+1-n_1)$$

Then the price becomes:

$$p = \frac{n_2(b+\gamma) + (N+1)c + (J+1)s + K_I + k_{J+1} - (Y^o + Y^f)}{2N+2-n_1}$$

<u>Stage 2:</u> Choice of capacity by the entrant:

Entrant chooses capacity k_{J+1} to maximize Π_{J+1}:

$$\Pi_{J+1}(p, k_{J+1}) = \frac{a^o\big(p, k_{J+1}\big)^2}{2} + k_{J+1}\big(z - r - k_{J+1}\big) - F$$

First Order condition: $s - k_{J+1} - p + z - r + \big(p - s - k_{J+1}\big)\frac{1}{2N+2-n_1} = 0$

$$(2N+3-n_1)k_{J+1} = (2N+1-n_1)s + (2N+2-n_1)(z-r) - (2N+1-n_1)p$$

$$k_{J+1} = \frac{(2N+1-n_1)(2N+2-n_1)s + (2N+2-n_1)^2(z-r)}{(2N^2+12N-4Nn_1-6n_1+{n_1}^2+7)} - \frac{(2N+1-n_1)\left(n_2(b+\gamma)+(N+1)c+(J+1)s+K_I-(Y^o+Y^f)\right)}{(2N^2+12N-4Nn_1-6n_1+{n_1}^2+7)}$$

$$(2N^2+12N-4Nn_1-6n_1+{n_1}^2+7)k_{J+1} = (2N+1-n_1)(2N+2-n_1)s + (2N+2-n_1)^2(z-r) - (2N+1-n_1)\left(n_2(b+\gamma)+(N+1)c+(J+1)s+K_I-(Y^o+Y^f)\right)$$

The price then becomes:

$$(2N^2+12N-4Nn_1-6n_1+{n_1}^2+7)p = (2N+3-n_1)[n_2(b+\gamma)+(N+1)c+(J+1)s+K_I-(Y^o+Y^f)] + (2N+1-n_1)s + (2N+2-n_1)(z-r)$$

$$p = \frac{(2N+3-n_1)[n_2(b+\gamma)+(N+1)c+(J+1)s+K_I-(Y^o+Y^f)]}{(2N^2+12N-4Nn_1-6n_1+{n_1}^2+7)} + \frac{(2N+1-n_1)s+(2N+2-n_1)(z-r)}{(2N^2+12N-4Nn_1-6n_1+{n_1}^2+7)}$$

<u>Stage 1:</u> Choice of capacity by the incumbents:

Incumbent oil producers choose capacity k_j to maximize Π_j:

First Order condition: $s - k_j - p + z - r + (y^o - s - k_j + p)\frac{1}{2N+2-n_1} = 0$

$$(2N+3-n_1)k_j = (2N+1-n_1)s + (2N+2-n_1)(z-r) + y^o - (2N+1-n_1)p$$

$$(2N+3-n_1)K_I = (2N+1-n_1)Js + (2N+2-n_1)J(z-r) + Y^o - (2N+1-n_1)Jp$$

$$K_I = \frac{(2N+1-n_1)Js}{(2N+3-n_1)} + \frac{(2N+2-n_1)J(z-r)}{(2N+3-n_1)} + \frac{(2N^2+12N-4Nn_1-6n_1+{n_1}^2+7)Y^o}{(2N+3-n_1)^2(2N+2-n_1)} - \frac{J(2N+1-n_1)[n_2(b+\gamma)+(N+1)c+(J+1)s-(Y^o+Y^f)]}{(2N+3-n_1)(2N+2-n_1)}$$

Results:

$$\widehat{K}_I^{***,iii} = \frac{(2N+1-n_1)Js}{(2N+3-n_1)} + \frac{(2N+2-n_1)J(z-r)}{(2N+3-n_1)} + \frac{(2N^2+12N-4Nn_1-6n_1+n_1{}^2+7)Y^o}{(2N+3-n_1)^2(2N+2-n_1)} - \frac{J(2N+1-n_1)[n_2(b+\gamma)+(N+1)c+(J+1)s-(Y^o+Y^f)]}{(2N+3-n_1)(2N+2-n_1)}$$

$$p^{***,iii} = \frac{(2N+3-n_1)[n_2(b+\gamma)+(N+1)c+(J+1)s-(Y^o+Y^f)]}{(2N^2+12N-4Nn_1-6n_1+n_1{}^2+7)} + \frac{(2N+1-n_1)(J+1)s+(2N+2-n_1)(J+1)(z-r)}{(2N^2+12N-4Nn_1-6n_1+n_1{}^2+7)} + \frac{Y^o}{(2N+3-n_1)(2N+2-n_1)} - \frac{J(2N+1-n_1)[n_2(b+\gamma)+(N+1)c+(J+1)s-(Y^o+Y^f)]}{(2N^2+12N-4Nn_1-6n_1+n_1{}^2+7)(2N+2-n_1)}$$

Section 2.3.iv: ($\boldsymbol{Q^o = Y^o, Q^l = Y^l, Q^f < Y^f}$)

Consider the case where the water constraints are binding for the oil producers and the local community, but the First Nation has more water than they choose to supply ($Q^f < Y^f$).

$$p = \frac{n_R b + n_2\gamma + (J+1)s + K_I + k_{J+1} - (Y^o + Y^l + Q^f)}{N+1}$$

Here, the first nation will chose their water to supply to maximize their profits to obtain:

$$Q^f = n_2(\gamma + b) - (N+1)c + p(N+1-2n_2)$$

Then the price becomes:

$$p = \frac{n_1 b + (J+1)s + (N+1)c + K_I + k_{J+1} - (Y^o + Y^l)}{(2N+2-2n_2)}$$

Stage 2: Choice of capacity by the entrant:

Entrant chooses capacity $k_{J+1}$to maximize Π_{J+1}:

$$\Pi_{J+1}(p, k_{J+1}) = \frac{a^o\left(p, k_{J+1}\right)^2}{2} + k_{J+1}\left(z - r - k_{J+1}\right) - F$$

First Order condition: $s - k_{J+1} - p + z - r + \left(p - s - k_{J+1}\right)\frac{1}{2N+2-2n_2} = 0$

$$(2N+1-2n_2)s - (2N+3-2n_2)k_{J+1} - (2N+1-2n_2)p + (2N+2-2n_2)(z-r) = 0$$

$$k_{J+1} = \frac{(2N+2-2n_2)(2N+1-2n_2)s + (2N+2-2n_2)^2(z-r)}{(4N^2+12N-8Nn_2-12n_2+4n_2{}^2+7)} - \frac{(2N+1-2n_2)[n_1b+(J+1)s+(N+1)c+K_I-(Y^o+Y^l)]}{(4N^2+12N-8Nn_2-12n_2+4n_2{}^2+7)}$$

The price then becomes:

$$p = \frac{(2N+3-2n_2)[n_1b+(J+1)s+(N+1)c+K_I-(Y^o+Y^l)]}{(4N^2+12N-8Nn_2-12n_2+4n_2{}^2+7)} + \frac{(2N+1-2n_2)s+(2N+2-2n_2)(z-r)}{(4N^2+12N-8Nn_2-12n_2+4n_2{}^2+7)}$$

<u>Stage 1:</u> Choice of capacity by the incumbents:

Incumbent oil producers choose capacity k_j to maximize Π_j:

First Order condition: $s - k_j - p + z - r + (y^o - s - k_j + p)\frac{1}{2N+2-2n_2} = 0$

$$(2N+3-2n_2)k_j = (2N+1-2n_2)s + (2N+2-2n_2)(z-r) + y^o - (2N+1-2n_2)p$$

$$K_I = \frac{(2N+1-2n_2)(2N+1-J-2n_2)Js}{(2N+2-2n_2)(2N+3-2n_2)} + \frac{(2N+2-2n_2)J(z-r)}{(2N+3-2n_2)} + \frac{(4N^2+12N-8Nn_2-12n_2+4n_2{}^2+7)Y^o}{(2N+2-2n_2)(2N+3-2n_2)^2} - \frac{(2N+1-2n_2)J[n_1b+(N+1)c-(Y^o+Y^l)]}{(2N+2-2n_2)(2N+3-2n_2)}$$

Results:

$$\widehat{K}_I{}^{***,iv} = \frac{(2N+1-2n_2)(2N+1-J-2n_2)Js}{(2N+2-2n_2)(2N+3-2n_2)} + \frac{(2N+2-2n_2)J(z-r)}{(2N+3-2n_2)} + \frac{(4N^2+12N-8Nn_2-12n_2+4n_2{}^2+7)Y^o}{(2N+2-2n_2)(2N+3-2n_2)^2} - \frac{(2N+1-2n_2)J[n_1b+(N+1)c-(Y^o+Y^l)]}{(2N+2-2n_2)(2N+3-2n_2)}$$

$$\hat{p}^{***,iv} = \frac{(2N+2-2n_2)[2s+(J+1)(z-r)]}{(4N^2+12N-8Nn_2-12n_2+4n_2{}^2+7)} + \frac{[n_1 b+(N+1)c-(Y^o+Y^l)]}{(2N+2-2n_2)}$$
$$+\frac{(2N+1-2n_2)(4N+4-J-4n_2)Js}{(4N^2+12N-8Nn_2-12n_2+4n_2{}^2+7)(2N+2-2n_2)}$$
$$+\frac{Y^o}{(2N+2-2n_2)(2N+3-2n_2)}$$

<u>Section 2.3.v:</u> ($\boldsymbol{Q^o < Y^o, Q^l = Y^l, Q^f = Y^f}$)

In the case where the oil producer's available water supply is not constrained by the initial endowment Y^o, there will be no strategic choice of capacity to deter entry, as in this case the water price does not depend on the incumbents choice of capacity.

$$p = \frac{n_R b + n_2\gamma + (J+1)s + K_I + k_{J+1} - (Q^o + Y^f + Y^l)}{N+1}$$

Where the incumbents will choose the following quantity of water to maximize profits:

$$Q^o = Jq_J^o = Js + K_I - J(N+1)c + NJp$$

As such, the price will be determined by

$$p = \frac{n_R b + n_2\gamma + s + k_{J+1} + J(N+1)c - (Y^f + Y^l)}{N+NJ+1}$$

<u>Stage 2:</u> Choice of capacity by the entrant:

Entrant chooses capacity $k_{J+1}$to maximize Π_{J+1}:

$$\Pi_{J+1}(p, k_{J+1}) = \frac{a^o(p, k_{J+1})^2}{2} + k_{J+1}(z-r-k_{J+1}) - F$$

First Order condition: $s - k_{J+1} - p + z - r + (p - s - k_{J+1})\frac{1}{N+NJ+1} = 0$

$$(N+NJ+2)(k_{J+1}) = (N+NJ)s + (N+NJ+1)(z-r) - (N+NJ)p$$

$$k_{J+1} = \frac{(N+NJ)^2 s + (N+NJ+1)^2(z-r)}{(N^2+2N^2J^2+N^2J+2NJ+2N+2)}$$
$$-\frac{(N+NJ)[n_R b + n_2\gamma + J(N+1)c - (Y^f+Y^l)]}{(N^2+2N^2J^2+N^2J+2NJ+2N+2)}$$

The price then becomes:

$$p = \frac{(N+NJ+2)[n_R b + n_2\gamma + J(N+1)c - (Y^f + Y^l)]}{(N^2 + 2N^2J^2 + N^2J + 2NJ + 2N + 2)} + \frac{s}{(N+NJ+1)} + \frac{(N+NJ)^2 s + (N+NJ+1)^2(z-r)}{(N+NJ+1)(N^2 + 2N^2J^2 + N^2J + 2NJ + 2N + 2)}$$

$$q_j^o = s + k_j - (N+1)c + \frac{N(N+NJ+2)[n_R b + n_2\gamma + J(N+1)c - (Y^f + Y^l)]}{(N^2 + 2N^2J^2 + N^2J + 2NJ + 2N + 2)} + \frac{Ns}{(N+NJ+1)} + \frac{N(N+NJ)^2 s + N(N+NJ+1)^2(z-r)}{(N+NJ+1)(N^2 + 2N^2J^2 + N^2J + 2NJ + 2N + 2)}$$

<u>Stage 1:</u> Choice of capacity by the incumbents:

Incumbent oil producers choose capacity k_j to maximize Π_j:

$$\Pi_j(p, q_j^o, k_j) = (p-c)q_j^o + \frac{a^o\left(p,k_j\right)^2}{2} + k_j\left(z - r - k_j\right) - F$$

First Order condition: $s - k_j - p + z - r + (q_j^o - s - k_j + p)\frac{1}{N+NJ+1} = 0$

$$(N+NJ+2)k_j = (N+NJ)s + (N+NJ+1)(z-r) + q_j^o - (N+NJ)p$$

$$k_j = s + (z-r) - \frac{(N+1)c}{(N+NJ+1)} - \frac{NJ(N+NJ+2)[n_R b + n_2\gamma + J(N+1)c - (Y^f + Y^l)]}{(N+NJ+1)(N^2 + 2N^2J^2 + N^2J + 2NJ + 2N + 2)} - \frac{NJ(2N^2 + 3N^2J^2 + 3N^2J + 2NJ + 2N + 2)s}{(N+NJ+1)^2(N^2 + 2N^2J^2 + N^2J + 2NJ + 2N + 2)} - \frac{NJ(z-r)}{(N^2 + 2N^2J^2 + N^2J + 2NJ + 2N + 2)}$$

Results:

$$\widehat{K_I}^{***,v} = Js + J(z-r) - \frac{J(N+1)c}{(N+NJ+1)} - \frac{NJ^2(N+NJ+2)[n_R b + n_2\gamma + J(N+1)c - (Y^f + Y^l)]}{(N+NJ+1)(N^2 + 2N^2J^2 + N^2J + 2NJ + 2N + 2)} - \frac{NJ^2(2N^2 + 3N^2J^2 + 3N^2J + 2NJ + 2N + 2)s}{(N+NJ+1)^2(N^2 + 2N^2J^2 + N^2J + 2NJ + 2N + 2)} - \frac{NJ^2(z-r)}{(N^2 + 2N^2J^2 + N^2J + 2NJ + 2N + 2)}$$

$$\hat{p}^{***,v} = \frac{(N+NJ+2)[n_R b + n_2\gamma + J(N+1)c - (Y^f + Y^l)]}{(N^2 + 2N^2J^2 + N^2J + 2NJ + 2N + 2)} + \frac{s}{(N+NJ+1)} + \frac{(N+NJ)^2 s + (N+NJ+1)^2(z-r)}{(N+NJ+1)(N^2+2N^2J^2+N^2J+2NJ+2N+2)}$$

$$k_{J+1}{}^{***,v} = \frac{(N+NJ)^2 s + (N+NJ+1)^2(z-r)}{(N^2+2N^2J^2+N^2J+2NJ+2N+2)} - \frac{(N+NJ)[n_R b + n_2\gamma + J(N+1)c - (Y^f + Y^l)]}{(N^2+2N^2J^2+N^2J+2NJ+2N+2)}$$

Section 2.4: Additional Analysis

Proposition 1: A binding water quota for the oil producers $Q^o = Y^o$ is both necessary and sufficient for the capacity to be a strategic variable in the water market.

In the absence of this condition holding, we shall show that the function for the water price will reduce when the incumbents choose their oil supply so that that price is no longer dependent on the choice of capacity, thus capacity is no longer strategic.

In the case of entry, we have producer demand function:

$$p = \frac{n_R b + n_2\gamma + (J+1)s + K_I + k_{J+1} - Q^l - Q^f - Q^o}{N+1}$$

We can see that the price is increasing in the total capacity of the incumbents K_I. However, when the water constraint is non-binding, the incumbents will choose their water supply to maximize profits, and will select the quantity according to:

$$Q^o = Js + K_I - J(\mathrm{N}+1)c + NJp$$

Since this contains the term K_I, as does the price function, the expression will reduce to obtain:

$$p = \frac{n_R b + n_2\gamma + s + J(\mathrm{N}+1)c + k_{J+1} - Q^l - Q^f}{(N+NJ+1)}$$

This is no longer a function of K_I, indicating that the choice of capacity is not strategic. Note that neither Q^l or Q^f is a function of Q^oor K_I, as such they will have no effect on whether or not the choice of capacity is strategic. When we have $Q^o = Y^o$ as a fixed value, the oil firms do not choose Q^o according to the formula above, since they sell their entire allocation. Therefore the capacity term will not cancel out of the price function and the choice of capacity will be strategic.

Now we compare the outcomes for 4 cases: No entry, Entry when the water quotas are not binding, entry when the water quota is binding, and entry deterrence when the water quota is binding. Note that here we consider case ii, where the quotas are binding for the oil producers only, not for the first nation or for the local community ($Q^o = Y^o, Q^l < Y^l, Q^f < Y^f$)

a. No entry vs Accommodated Entry:

$$p^* = c$$
$$K_I^* = J(z-r) + Js - Jc$$

$$K_I^{**} = J(z-r) + Js - Jc$$

$$p^{**} = \frac{s + (J+2)(N+1)c}{JN + 2N + J + 3} + \frac{(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} + \frac{N(JN+2N+J+2)[s-c]}{(JN+2N+J+3)[(N+2)(JN+2N+J+3)+N]}$$

Here we see that $p^* < p^{**}$ and $K_I^{**} = K_I^*$, since $p^* < p^{**} \Leftrightarrow$

$$c < \frac{s + (J+2)(N+1)c}{JN + 2N + J + 3} + \frac{(N+1)(z-r)}{(N+2)(JN+2N+J+3)+N} + \frac{N(JN+2N+J+2)[s-c]}{(JN+2N+J+3)[(N+2)(JN+2N+J+3)+N]}$$

This is true since the second two terms in the RHS are positive since $z > r$ and $s > c$ by assumption and

$$c < \frac{s + (J+2)(N+1)c}{JN + 2N + J + 3} = \frac{s}{JN + 2N + J + 3} + \frac{(NJ + 2N + J + 2)c}{JN + 2N + J + 3}$$

We can see that $\frac{(NJ+2N+J+2)c}{JN+2N+J+3} \approx c$ and for s sufficiently large, the above will be true.

b. Accommodation in the absence of a binding constraint vs accommodation where there is a binding water constraint: $Q^o = Y^o$. We can see that both the capacity and the price are greater when the water constraint is binding:

$$\widehat{K}_I^{***,ii} > K_I^{**} \leftrightarrow$$

$$\frac{(2N+J+2)Js}{(2N+J+4)}+\frac{(2N+J+3)J(z-r)}{(2N+J+4)}+\frac{\big((2N+J+3)(2N+J+4)+(2N+J+2)\big)Y^o}{(2N+J+3)(2N+J+4)^2} - \frac{(2N+J+2)J[(J+1)s-Y^o+2(\mathrm{N}+1)\mathrm{c}]}{(2N+J+3)(2N+J+4)} > J(z-r)+Js-Jc$$

We can see that this will be true for Y^o sufficiently large. If Y^o is not sufficiently large, we will see in the next case that entry deterrence will be optimal when there is a constraint on the water price. We also predict that when the water constraint is binding, the water price will be higher:

$$\hat{p}^{***,ii} > p^{**}$$

$$\begin{aligned}(2N+J+4)&[(J+1)Js-JY^o+2J(N+1)c]\\ &+(J+1)[(2N+J+2)s+(2N+J+3)(z-r)]\\ &+\frac{(4N^2+4NJ+16N+J^2+8J+14)Y^o}{(2N+J+3)(2N+J+4)}\\ &>\frac{(4N^2+4NJ+16N+J^2+8J+14)[s+(J+2)(N+1)c]}{JN+2N+J+3}\\ &+\frac{(N+1)(4N^2+4NJ+16N+J^2+8J+14)(z-r)}{[N^2J+2N^2+3NJ+8N+2J+6]}\\ &+\frac{N(JN+2N+J+2)(4N^2+4NJ+16N+J^2+8J+14)[s-c]}{(JN+2N+J+3)[N^2J+2N^2+3NJ+8N+2J+6]}\end{aligned}$$

c. Accommodation vs deterrence where there is a binding water constraint: $Q^o = Y^o$. We can see that both the capacity and the price are greater when entry is deterred:

$$p^{***,ii} > \hat{p}^{***,ii} \leftrightarrow$$

$$\begin{aligned}s+\frac{\alpha}{2}-\frac{(\alpha^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)^2}&\\ >\frac{(2N+J+4)J[(J+1)s-Y^o+2(\mathrm{N}+1)\mathrm{c}]}{(4N^2+4NJ+16N+J^2+8J+14)}&\\ +\frac{(2N+J+2)(J+1)s+(2N+J+3)(J+1)(z-r)}{(4N^2+4NJ+16N+J^2+8J+14)}&\\ +\frac{Y^o}{(2N+J+3)(2N+J+4)}&\end{aligned}$$

We see that this is indeterminate and the greater price will be determined by both the value of the water quota and by the fixed cost

$$K_I^{***,ii} > \widehat{K}_I^{***,ii} \leftrightarrow$$

$$Y^o + 2(\mathrm{N}+1)(\mathrm{s}-\mathrm{c}) + \frac{(2N+J+2)(z-r)}{2} - \frac{(\alpha^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)} > \frac{(2N+J+2)Js}{(2N+J+4)} + \frac{(2N+J+3)J(z-r)}{(2N+J+4)} + \frac{\big((2N+J+3)(2N+J+4)+(2N+J+2)\big)Y^o}{(2N+J+3)(2N+J+4)^2} - \frac{(2N+J+2)J[(J+1)s - Y^o + 2(\mathrm{N}+1)\mathrm{c}]}{(2N+J+3)(2N+J+4)}$$

We can see here that, given the size of the denominators on the right hand side above, the accommodation capacity will be lower, again dependent on the prior assumption about the fixed cost being sufficiently low.

<u>Proposition 2</u>: Under entry deterrence, the choice of capacity is greater than the efficient capacity such that $MB + \sigma = MC$

Proof: $$MB + \sigma = MC \leftrightarrow$$

$$s + k_j - p + z - r + \sigma = 2k_j \leftrightarrow$$

$$s + z - r - k_j - p < 0$$

$$s + z - r - Y^o - 2(\mathrm{N}+1)(\mathrm{s}-\mathrm{c}) - \frac{(2N+J+2)(z-r)}{2} + \frac{(2N+J+2)((z-r)^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)^2} - s - \frac{z-r}{2} + \frac{((z-r)^2+4\beta)^{\frac{1}{2}}(4N^2+4NJ+16N+J^2+8J+14)}{2(2N+J+2)^2} < 0$$

$$\left[\frac{4F-(z-r)^2}{(4N^2+4NJ+16N+J^2+8J+14)}\right]^{1/2} < \frac{2(2N+J+2)[Y^o+2(\mathrm{N}+1)(\mathrm{s}-\mathrm{c})]}{(2N+J+3)(4N^2+4NJ+16N+J^2+8J+14)} + \frac{(2N+J+2)(2N+J+1)(z-r)}{(2N+J+3)(4N^2+4NJ+16N+J^2+8J+14)}$$

Given the restriction that we have a sufficiently small fixed cost F in the industry (in proportion to $(z-r)$) to ensure that the firms will earn positive profits in equilibrium, we can see that the marginal cost here in fact does exceed the benefits from the entry deterrence level of capacity.